Essays in Industrial Organization

DISSERTATION
of the University of St. Gallen,
School of Management,
Economics, Law, Social Sciences
and International Affairs
to obtain the title of
Doctor of Philosophy in Economics and Finance

submitted by

Philemon Krähenmann
from Aadorf (Thurgau)

Approved on the application of

Prof. Dr. Stefan Bühler
and
Prof. Dr. Armin Schmutzler

Dissertation no. 4930

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The University of St. Gallen, School of Management, Economics, Law, Social Sciences and International Affairs hereby consents to the printing of the present dissertation, without hereby expressing any opinion on the views herein expressed.

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The President:

Prof. Dr. Thomas Bieger

Abstract

This dissertation contains three essays in industrial organization.

Chapter 1 studies monopoly pricing in a situation of two-sided asymmetric information. In each perfect Bayesian equilibrium that survives the D1 criterion, the price must fully reveal the monopolist's private information. Public information is shown to reduce the cost of this price signaling by revealing a part of the monopolist's private information for free. Further, it allows her to form a more accurate belief about the buyer's private information and, therefore, to extract a larger surplus in the absence of signaling issues. Thus, while public information increases welfare and profit, it has an ambiguous effect on consumer surplus. Similar results hold, if the monopolist is either fully informed, or not informed at all.

Chapter 2 examines whether a position in a search list (e.g. on a sales platform) can signal the quality of an experience good, if firms are allowed to pay for it. We show that it can do so, if vertical differentiation between firms is high, while it cannot, if it is low. Intuitively, if uninformed consumers believe the ranking, the 'correct' ranking induces homogeneous beliefs among informed and uninformed consumers. In doing so, it facilitates market segmentation. Meanwhile, the 'wrong' ranking induces heterogeneous beliefs among consumers and, therefore, softens competition. The first effect is important if differentiation, and therefore the price gap, between firms is high. The second one is more relevant if differentiation is low and competition between firms intense.

Chapter 3 deals with peer-to-peer platforms. There, participants can trade among each other via a platform specific currency, and price setting is often restricted to some extent. We model such platforms as pure exchange economies and characterize all fixed price equilibria. We discuss the inherent inefficiency following from the combination of fixed prices and voluntary trade and show that under the fix price regime simple additional Pareto improving trades exist. Our theoretical analysis predicts that fixed prices lead on the one hand to less trade, but on the other hand to lower inequality than flexible prices. We illustrate our findings with transaction data from several time exchange platforms.

Zusammenfassung

Die vorliegende Dissertation umfasst drei Kapitel aus dem Forschungsfeld der Industrieökonomik.

Kapitel 1 studiert die optimale Preissetzung eines Monopolisten unter zweiseitiger asymmetrischer Information. In jedem perfekten bayesschen Gleichgewicht welches das D1 Kriterium überlebt, legt der Preis die private Information des Monopolisten offen. Es wird gezeigt, dass Kosten, die dadurch entstehen, dass die private Information des Monopolisten über den Preis signalisiert wird, durch öffentliche Information reduziert werden. Sozusagen, weil dadurch ein Teil der Information gratis offengelegt wird. Sie erlaubt es dem Monopolisten auch eine genauere Vorstellung über die private Information des Käufers zu bilden. Öffentliche Information erhöht deshalb den Gewinn des Monopolisten und auch die Gesamtwohlfahrt, der Effekt auf die Konsumentenwohlfahrt ist aber uneindeutig. Ähnliche Resultate gelten, wenn der Monopolist vollständig oder überhaupt nicht informiert ist.

Kapitel 2 ergründet die Frage, ob die Position in einer Suchliste (z.B. auf einer Verkaufsplattform) die Qualität eines Erfahrungsgutes signalisieren kann, wenn Firmen dafür bezahlen können. Gezeigt wird, dass dies möglich ist, wenn die Differenzierung zwischen den Firmen gross ist, nicht aber, wenn sie klein ist. Wenn uninformierte Konsumenten der Rangliste glauben, so führt eine 'richtige' Liste zu homogenen Erwartungen zwischen informierten und uninformierten Konsumenten, und erleichtert so die Marktsegmentierung. Eine 'falsche' Liste führt hingegen zu heterogenen Erwartungen zwischen Konsumenten und damit zu einer geringeren Wettbewerbsintensität. Der erste Effekt dominiert, wenn die Differenzierung, und damit auch der Preisunterschied zwischen den Firmen, hoch ist. Der Zweite hingegen, wenn die Differenzierung gering, und somit die Wettbewerbsintensität hoch ist.

Kapitel 3 beschäftigt sich mit Tauschbörsen. In diesen wird mit einer plattform-spezifischen Währung getauscht und die Preise können oft nicht völlig frei gewählt werden. Wir modellieren diese Börsen als Tauschökonomien und charakterisieren die Menge der Fixpreisgleichgewichte. Wir diskutieren die Ineffizienz, die sich immanent aus der Kombination von fixen

Preisen und Freiwilligkeit ergibt. Wir zeigen, dass es Pareto verbessernde Tauschgeschäfte gibt, und zwar unter den gegebenen fixen Preisen. Unsere theoretische Analyse sagt voraus, dass im Vergleich zu flexiblen Preisen, fixierte Preise auf der einen Seite zu weniger Tausch führen, auf der anderen Seit aber zu mehr Gleichheit. Wir illustrieren diese Resultate mit Transaktionsdaten von verschiedenen Zeittauschbörsen.

Contents

Chapter 1

Monopoly Pricing and the Value of Public Information Under Two-Sided Asymmetric Information

Abstract

This paper studies monopoly pricing under two-sided asymmetric information and shows how public information increases welfare and profit, while having an ambiguous effect on consumer surplus. The price fully reveals the seller's private information in each perfect Bayesian equilibrium surviving the D1 criterion. Public information reduces the cost of this price signaling by revealing a part of the seller's private information for free. It further allows the seller to form a more accurate belief about the buyer's private information and, therefore, to extract a larger share of the potential gains from trade. A similar result holds when the seller is fully informed.

1.1 Introduction

Sellers and potential buyers frequently do not share common prior information about the buyer's true valuation of an item that is for sale. Instead, they may each possess independent limited first-hand information about it. For example, an inexperienced seller may observe most product characteristics yet might be unsure about their importance to a potential buyer. Meanwhile, the prospective buyer may know what weights he attaches to various characteristics but might not observe all of them. This paper studies the effect of public information on welfare and profit under such two-sided private information when the seller is a monopolistic price setter.

Understanding these effects is relevant for two reasons. First, many situations of bilateral trade are characterized by private information on both sides combined with some public information. Consider for example trade on online marketplaces. Here, public information is ubiquitous in the form of customer reviews, ratings, and selling histories.[1] At the same time, two-sided asymmetric information is likely to persist in such marketplaces due to a lack of direct interaction between sellers and potential buyers. Second, two-sided asymmetric information is arguably the richest form of information asymmetry in a seller/buyer setting. In particular, it allows the price to reflect the monopolist's belief. Therefore, pricing is strategic, and it is no longer clear whether the standard argument, according to which public information increases a monopolist's profit by reducing the consumer's information rent [Milgrom and Weber, 1982; Ottaviani and Prat, 2001], remains valid.

I examine a model where a monopolistic seller wants to sell a single good to one prospective buyer. Both the buyer and the seller are partially yet privately informed about the buyer's true valuation of the good, which can be either high or low. The belief distributions are common knowledge, and private signals are

[1] Compared to other sources of information, these are not only available to everyone but are also common knowledge. For instance, a seller on Amazon is perfectly aware that any potential buyer will observe her star rating prior to purchase. Similarly, any potential buyer knows that the seller may have conditioned the price of the good on the rating. Fan, Ju and Xiao [2016], Jolivet, Jullien and Postel-Vinay [2016] and Eschenbaum and Liebert [2018] show that sellers indeed condition prices on the realization of these public signals.

drawn independently. The starting point of the analysis is the observation that a seller who has favorable prior information about the buyer's true valuation for the good will also be optimistic about the buyer's prior belief.[2] Therefore, the optimal price depends on the seller's initial belief. This in turn allows the seller to signal private information to the buyer through the price. I first show that in each perfect Bayesian equilibrium surviving the D1 criterion, the price must fully reveal the seller's private information (Proposition 1.1). Yet, this signaling can be costly because it might require the seller to set a price that is distorted upwards relative to the case where her private information was public. Public information reduces this cost by revealing a part of the seller's private information for free. Indeed, it allows her to lower the price relative to the separating price under no public information, without signaling a pessimistic prior belief (Proposition 1.2). Consequently, the overall welfare increases. However, the potential buyer does not necessarily benefit because public information permits the seller to form a more accurate expectation about the buyer's prior belief. This allows her, in the absence of signaling issues, to extract a larger share of the potential gains from trade (Proposition 1.3). Together with the fact that she will also set a more efficient price, this implies an increase in profit (Proposition 1.4). While the buyer benefits from a more efficient price, he may be hurt by a smaller information rent. Therefore, the overall effect on consumer welfare is ambiguous.

Taken together, the positive value of public information for a monopolist is shown to be robust in a situation of two-sided asymmetric information. However, this is based on the assumption that out-of-equilibrium beliefs are in line with the D1 criterion and that the public signal is sufficiently precise. In the first extension of the paper I will examine the case where the seller is fully informed about the buyer's true valuation, while the buyer remains incompletely informed. There, I can fully characterize the set of perfect Bayesian equilibria and I show that in the seller's most favorable equilibrium, she always benefits from the release of a public signal, regardless of its precision (Proposition 1.6). The second

[2] In the above described situation, the inexperienced seller is unsure about what characteristics are important for the buyer and which of them are observed by him. Nevertheless, observing superior characteristics will clearly make her more optimistic about the buyer's true valuation for the good, and also about the buyer's prior belief.

extension deals with an uninformed seller. Signaling is in this case not an issue and public information then increases the seller's profit unambiguously (Proposition 1.7). The final extension studies the case where the prospective buyer does not possess private information. Here, the seller can always extract full consumer surplus. Consequently, public information has no effect on the seller's expected profit (Proposition 1.8).

Price signaling in monopoly has been studied extensively [e.g. Adriani and Deidda, 2009; Bagwell and Riordan, 1991; Ellingsen, 1997; Judd and Riordan, 1994; Laffont and Maskin, 1987; Voorneveld and Weibull, 2011]. In most of these papers, it is the seller who is fully informed about quality, while the buyer remains uninformed.[3] An exception is Judd and Riordan [1994], who examine price signaling caused by two-sided asymmetric information. These authors study a separating equilibrium where prices are always distorted upwards. I contribute to this strand of literature by showing how public information affects the cost of signaling.

Many authors have studied the effect of public information in a monopoly when it is either informative for the seller or the buyer.[4] However, only few have considered the case where public information affects both the monopolist's and the buyer's beliefs. Schlee [1996] examines a model where neither the seller nor the buyer possess private information about quality of a good. His assumptions about the buyer's preferences ensure that the profit function of the seller is convex in expected quality. Therefore, the seller always benefits from public information. In Ottaviani and Prat [2001], buyers have incomplete private information about quality (their valuation), while the seller remains completely uninformed. These authors show that the value of public information for the monopolist is always positive. Roughly speaking, this is because public information undermines private information and, therefore, allows the monopolist to extract a larger share of the potential gains from trade. This finding is closely related to the linkage principle [Milgrom and Weber, 1982]. I contribute to this strand of literature by studying

[3]In Voorneveld and Weibull [2011], the buyer also possesses private information. Their setting is therefore similar to the first extension of this paper.

[4]For example, in the quality disclosure literature, it is only the buyer who learns from disclosure.

the case where both, the seller and the buyer, possess valuable private information. Public information is then valuable for an additional reason: its effect on signaling cost.[5]

In the case of a fully informed monopolist, public information only affects the buyer's belief distribution. Several authors have studied the buyer's optimal belief distribution in a monopoly setting from the seller's viewpoint [Lewis and Sappington, 1994; Li and Shi, 2017], and the buyer's viewpoint [Roesler and Szentes, 2017]. However, in these papers, the seller does not possess private information and pricing is therefore not strategic.

Other papers have studied public information in more general settings where an informed sender wants to persuade an uninformed receiver [Alonso and Camara, 2018; Gill and Sgroi, 2012; Hedlund, 2017; Perez-Richet, 2014]. The sender here affects the amount of public information available through the choice of an experiment. Because this choice can already be informative about the seller's private information, there is an interplay between signaling and public information. The most similar to this paper is Hedlund [2017], who examines the case of an incompletely informed sender and solves for equilibria surviving the D1 criterion. Then, either the choice of the experiment fully reveals the sender's private information, or the chosen experiment is fully informative. In contrast to this paper, the receiver (the buyer) is fully uninformed.

The remainder of this paper is structured as follows. Section 1.2 presents the model and discusses the equilibrium concept. Section 1.3 presents results for two-sided asymmetric information. Section 1.4 examines cases where one party is either fully informed or not informed at all. Section 1.5 concludes.

[5]Note that Ottaviani and Prat [2001] also discuss the case of an informed monopolist. In particular they show that the seller would always benefit from signaling private information through a costless signal instead of signaling it through the price. Consequently, she would benefit from a fully informative public signal. In this paper instead, I examine the value of a public signal which is not fully informative. Hence, the price is still allowed to signal private information.

1.2 Model

1.2.1 Setup

Consider a risk neutral monopolistic seller (she) M who wants to sell a single good to one prospective buyer (he) B. The buyer's valuation of the good is drawn at the beginning of the game by nature.[6] It is either high (H) or low (L) and, without loss of generality, normalized such that $H - L = 1$. The marginal cost is normalized to zero and I assume that $H > 0$. Thus, the high valuation is higher than marginal cost, while the low valuation is allowed to be below marginal cost. In this case ($L < 0$), selling the good reduces welfare. Neither the seller nor the buyer observe the buyer's true valuation of the good. The seller's prior belief that it is high is μ_0, the buyer's prior belief is β_0. Both share a common prior belief $\mu_0 = \beta_0 \in (0, 1)$.

As in Hedlund [2017], at the beginning of the game M observes a discrete private signal and updates her prior belief μ_0 to $\mu_t \in T := \{\mu_1, \mu_2, ..., \mu_N\}$ according to Bayes rule. μ_t (or just t) is said to be M's type (μ_0 is her ex-ante type). I assume that no outcome of this signal is fully informative, so $\mu_t \in (0, 1)$. Types are ordered according to their belief that the buyer's true valuation is H, hence $\mu_1 < \mu_2 < ... < \mu_N$. The set T is common knowledge. M can then set a price p, where p_t indicates the price when her type is μ_t. A pure strategy of the seller is a price vector $\pi = (p_1, ..., p_N) \in \mathbb{R}^N$.

The potential buyer observes p_t and an outcome $s \in S := (\underline{s}, \bar{s})$ of a private signal. I will often refer to s as the buyer's type. I assume that the buyer's interpretation of p does not depend on s. It is convenient for what follows to distinguish between the belief updating from observing p and the updating from observing s. I denote the updated belief after observing p as the interim belief $\beta(p) \in [\mu_1, \mu_N]$ and the belief after observing p and s as the final belief $\beta^F(s, \beta(p))$.[7] Having observed p and s, B chooses an action $a \in A := \{0, 1\}$, where $a = 1$ is buying and $a = 0$ is not buying. His pure strategy is a function $\alpha : \{\mathbb{R} \times S\} \to A$. B

[6] Because there is only one prospective buyer, one could equally well interpret the buyer's valuation as a quality level.

[7] Following Hedlund [2017], I model the interim belief as a belief about the true valuation. Yet, it is actually a belief about the seller's type, and therefore in the interval $[\mu_1, \mu_N]$.

is assumed to maximize expected utility, and the outside payoff is set to zero. Because $H = L + 1$, the buyers expected valuation of the good when observing p and s is $\beta^F(s, \beta(p)) + L$, and his expected utility then is $a(\beta^F(s, \beta(p)) + L - p)$.

I make the following assumptions about the private signal. First, conditional on the buyer's true valuation, s is drawn independently from the seller's private information. The conditional density function of s when the buyer's true valuation is high is $f_H(s)$ and $f_L(s)$ in case it is low, and is assumed to be continuous with full support on the interval $(\underline{s}, \bar{s})$. The corresponding cumulative distribution functions are $F_H(s)$ and $F_L(s)$. The signal distribution is common knowledge. In the spirit of Milgrom [1981], s is good news whenever $f_H(s) \geq f_L(s)$, otherwise it is bad news. Without loss of generality, signals are ordered such that the monotone likelihood ratio property (MLRP) holds. I further assume that the strict version holds. Hence, for any two signals $s, s' \in (\underline{s}, \bar{s}) : s < s' \iff \frac{f_H(s)}{f_L(s)} < \frac{f_H(s')}{f_L(s')}$. This guarantees that each buyer type s has a different final belief. Finally, I assume that the private signal is fully informative in the limit: $\lim_{s \to \underline{s}} \frac{f_H(s)}{f_L(s)} = 0$ and $\lim_{s \to \bar{s}} \frac{f_H(s)}{f_L(s)} = \infty$. The first assumption is just for convenience, while the latter is crucial for the result to hold if $L < 0$. It ensures that for a given interim belief, a seller type with a positive willingness to pay always exists.

1.2.2 Equilibrium Concept

I solve this game using the concept of perfect Bayesian equilibrium (PBE).

Definition 1.1 (PBE). *A perfect Bayesian equilibrium in this game is a strategy profile (π, α) and a system of beliefs β and β^F requiring that*

(i) $\beta^F(s, \beta(p))$ is formed according to Bayes rule, for all $(p, s) \in \{\pi \times S\}$,

(ii) for any price p and signal $s \in S$, $a = 1$ if $p < \beta^F(s, \beta(p)) + L$ (and only if in case the inequality is weak),

(iii) $p_t = \arg\max_p \int_{\underline{s}}^{\bar{s}} (\mu_t f_H(s) + (1 - \mu_t) f_L(s))\, \alpha(p, s) p ds$, for any $\mu_t \in T$.

(i) Requires the buyer's belief to be consistent on the equilibrium path. According to *(ii)*, B should buy the good only if his expected valuation is larger

than the price. Finally, *(iii)* requires each seller type to have consistent expectations about the buyer's private information, and to choose a price that maximizes expected profit given these expectations and the buyer's strategy α. I denote by $\alpha(p, \hat{\beta}, s)$ B's strategy in line with requirements *(i)* and *(ii)*, if the price is p, the outcome is s, and the interim belief is set to $\hat{\beta}$. Hence, the final belief must be formed according to Bayes' rule for the given interim belief $\hat{\beta}$, and B's action should be rational given his final belief. M's profit when charging a price p, given that B observes the outcome s and has an interim belief $\hat{\beta}$ is then $\Pi(p, \hat{\beta}, s) = \alpha(p, \hat{\beta}, s)p$. The expected profit of the seller type t when charging the price p, if the interim belief is $\beta(p) = \hat{\beta}$, and otherwise everything is in line with *(i)* and *(ii)*, then is

$$\Pi_t(p, \hat{\beta}) = \int_{\underline{s}}^{\bar{s}} (\mu_t f_H(s) + (1 - \mu_t) f_L(s)) \, \Pi(p, \hat{\beta}, s)) ds.$$

I denote the expected equilibrium profit of the seller type t as $\Pi_t(\pi, \beta) = \Pi_t(p_t, \beta(p_t))$.

The equilibrium concept does not put any restrictions on out-of-equilibrium beliefs. However, the outcome of the game is very sensitive with respect to these beliefs. To focus on a specific equilibrium, I restrict these beliefs to be in line with the D1 criterion [Cho and Kreps, 1987].[8] This refinement eliminates any pooling and semi-pooling equilibrium, and it selects the seller optimal separating equilibrium. I take the formulation of the D1 criterion from Cho and Kreps [1987], but define it as Hedlund [2017] does in terms of interim beliefs. I denote the set of out-of-equilibrium interim beliefs $\beta(p')$ that make type t indifferent between choosing p_t and p' as $D^0(p', \mu_t) := \{\beta(p') \in [\mu_1, \mu_N] : \Pi_t(p', \beta(p')) = \Pi_t(p_t, \beta(p_t))\}$. The set of interim beliefs that make her strictly benefit from this deviation are denoted by $D^1(p', \mu_t) := \{\beta(p') \in [\mu_1, \mu_N] : \Pi_t(p', \beta(p')) > \Pi_t(p_t, \beta(p_t))\}$.

Definition 1.2 (Criterion D1 [Cho and Kreps, 1987]). *If for some type t there exists a second type t' with $D^1(p', \mu_t) \cup D^0(p', \mu_t) \subseteq D^1(p', \mu_{t'})$, then (p', μ_t) may be pruned from the game.*

[8] In section 1.4.1 I assume that M is fully informed about B's true valuation. This allows for a characterization of the whole set of PBE.

Hence, if a type t' exists, who strictly benefits from defecting from the equilibrium price $p_{t'}$ to an out-of-equilibrium price p' whenever the interim belief $\beta(p')$ is such that type t benefits weakly from choosing p' instead of the equilibrium price p_t, then the buyer must not associate p' with type t. Intuitively, a deviation should be associated with the seller type, that has the largest incentive to deviate.

1.3 Results

1.3.1 The seller's optimization problem

I start with the seller's pricing problem. The price affects demand directly and, via the interim belief, also the buyer's willingness to pay. When B observes the price p_t and the private signal s, her final belief is

$$\beta^F(s, \beta(p_t)) = \frac{\beta(p_t) f_H(s)}{\beta(p_t) f_H(s) + (1 - \beta(p_t)) f_L(s)}.$$

B's willingness to pay, which is $\beta^F(s, \beta(p_t)) + L$, is strictly increasing in the interim belief and in the likelihood ratio $\frac{f_H(s)}{f_L(s)}$. Because s does not affect the interim belief, the final belief must be strictly increasing in s. Consequently, at each price, at most one buyer type s can be indifferent between buying and not buying. Furthermore, all buyer types larger than s will have a strictly higher willingness to pay than s, while the willingness to pay of any lower type will be strictly below.

Any price $p \in (L, H)$ must make one buyer type indifferent between buying and not buying. The reason is that every price induces one interim belief $\beta(p) \in [\mu_1, \mu_N]$, and that the outcome of the buyer's private signal is continuously distributed and perfectly informative in the limit. Consequently, for each interim belief $\beta(p)$ a private signal s exists, such that $\beta^F(s, \beta(p)) + L = p$. If a price p makes type s indifferent between buying and not buying, I denote this price as p^s. Note that this price neither has to be unique, nor does it have to exist for any buyer type.

Expected demand when $p_t = p^s$ is

$$D_t(s) = \mu_t(1 - F_H(s)) + (1 - \mu_t)(1 - F_L(s)).$$

$D_t(s)$ is linearly increasing in the prior μ_t because $F_L(s) > F_H(s)$. Further, it is strictly decreasing in s. In absolute terms, making a higher buyer type s indifferent reduces demand more for high (low) seller types if s is good (bad) news. I will show in Lemma 1.1 that the relative loss in demand from making a higher buyer type indifferent is always strictly decreasing in μ_t. The seller's optimization problem can be written as

$$\max_{s \in S} \Pi_t(p^s, \beta(p^s)) = p^s D_t(s).$$

I will often write $\Pi_t(s, \beta(p^s))$ instead of $\Pi_t(p^s, \beta(p^s))$. Note that this profit function is defined only for buyer types s, which are indifferent at least at one price. Analogously, I will denote t's expected profit when making consumer type s indifferent for a given interim belief $\hat{\beta}$ as $\Pi_t(s, \hat{\beta})$. This profit function is defined for any buyer type $s \in S$, because for a given interim belief, any buyer type $s \in S$ can be made indifferent with a price $p^s \in (L, H)$. For convenience, in what follows I will refer to $\Pi_t(s, \beta(p^s))$ and $D_t(s)$ as profit and demand, instead of expected profit and expected demand.

1.3.2 D1 Equilibrium

While making a higher buyer type $s' > s$ instead of s indifferent always reduces demand, it does not necessarily increase the indifferent type's willingness to pay. It does so only if the interim belief $\beta(p^{s'})$ is not too low compared to $\beta(p^s)$. Hence, a defection from a candidate equilibrium price p^s to $p^{s'}$ can only be profitable if $\beta(p^{s'})$ is sufficiently large. The first Lemma states that whenever $\beta(p^{s'})$ is sufficiently large such that a type μ_t weakly benefits from defecting from p^s to $p^{s'}$, then any higher type $\mu_{t'} > \mu_t$ would strictly benefit from doing so. All proofs are relegated to the Appendix.

Lemma 1.1. *For any two seller types $t, t' \in T$ with $\mu_{t'} > \mu_t$, signals $s, s' \in S$*

with $s' > s$ *and interim beliefs* $\hat{\beta}_1, \hat{\beta}_2 \in (0,1)$ *the following holds: If* $\Pi_t(s', \hat{\beta}_2) \geq \Pi_t(s, \hat{\beta}_1)$*, then* $\Pi_{t'}(s', \hat{\beta}_2) > \Pi_{t'}(s, \hat{\beta}_1)$*.*

Lemma 1.1 follows from the fact that the relative cost from making a higher buyer type indifferent, in terms of lower demand, is strictly decreasing in the seller's type. The reason is that the mass of marginal buyer types relative to inframarginal buyer types is strictly decreasing in μ_t for any possible s. Consequently, the relative gain from a higher price paid by inframarginal buyers, compared to the loss of marginal buyers, is strictly increasing in M's type. Therefore, if p^s is a candidate equilibrium price for types t and t', then the D1 criterion immediately puts restrictions on the out-of-equilibrium belief $\beta(p^{s'})$. It requires the buyer not to associate $p^{s'}$ with the seller type t. Hence, $\beta(p^{s'})$ must not be affected by the value μ_t.

An immediate consequence of this result is that no (partial) pooling equilibrium can survive the D1 criterion. Any deviation from a pooling price to a higher price must be associated with the highest M-type (out of the whole set of types, who choose the pooling price). Making a marginally higher buyer type indifferent then would always result in a non-marginal increase in the interim belief and, therefore, in a non-marginal increase in the willingness to pay of all inframarginal buyer types. Because such a deviation lowers demand only marginally, all seller types would have an incentive to deviate from the pooling price. Thus, each seller type must choose a different price in equilibrium. This price must then fully reveal the seller's type, such that $\beta(p_t) = \mu_t$. Such a separating equilibrium exists, as stated in the first Proposition.

Proposition 1.1. *There exists a fully separating PBE that survives the D1 criterion. Moreover, each equilibrium surviving the D1 criterion is fully separating. Further, if* $p_t = p^s$ *and* $p_{t'} = p^{s'}$*, then* $\mu_t < \mu_{t'} \iff s < s'$*.*

Proposition 1.1 shows that a fully separating equilibrium exists.[9] The separating equilibrium is not necessarily unique. However, each equilibrium that survives

[9] In contrast to Adriani and Deidda [2009], a separating equilibrium exists even if $L \leq 0$. Crucial for this is that first, no seller type is fully informed about the buyer's true valuation, and second, the buyer's private signal has a sufficient favorable outcome. Then it is guaranteed that even the lowest seller type can target a buyer type such that her expected profit when doing so is strictly positive.

the D1 criterion must be fully separating. Finally, a higher seller type will set a price that makes a strictly higher buyer type indifferent than a lower seller type.

This kind of price signaling is typically costly. Compared to the situation where the seller's type was public, the seller might have to distort the price upwards to signal her type. The D1 criterion selects a separating equilibrium that is most favorable to the seller. To see this, note that the lowest seller type will not distort her price, because the interim belief can never be worse than μ_1. She will therefore make a buyer type $s^1 = \arg\max_{s \in S} \Pi_1(s, \mu_1)$ indifferent. Let then S^2 be the set of buyer types that profitably separate μ_2 from μ_1. That is: $\Pi_1(s^1, \mu_1) > \Pi_1(s', \mu_2)$ and $\Pi_2(s', \mu_2) \geq \Pi_2(s^1, \mu_1)$, $\forall\ s' \in S^2$. The D1 criterion ensures that $s^2 = \arg\max_{s' \in S^2} \Pi_2(s', \mu_2)$. The reason is that making s' indifferent will never be associated with μ_1. Hence, out-of-equilibrium beliefs are bounded, $\beta(p^{s'}) \geq \mu_2$. Consequently, there is no reason why μ_2 should target another type than s^2, except to mimic a higher seller type. However, any higher seller type will target a sufficiently high buyer type such that mimicking is unprofitable. Hence, the equilibrium is efficient in the sense that every seller type makes the profit-maximizing buyer type indifferent out of all buyer types separating her from lower seller types.[10]

Prices depend on the whole set of seller types T and the distribution of buyer types. The exact distribution of seller types is not important though. This follows from the extreme assumptions made about out-of-equilibrium beliefs. B must fully associate any off-equilibrium price with the seller type having the largest incentive to deviate to that price, regardless of the likelihood that the seller has this type in the first place.

1.3.3 Public information

Having established that a PBE surviving the D1 criterion exists and that it is fully separating, I now proceed by studying the value of public information in this equilibrium. Let a_i (b_i) be the likelihood that a particular outcome $i \in I$ of a public observable signal z is realized if the true valuation is H (L). If z gets

[10] In contrast to Judd and Riordan [1994], separation is not necessarily costly at all, because undistorted prices may already be separating.

released, then one outcome i must be observed, hence $\sum_{i\in I} a_i = \sum_{i\in I} b_i = 1$. I assume that the signal outcome is independent of the realizations of the private signals and that no outcome is fully informative. An outcome is good news whenever the likelihood ratio $\gamma_i = \frac{a_i}{b_i}$ is larger than one. I denote the updated belief of seller type t as μ_t^i, the corresponding price as p_t^i, and the corresponding profit as Π_t^i. The buyer's updated prior belief is β_i^0. His interim belief, having observed i and p, is $\beta_i(p)$, and his final belief after observing i, p and s is $\beta_i^F(s, \beta_i(p))$. Because i is observed by both players, conditional on the updated seller type μ_t^i and the updated buyer's prior belief β_i^0, the equilibrium definition remains the same. A PBE therefore just requires that μ_t^i and β_i^0 are updated according to Bayes rule and that Definition 1.1 holds for every outcome $i \in I$. From Proposition 1.1, $\beta_i(p_t^i) = \mu_t^i$ must hold in the equilibrium. I first examine the effect of such an outcome on the cost of signaling.

1.3.3.1 Signaling cost

Consider the effect of public information on the mimicking incentives in a case with only two seller types t and t'. Let $p_t = p^s$ and $p_{t'} = p^{s'} > p^s$ be the equilibrium prices in the case without public information. An outcome i affects t's relative cost and her relative gain from mimicking t' in several ways.

The effect of i on the relative demand loss is straightforward. By Lemma 1.1 this loss is strictly decreasing in the seller's belief. Consequently, it decreases when i is good news and increases when i is bad news. Therefore, choosing $p^{s'}$ becomes relatively less costly for t if i is good news, and it becomes relatively more costly if i is bad news. Note that this cost does not get arbitrarily small as i becomes more favorable.

The effect of i on the relative gain from choosing $p^{s'}$ is less direct. The overall direction of the effect is stated in the following Lemma.

Lemma 1.2. *Far any $s, s' \in S$ with $s < s'$ and $t, t' \in T$ with $\mu_t < \mu_{t'}$, the ratio $\frac{\beta_i^F(s', \mu_{t'}^i)+L}{\beta_i^F(s, \mu_t^i)+L}$ is increasing in γ_i if and only if*

$$\frac{L}{1+L} > \frac{\mu_t}{1-\mu_t}\frac{\mu_{t'}}{1-\mu_{t'}}\frac{f_H(s)}{f_L(s)}\frac{f_H(s')}{f_L(s')}\gamma_i^2.$$

Lemma 1.2 shows how the relative gain from mimicking t' by setting the price $p^{s'}$ instead of p^s changes, depending on the outcome i. Roughly speaking, if i is bad news ($\gamma_i < 1$), it increases the gain from mimicking a higher type, when seller types and indifferent buyer types are sufficiently high. The reason for this is that favorable private signal outcomes on the buyer and the seller side, contain more information if the outcome i is bad news. The opposite is the case if i is good news. Then unfavorable private signal outcomes become more informative.

Three cases should be considered in more detail. First, if $L \leq 0$ the inequality never holds. Hence, in this case the relative gain from mimicking a higher type always increases if i is bad news and decreases if i is good news. Second, if $L > 0$ and γ_i is sufficiently large, then the inequality does not hold, either. In this case, i reduces the relative gain from mimicking a higher type. Finally, if $L > 0$ and γ_i is sufficiently low, the inequality must hold. Hence, i then reduces the gain from mimicking as well. Therefore, when $L > 0$ the relative gain from mimicking is hump-shaped in γ_i, and it becomes (arbitrarily) small as i becomes precise. Because the cost from mimicking is bounded from below, the next result follows.

Proposition 1.2. *Consider any two seller types $t, t' \in T$ with $\mu_t < \mu_{t'}$ and signals $s, s' \in S$ with $s < s'$. If $L > 0$ and γ_i is sufficiently small or sufficiently large, then $\Pi_t^i(s, \mu_t^i) > \Pi_t^i(s', \mu_{t'}^i)$. If $L \leq 0$, then this inequality holds whenever γ_i is sufficiently large.*

Proposition 1.2 shows that public information reduces the seller's incentive to mimic a higher type. Moreover, it establishes more generally that public information lowers the gain from making high buyer types indifferent. Roughly speaking, this happens because it reduces the effect private information (on both sides) has on the buyer's final belief. For $L > 0$, the result holds whenever i is sufficiently good or sufficiently bad news. For $L \leq 0$, it is required that i is good news. Because Proposition 1.2 holds for all seller types, the next result follows immediately.

Corollary 1.1. *A sufficiently favorable outcome of a public signal makes every seller type target a strictly lower buyer type. A sufficiently unfavorable outcome of a public signal makes every seller type target a strictly lower buyer type if $L > 0$.*

Corollary 1.1 shows that all seller types will make lower buyer types indifferent in response to public information (at least if the outcome is sufficiently favorable). Hence, public information reduces the cost of signaling, because it allows the seller to lower the price, without signaling unfavorable private information. Corollary 1.1 has strong welfare implications. If $L > 0$, then buying is always welfare increasing. The welfare effect of targeting a lower buyer type is therefore positive. Further, B benefits from i when his type is sufficiently low. Instead of being below the marginal buyer type he might become strictly above because the seller might make a strictly lower type indifferent. Higher buyer types benefit only if the absolute price decreases, which is always the case if the outcome of the public signal is sufficiently unfavorable. This is intuitive. If the public signal is bad news, favorable private information becomes relatively more informative. Consequently, a higher buyer type will benefit from a higher information rent.

1.3.3.2 Rent extraction

Here I show that for a given separating price p^s, public information allows M to realize a higher share of the potential gains from trade. Proposition 1.3 states that conditional on making the same buyer type s indifferent, profit is strictly convex in the seller's type μ_t.

Proposition 1.3. $\Pi_t(s, \mu_t)$ *is strictly convex in* μ_t.

To understand this result consider first the case where s is bad news. A low interim belief $\beta(p^s)$ then affects the final belief of the s type less than a higher interim belief. The final belief of a low buyer type is therefore convex in the interim belief which is equal to the seller's updated belief μ_t (or μ_t^i if an outcome i was released), and so is the seller's profit. By the same logic, the final belief of a high buyer type is concave in the interim belief. Hence, p^s is concave in μ_t if s is good news. Yet, the profit remains convex because expected demand when making s indifferent is linearly increasing in μ_t. The positive demand effect of a higher μ_t becomes larger as higher the indifferent buyer type s is. Hence, if s is a high buyer type, then the effect of a higher μ_t on the willingness to pay is small but its positive demand effect becomes large. Together, both effects ensure that the profit function $\Pi_t(s, \mu_t)$ is convex in the seller's type.

The public signal imposes a mean preserving spread in each seller type's belief. Therefore, as long as the seller makes the same buyer type indifferent it increases her profit.[11] Clearly, public information then also increases the total surplus. Expected demand and the expected valuation is not affected by public information, but demand is higher in cases where the expected valuation is higher. The buyer's information rent does not necessarily increase because the seller can extract at least a part of this additional surplus. Indeed, the public signal is informative about the buyer's willingness to pay and permits the seller to form a more accurate belief about it. As Proposition 1.3 shows, this allows her to make a higher profit just by targeting a fix buyer type.

1.3.3.3 Total effect

The seller benefits from public information through two different channels: by reduced signaling cost, as described in Proposition 1.2, and by extracting a larger surplus from a given set of buyer types, as stated in Proposition 1.3. Let type t's expected profit when a public signal z is released be

$$\Pi_t(\pi, \beta, z) = \sum_{i \in I} \left(a^i \mu_t + b^i (1 - \mu_t) \right) \Pi_t^i(p_t^i, \mu_t^i),$$

to formulate the main result.

Proposition 1.4. *If $L > 0$ and if every outcome $i \in I$ is sufficiently precise (i.e. γ_i is sufficiently high or low), then public information increases the profit of any seller type, that is, $\Pi_t(\pi, \beta, z) > \Pi_t(p_t, \mu_t)$. The inequality always holds for the lowest seller type, that is, $\Pi_1(\pi, \beta, z) > \Pi_1(p_1, \mu_1)$.*

Proposition 1.4 shows that any seller type benefits from the release of a publicly observable signal that is informative about the buyer's true valuation. It is then obvious that the ex-ante seller type μ_0 does so as well. Hence, the general principle, according to which the value of public information is positive for a

[11] Bose et al. [2006] derive the same result when B's private signal outcome is discrete.

monopolist, also applies when the monopolist possesses private information. The conditions stated in the Proposition are sufficient but not necessary for the seller to benefit. They guarantee that to separate, observing the outcome of the public signal does not require the seller to target a higher buyer type. Because the lowest type (μ_1) does not face any cost from signaling, she unambiguously benefits from the public signal.

A related result of Ottaviani and Prat [2001] shows that the profit from a monopolistic seller can never be higher than in the full information case. Consequently, she would always benefit from a fully informative public signal. This is intuitive because in the full information case the monopolist can extract full consumer surplus and she can choose a price that is not distorted. In contrast, the price in Proposition 4 still contains information. Due to the discontinuous nature of the D1 refinement, it is not apparent that the D1-refined equilibrium outcome and the full information equilibrium outcome coincide when the public signal becomes precise. Indeed, the D1-refined equilibrium outcome does, for example, not approach the full information equilibrium outcome, even if B's private signal becomes arbitrarily precise. This is also true for a public signal that changes the distribution of seller types but not its support T. Such an outcome would not affect prices at all and could easily lower the seller's profit.

1.4 Extensions

1.4.1 Fully informed seller

In this section, I will study the situation where the seller is fully informed about the buyer's true valuation of the good. Everything else remains as described in Section 1.2. Her type is then either $\mu_l = 0$ or $\mu_h = 1$. This information asymmetry is comparable to the standard case [Akerlof, 1970], except that the prospective buyer is allowed to possess private information. Information asymmetry remains two-sided because the seller cannot observe the buyer's private information. Yet, the price can no longer fully reveal the seller's type if it is high. Otherwise, the buyer's private information would not have any effect on his final belief. Then any buyer type would have a willingness to pay of H and buy the good when observing the revealing price. But then, choosing this price would be a strictly dominant strategy of both seller types. Hence, no fully separating equilibrium exists. However, by the logic of Proposition 1.1, no (semi-) pooling equilibrium can survive the D1 criterion. Consequently, no equilibrium exists that survives the D1 criterion.

Instead, the limitation to only two seller types facilitates the characterization of the whole set of PBE. For this, I allow the seller to play a mixed strategy and denote by q_t^s the probability that the seller type t sets a price that makes buyer type s indifferent, that is, $p_t = p^s$. For convenience, I assume that $L > 0$. Proposition 1.5 fully characterizes the set of PBE.[12]

Proposition 1.5. *In any perfect Bayesian equilibrium:*

(a) The high seller type plays a pure strategy $q_h^s \in \{1, 0\}$, and makes one buyer type indifferent. If $q_h^s = 1$, then $q_l^s > 0$, while with probability $1 - q_l^s$ the low type sets the price $p_l = L$.

(b) The price is pooling when β_i^0 is sufficiently high and semi-pooling otherwise.

In addition, a PBE where buyer type s is indifferent between buying and not buying ($q_t^s > 0$) exists if an only if $(1 - F_L(s))H > L$.

Proposition 1.5 shows that no fully separating equilibrium exists. Further, each equilibrium is pooling if the buyer's updated belief is high, and semi-pooling if it is low. Finally, whether an equilibrium exists where a type s is made indifferent, is not affected by the buyer's updated belief.

Consider the statements in turn. Part *(a)* shows that no fully separating equilibrium exists because a price can never fully reveal the high seller type. Furthermore, the high seller type will never randomize between different prices because this would require the low type to mix among those prices as well. However, both types cannot be indifferent between the same prices due to the MLRP.

[12] Voorneveld and Weibull [2011] characterize the set of PBE in a closely related setting. They exclude mixed strategies, but allow the high seller type to have higher marginal cost. Therefore, separating equilibria exist in their model.

Consequently, whenever the low seller type mixes between two prices, one of them must fully reveal her type. This price is then $p_l = L$. When choosing $p_l = L$, the prospective buyer buys with probability one, while expected demand when choosing $p_l = p^s$ is D_l^s. Thus, the low seller type will randomize between prices only if $p^s D_l^s = L$. Because D_l^s is unaffected by the buyer's belief ($D_l^s = 1 - F_L(s)$), in such a semi-pooling equilibrium, the price p^s is not affected by the outcome of the public signal. All information that is revealed through the public signal is already contained in the semi-pooling price.

Part *(b)* states that a PBE is always semi-pooling if the updated prior belief is low, and pooling otherwise. Intuitively, if β_i^0 is high, the pooling price must be high as well. The low seller type then does not have an incentive to set a different price. Meanwhile, if β_i^0 is low, p^s gets close to L in case of pooling. Revealing to be the low seller type becomes then relatively cheap. The equilibrium must then be semi-pooling.

The final statement shows that targeting buyer type s can be an equilibrium strategy if and only if the expected demand for the low type when doing so is not too low. Consequently, the outcome of the public signal does not affect what buyer types can be made indifferent in equilibrium.

To make a statement about the value of public information in this setting, I focus on the equilibrium most favorable to the seller.[13] Because the profit maximizing equilibrium differs between seller types, I consider the profit maximizing equilibrium for the ex-ante seller type μ_0. Her expected profit is

$$\Pi_0(\pi, \beta, z) = \mu_0 \sum_{i \in I} a^i \Pi_h^i(p_h^i, \beta_i(p_h^i)) + (1 - \mu_0) \sum_{i \in I} b^i \Pi_l^i(p_l^i, \beta_i(p_l^i))$$

if a public signal is released. If no public signal is released it is

$$\Pi_0(\pi, \beta) = \mu_0 \Pi_h(p_h, \beta(p_h)) + (1 - \mu_0) \Pi_l(p_l, \beta(p_l)).$$

Proposition 1.6. *If the seller is fully informed, then public information always increases the seller's profit in the seller optimal PBE, that is* $\Pi_0(\pi, \beta, z) \geq$

[13] This is standard and analogous to the previous section, where the seller optimal separating equilibrium was studied.

$\Pi_0(\pi, \beta)$. *The inequality is strict, if at least one outcome i is sufficiently favorable (i.e. γ_i sufficiently high).*

Proposition 1.6 shows that the seller's profit in the seller optimal PBE when public information is released is larger than her profit in the seller optimal PBE when no public information is released. In other words, if the ex-ante seller type could commit to an equilibrium strategy, then she would always benefit from additional public information. To understand this result, assume that without public information the ex-ante profit is maximized when $p_h = p^s$. Analogous to Proposition 1.3, if p^s is a pooling price, the seller's ex-ante profit is convex in the buyer's prior belief β_0 (which is equal to the seller's ex-ante type). The result then immediately follows from the fact that each public signal imposes a mean preserving spread in the buyer's prior belief, and from Proposition 1.5, stating that targeting type s is still an equilibrium strategy of a PBE after the release of public information. If p^s is a semi-pooling price, then it is not affected by the public signal. Observing p^s is already informative about the seller's type, and the public signal does not add any additional information. Hence, it does not have any effect on the profit.

It is worth mentioning two counter-intuitive features of the semi-pooling equilibrium. First, the low seller type's expected profit in any semi-pooling equilibrium is L. This is the lowest possible profit. Meanwhile, each PBE must be pooling when sufficiently favorable public information gets released. In this case, the low seller type's profit is strictly larger than L. Consequently, regardless of equilibrium selection, if the equilibrium without the public signal is semi-pooling, the low seller type would always (weakly) benefit from the release of a public signal. Second, in the seller optimal semi-pooling equilibrium, releasing bad news will never lower the seller's profit. This is a consequence of Proposition 1.5(b), which states that the equilibrium remains semi-pooling whenever β_i^0 falls. As the equilibrium remains semi-pooling, bad news cannot affect the buyer's willingness to pay. Meanwhile, sufficiently favorable news will make the equilibrium pooling, and then profit increases. This unexpected result follows from a positive externality in the semi-pooling case. The high seller type benefits when the low seller type reveals herself as the low type. This probability must increase, when the outcome of the public signal is bad news.

1.4.2 Uninformed seller

In this section, I will study the case where the seller does not posses private information, while the buyer remains privately (but incompletely) informed. There is only one seller type μ_0, and her expected profit is

$$\Pi_0(\pi, \beta, z) = \mu_0 \sum_{i \in I} a^i \Pi_0^i(p_0^i, \beta_i(p_0^i)) + (1 - \mu_0) \sum_{i \in I} b^i \Pi_0^i(p_0^i, \beta_i(p_0^i)).$$

The next result follows immediately from Proposition 1.3 and the fact that the interim belief is always equal to the seller's type because there is only one type.

Proposition 1.7. *If the seller does not posses private information about the buyer's true valuation, then public information always increases her expected profit, that is,* $\Pi_0(\pi, \beta, z) > \Pi_0(\pi, \beta)$.

This result is a special case of Theorem 1 in Ottaviani and Prat [2001]. It shows that if signaling is not an issue, the value of public information is always positive for the seller.

1.4.3 Uninformed buyer

In this final section, I will examine the case where the buyer does not possess private information about his true valuation. For convenience, I again assume that $L > 0$. Because there is only one buyer type, the seller's private information is no longer informative about the buyer's belief. The seller's profit when setting a price p is p if $L + \beta_i(p) \geq p$, respectively 0 otherwise. Consequently, each seller type must set the same price p^i, which then cannot contain any information about the seller's type. Each PBE must therefore be pooling but there is a whole continuum of equilibria. I again focus on the equilibrium most favorable to the seller. The price in this equilibrium makes the buyer indifferent between buying and not buying, hence $p^i = \beta_i^0 + L$. The seller's profit is

$$\Pi_0(\pi, \beta, z) = \mu_0 \sum_{i \in I} a^i (L + \beta_i^0) + (1 - \mu_0) \sum_{i \in I} b^i (L + \beta_i^0).$$

Because the public signal just imposes a mean preserving spread into β_0, the next result follows.

Proposition 1.8. *If the buyer does not possess private information about his true valuation, then, in the seller optimal PBE, public information does not affect the seller's profit, that is,* $\Pi_0(\pi, \beta, z) = \Pi_0(\pi, \beta)$.

Proposition 1.8 shows that public information does not affect the seller's expected profit if the prospective buyer does not possess private information. This result is contrary to Schlee [1996], where the seller profits from public information in the absence of any private information (on both sides). The difference arises from the unit demand assumption in my model, which makes the profit function linear in the buyer's expected valuation. Meanwhile, in Schlee [1996] demand is elastic. Hence, if the expected valuation is high, the monopolist will charge a higher price and sell a larger amount. This makes the profit function convex.

1.5 Conclusion

This paper has shown how public information increases welfare and profit in a situation of two-sided asymmetric information. By revealing a part of the seller's and the buyer's private information, it reduces the cost of price signaling and it also allows the seller to extract a larger surplus from a given set of buyer types. A limitation of this result is that the positive value of public information can only be established for signals that are sufficiently precise. However, it was shown that in the case of a fully informed monopolist, any public signal is profit increasing in the seller optimal equilibrium. It seems likely that this finding also holds for the incompletely informed monopolist.

The model makes clear empirical predictions about the evolution of monopoly prices and sales if public information is released, for example, in the form of additional consumer reviews on online marketplaces. First, if the low valuation of the good is higher than marginal cost, additional information (when sufficiently precise) should always increase sales, regardless of being bad news or good news. This follows directly from Corollary 1.1. The intuition for this result is that additional public information reduces the gain from targeting a high buyer type because it lowers the effect private information has on the final belief. Second, the price should decrease if bad news is released because this lowers the willingness to pay of all buyer types and, additionally, the seller will target a lower buyer type. The price effect from the release of good news is ambiguous. Good news increases the willingness to pay of all buyer types, but the seller will target a lower buyer type.

These predictions differ from those in the literature about seller reputation, where the seller is fully informed about the buyer's true valuation while the buyer does not possess private information.[14] There, good news (e.g. a favorable customer review) should increase sales and prices. Meanwhile, bad news should decrease both. An interesting suggestion for future research would be to empirically evaluate how the effect of ratings and reviews differs with the underlying kind of information asymmetry. In particular, a systematical difference should arise between situations where prospective buyers possess private information, respectively do not possess private information.

[14] See Tadelis [2016] for an overview.

1.6 Appendix

1.6.1 Proof of Lemma 1.1

$\beta^F(s, \hat{\beta})$ does not depend on the seller type. Therefore, the statement is true if $\frac{D_t(s)}{D_t(s')} > \frac{D_{t'}(s)}{D_{t'}(s')}$. Hence, it is sufficient to show that relative demand $\frac{D_t(s)}{D_t(s')}$ is decreasing in μ_t. This is indeed the case because $\frac{D_t(s)}{D_t(s')} = \frac{\mu_t(1-F_H(s)+(1-\mu_t)(1-F_L(s))}{\mu_t(1-F_H(s')+(1-\mu_t)(1-F_L(s'))} = 1 + \frac{\mu_t(F_H(s')-F_H(s)+(1-\mu_t)(F_L(s')-F_L(s))}{\mu_t(1-F_H(s')+(1-\mu_t)(1-F_L(s'))}$ and therefore

$$\frac{\partial\left(\frac{D_t^s}{D_t^{s'}}\right)}{\partial\mu} < 0 \iff \frac{F_L(s')}{F_L(s)} < \frac{F_H(s')}{F_H(s)}.$$

This final statement is true due to the MLRP. □

1.6.2 Proof of Proposition 1.1

I prove the statements consecutively.

Existence: I start with the following claims.

Claim 1.1. *For any two seller types $t, t' \in T$ with $\mu_t < \mu_{t'}$ and price $p_t = p^s$, a nonempty set of buyer types $S' \subset S$ exists, such that for each type $s' \in S'$:*

(a) $\Pi_t(s, \mu_t) > \Pi_t(s', \mu_{t'})$, *and*

(b) $\Pi_{t'}(s, \mu_t) \leq \Pi_{t'}(s', \mu_{t'})$.

Proof. First, it is obvious that $\Pi_{t'}(s, \mu_t) < \Pi_{t'}(s, \mu_{t'})$. Second, $\Pi_{t'}(s', \mu_{t'})$ is continuous in s' because for a given interim belief $\mu_{t'}$, the final belief is continuously increasing in s', and demand is continuously decreasing in s'. Third, $\lim_{s'\to\bar{s}} \Pi_{t'}(s', \mu_{t'}) = 0$ because $\lim_{s'\to\bar{s}} D_{t'}(s') = 0$, while the willingness to pay cannot be higher than H. Hence, it is possible to choose $s' > s$ such that $\Pi_{t'}(s, \mu_t) = \Pi_{t'}(s', \mu_{t'})$. By Lemma 1.1, $\Pi_t(s, \mu_t) > \Pi_t(s', \mu_{t'})$ then holds. □

Claim 1.2. *For any three seller types $t, t', t'' \in T$ with $\mu_t < \mu_{t'} < \mu_{t''}$, signals $s, s', s'' \in S$ with $s < s' < s''$, and interim beliefs $\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3$, the following holds:*

(a) If $\Pi_{t'}(s', \hat{\beta}_2) \geq \Pi_{t'}(s'', \hat{\beta}_3)$ *and* $\Pi_t(s, \hat{\beta}_1) \geq \Pi_t(s', \hat{\beta}_2)$*, then* $\Pi_t(s, \hat{\beta}_1) > \Pi_t(s'', \hat{\beta}_3)$*.*

(b) If $\Pi_{t''}(s'', \hat{\beta}_3) \geq \Pi_{t''}(s', \hat{\beta}_2)$ *and* $\Pi_{t'}(s', \hat{\beta}_2) \geq \Pi_{t'}(s, \hat{\beta}_1)$*, then* $\Pi_{t''}(s'', \hat{\beta}_3) > \Pi_{t''}(s, \hat{\beta}_1)$*.*

Proof. I prove both parts in turn.

(a) Assume the inequality does not hold and get the following contradiction. If $\Pi_t(s, \hat{\beta}_1) \leq \Pi_t(s'', \hat{\beta}_3)$, then $\Pi_t(s', \hat{\beta}_2) \leq \Pi_t(s'', \hat{\beta}_3)$ because $\Pi_t(s', \hat{\beta}_2) \leq \Pi_t(s, \hat{\beta}_1)$ holds by assumption. But then, $\Pi_{t'}(s', \hat{\beta}_2) < \Pi_{t'}(s'', \hat{\beta}_3)$ by Lemma 1.1 and the fact that $\mu_t < \mu_{t'}$.

(b) Assume the inequality does not hold and get the following contradiction. If $\Pi_{t''}(s'', \hat{\beta}_3) \leq \Pi_{t''}(s, \hat{\beta}_1)$, then $\Pi_{t''}(s', \hat{\beta}_2) \leq \Pi_{t''}(s, \hat{\beta}_1)$ because $\Pi_{t''}(s', \hat{\beta}_2) \leq \Pi_{t''}(s'', \hat{\beta}_3)$ holds by assumption. But then, $\Pi_{t'}(s', \hat{\beta}_2) < \Pi_{t'}(s, \hat{\beta}_1)$ by Lemma 1.1 and the fact that $\mu_{t'} < \mu_{t''}$..

□

Therefore, we can construct a simple equilibrium where μ_1 targets the buyer type s^1 such that $s^1 = \arg\max_{s \in S} \Pi_1(s, \mu_1)$. Denote by S^2 the set of buyer types such that for any type $s'' \in S^2$ we have *(a)* $\Pi_1(s^1, \mu_1) > \Pi_1(s'', \mu_2)$ and *(b)* $\Pi_2(s^1, \mu_1) \leq \Pi_2(s'', \mu_2)$. By Claim 1.1 this set is nonempty. Let then μ_2 target the buyer type s^2 such that $s^2 = \arg\max_{s'' \in S^2} \Pi_2(s'', \mu_2)$. Denote by S^3 the set of buyer types such that for any type $s''' \in S^3$ we have *(a)* $\Pi_2(s^2, \mu_2) > \Pi_2(s''', \mu_3)$ and *(b)* $\Pi_3(s^2, \mu_2) \leq \Pi_3(s''', \mu_3)$. Again, Claim 1.1 ensures that this set is nonempty. Let then μ_3 target the buyer type s^3 such that $s^3 = \arg\max_{s''' \in S^3} \Pi_3(s''', \mu_3)$. From Claim 1.2 we then also know that $\Pi_1(s^1, \mu_1) > \Pi_1(s^3, \mu_3)$ and $\Pi_3(s^3, \mu_3) > \Pi_3(s^1, \mu_1)$. We can continue in the same way until we have constructed a fully separating equilibrium.

No (semi)-pooling: Suppose an equilibrium is not separating and a set of types T' that consists of strictly more than one seller type chooses the same price $p^{T'} = p^s$. Denote the interim belief that is consistent with observing $p^{T'}$ by $\beta(p^{T'})$

and the highest type in T' by $\bar{t}'$. Consider now a deviation from p^s to a price $p^s + d > p^s$. As long as $p^s + d < H$, this price must make one buyer type s' indifferent between buying and not buying. Then

$$p^s + d = \frac{\beta(p^s + d) f_H(s')}{\beta(p^s + d) f_H(s') + (1 - \beta(p^s + d)) f_L(s')}.$$

Because $d > 0$ it must be that $s' > s$, otherwise all seller types $t' \in T'$ would benefit from this deviation. Therefore, by Lemma 1.1, the highest type $\bar{t}'$ benefits strictly from this deviation, whenever another seller types $t' \in T'$ benefits weakly. That is, $D^1(\beta(p^s + d), t') \cup D^0(\beta(p^s + d), t') \subseteq D^1(\beta(p^s + d), \overline{t'})$, $\forall t' \neq \overline{t'} \in T'$. Hence, the D1 criterion requires that $\beta(p^s + d) = \mu_{\bar{t}'}$ and, therefore,

$$p^s + d = \frac{\mu_{\bar{t}'} f_H(s')}{\mu_{\bar{t}'} f_H(s') + (1 - \mu_{\bar{t}'}) f_L(s')}.$$

By lowering d, s' becomes arbitrarily close to s. However, even if s' approaches s, d remains strictly positive because $\mu_{\bar{t}'} - \beta(p^{T'}) > 0$ does not get smaller. Thus, a deviation from p^s exists, that reduces demand only marginally but results in a non marginal increase in the willingness to pay. But then, this deviation would be profitable for each type $t' \in T'$. Hence, $p^{T'}$ cannot be a best response.

Order: Follows directly from Lemma 1. □

1.6.3 Proof of Lemma 1.2

$\frac{p(s', \mu^i_{t'})}{p(s, \mu^i_t)} = \frac{\frac{m'\sigma'\gamma_i}{m'\sigma'\gamma_i + 1} + L}{\frac{m\sigma\gamma_i}{m\sigma\gamma_i + 1} + L}$, where $m' = \frac{\mu_{t'}}{1-\mu_{t'}}$, $m = \frac{\mu_t}{1-\mu_t}$, $\sigma' = \frac{f_H(s')}{f_L(s')}$ and $\sigma = \frac{f_H(s)}{f_L(s)}$. Taking the first derivative yields

$$\frac{\partial \left(\frac{p(s', \mu^i_{t'})}{p(s, \mu^i_t)} \right)}{\partial \gamma_i} = \frac{\left(\frac{m\sigma\gamma_i}{m\sigma\gamma_i + 1} + L \right) \frac{m'\sigma'}{(m'\sigma'\gamma_i + 1)^2} - \left(\frac{m'\sigma'\gamma_i}{m'\sigma'\gamma_i + 1} + L \right) \frac{m\sigma}{(m\sigma\gamma_i + 1)^2}}{\left(\frac{m\sigma\gamma_i}{m\sigma\gamma_i + 1} + L \right)^2}.$$

Because the denominator is always positive

$$\frac{\partial \frac{p(s',\mu^i_{t'})}{p(s,\mu^i_t)}}{\partial \gamma_i} > 0 \iff \frac{L}{1+L} > \frac{\mu_t}{1-\mu_t}\frac{\mu_{t'}}{1-\mu_{t'}}\frac{f_H(s)}{f_L(s)}\frac{f_H(s')}{f_L(s')}\gamma_i^2.$$

□

1.6.4 Proof of Proposition 1.2

If $L > 0$, the ratio $\frac{\beta_i^F(s',\mu^i_{t'})+L}{\beta_i^F(s,\mu^i_t)+L}$ is decreasing whenever γ_i is sufficiently high or low. In this case, this ratio therefore monotonically converges to 1 as $\lim_{\gamma_i\to 0}\frac{p(s',\mu^i_{t'})}{p(s,\mu^i_t)} = \lim_{\gamma_i\to\infty}\frac{p(s',\mu^i_{t'})}{p(s,\mu^i_t)} = 1$. Meanwhile, the relative demand loss is strictly decreasing in γ_i (by Lemma 1.1), and therefore bounded from below $\frac{D^i_t(s)}{D^i_t(s')} > \frac{1-F_H(s)}{1-F_H(s')} = 1+\frac{F_H(s')-F_H(s)}{1-F_H(s')}$. Hence, if γ_i gets sufficiently large or small, the inequality $\left(\beta_i^F(s,\mu^i_t)+L\right)D^i_t(s) > \left(\beta_i^F(s',\mu^i_{t'})+L\right)D^i_t(s')$ must hold. If $L \leq 0$ it holds when γ_i is sufficiently large. □

1.6.5 Proof of Corollary 1.1

Denote the seller type targeted by t when i is realized as s^{ti}, respectively, s^t when no public signal is released. S^{ti} and S^t are defined analogously. In any equilibrium $s^{1i} = \arg\max_{s\in S}\Pi_1(s,\mu^i_1)$, because out-of-equilibrium interim beliefs cannot be worse than μ^i_1. Proposition 1.2 then directly implies that when i is precise $s^{1i} < s^1$ (even if s^1 and s^{1i} are not unique) and that $S^2 \subset S^{2i}$. In any equilibrium $s^{2i} = \arg\max_{s''\in S^2}\Pi_2(s'',\mu^i_2)$ because targeting any buyer type $s'' \in S^{2i}$ will never be associated with a lower type than μ^i_2. Proposition 1.2 then implies that when i is precise $s^{2i} < s^2$ and $S^3 \subset S^{3i}$. The same argument can be made for all seller types. □

1.6.6 Proof of Proposition 1.3

When $p_t = p^s$ is a separating price, the interim belief is $\beta(p^s) = \mu_t$. The expected profit is then

$$\Pi_t(s, \mu_t) = (\mu_t(1 - F_H(s)) + (1 - \mu_t)(1 - F_L(s))) \left(\frac{\mu_t f_H(s)}{\mu_t f_H(s) + (1 - \mu_t) f_L(s)} + L \right).$$

This is clearly increasing in μ_t. Moreover

$$\frac{\partial^2 \Pi_t(s, \mu_t)}{\partial \mu_t \partial \mu_t} = \frac{2 f_L(s) f_H(s) \left(f_L(s)(1 - F_H(s)) - f_H(s)(1 - F_L(s)) \right)}{\left(\mu_t f_H(s) + (1 - \mu_t^i) f_L(s) \right)^3}$$

is strictly positive if and only if

$$\frac{1 - F_H(s)}{f_H(s)} > \frac{1 - F_L(s)}{f_L(s)}.$$

This inequality always holds because the strict version of the MLRP is assumed to hold. □

1.6.7 Proof of Proposition 1.4

Assume $p_t = p^s$. If every outcome $i \in I$ is sufficiently precise

$$\begin{aligned}\Pi_t(\pi, \beta, z) = \sum_{i \in I} \left(a^i \mu_t + b^i (1 - \mu_t) \right) \Pi_t^i(p_t^i, \mu_t^i) \geq \sum_{i \in I} \left(a^i \mu_t + b^i (1 - \mu_t) \right) \Pi_t^i(p^s, \mu_t^i) \\ > \Pi_t \left(p^s, \mu_t \sum_{i \in I} a^i \mu_t^i + (1 - \mu_t) \sum_{i \in I} b^i \mu_t^i \right) = \Pi_t(p_t, \mu_t)\end{aligned}$$

The first inequality follows from Proposition 2 and the argument made in Corollary 1, stating that a separating price p_t remains separating when i is sufficiently precise. Consequently, if $p_t^i \neq p^s$, the profit $\Pi_t^i(p_t^i, \mu_t^i) \geq \Pi_t^i(p^s, \mu_t^i)$, because setting $p_t^i = p^s$ would not lower the interim belief. The second inequality follows from the strict convexity of the profit function in the type conditional on a given separating price p^s (Proposition 1.3). Finally, for μ_1 the first inequality holds for any value of i, because she always chooses the profit maximizing price out of the whole set of possible prices. □

1.6.8 Proof of Proposition 1.5

I will prove the statements consecutively.

(a) *First*, if $q_h^s > 0$, then $q_l^s > 0$. If this is not true, then the final belief when the price is p^s would be $\beta_i^F(s, \beta(p^s)) = 1 \ \forall s \in S$. Hence, $\Pi_h(s, \beta(p^s)) = \Pi_l(s, \beta(p^s)) = H$. Consequently, $q_l^s = 1$ would be the unique best response. Clearly, the high seller type might also choose a separating price $p < H$ such that all buyer types would strictly benefit from buying. The profit when choosing the price p then is $\Pi_h(s, \beta(p)) = \Pi_l(s, \beta(p)) = p$. Hence, $p_l = p$ is then still the unique best response. Setting a lower price $p_l < p$ would strictly lower the low seller type's profit. Setting a price $p_l > p$ also, because otherwise $p_h = p$ could not be a best response by Lemma 1.1.
Second, from above it must be the case that $\beta(p_h) < 1$. Further, $\beta(p_h) > 0$. Hence, for any price $p_h \in (L, H)$, exactly one buyer type must be indifferent between buying and not buying.
Third, if $q_h^s > 0$, then $q_h^s = 1$. If not, a price $p^{s'}$ must exist such that $q_h^{s'} > 0$ and $q_l^{s'} > 0$. This requires that $p^s(1 - F_H(s)) = p^{s'}(1 - F_H(s'))$ and $p^s(1 - F_L(s)) = p^{s'}(1 - F_L(s'))$. Both conditions cannot hold simultaneously if $s \neq s'$ due to the strict version of the MLRP.
Fourth, if a price p_l is uniquely chosen by the low type, then $\beta_i^F(s, \beta(p_l)) = 0, \forall s \in S$. Therefore, the prospective buyer will buy only if $p_l \leq L$. Hence, $p_l \neq p_h$ can be a best response only if $p_l = L$.

(b) If $p_h = p^s$, the low seller type is indifferent between choosing $p_l = L$ and $p_l = p^s$ if and only if $L = (\beta_i^F(s, \beta_i(p^s)) + L)(1 - F_L(s))$. Bayesian updating requires

$$\beta_i^F(s, \beta_i(p^s)) = \frac{\beta_i^0 f_H(s)}{\beta_i^0 f_H(s) + (1 - \beta_i^0) q_l^s f_L(s)}.$$

μ_l is therefore indifferent if $L = (1 - F_L(s))(L + \frac{\beta_i^0 f_H(s)}{\beta_i^0 f_H(s) + (1 - \beta_i^0) q_l^s f_L(s)})$.

Solving for q_l^s yields

$$q_l^s = \frac{\beta_i^0 f_H\left((1-F_L(s))H - L\right)}{(1-\beta_i^0) f_L F_L L}$$

Hence, in equilibrium $q_l^s = \min\{1, \frac{\beta_i^0 f_H((1-F_L(s))H-L)}{(1-\beta_i^0)f_L F_L L}\}$ which is always positive as long as $(1-F_L(s))H > L$ which must be the case in any equilibrium (as I will show at the end of the proof). An equilibrium is semi-pooling if and only if $q_l^s < 1$. That is if

$$\frac{\beta_i^0}{1-\beta_i^0} < \frac{f_L F_L L}{f_H\left((1-F_L(s))H - L\right)}.$$

Thus, it is semi-pooling when β_i^0 is sufficiently small, and pooling otherwise.

Finally, I show that an equilibrium where $p_t = p^s$ exists, iff $(1-F_L(s))H > L$. *Only if:* Assume $p_t = p^s$ and $(1-F_L(s))H \leq L$. Then, whenever $q_l^s > 0$, $p^s = \beta_i^F(s, \beta_i(p^s)) + L < H$. But then $(1-F_L(s))p^s < L$ and $p_l = L$ strictly dominates $p_l = p^s$. But then it must be the case that $q_l^s = 0$ and $q_h^s = 0$ by (a). *If:* Take any arbitrary updated prior $\beta_i^0 \in (0,1)$ and a signal s such that $(1 - F_L(s))H > L$. Set the interim belief $\beta_i(p^s) = \frac{\beta_i^0}{\beta_i^0 + (1-\beta_i^0)q_l^s}$, and $\beta_i(p') = 0$, $\forall p' \neq p^s$. Set the corresponding final belief to $\beta_i^F(s, \beta_i(p^s)) = \frac{\beta_i(p^s) f_H(s)}{\beta_i(p^s) f_H(s) + (1-\beta_i(p^s)) f_L(s)}$. Set $q_h^s = 1$ and $q_l^s = \min\{1, \frac{\beta_i^0 f_H((1-F_L(s))H-L)}{(1-\beta_i^0)f_L F_L L}\}$, while $p_l = L$ with probability $1 - \min\{1, \frac{\beta_i^0 f_H((1-F_L(s))H-L)}{(1-\beta_i^0)f_L F_L L}\}$. This is a PBE if B buys at price p^s if and only if his type is higher than s.

□

1.6.9 Proof of Proposition 1.6

Denote the high seller type's equilibrium price as p^s. Then

$$
\begin{aligned}
&\Pi_0(\pi,\beta)\\
&= \beta_0 p^s(1-F_H(s)) + (1-\beta_0)q_L^s p^s(1-F_L(s)) + (1-\beta_0)(1-q_L^s)L\\
&= \beta_0 p^s(1-F_H(s)) + (1-\beta_0)p^s(1-F_L(s)) + (1-\beta_0)\underbrace{(1-q_L^s)(L-p^s(1-F_L(s)))}_{=0}\\
&= (\beta_0(1-F_H(s)) + (1-\beta_0)(1-F_L(s)))\left(\frac{\beta_0 f_H(s)}{\beta_0 f_H(s)+(1-\beta_0)q_L^s f_L(s)} + L\right)\\
&= \Pi_0(p^s,\beta(p^s)).
\end{aligned}
$$

The last term in the third line is zero because either $q_L^s = 1$ (in the pooling case) or $L = p^s(1-F_L(s))$ (in the semi-pooling case). This profit function is

- Strictly convex in β_0 in the pooling case ($q_L^s = 1$) as shown in Proposition 3.

- Linear in β_0 in the semi-pooling case because then $p^s = \frac{L}{1-F_L(s)}$ is independent of β_0.

Therefore

$$
\begin{aligned}
\Pi_0(\pi,\beta) = \Pi_0(p^s,\beta(p^s)) &\leq \sum_{i\in I}\left(a^i\mu_0 + b^i(1-\mu_0)\right)\Pi_0^i(p^s,\beta_i(p^s))\\
&\leq \sum_{i\in I}\left(a^i\mu_0 + b^i(1-\mu_0)\right)\Pi_0^i(p_h^i,\beta_i(p_h^i)) = \Pi_0(\pi,\beta,z).
\end{aligned}
$$

The first inequality follows from the convexity of the profit function. Further, if p^s can be an equilibrium price without a public signal, it can also be an equilibrium price if a public signal is released (Proposition 1.5). This establishes the second inequality. Finally, if γ_i (and therefore β_i^0) is sufficiently large, the equilibrium must be pooling by Proposition 1.5(b). In this case the first inequality holds strict. □

1.6.10 Proof of Proposition 1.7

Denote the profit maximizing price without public information as $p_0 = p^s$. Then

$$\begin{aligned}\Pi_0(\pi,\beta) = \Pi_0(p^s,\beta_0) = \Pi_0\left(p^s, \sum_{i\in I}\left(\mu_0 a^i + (1-\mu_0)b^i\right)\mu_0^i\right) \\ \leq \sum_{i\in I}\left(\mu_0 a^i + (1-\mu_0)b^i\right)\Pi_0^i(p^s,\beta_i^0) \\ \leq \sum_{i\in I}\left(\mu_0 a^i + (1-\mu_0 b^i)\right)\Pi_0^i(p_0^i,\beta_i^0) = \Pi_0(\pi,\beta,z).\end{aligned}$$

The first inequality follows from Proposition 1.3. The second one from the fact that each price is separating because there is only one type. □

1.6.11 Proof of Proposition 1.8

The expected profit of M is

$$\Pi_0(\pi,\beta,z) = \sum_{i\in I}\left(\mu_0 a^i + (1-\mu_0)b^i\right)\left(\frac{\beta_0 a^i}{\beta_0 a^i + (1-\beta_0)b^i} + L\right) = \mu_0 + L = \Pi_0(\pi,\beta).$$

The equality follows from $\beta_0 = \mu_0$. □

Chapter 2

Signaling Quality Through Visibility

joint with Maximilian Conze

Abstract

We ask whether positions in a search list can signal quality of an experience good if vertically differentiated firms can pay for it. We show that this is possible if only if the correct ranking maximizes the aggregate profit. If uninformed consumers believe the ranking, the 'correct' ranking induces homogeneous beliefs among informed and uninformed consumers. In doing so, it facilitates market segmentation. Meanwhile, the 'wrong' ranking induces heterogeneous beliefs among consumers and, therefore, softens competition. The market segmentation effect dominates when vertical differentiation is high, while the competition softening effect dominates when it is low. Therefore, positions can reveal quality in the first, but not in the second case.

2.1 Introduction

Most intermediaries and retailers carry products from competing upstream firms. In many instances, the downstream firm offers those upstream firms the possibility to increase the visibility of their products in exchange for some payment. Conventional retail stores, for example, sometimes charge a placement fee for premium shelf spots [see e.g. Rivlin, 2016]. Similarly, on e-commerce sites, payments made by an upstream firm are often an important determinant for its position in the list of products that is shown to consumers. Consider so-called 'online travel agencies'. On Booking.com, hotels can pay for a 'Visibility Booster' to get listed at a 'better' position. On Expedia.com, after entering the details of your planned trip you are presented with a list that is not solely sorted with respect to usual criteria, such as ratings and prices.[1] Digging through the terms of use, one finds the following quote about the sort order of their 'Lodging' category: "[...] The compensation which a property pays us for bookings made through our sites is also a factor for the relative ranking of properties with similar offers, [...]."[2]

As firms are willing to pay for certain positions on e-commerce sites and particular shelf spots in brick and mortar stores, those seem to be 'better' in some regard, i.e. they appear to have a positive effect on demand. A classical explanation for such a positive demand effect is based on search costs [Athey and Ellison, 2011]. If prospective consumers follow some search strategy (e.g. examine the list from the top to the bottom), firms who meet consumers' needs best are willing to pay most for good positions. This line of argument is plausible for search goods. Yet, it is not fully convincing for experience goods because it relies on the ability of consumers to become fully informed about an offer by paying the search costs and inspecting the product. In the 'online travel agencies' example given above, products are better characterized as experience goods than as search goods. The aim of this paper is to explain why firms pay for the increased visibility of their products in situations where search costs are not an issue. More precisely, we examine if and under what conditions ranks or positions in a list can serve as a signal for the quality of an experience good in the spirit of Nelson [1974].

[1] Clearly, you can sort the results also with respect to those criteria.
[2] See `https://www.expedia.com/p/info-other/legal.htm`, last accessed on 2019/01/08.

Our setting consists of a vertically differentiated upstream duopoly and a monopolistic intermediary downstream. Both upstream firms sell their good through the intermediary to final consumers. The intermediary's role is very limited. He only displays the goods in a list to consumers. He can decide which good to put on which position, and he can charge a fee depending on the position the product is displayed on. However, prices are set by the upstream firms. Consumers differ in their valuation for quality, but not all of them can observe it. Uninformed consumers form beliefs about the quality based on the firms' positions in the list. Finally, consumers buy the good that yields them the highest expected net utility.

We derive two key results. First, we show that a separating equilibrium, where each firm gets assigned to a specific position with probability one, exists if and only if for given beliefs this assignment maximizes the aggregate profit. Otherwise, the low quality firm always has a larger willingness to pay for the high quality firm's position and vice versa. On the other hand, if the 'correct' ranking maximizes industry profit, the intermediary does not have an incentive to charge fees such that both firms choose the same position.[3]

Second, we show that the ranking can fully reveal the firms' qualities if differentiation between them is sufficiently high. It cannot, if differentiation is low. Two opposing effects are at play. The first is a market segmentation effect. All consumers share the same belief if the 'correct' ranking is displayed. Consequently, any consumer with a high valuation for quality will buy the high quality good for a high price. If, however, consumers trust the ranking and the 'wrong' ranking was displayed, uninformed and informed consumers would no longer share the same belief. In that case not all consumers with a high valuation for quality would buy the same (more expensive) good. Thus, the 'wrong' ranking would hinder market segmentation, which lowers the aggregate profit. The second is a competition softening effect. Heterogeneous beliefs would cause the low price firm to set a higher price. The reason is that consumers believing it was the high quality firm would never buy from the more expensive competitor, regardless of how large (or

[3]The crucial assumption for this to hold is that the intermediary charges a lump sum fee, and that firms pay for their realized position and not for the position they have asked for. We show in an extension that if firms pay for their asked position, conditions for a separating equilibrium to exist are harder.

small) the price gap between the firms was. Heterogeneous beliefs, resulting from the 'wrong' ranking, would therefore soften competition by reducing incentives to lower prices. This would increase the aggregate profit. We show that the market segmentation effect dominates when differentiation between firms is high, while the competition softening effect dominates in case it is low.

Our model contributes to several strands of literature. Most closely related are papers studying dissipative advertising [for an overview see Bagwell, 2007]. The main issues in these papers are whether advertising is necessary to signal quality, and whether it makes signaling less costly (in case it is not necessary) than signaling via prices. Fluet and Garella [2002] and Hertzendorf and Overgaard [2001] examine the case of a vertically differentiated duopoly. They show that advertisement is necessary to signal quality when vertical differentiation between firms is low. Yehezkel [2008] shows in a similar model that signaling through advertisement is relatively cheap if the share of informed consumers is low, while price signaling becomes cheaper if the share gets large.

Paying for visibility differs from these approaches in two ways. First, visibility is rivalrous. If one firm is at a better position, the competitor must be at a worse one. This fundamentally changes incentives. In the non-rivalrous case, advertising works whenever the low quality firm does not have an incentive to advertise, given that the high quality firm advertises. In contrast, the low quality firm in our setting can make the high firm less visible by also paying for visibility. Therefore, signaling only works if the high quality firm is willing to pay more for visibility than the low quality firm. A second difference, which turns out to be minor, is the presence of a profit maximizing intermediary who is selling visibility. This is analogous to introducing incentives from a billboard seller in the dissipative advertising literature.

Some papers on position auctions have examined the signaling role of positions in search lists [Athey and Ellison, 2011; Chen and He, 2011; Jerath et al., 2011]. These signals are also rivalrous. Therefore, which firm has a larger willingness to pay for a signal matters. Yet, the logic of these models only applies to search goods. Search is costly for consumers, therefore, they will start inspecting those sellers they believe meet their needs best. Because consumers always find out quality prior to purchase, the willingness to pay for positions that are examined first differs between firms. Contrary to this, in our model both firms may have a different willingness to pay for positions because some consumers can observe quality. Hence, prices and associated profits will never depend merely on the list position.

The effect of information on competition has been studied in some recent information disclosure papers. Bouton and Kirchsteiger [2015] argue that an informative ranking can harm consumers by reducing competition between vertically differentiated firms. Similarly, Canidio and Gall [2018] show that public information about vertically differentiated firms can soften competition. They derive conditions for firms to disclose too much information from a social perspective. In both papers, consumers share a common belief. Thus, reduced competition does not follow from belief heterogeneity among consumers, but from a more accurate belief about which firm is the high quality firm. The effect of belief heterogeneity on competition in turn has been studied by Hefti, Liu and Schmutzler [2018] in case of a horizontally differentiated duopoly. These authors show that confusion among consumers can soften competition, because this might reduce the number of indifferent consumers. This is analogous to what happens with heterogeneous beliefs in our model. Consumers believing the low price firm is of high quality are never indifferent between buying from one or the other firm, regardless of their valuation for quality. Firms do therefore not compete for these consumers.

Finally, some authors have argued that the willingness to pay slotting fees for premium shelf spots can serve as a signal for quality [see e.g. Chu, 1992; Lariviere and Padmanabhan, 1997]. Slotting allowances are similar to lump sum fees in our model. However, in these models, the payment allows the upstream firm to signal its quality to the downstream firm, while in our case it signals quality to final consumers. Garella and Peitz [2000] show that paying a fee for selling through an intermediary can serve as a signal for quality. The result crucially relies on the possibility of the high quality firm to (costly) disclose its quality, and selling through an intermediary is not rivalrous. Hence, their model is very different to the model presented in this paper.

We proceed as follows. Section 2.2 presents the model. In Section 2.3 we derive a sufficient and necessary reduced form condition under which the position can signal quality. Section 2.4 shows when this condition holds in our model of a vertically differentiated duopoly. Section 2.5 studies the case where firms do not have to pay for the received position, but for the position they have asked for. Section 2.6 concludes.

2.2 Model

Consider a high quality firm H and a low quality competitor L (both she, and indexed by i). Both observe their own and the competitor's type, and produce with zero marginal cost. These firms can sell their product through an intermediary I (he), for example a sales platform, to final consumers. The only role of the intermediary is to display the products in a list to consumers prior to purchase. Hence, he has to put one firm on the high position $r = h$, and the other one on the low position $r = \ell$. He can charge lump sum fees F^r for each attached position. The intermediary knows the type distribution of firms, but does not observe which firm is H and which is L.[4]

Each firm i can ask to be at the low position $\tilde{r}_i = \ell$, or to be at the high position $\tilde{r}_i = h$. Instead of modeling an outside option, we assume that they also have the possibility to choose $\tilde{r}_i = 0$, and no fee can be charged when this option is chosen. Hence, a firm always has the possibility to sell the good through the intermediary without paying a fee.[5] The intermediary then assigns to each firm a position $r_i \in \{\ell, h\}$. We denote this assignment as $r(\tilde{r}_i, \tilde{r}_j) = (r_i, r_j)$. Because the intermediary does not observe types, we assume that if both choose the same position $\tilde{r}_i = \tilde{r}_j$, positions are assigned randomly with equal probability. Further, with lump sum fees the intermediary does not care about which firm pays for which position. Hence, it is without loss of generality to assume that he can commit to an assignment rule. We specify this rule as follows. If both firms

[4]With lump sum fees, the intermediary's optimal strategy does not depend on the true type. The assumption is therefore unimportant with lump sum fees, but would be important in case of proportional fees.

[5]This assumption can also be interpreted as describing a situation where every firm can join the intermediary for the same fee and firms can additionally invest in increased visibility.

ask for different positions that are not $\tilde{r}_i = 0$, both are assigned to the position they asked for. This is without loss of generality because the choices $\tilde{r}_i = \ell$ and $\tilde{r}_i = h$ are just labels. This is not the case for the choice $\tilde{r}_i = 0$ because this requires that firm i does not pay any fee, regardless of the assigned position. Specifying a particular assignment for this case would therefore not be without loss of generality. Thus, we denote by n the probability with which a firm asking for position ℓ gets assigned to position ℓ if the competing firm chooses $\tilde{r}_i = 0$. Equivalently, m is the probability that a firm choosing position h gets assigned to position h if the competing firm chooses $\tilde{r}_i = 0$.

Finally, each firm has to pay for the realized position if its choice was not $\tilde{r}_i = 0$. This assumption is not without loss of generality. It ensures that both firms choose their preferred position instead of strategically choosing the cheaper one, if the competing firm does so as well. Furthermore, it states that each fee is paid at most by one firm. It therefore makes a separating equilibrium easier to exist, compared to the rule where both firms pay for the position they asked for. We examine the alternative case as an extension.

Firms observe the realized positions and simultaneously set their prices p_i to maximize their profits Π_i, which depend on both, the own and the opponent's price as well as (perceived) qualities.

Demand is modeled as in Bagwell and Riordan [1991]. There is a total mass one of consumers each of whom buys at most one good. Consumers are homogeneous in their valuation for the low quality good, but value the high quality one differently. All consumers' valuation for the low quality good is given by V, while the valuation for the high quality good of a consumer is equal to his type, which is a draw from a uniform distribution between V and $V + 1$. Because the quality difference is normalized to one, V is a measure of vertical product differentiation: differentiation becomes larger as V gets smaller and vice versa. A share $\alpha \geq 0$ of consumers observe the quality of the firms, while the remaining consumers only know the quality distribution. We assume that a consumer's information and her type are not correlated. All consumers observe both prices and attached positions. We denote the uninformed consumers' beliefs that $i = H$ when observing prices and positions as $b((p_i, r_i), (p_j, r_j))$.

To sum up, the timing is as follows.

i) I sets lump sum fees $(F^{\ell}, F^{h}, F^{0} = 0)$.

ii) Firms observe fees, and simultaneously make their choice $\tilde{r}_i \in \{0, \ell, h\}$.

iii) I observes $(\tilde{r}_i, \tilde{r}_j)$ and assigns positions (r_i, r_j), which are observed by all players, as follows:

- If $\tilde{r}_i \neq \tilde{r}_j$ and $\tilde{r}_i, \tilde{r}_j \neq 0$:
  $r(\tilde{r}_i, \tilde{r}_j) = (\tilde{r}_i, \tilde{r}_j)$.
- $r(0, \ell) = (h, \ell)$ with probability n, $r(0, h) = (\ell, h)$ with probability m
- If $\tilde{r}_i = \tilde{r}_j$:
  $r(\tilde{r}_i, \tilde{r}_j) = (\ell, h)$ or $r(\tilde{r}_i, \tilde{r}_j) = (h, \ell)$, each with probability 1/2.

iv) Each firm i pays F^{r_i}. Firms then simultaneously set prices conditional on attached positions $p_i(r_i, r_j)$.

v) Uninformed consumers form beliefs $b((p_i, r_i), (p_j, r_j))$. Each consumer buys the good providing him the higher net utility, or none if both net utilities are negative.

We solve for perfect Bayesian equilibria (PBE), which require strategies to be sequentially rational and beliefs to be consistent on the equilibrium path.

2.3 Conditions for separation

We are looking for a separating equilibrium. Because the actual positions are just labels, we assume without loss of generality that in such an equilibrium $r_H = h$ and $r_L = \ell$. Consistent beliefs then require $b((h, p_H), (\ell, p_L)) = 1$ and $b((\ell, p_L), (h, p_H)) = 0$. We start defining profits and deviation profits of the pricing game (iv) and then turn to the positioning game (i and ii). Two final subgames exist. The 'correct' one where $r_H = h$ and $r_L = \ell$, and the 'wrong' one where $r_H = \ell$ and $r_L = h$. Sequential rationality requires that both firms play mutual best responses in both of these subgames. We denote best responses in the ‘correct’ subgame as

$$p_H^h := p_H(h,\ell) \in \arg\max_p \Pi_H[(h,p),(\ell,p_L),b((h,p),(\ell,p_L)),b((\ell,p_L),(h,p))] \text{ and}$$
$$p_L^\ell := p_L(\ell,h) \in \arg\max_p \Pi_L[(\ell,p),(h,p_H),b((\ell,p),(h,p_H)),b((h,p_H),(\ell,p))],$$

and corresponding gross profits as

$$\Pi_H^h = \Pi_H[(h,p_H^h),(\ell,p_L^\ell),1,0] \text{ and } \Pi_L^\ell = \Pi_L[(\ell,p_L^\ell),(h,p_H^h),0,1]\ .$$

Equivalently, in the ‘wrong’ subgame best responses and gross profits are

$$p_H(\ell,h) = p_H^\ell \in \arg\max_p \Pi_H[(\ell,p),(h,p_L),b((\ell,p),(h,p_L)),b((h,p_L),(\ell,p))] \text{ and}$$
$$p_L(h,\ell) = p_L^h \in \arg\max_p \Pi_L[(h,p),(\ell,p_H),b((h,p),(\ell,p_H)),b((\ell,p_H),(h,p))],$$

respectively,

$$\Pi_H^\ell = \Pi_H[(\ell,p_H^\ell),(h,p_L^h),b((\ell,p_H^\ell),(h,p_L^h)),b((h,p_L^h),(\ell,p_H^\ell))] \text{ and}$$
$$\Pi_L^h = \Pi_L[(h,p_L^h),(\ell,p_H^\ell),b((h,p_L^h),(\ell,p_H^\ell)),b((\ell,p_H^\ell),(h,p_L^h))].$$

We turn to the positioning game, which is illustrated in Figure 2.1. Separation requires $\tilde{r}_L \neq \tilde{r}_H$. Because the intermediary can always set $F^\ell = 0$ and/or $F^h = 0$, it is without loss of generality to assume that $\tilde{r}_L \neq 0$ and $\tilde{r}_H \neq 0$ in any candidate equilibrium. We thus denote choices in the separating equilibrium as $\tilde{r}_H = h$ and $\tilde{r}_L = \ell$. Expected profits when $\tilde{r}_L = \ell$ and $\tilde{r}_H = h$ are then (Π_L^ℓ, Π_H^h). Expected deviation profits are $\left(\frac{1}{2}(\Pi_L^\ell + \Pi_H^\ell), \frac{1}{2}(\Pi_H^h + \Pi_H^\ell)\right)$ when deviating to the choice of the competitor, and (Π_L^0, Π_H^0) when deviating to $\tilde{r} = 0$. Clearly we have $\Pi_i^0 \in [\Pi_i^\ell, \Pi_i^h]$, because the final gross profit is always either Π_i^ℓ or Π_i^h.

			L	
		h	ℓ	0
H	h	$\frac{1}{2}\left(\Pi_H^h + \Pi_H^\ell - F^h - F^\ell\right)$ $\frac{1}{2}\left(\Pi_H^h + \Pi_H^\ell - F^h - F^\ell\right)$	$\Pi_H^h - F^h$ $\Pi_L^\ell - F^\ell$	$m\left(\Pi_H^h - F^h\right) + (1-m)\left(\Pi_H^\ell - F^\ell\right)$ $(1-m)\Pi_L^h + m\Pi_L^\ell$
	ℓ	$\Pi_H^\ell - F^\ell$ $\Pi_L^h - F^h$	$\frac{1}{2}\left(\Pi_H^h + \Pi_H^\ell - F^h - F^\ell\right)$ $\frac{1}{2}\left(\Pi_H^h + \Pi_H^\ell - F^h - F^\ell\right)$	$(1-n)\left(\Pi_H^h - F^h\right) + n\left(\Pi_H^\ell - F^\ell\right)$ $n\Pi_L^h + (1-n)\Pi_L^\ell$
	0	$(1-m)\Pi_H^h + m\Pi_H^\ell$ $m\left(\Pi_L^h - F^h\right) + (1-m)\left(\Pi_L^\ell - F^\ell\right)$	$n\Pi_H^h + (1-n)\Pi_H^\ell$ $(1-n)\left(\Pi_L^h - F^h\right) + n\left(\Pi_L^\ell - F^\ell\right)$	$\frac{1}{2}\left(\Pi_H^h + \Pi_H^\ell\right)$ $\frac{1}{2}\left(\Pi_H^h + \Pi_H^\ell\right)$

Figure 2.1: The positioning game

If $\tilde{r}_H = h$, then $\tilde{r}_L = \ell$ is a best response only if $\Pi_L^\ell - F^\ell \geq \Pi_L^h - F^h$. Similarly, $\tilde{r}_H = h$ is a best response to $\tilde{r}_L = \ell$ only if $\Pi_H^h - F^h \geq \Pi_H^\ell - F^\ell$. These incentive compatibility constraints set upper and lower bounds for F^ℓ and F^h. Both conditions can simultaneously hold only if $\Pi_H^h + \Pi_L^\ell \geq \Pi_H^\ell + \Pi_L^h$. The first result states that this condition is also sufficient for a separating equilibrium to exist. All proofs are relegated to the Appendix.

Proposition 2.1. *A separating equilibrium exists if and only if* $\Pi_H^h + \Pi_L^\ell \geq \Pi_H^\ell + \Pi_L^h$. *Prices in this equilibrium are* p_H^h *and* p_L^ℓ *and fees are* $F^h = \max\{\Pi_H^h - \Pi_H^l, 0\}$ *and* $F^\ell = \max\{\Pi_L^\ell - \Pi_L^h, 0\}$.

Proposition 2.1 shows that a separating equilibrium exists if and only if, for beliefs that are consistent with a separating equilibrium, the aggregate profit is maximized when the 'correct' ranking and thus the correct beliefs are imposed. Furthermore, the intermediary can extract at least the whole surplus resulting from displaying the 'correct' ranking instead of the 'wrong' ranking. In particular, if both firms benefit from the 'correct' ranking, the intermediary can charge a strictly positive fee for both positions.

Charging fees such that both firms ask for the same position is never profitable for the intermediary. It would require the fee to be sufficiently low such that the firm with the lower willingness to pay is willing to pay it. Because only one firm would finally pay the fee, and because it does not matter for the intermediary which one, this can never be beneficial for the intermediary. This would be different in case of proportional fees, or the rule that firms have to pay for the position they asked for. In the first case, because it would matter which firm paid the fee. In the latter case, because both firms might pay the fee. In these alternative settings, the intermediary's incentive compatibility constraint would, therefore, not be fulfilled for free.

Proposition 2.1 simplifies the question of whether or not a separating equilibrium exists. We merely have to check if for a given belief system that is consistent with a putative separating equilibrium, the aggregate profit is maximized if the 'correct' or the 'wrong' ranking is displayed.

2.4 Fully informative ranking

We are interested in a separating equilibrium where types are signaled by the position alone. Beliefs of uninformed consumers should therefore not depend on prices, but only on the observed rank position. Hence $b((h, p_i), (\ell, p_j) = 1$ $\forall p_i, p_j$, and $b((\ell, p_i), (h, p_j) = 0$ $\forall p_i, p_j$. In this candidate equilibrium, beliefs are homogeneous if the 'correct' ranking is imposed. If an agent deviated such that the 'wrong' ranking was imposed, beliefs would have to be heterogeneous. Indeed, informed consumers would believe that firm H, being on position ℓ, was the high quality firm. Uninformed consumers would believe that firm L, being on position h, was the high quality firm.

We start with the observation that if $p_H < p_L$ no informed consumer will buy the low quality good. Equally, if $p_i^h < p_j^l$ no uninformed consumer with belief $b((h, p_i^h), (\ell, p_j^l)) = 1$ will buy from the firm on position ℓ. Hence, if uninformed and informed consumers do not share the same belief, there is a discontinuity in the demand function at $p_H = p_L$.

We denote the firm choosing the weakly lower price as A and the other one as B, hence $p_A \leq p_B$. Let a be the share of consumers believing $A = H$ and $1 - a$ the share believing $B = H$. All consumers believing A is the high quality firm will buy from A as long as $V \geq p_A$. Consumers believing B is the high quality firm buy from A only if their valuation for quality is sufficiently low. These consumers will however never buy from A if $V < p_A$. In this case p_B does not affect the demand of firm A. Hence, firm A's demand is given by

$$D_A(p_A, p_B) = \begin{cases} a + (1-a)(p_B - p_A) & \text{if } V \geq p_A \\ a(1 + V - p_A) & \text{if } V < p_A. \end{cases}$$

Consumers buy from firm B only if they believe it is the high quality firm and if their valuation for quality is sufficiently high. Demand of firm B is therefore

$$D_B(p_B, p_A) = \begin{cases} (1-a)(1 + p_A - p_B) & \text{if } V \geq p_A \\ (1-a)(1 + V - p_B) & \text{if } V < p_A. \end{cases}$$

The optimization problem of firm i is

$$\max_{p_i} \Pi_i = p_i D_i(p_i, p_j).$$

In the proof of Proposition 2.2 we show that if there is an equilibrium with $p_A < p_B$, it must be that $a \leq \frac{1}{2}$. Hence, if beliefs are homogeneous we have $a = 0$ and A must be the low quality firm. If beliefs are heterogeneous ($a > 0$), then A is the low quality firm if the share of informed consumers is large ($\alpha > \frac{1}{2}$), and the high quality firm if the share of informed consumers is small ($\alpha < \frac{1}{2}$). According to Proposition 1, a separating equilibrium does therefore exist if and only if the aggregate profit when $a = 0$ is not lower than the aggregate profit when $a = \min\{\alpha, 1-\alpha\}$. Proposition 2 shows that this is the case whenever V is sufficiently low.

Proposition 2.2.

(a) *The position can never fully reveal quality if $V \geq 1$.*

(b) *If $V \in \left(\frac{1}{3}, 1\right)$, the position can fully reveal quality if the share of informed consumers is sufficiently close to $\frac{1}{2}$ and V is sufficiently small. It can never fully reveal quality if either V is sufficiently high, or the share of informed consumers is close to 1 or 0.*

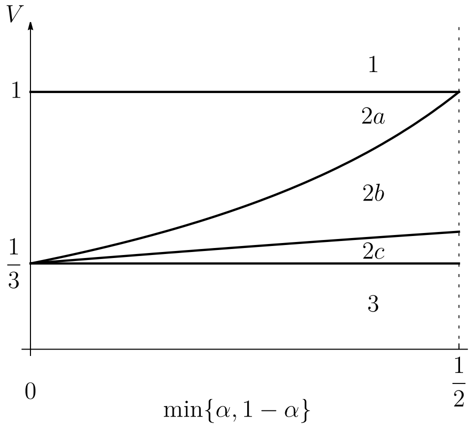
(c) *The position can fully reveal quality for any share of informed consumers if* $V \leq \frac{1}{3}$.

Proposition 2.2 shows that the ranking cannot fully reveal quality if differentiation between firms is low, while it can, if differentiation is high. To understand this result consider the two opposing effects belief heterogeneity has on the aggregate profit. First, with homogeneous beliefs all consumers with a high valuation for quality buy the good from the high quality firm, which charges a higher price than the low quality firm. However, this is no longer the case if beliefs are heterogeneous. In this case, some consumers belief that the cheaper good is from the high quality firm. Hence, some high valuation consumers will buy the cheaper good, while low valuation consumers always buy the cheaper good. This effect of belief heterogeneity lowers the aggregate profit. Second, consumers believing the low price firm to be the high quality firm, are never indifferent regarding buying from one or the other firm. Thus, for given prices, elasticity of demand is smaller with heterogeneous beliefs as less consumers are willing to change their purchase decision in response to small price changes. Hence, the low price firm's demand becomes less elastic, which reduces her incentive to undercut the competitor's price. This effect increases the aggregate profit. Put differently, heterogeneous beliefs hinder market segmentation, which is negative for the aggregate profit, but also soften competition, which is positive for the aggregate profit.

The competition softening effect dominates when vertical differentiation is low (V large), while the market segmentation effect dominates when vertical differentiation is high (V small). This is intuitive. If vertical differentiation is low, competition is intense and the price difference between firms is relatively small. If differentiation is high, competition between firms is less intense, but the price difference between firms is relatively large. In the first case, softening competition has a large impact on the aggregate profit, while in the latter case, hindering market segmentation has a large (negative) impact on the aggregate profit.

Proposition 2.2 is illustrated in Figure 2.2. In region 3, firm B always sets the monopoly price. Then, no competition softening can arise and aggregate profit is higher if beliefs are homogeneous. Hence, a separating equilibrium exists. In region 1 and 2a, both firms set a smaller price than the monopoly price. Surpris-

Figure 2.2: Illustration of Proposition 2.2



Note: A separating equilibrium exists only in regions 2c and 3.

ingly, then the competition softening effect always dominates and no separating equilibrium can exist. In regions 2b and 2c, both prices are below the monopoly price if beliefs are homogeneous, while firm B will choose the monopoly price if beliefs are heterogeneous. The competition softening effect dominates in region 2b, while the market segmentation effect dominates in region 2c. Consequently, a separating equilibrium exists in region 2c, but not in region 2b. Intuitively, incomplete market segmentation is costly if the price gap between firms is large (this is the case if V is small and differentiation therefore high), and/or if heterogeneity is large (this is the case when the share of informed consumers is close to $\frac{1}{2}$).

2.5 Extension: Pay for the asked position

We now consider the case where firms have to pay for their asked instead of the assigned position. Everything else remains as described in the previous sections.

Proposition 2.3. *A separating equilibrium when firms have to pay the lump sum fees for the position they ask for (and not for the position they get) exists if and* *only if either* $\Pi_H^h + 2\Pi_L^\ell \geq \Pi_H^\ell + 2\Pi_L^h$ *and* $\Pi_H^h \geq \Pi_H^\ell$*, and/or if* $2\Pi_H^h + \Pi_L^\ell \geq 2\Pi_H^\ell + \Pi_L^h$ *and* $\Pi_L^\ell \geq \Pi_L^h$ *holds.*

It is harder for a separating equilibrium to exist than in the situation where firms have to pay for the realized position. The reason is that if both firms ask for the same position, then both firms have to pay the same fee. This makes it possible that the intermediary benefits from choosing fees such that both firms ask for the same position. Hence, the intermediary's incentive compatibility constraint is no longer fulfilled for free. Furthermore, the intermediary cannot extract to whole surplus resulting form displaying the 'correct' instead of the 'wrong' ranking. The reason is that the firm which asks for the more expensive position, has an additional incentive to choose the cheaper one. By doing so, she might get the preferred position but for the lower fee.

An interesting observation can be made with regards to a situation where profits only depend on the ranking but not on the firms' types such that $\Pi_H^h = \Pi_L^h$ and $\Pi_H^\ell = \Pi_L^\ell$. Then, a separating equilibrium exists under this rule only if the profit on both positions is the same. Consequently, the intermediary could never charge a positive fee. That is in sharp contrast to Hertzendorf and Overgaard [2001] and Fluet and Garella [2002]. In these settings, a firm's incentive to pay for advertisement becomes smaller if the competing firm advertises. Advertising is a strategic substitute. In the symmetric case, where ex-ante both firms benefit in the same way from advertising, a separating equilibrium therefore exists. In our setting, exactly the opposite is the case. Paying for visibility is a strategic complement. The willingness to pay for visibility increases, when the competing firm pays for it. Therefore, in the symmetric case, where ex-ante both firms benefit in the same way from visibility, no separating equilibrium exists.

Revisiting the setting from Section 2.4, the next result shows the implications of Proposition 2.3 in our setup.

Proposition 2.4. *When firms have to pay for the position they ask for, then:*

(a) The rank can never fully reveal quality if $\alpha < \frac{1}{2}$.

(b) The rank can fully reveal quality if α *is sufficiently high and* $V \leq \frac{1}{3}$.

Proposition 2.4 shows that a separating equilibrium only exists if the share of informed consumers is high. If $\alpha < 0.5$, then Π_L^h is relatively high. It is therefore difficult for the condition to hold. The intuition for this result is as follows. The intermediary's incentive to choose fees such that both firms pool is high if the low type's willingness to pay for the high position is high. This is the case when the share of informed consumers (α) is small. Hence, if α is small, no separating equilibrium can exist because the intermediary benefits from choosing pooling fees.

2.6 Conclusion

We have shown under which conditions positions in a list can serve as a signal for the quality of an experience good, if competing firms can pay for it. We have derived two key results. First, given that consumers trust the ranking, this kind of signaling is possible if and only if the 'correct' ranking maximizes industry profit. Second, the 'correct' ranking maximizes industry profit if firms are sufficiently differentiated. Consequently, positions can signal quality if the quality differentiation between firms is high, but not if differentiation is low.

This provides an explanation why some positions in a search list might be 'better' than others in case of experience goods. The condition for signaling to work is however very strict. This does partly rely on our extreme equilibrium refinement. We assumed that beliefs only depend on the displayed position, but not on the price. It would be an interesting extension to allow the price to contain information as well. If differentiation between firms is low, prices alone cannot signal quality as shown by Hertzendorf and Overgaard [2001] and Fluet and Garella [2002]. It is therefore not clear, whether or not a price-rank combination could do so.

The effect of other possible extensions is more straightforward. First, with more than two firms, a separating equilibrium would still require the aggregate profit under the 'correct' ranking (and therefore 'correct' beliefs) to be higher than under every other ranking (with 'wrong' beliefs). The necessary condition would therefore look similar to the condition in Proposition 1. Second, it is standard to think about marginal cost being positively correlated with a firms quality. In our model this would make a separating equilibrium harder to exist, because under 'correct' beliefs, it is the high quality firm which faces a higher demand in equilibrium.

2.7 Appendix

2.7.1 Proof of Proposition 2.1

Only if: Follows directly form the text.
If: We have to make a case distinction. The necessary condition implies that either:

(A) $\Pi_H^h \geq \Pi_H^\ell$ and $\Pi_L^\ell \geq \Pi_L^h$,

(B) $\Pi_H^h > \Pi_H^\ell$ and $\Pi_L^\ell \leq \Pi_L^h$,

(C) $\Pi_H^h \leq \Pi_H^\ell$ and $\Pi_L^\ell > \Pi_L^h$.

Recall the assignment rule in case one firm reports $\tilde{r}_i = 0$: $r(0,h) = (\ell,h)$ with probability m, $r(0,\ell) = (h,\ell)$ with probability n.

$\tilde{r}_L, \tilde{r}_H \neq 0$ requires both

$$\Pi_L^\ell - F^\ell \geq m\Pi_L^\ell + (1-m)\Pi_L^h \tag{IR_L}$$

and

$$\Pi_H^h - F^h \geq (1-n)\Pi_H^\ell + n\Pi_H^h \tag{IR_H}$$

to hold. Hence

$$\Pi_I = F^\ell + F^h \leq (1-m)(\Pi_L^\ell - \Pi_L^h) + (1-n)(\Pi_H^h - \Pi_H^\ell).$$

The intermediary's maximal profit if $\tilde{r}_L, \tilde{r}_H \neq 0$ (obtained by optimally choosing n and m) in the different situations is therefore

(A) $\Pi_I = \Pi_L^\ell - \Pi_L^h + \Pi_H^h - \Pi_H^\ell$;

(B) $\Pi_I = \Pi_H^h - \Pi_H^\ell$;

(C) $\Pi_I = \Pi_L^\ell - \Pi_L^h$.

The intermediary's maximal profit if either $\tilde{r}_L = 0$ or/and $\tilde{r}_H = 0$, is equivalent to setting either $F^\ell = 0$ or/and setting $F^h = 0$. We show that a separating equilibrium where $\tilde{r}^*_L \neq \tilde{r}^*_H$ and $r(\tilde{r}^*_L, \tilde{r}^*_H) = (\ell, h)$ exists, where these profits can be achieved. In *(A)* $F^\ell = \Pi^\ell_L - \Pi^h_L$, $F^h = \Pi^h_H - \Pi^\ell_H$, $m = n = 0$; in *(B)* $F^\ell = 0$, $F^h = \Pi^h_H - \Pi^\ell_H$, $m = 1$, $n = 0$; in *(C)* $F^\ell = \Pi^\ell_L - \Pi^h_L$, $F^h = 0$, $m = 0$, $n = 1$. It is straightforward to check that $(\tilde{r}^*_L, \tilde{r}^*_H)$ are then indeed mutual best responses and that the profit is maximized. □

2.7.2 Proof of Proposition 2.2

The proof consists of a succession of claims, that together establish the result.

Claim 2.1. *If $a \leq \frac{1}{2}$ then $p_A \leq p_B$ in equilibrium.*[6]

Proof. Suppose not, i.e. assume $a \leq 0.5$ and $p_A > p_B$.

- $p_A \leq 1$ must hold in equilibrium, because otherwise A could always profit from lowering the price by a sufficiently low amount ε, (such that $p_A - \varepsilon > p_B$) because then

$$\Pi_A(p_A - \varepsilon, p_B) - \Pi_A(p_A, p_B) = p_A a\varepsilon - a\varepsilon(1 + p_B - p_A + \varepsilon) > 0,$$

- Consider the case where $p_A \leq V$. Then B's profit when $p_B < p_A$ is $\Pi_A(p_B < p_A, p_A) = p_B\left((1-a) + a(p_A - p_B)\right)$. By choosing $p_B = p_A$ her profit would however be $\Pi_A(p_B = p_A, p_A) = p_A(1-a)$, and therefore (strictly) larger when $a \leq (<)\frac{1}{2}$ and $p_A \leq 1$.

  Consequently, if $p_A \leq V$, $p_B < p_A$ cannot happen in equilibrium.

- In case $p_A \geq V$ and $p_B < V$, B's profit when $p_B < p_A$ is $\Pi_B(p_B < p_A, p_A) = p_B\left((1-a) + a(p_A - p_B)\right)$. By choosing $p_B = V$ her profit would however be $\Pi_A(p_B = p_A, p_A) = V(1-a)$, and therefore (strictly) larger when $a \leq (<)\frac{1}{2}$ and $p_A \leq 1$.

[6] In all Claims we assume that a separating equilibrium exists and derive properties that must then be true.

- If $p_A \geq V$ and $p_B \geq V$, prices do not depend on a and are $p_A = p_B = \frac{1+V}{2}$.

$\square$

Claim 2.2. *If $p_A < p_B$, then $a \leq \frac{1}{2}$.*

Proof. Because of symmetry, the above claim also implies that $p_B \leq p_A$ if $a \geq 0.5$ and the claim must hold. $\square$

Claim 2.3. *If $V \geq \frac{1+a}{3(1-a)}$, then $p_A = \frac{1+a}{3(1-a)} \leq V$ in equilibrium.*

Proof. Under the assumption that $p_A \leq V$, best responses are calculated as $p_A(p_B) = \frac{a+(1-a)p_B}{2(1-a)}$ and $p_B(p_A) = \frac{1+p_A}{2}$. Prices in the putative equilibrium are then $p_A = \frac{1+a}{3(1-a)}$ and $p_B = \frac{2-a}{3(1-a)}$. Those equations thus characterize optimal prices if and only if $V \geq \frac{1+a}{3(1-a)}$. $\square$

Claim 2.4. *If $V < \frac{1+a}{3(1-a)}$, then p_A is either $p_A = V$ or $p_A = \frac{1+V}{2} > V$.*

Proof. A's profit when choosing a price $p_A \leq V$ is $p_A\left(a + (1-a)(p_B - p_A)\right)$ and strictly increasing in p_A as long as $p_B \geq \frac{1+p_A}{2}$, which is always true if $p_A \leq V$. If $p_A > V$, A cannot attract any consumers who believes A is the low type. Consequently, she will set the monopoly price with respect to those consumers, believing she is the high type. A's optimal price must therefore be either $p_A = V$ or the monopoly price $p_A = \frac{1+V}{2}$ A's profit is therefore $V\left(a + (1-a)\frac{1-V}{2}\right)$ when $p_A = V$ and $a\left(\frac{1+V}{2}\right)^2$ if $p_A > V$. $\square$

Claim 2.5. *The industry profit is strictly increasing in a if $V \geq \frac{1+a}{3(1-a)}$.*

Proof. Using the prices from Claim 2.3, the industry profit is calculated as

$$\Pi_A + \Pi_B = \frac{(1+a)^2}{9(1-a)} + \frac{(2-a)^2}{9(1-a)},$$

and therefore strictly increasing in a. $\square$

Claim 2.6. *The industry profit is maximized for $a = 0$ if $V < \frac{1+a}{3(1-a)}$.*

Proof. From Claim 2.4, the optimal price is either $p_A = V$ or the monopoly price, so the following case distinction is needed:

- If $a \leq \frac{2V(1-V)}{1-V^2}$, the profit when $p_A = V$ is larger. In that case $p_B = \frac{1+V}{2}$ the and industry profit is

$$\Pi_A + \Pi_B = V\left(a + (1-a)\frac{1-V}{2}\right) + (1-a)\left(\frac{1+V}{2}\right)^2,$$

  and therefore strictly decreasing in a because $V < 1$ (otherwise we are in Case 1).

- If $a > \frac{2V(1-V)}{1-V^2}$, the optimal price is $p_A = \frac{1+V}{2}$. In that case $p_B = \frac{1+V}{2}$ and the industry profit is

$$\Pi_A + \Pi_B = \left(\frac{1+V}{2}\right)^2,$$

  This industry profit is strictly smaller than the profit when $a = 0$, which is $V\frac{1-V}{2} + \left(\frac{1+V}{2}\right)^2$.

□

If $V \geq 1$ (region 1) Claim 2.3 applies whereas Claim 2.4 applies if $V \leq \frac{1}{3}$ (region 3).

If $V \in \left(\frac{1}{3}, 1\right)$ depending on a, either case might apply. We are at the interior solution of Claim 2.3 when a is sufficiently low and at the corner or monopoly solutions of Claim 2.4 otherwise. Thus, if beliefs are homogeneous ($a = 0$) the industry profit is

$$\Pi_A + \Pi_B = \frac{5}{9}.$$

In case a is not too high with heterogeneous beliefs, $V \geq \frac{1+a}{3(1-a)}$ (region 2a) still holds (Claim 2.3 applies) and the industry profit is then higher with heterogeneous beliefs than with homogeneous beliefs. If a is sufficiently high with heterogeneous beliefs such that $V < \frac{1+a}{3(1-a)}$, industry profit is either (Claim 2.4):

$$\Pi_A + \Pi_B = V\left(a + (1-a)\frac{1-V}{2}\right) + (1-a)\left(\frac{1+V}{2}\right)^2,$$

or

$$\Pi_A + \Pi_B = \left(\frac{1+V}{2}\right)^2.$$

Both of these profits are smaller than $\frac{5}{9}$ if V is sufficiently small (region 2c), and larger than $\frac{5}{9}$ if V is sufficiently high (region 2b). The profit with homogeneous beliefs ($a = 0$) is in this case therefore larger than the profit with heterogeneous beliefs if V is sufficiently small (sufficiently close to $\frac{1}{3}$) and smaller than the profit with heterogeneous beliefs if V is sufficiently large (close to 1). That is, if V is close to 1 and the share of informed consumers is close to $\frac{1}{2}$.

2.7.3 Proof of Proposition 2.3

Only if: As in the previous case, firms' incentive compatibility constraints hold iff $\Pi_H^h + \Pi_L^\ell \geq \Pi_H^\ell + \Pi_L^h$.

Consider the case where $\Pi_H^h > \Pi_H^\ell$ and $\Pi_L^h \geq \Pi_L^\ell$ and assume that the condition does not hold, that is, $\Pi_H^h + 2\Pi_L^\ell < 2\Pi_L^h + \Pi_H^\ell$. We show that the Intermediary then has an incentive to set fees such that both firms ask for the same position. First, $\tilde{r}_L = \ell$ requires $\Pi_L^\ell - F^\ell \geq m\Pi_L^\ell + (1-m)\Pi_L^h$, and therefore $F^\ell \leq 0$. Second, $\tilde{r}_H \neq \ell$ requires $\Pi_H^h - F^h \geq \frac{1}{2}\left(\Pi_H^h + \Pi_H^\ell - F^\ell\right)$, and therefore $F^h \leq \frac{1}{2}\left(\Pi_H^h - \Pi_H^\ell\right)$. The intermediaries profit is therefore

$$\Pi_I = F^\ell + F^h \leq \frac{1}{2}\left(\Pi_H^h - \Pi_H^\ell\right).$$

Consider the fees $F_L > 0$ and $F_H = \frac{1}{2}(\Pi_L^h - \Pi_L^\ell)$ and the assignment rule $r(0,h) = (\ell, h)$ (that is $m = 1$). Then both firms would choose $\tilde{r} = h$ (note that IR_H holds because $\Pi_H^h - \Pi_H^\ell \geq \Pi_L^h - \Pi_L^\ell$). The intermediary's profit in this pooling case is

$$\Pi_I = 2F^h = \Pi_L^h - \Pi_L^\ell > \frac{1}{2}\left(\Pi_H^h - \Pi_H^\ell\right).$$

Hence, the intermediary would benefit from choosing fees such that firms pool. The case where $\Pi_L^\ell > \Pi_L^h$ and $\Pi_H^\ell \geq \Pi_H^h$ works equivalently.

If: From the previous part we again have $\Pi_H^h + \Pi_L^\ell \geq \Pi_H^\ell + \Pi_L^h$, and therefore three different cases.

(A) $\Pi_H^h \geq \Pi_H^\ell$ and $\Pi_L^\ell \geq \Pi_L^h$.

(B) $\Pi_H^h > \Pi_H^\ell$ and $\Pi_L^\ell \leq \Pi_L^h$,

(C) $\Pi_H^h \leq \Pi_H^\ell$ and $\Pi_L^\ell > \Pi_L^h$.

We show that a separating equilibrium where $\tilde{r}_L^* \neq \tilde{r}_H^*$ and $r(\tilde{r}_L^*, \tilde{r}_H^*) = (\ell, h)$ exists. In *(A)* $F^\ell = \frac{1}{2}\left(\Pi_L^\ell - \Pi_L^h\right)$, $F^h = \frac{1}{2}\left(\Pi_H^h - \Pi_H^\ell\right)$, $m = n = 0$; in *(B)* $F^\ell = 0$, $F^h = \frac{1}{2}\left(\Pi_H^h - \Pi_H^\ell\right)$, $m = 1$, $n = 0$; in *(C)* $F^\ell = \frac{1}{2}\left(\Pi_L^\ell - \Pi_L^h\right)$, $F^h = 0$, $m = 0$, $n = 1$. It is straightforward to check that $(\tilde{r}_L^*, \tilde{r}_H^*)$ are then indeed mutual best responses. The profit is maximized because the intermediary's maximal profit if either $\tilde{r}_L = 0$ or/and $\tilde{r}_H = 0$, is equivalent to setting either $F^\ell = 0$ or/and setting $F^h = 0$. Moreover, by the first part of the proof, the intermediary's profit cannot be higher when setting fees such that $\tilde{r}_L = \tilde{r}_H$. Finally, setting fees such that $\tilde{r}_L^* \neq \tilde{r}_H^*$ and $r(\tilde{r}_L^*, \tilde{r}_H^*) = (h, \ell)$ is only possible if $\Pi_H^h = \Pi_H^\ell$ and $\Pi_L^\ell = \Pi_L^h$. But then, all fees must be 0. □

2.7.4 Proof of Proposition 2.4

The condition in Proposition 2.3 can never hold if the condition in Proposition 2.1 does not hold. We therefore only have to consider region 2.c and region 3.

Case 1 $\alpha < \frac{1}{2}$**:** In region 2.c, profits are as described in Claim 5 if $a = 0$ and as described in Claim 6 if $a > 0$.

- If $a = 0$, then by Claim 5: $\Pi_H^h = \Pi_B = \frac{4}{9}$ and $\Pi_L^\ell = \Pi_A = \frac{1}{9}$ and therefore $\Pi_H^h + 2\Pi_L^\ell = \frac{2}{3}$.
- If $a > 0$ we have $a = \alpha$, then by Claim 6: $\Pi_L^h = \Pi_B = (1 - \alpha)\left(\frac{1+V}{2}\right)^2$ and $\Pi_H^\ell = \Pi_A \geq \alpha\left(\frac{1+V}{2}\right)^2$ and therefore $\Pi_H^\ell + 2\Pi_L^h \geq (2 - \alpha)\left(\frac{1+V}{2}\right)^2$.

But this violates the condition because $\Pi_H^h + 2\Pi_L^\ell = \frac{2}{3} < (2 - \alpha)\left(\frac{1+V}{2}\right)^2 \leq 2\Pi_L^h + \Pi_H^\ell$.

In region 3, profits are as described in Claim 6.

- If $a=0$: $\Pi_H^h=\Pi_B=\left(\frac{1+V}{2}\right)^2$, $\Pi_L^\ell=\Pi_A=V\frac{1-V}{2}$ and therefore $\Pi_H^h+2\Pi_L^\ell=\left(\frac{1+V}{2}\right)^2+V(1-V)$.

- If $a>0$ we have $a=\alpha$: $\Pi_L^h=\Pi_B=(1-\alpha)\left(\frac{1+V}{2}\right)^2$ and $\Pi_H^\ell=\Pi_A\geq\alpha\left(\frac{1+V}{2}\right)^2$ and therefore $\Pi_H^\ell+2\Pi_L^h\geq(2-\alpha)\left(\frac{1+V}{2}\right)^2$.

Again, this violates the condition because $\Pi_H^h+2\Pi_L^\ell=\left(\frac{1+V}{2}\right)^2+V(1-V)<(2-\alpha)\left(\frac{1+V}{2}\right)^2\leq 2\Pi_L^h+\Pi_H^\ell$.

Case 2 $\alpha>\frac{1}{2}$: In region 2.c, profits are as described in Claim 5 if $a=0$ and as described in Claim 6 if $a>0$.

- If $a=0$, then by Claim 5: $\Pi_H^h=\Pi_B=\frac{4}{9}$ and $\Pi_L^\ell=\Pi_A=\frac{1}{9}$ and therefore $\Pi_H^h+2\Pi_L^\ell=\frac{2}{3}$.

- If $a>0$ we have $a=1-\alpha$, then by Claim 6: $\Pi_H^\ell=\Pi_B=\alpha\left(\frac{1+V}{2}\right)^2$, $\Pi_L^h=\Pi_A=\max\left\{V\left((1-\alpha)+\alpha\frac{1-V}{2}\right),(1-\alpha)\left(\frac{1+V}{2}\right)^2\right\}$, where $V\left((1-\alpha)+\alpha\frac{1-V}{2}\right)>(1-\alpha)\left(\frac{1+V}{2}\right)^2$ if α is sufficiently high. In this case we therefore have $\Pi_H^\ell+2\Pi_L^h=\alpha\left(\frac{1+V}{2}\right)^2+2V(1-\alpha)+\alpha V(1-V)$.

But then the condition always holds if α is sufficiently high and V sufficiently small (close to $\frac{1}{3}$) because then $\Pi_H^h+2\Pi_L^\ell=\frac{2}{3}\geq\alpha\left(\frac{1+V}{2}\right)^2+2V(1-\alpha)+\alpha V(1-V)=\Pi_H^\ell+2\Pi_L^h$.

In region 3 ($V\leq\frac{1}{3}$) profits are as described in Claim 6.

- If $a=0$: $\Pi_H^h=\Pi_B=\left(\frac{1+V}{2}\right)^2$, $\Pi_L^\ell=\Pi_A=V\frac{1-V}{2}$ and therefore $\Pi_H^h+2\Pi_L^\ell=\left(\frac{1+V}{2}\right)^2+V(1-V)$.

- If $a>0$ we have $a=1-\alpha$: $\Pi_H^\ell=\Pi_B=\alpha\left(\frac{1+V}{2}\right)^2$, $\Pi_L^h=\Pi_A=\max\left\{V\left((1-\alpha)+\alpha\frac{1-V}{2}\right),(1-\alpha)\left(\frac{1+V}{2}\right)^2\right\}$, where $V\left((1-\alpha)+\alpha\frac{1-V}{2}\right)>(1-\alpha)\left(\frac{1+V}{2}\right)^2$ if α is sufficiently high. In this case we therefore have $\Pi_H^\ell+2\Pi_L^h=\alpha\left(\frac{1+V}{2}\right)^2+2V(1-\alpha)+\alpha V(1-V)$.

But then the condition always holds if $V\leq\frac{1}{3}$ because $\Pi_H^h+2\Pi_L^\ell=\left(\frac{1+V}{2}\right)^2+V(1-V)\geq\alpha\left(\frac{1+V}{2}\right)^2+2V(1-\alpha)+\alpha V(1-V)=\Pi_H^\ell+2\Pi_L^h$. $\square$

Chapter 3

Fixed Price Equilibria on Peer-to-Peer Platforms: Lessons from Time-Based Currencies

joint with Berno Buechel

Abstract

Online, there are many platforms for peer-to-peer exchange, on which participants can trade certain goods or services among each other. Typically, these platforms introduce a platform-specific currency and fix prices to some extent. We model such platforms as pure exchange economies and characterize all fixed price equilibria. We discuss the inherent inefficiency following from the combination of fixed prices and voluntary trade and show that simple additional Pareto improving trades exist. Our theoretical analysis predicts that fixed prices lead on the one hand to less trade, but on the other hand to lower inequality than flexible prices. An empirical investigation of several platforms covering around 100k transactions illustrates that the observed patterns are fully in line with our predictions. This is informative for the market design of peer-to-peer platforms and for markets with price restrictions more generally.

3.1 Introduction

Platforms for peer-to-peer exchange have recently popped up all around the world and for various kinds of goods. Members can trade there as on conventional marketplaces, except that one cannot solely be a buyer or a seller. Instead, every participant must be buyer and seller to some extent. This feature is typically guaranteed by a platform-specific currency that can be earned only through sales on the platform and that can be used only for purchases on it. There are different reasons why a platform operator might want to create such a closed exchange marketplace. First, because it provides incentives for interested buyers to also contribute as a seller. Second, because it commits sellers to spend their earnings among the participants.[1] Third, it is a way of excluding certain sellers, e.g. some platforms exclude professional sellers in order to differentiate themselves from other platforms. Examples for platforms of peer-to-peer exchange are guestoguest.com, where members can rent homes with guestpoints. These points can only be earned by renting one's own home to other members, while the maximal price one is allowed to charge depends on defined house characteristics. On bookmooch.com members can swap goods, where each book costs exactly one point. Further, so-called time banks allow for local exchange of services, where one hour of service is typically fixed to cost one hour of a time currency. As these examples show, many of these platforms restrict price setting. Their motivation to do so could be to guarantee some price stability on the platform, to increase market transparency, or for some kind of fairness considerations.[2] However, the consequences of the platform operator's decision to keep prices rather fixed or rather flexible are not well understood.

In this paper we model such marketplaces and study the effect of price setting restrictions on efficiency, extent of trade, and equality. We believe this is interesting for at least two reasons. First, peer-to-peer exchange has become common in the Internet. Even though marketplaces where members have to be active on the demand and supply side have existed at least since the nineteenth century [see

[1] See Mailath, Postlewaite and Samuelson [2016] for a formalization of that argument.

[2] We will discuss the reasons to keep prices fixed in some more detail in section 3.5, i.e. when we can relate to our results.

e.g. Warren, 1852], such systems have become more popular when internet lowered transaction costs. This development is similar to the increased use of online platforms such as eBay, Amazon, and Alibaba to trade more goods from consumer to consumer than it would have been possible with garage sales. Shedding light on the workings of such platforms is informative for their market design. In particular, rules of price setting, which we can study with our framework, seem to be a crucial feature of a platform's market design.

Second, economists have been interested in general equilibrium effects in closed exchange economies for a long time. Peer-to-peer exchange platforms are wonderful real-world examples for such closed exchange economies. Hence, we can make use of a rich body of theoretical work, in particular, on the properties of equilibrium allocations with and without Walrasian prices, and link this theoretical work to recent empirical observations.

We model a simple exchange economy with fixed prices. Each agent can offer his endowment and consume goods that are offered by others. Goods can only be traded for a platform-specific currency. To keep the model simple, agents are assumed to have additively separable preferences, which are quasi-linear in the currency and strictly convex.[3] We look for fixed price equilibria. The corresponding equilibrium concept is provided by Maskin and Tirole [1984] and refers back to Grandmont [1977], among others. These authors call it K-equilibrium and show that it naturally incorporates the properties of the formerly introduced Dréze equilibrium and Bénassy equilibrium. In particular, a fixed price equilibrium requires that no agent can be forced to trade ("voluntariness") and that there is no pair of agents who can improve by trading some good ("weak order"). When the fixed prices happen to coincide with Walrasian equilibrium prices, then the fixed price equilibrium and the Walrasian equilibrium allocations coincide. Otherwise, fixed prices necessitate that some agents are constrained from buying or from selling certain goods.

Assuming quasi-linearity of preferences allows us to characterize all fixed price equilibria and to derive empirical predictions about the effect of price setting

[3] We tailor the assumptions to the application and keep the model simple. This buys us clear-cut results that make the underlying effects transparent. We study robustness to relaxing the assumptions in Appendix 3.8.

restrictions in these markets. The starting point of our analysis is the distinction between *scarce* goods and *non-scarce* goods. The former ones are goods for which market demand at given prices is larger than the total endowment. For non-scarce goods market demand is smaller. We show that in any fixed price equilibrium, sellers providing a scarce good keep their optimal amount of that particular good (while all buyers receive at most their desired amount). Further, all buyers receive their optimal amount of each non-scarce good, while the seller of the non-scarce good keeps the rest, which is more than this agent desired. In other words, the seller of a non-scarce good is constrained from selling the desired amount, and at least one of the buyers of a scarce good is constrained from buying the optimal amount. The rationing scheme therefore only affects the allocation of scarce goods, but not the allocation of non-scarce goods, which must be the same in every fixed price equilibrium.

The first implication of this characterization is that, under very weak conditions any fixed-price equilibrium is not only Pareto *in*efficient, but also *constrained inefficient*. Indeed, we can construct simple chains of bilateral trades that are Pareto improvements within the given price system, under weak conditions on the existence of either strictly scarce or strictly non-scarce goods. Thereby, each bilateral trade either involves agents who are constrained sellers of a non-scarce good and can sell more of their good, or constrained buyers of a scarce good who can buy more of this good. In the simplest case there are two suppliers of non-scarce goods who have a non-zero demand for each other's good. Then they can both improve by exchanging their services. However, in a market with fixed prices this will not occur because both value the numeraire good (currency) more than the consumption of the other's good. In that sense, the price of their goods is "too high." The case with "too low" prices works similarly, and there are also combinations of the two.

We then proceed by comparing fixed price equilibria with Walrasian equilibria. It turns out that the extent of trade in the Walrasian equilibrium is larger than in any fixed price equilibrium. That is, every agent can sell weakly less in a fixed price equilibrium than in the Walrasian equilibrium. In the generic case that a good is strictly scarce or strictly non-scarce, in any fixed price equilibrium the amount traded of any good is even strictly smaller than in the Walrasian equilibrium. Hence, it becomes apparent that fixed prices hamper trade, which is a clear downside of most such platforms. However, Walrasian equilibria do not Pareto dominate fixed price equilibria in general, such that both regimes generate their "winners" and "losers." The winners of flexible prices are typically suppliers of scarce goods because they sell more and at a higher price. As a consequence, inequality is often larger under flexible prices than under fixed prices. We finally investigate data from seven time exchange markets, covering almost 100,000 transactions. These are peer-to-peer exchange platforms, facilitating decentral trade typically through a time-based currency. Prices are fixed to different degrees. We observe that those platforms with fixed prices indeed have lower trade volume and tend to exhibit lower income inequality than those with rather flexible prices. Hence, the empirical patterns are perfectly in line with our model predictions.

Our paper makes three contributions. First, we show that price restrictions, which are a very common feature of peer-to-peer platforms, come at a very high cost. We show theoretically and illustrate empirically that under fixed prices participants leave out many Pareto improving trades, even within the given price regime. The relatively low number of transactions and the correspondingly low trade volume indicate that price restrictions seriously hamper the working of the market.

Second, we show that a potential benefit of price restrictions is a that they may lead to more equal market outcomes. Equality of the income distribution is strongly related to the perceived fairness of allocations [e.g. Alesina and Angeletos, 2005; Almås et al., 2010; Bénabou and Tirole, 2006]. Hence, it may well be that platform operators and market participants who consider the fixed prices of a given platform as more "fair" have a point.

Third, we apply general equilibrium theory, in particular on Walrasian and fixed prices in exchange economies, to a new setting and derive predictions that can be empirically tested. It is well known that non-Walrasian market allocations are generally not Pareto efficient. Moreover, it has been shown that such allocations typically do not even satisfy *constrained efficiency*, that is, there exist Pareto improving trades within the given, non-Walrasian price regime [Herings and Konovalov, 2009; Maskin and Tirole, 1984; Younés, 1975]. We do not only show for the application of peer-to-peer platforms that this insight applies, but we characterize the inefficiency more specifically by showing how "too high" or "too low" prices prevent some simple Pareto improving trades. In comparison to Herings and Konovalov [2009], we make more simplifying assumptions on the utility functions of the market participants, but stay more general in terms of admitting boundary solutions and not imposing a particular rationing scheme. We think that in our application and in many others it is an important feature that a given participant need not buy all products that are in the market and that equal rationing is a very stylized assumption.

We think that our results are also informative for market design outside of peer-to-peer platforms. In many real-world markets prices are (at least in the short run) non-Walrasian. There are several causes of price stickiness, such as costs of changing marketing activities, consumers' perceptions of clear or "fair" prices, or governmental regulations. Our analysis of closed exchange economies suggests that on many more markets price restrictions hamper trade, induce an inefficiency even in the given price regime, but can contribute to the equality of the market outcomes.

In the next section, we introduce the model. Section 3.3 presents the results. The empirical illustration follows in section 3.4. In section 3.5 we discuss advantages and disadvantages of fixed prices for peer-to-peer exchange platforms, before we conclude in section 3.6. All proofs are relegated to the Appendix.

3.2 Model

Consider a pure exchange economy with $n \geq 2$ agents indexed by i ($i = 1, 2, ..., n$) and $m + 1$ goods indexed by h ($h = 0, 1, ..., m$). A price vector $p \in \mathbb{R}^{m+1}$ with $p_0 = 1$ and $p_h > 0$ is exogenously fixed. Each agent i is characterized by a convex consumption set $X^i \subseteq \mathbb{R}^{m+1}$ and an endowment $\omega^i \in X^i$. Each agent i has complete and transitive preferences $\succsim^i$ over consumption bundles X^i, represented by a utility function $U^i : X^i \to \mathbb{R}_+$. We assume that preferences are continuous and strictly convex.

For the main part of our analysis, we make the simplifying assumption that each agent is endowed with exactly one good such that $\omega_i^i > 0$, while $\omega_h^i = 0$ for $h \neq i$ and $m = n$. This assumption is tailored to the example of house exchange and of service exchange, while the intuition easily extends to the more general case.[4]

For the application of service exchange with a time-based currency, we consider the following interpretation of the model. Each agent j can provide one service $h = j$. A service j is quantified by the amount of time agent j needs to provide that service. Thus, agent i receives one hour of another agent j means that agent j provides an amount of service to agent i, which costs him one hour. Let x_j^i be the amount of time that j stands in the service of i. We denote by $u_j^i(x_j^i)$ the utility agent i derives from service of agent j. Services are priced on that basis. Each hour of service costs one amount of the numeraire good $h = 0$, so $p \equiv (1, ..., 1)$. The numeraire good is not a service but a time-based currency. For the application of goods that are not services we can immediately interpret x_j^i as the quantity i consumes of the good bought from agent j.

We focus on preferences that are additively separable and quasi-linear in the numeraire.[5] Utility of agent i is given by:

$$U^i(x^i) = x_0^i + u_1^i(x_1^i) + ... + u_i^i(x_i^i) + ... + u_n^i(x_n^i).$$

We assume that u_h^i is twice differentiable with marginal utility $mu_h^i(x_h^i) > 0$ and $\frac{\partial mu_h^i(x_h^i)}{\partial x_h^i} < 0$ for all i, h and x_h^i.

Let us now turn to the equilibrium concept. As is well-known, for fixed prices we can in general not expect the feature of Walrasian equilibrium that individual optimal decisions are consistent with market clearing. Instead some agents are constrained from selling or from buying on certain markets. The corresponding equilibrium concepts for fixed prices (i.e. in general non-Walrasian prices) are based on two fundamental principles:

[4] Relaxing this assumption is straightforward (see Appendix 3.8.1). The consequences for the results are not severe, but the simple exposition would suffer.

[5] This assumption simplifies the analysis by making demand in one market independent from constraints in other markets. We relax the assumption in section 3.8.2 in the Appendix.

(i) *voluntariness*: no agent can be forced to trade. (Otherwise, his choice could be inconsistent with his preferences.)

(ii) *weak order*: no two agents can be constrained on two different sides of the same market. (Otherwise, they could improve by trading.)

We precisely follow Maskin and Tirole [1984] by defining a fixed price equilibrium based on these two principles. For this purpose, we need some additional notation. Agent i's consumption bundle x^i, can be captured by his net trades t^i:

$$x^i = \omega^i + t^i$$

and likewise we can construct the set of possible trades $T^i = \{x^i - \omega^i | x^i \in X^i\}$ of agent i. Since in our context there is only one seller on each market, the endowments are of the form $\omega^i = (0, ..., 0, 1, 0, ..., 0)$. For all $i \neq h$ we therefore have $x^i_h = t^i_h$ with weakly positive t^i_h; and for $i = h$ we have $x^i_h = \omega^i_i + t^i_h$ with weakly negative t^i_h. Let $\tilde{T}^i := T^i \cap \{t^i | p \cdot t^i = 0\}$ be the set of (with respect to budget) feasible net trades of agent i. $\tilde{T} = \{(t^1, ..., t^n) \in (\tilde{T}^i, ..., \tilde{T}^n) | \sum_i t^i = 0\}$ is then the set of feasible net trades in the economy. We define $\tau^i_h(t^i) := \{\tilde{t}^i \in \tilde{T}^i | \tilde{t}^i_k = t^i_k \ \ \forall k \neq 0, h\}$, as the (budget) feasible net trades of agent i that coincide with the net trades t^i on all markets, but on market h and 0. Finally, let $Z = ((\underline{Z}^1, \bar{Z}^1), ..., (\underline{Z}^n, \bar{Z}^n))$ be a vector of quantity constraints such that $\underline{Z}^i \leq 0$ and $\bar{Z}^i \geq 0$ and $\underline{Z}^i_0 = -\infty$ and $\bar{Z}^i_0 = \infty$ for all i.

We can now define equilibrium allocations x under fixed prices p by defining the corresponding equilibrium trades t.

Definition 3.1 (Fixed Price Equilibrium, Maskin and Tirole, 1984)**.** *A fixed price equilibrium (FPE) is a vector of (fully) feasible net trades $t \in \tilde{T}$ associated with a vector of quantity constraints Z such that for all i,*

(V) exchange is "voluntary:" t^i is the $\succsim^i$-maximal element among the (budget) feasible net trades $\tilde{t}^i \in \tilde{T}^i$ that satisfy the constraints $\underline{Z}^i \leq \tilde{t}^i \leq \bar{Z}^i$.

(WO) exchange is "weakly orderly:" if for some commodity h, and some agents i, j, there is a trade $(\tilde{t}^i, \tilde{t}^j) \in \tau^i_h(t^i) \times \tau^j_h(t^j)$ such that $\tilde{t}^i \succ^i t^i$ and $\tilde{t}^j \succ^j t^j$, then $(\tilde{t}^i_h - \bar{Z}^i_h)(\tilde{t}^j_h - \underline{Z}^j_h) \geq 0$. In words: if there is a feasible trade that only differs from trade t on market h and on market 0 and both traders i

and j would benefit from that trade, then it cannot be that the two traders are at different sides of the market in the sense of one wanting to buy less (respectively to sell more) and the other wanting to buy more (respectively to sell less).

Voluntariness (V) captures that individual agents optimize across all markets, given their constraints Z^i. $\underline{Z}^i \leq 0$ ($\bar{Z}^i \geq 0$) then ensures that i cannot be forced to buy (sell). Weak order (WO) captures that there is no pair of agents i, j who can both strictly improve by making an (additional) trade on a single market h, when the constraints on this market are relaxed. Such a trade can either be between a seller and a buyer who exchange good h for money; or between two buyers who change the amount they buy of good h without changing the total demand (for seller h).

Weak order (WO) is equivalent to the following property, which is actually used in Maskin and Tirole [1984]: there is no market $h (\neq 0)$ in which a Pareto improvement can be reached when ignoring the constraints on this market and keeping all other markets (except the market for the numeraire 0) fixed.[6]

3.3 Results

An agent i can only afford consumption bundles x^i that are in his budget set $\tilde{X}^i(p) = \{x^i | p \cdot x^i \leq p \cdot \omega^i = p_i \omega^i_i\}$. For fixed prices p, compute demand $\hat{x}^i$ of an agent i as $\hat{x}^i := \arg\max_{x^i \in \tilde{X}^i(p)} U^i(x^i)$, i.e. the consumption bundle that maximizes agent i's utility within the budget set.

Definition 3.2 (scarce and non-scarce goods)**.** *Good h is called* scarce *if there is no excess supply (at fixed prices p), i.e. if $\sum_{i \in N} \hat{x}^i_h \geq \sum_{i \in N} \omega^i_h = \omega^h_h$. Otherwise, it is called* non-scarce.

Scarce goods are in high demand, relative to their supply, while non-scarce goods are not. The following lemma shows that scarcity of a good h can be

[6]This notion is called "weak order (O")" in Maskin and Tirole [1984]. We show the equivalence of the two notions in Section 3.7.1.

inferred by comparing the given fixed price p_h with the Walrasian equilibrium price p_h^*.[7]

Lemma 3.1. *Let p^* denote the price vector of the Walrasian market equilibrium. Good h is scarce (at fixed prices p) if and only if $p_h^* \geq p_h$.*

3.3.1 Characterization of fixed price equilibria

Proposition 3.1 (Characterization). *In every FPE, each good $h \neq 0$ is allocated as follows:*

(a) If h is non-scarce, every buyer receives the desired amount, while the seller keeps the rest. That is: $\forall i \neq h$, $x_h^i = \hat{x}_h^i$ and $x_h^h = \omega_h^h - \sum_{i \neq h} \hat{x}_h^i (> \hat{x}_h^h)$.

(b) If h is scarce, every buyer receives at most his desired amount, while the seller keeps (exactly) the desired amount. That is: $\forall i \neq h$, $x_h^i \leq \hat{x}_h^i$ and $x_h^h = \hat{x}_h^h$.

Proposition 3.1 provides a clear-cut characterization of all FPE. It fully determines the allocation of all non-scarce goods and it determines the allocation of all scarce goods up to a rationing scheme. In the literature equal rationing is sometimes imposed [e.g. Herings and Konovalov, 2009]. Our results hold for all FPE and hence for all rationing schemes. Note also that Proposition 3.1 holds without any assumption on $\hat{x}_h^i$ being interior. In particular, $\hat{x}_h^i \in \{0, \omega_h^h\}$ is admitted and does not change the statement. Such a clear characterization of all FPE is due to our assumptions on the utility function. Demand in one market is not affected by quantity constraints in another market. The rationing scheme for good h therefore only affects demand of good h and the numeraire. The asymmetry in the strength of the two statements (a) and (b) follows from the assumption that every agent is only endowed with one good, which means that every agent can only sell one good, while he can buy any good.[8]

[7] Due to our assumptions on preferences, the Walrasian equilibrium is unique.

[8] Relaxing this assumption, would lead to results for non-scarce goods that are fully analogous to the results with scarce goods (see Appendix 3.8.1). Such results are slightly weaker since the allocation of non-scarce goods then also depends on the rationing scheme. However, loosening this assumption would not undermine the substance of the results.

If a good h is scarce but not strictly scarce, then the inequality of the second statement of the Proposition 3.1 holds in fact with equality. Since Walrasian prices p^* have the feature that each good h is scarce, but not strictly scarce, it follows that in Walrasian equilibrium, which is a special case of a FPE, no agent is constrained, while markets clear. However, for generic prices $p_h \neq p_h^*$ at least one agent is constrained from buying the desired amount of a scarce good h and the seller of a non-scarce good h is constrained from selling the desired amount.

3.3.2 Inefficiency of Fixed Price Equilibria

We now turn to efficiency.

Definition 3.3 (Pareto Efficiency and Constrained Efficiency). *An allocation x is* Pareto efficient *(PE) if $\nexists x' = x + t$ with $\sum_i t^i = 0$ which Pareto dominates x. An allocation x is* constrained efficient *(cPE) if $\nexists x' = x + t$ with $t \in \tilde{T}$ which Pareto dominates x.*

The notion of Pareto efficiency is stronger than the notion of constrained efficiency because it admits more general improvements. For Pareto efficiency we consider any other allocation that is feasible, while constrained efficiency only considers allocations that obey the budget feasibility for the fixed prices p. Instead of requiring that every agent wants to consume a strictly positive amount of every good, i.e. interiority, we make a much weaker assumption on the attractiveness of different goods.

Definition 3.4 (Weak Interiority). *An economy satisfies* weak interiority *if the following holds for every market h.*

(i) If h is non-scarce, then there is another non-scarce good $k \neq h$ such that $\hat{x}_k^h > 0$, i.e. the seller of a non-scarce good h demands at least one other non-scarce good.

(ii) If h is scarce, then $\hat{x}_h^h > 0$, i.e. the seller of a scarce good h demands a positive amount of it.

With these notions in hand, we can formalize the inefficiency, not only with respect to Pareto efficiency, but also with respect to constrained efficiency.

Proposition 3.2 (Inefficiency). *Suppose a non-scarce good h and at least one agent $i \neq h$ exist such that $\hat{x}_h^i > 0$. Then no FPE is Pareto efficient. Suppose $p_h^* \neq p_h$, $\forall h$ and weak interiority is satisfied. Then no FPE is constrained efficient.*

The first statement of Proposition 3.2 is a standard inefficiency result. In the proof of the second part, we show that under the condition of weak interiority, there is a chain of agents such that each pair in the chain can strictly improve by bilateral trade on a single market.[9] The inherent type of inefficiency emerging from the combination of fixed prices and decentralized trade is easiest to see by assuming prices fixed to $p \equiv (1, ..., 1)$ and interiority. By Proposition 3.1 a supplier i of a non-scarce good derives then a marginal utility of 1 from each non-scarce good $h \neq i$. However, his marginal utility from good i is strictly smaller. Therefore, any two suppliers of a non-scarce good could improve by exchanging some amount of their goods directly, without using the numeraire good in the transactions. This will not occur because both value the numeraire good (currency) more than the consumption of the other's good. In some sense the prices of the two goods are "too high." A similar issue occurs for scarce goods: prices are "too low" such that despite the high demand, a supplier of the scarce good is not willing to offer a sufficient amount of it, while she would do so in exchange for another good that she values highly. This shows how decentralized trade fails to enable even simple Pareto improving trades when prices are fixed.

3.3.3 Fixed price vs. Walrasian equilibrium

We now compare the Walrasian equilibrium and FPE, first with respect to the amount traded and then with respect to inequality.

Proposition 3.3 (less trade). *In every FPE t, the total amount traded of any good $h \neq 0$ is smaller than in the Walrasian equilibrium t^*, i.e. $\sum_{i \neq h} t_h^i \leq \sum_{i \neq h} t_h^{i,*}$. For non-scarce goods h, every single buyer $i \neq h$ buys less than in the Walrasian equilibrium, i.e. $t_h^i \leq t_h^{i,*}$, $\forall i \neq h$.*

[9] In the terminology of Herings and Konovalov [2009], this means that no fixed price equilibrium is "B-p efficient", which is an even weaker notion of efficiency than constrained efficiency.

This result follows from Proposition 3.1 and the fact that the demand of each good is decreasing in its own price. For the interpretation, suppose that the fixed price p_h of a good h does not coincide with the Walrasian price p_h^*. If h is non-scarce, $p_h^* < p_h$ (Lemma 1). Since buyers of non-scarce goods are not constrained (neither in the FPE nor in the Walrasian equilibrium), they would buy more in the Walrasian equilibrium. If good h is scarce, $p_h^* > p_h$ (Lemma 1). Since sellers of scarce goods are not constrained (neither in the FPE nor in the the Walrasian equilibrium), they would sell more in the Walrasian equilibrium.

The result on less trade also has implications for inequality of incomes. Let $y = (y^1, ..., y^n)$ denote the income distribution with $y^i = |t_i^i| \cdot p_i$. Since suppliers of scarce goods sell less with fixed prices (by Proposition 3.3) and fixed prices are lower than flexible prices in equilibrium (by Lemma 3.1), their income is lower under fixed prices, i.e. $y^i = |t_i^i| \cdot p_i < |t_i^{i,*}| \cdot p_i^* = y^{i,*}$. Suppliers of non-scarce goods also sell less in the FPE, but fixed prices for their goods are higher than Walrasian prices. Whether the overall effect on income is positive or negative depends on the price elasticity of demand. The relevant prices are $p = (1, p_1, ..., p_n)$ and $p^* = (1, p_1^*, ..., p_n^*)$ and the corresponding demand is $Q_h := \sum_{i \neq h} \hat{x}_h^i$ and $Q_h^* := \sum_{i \neq j} x_h^{i,*}$. Hence, we define the (discrete) price elasticity of demand as $\varepsilon_h := \frac{\Delta Q_h}{\Delta p_h} \cdot \frac{p_h}{Q} = \frac{Q_h^* - Q_h}{p_h^* - p_h} \cdot \frac{p_h}{Q_h}$. With two strong conditions that we define next, we can compare FPE with Walrasian equilibria in terms of inequality.

Assumption 3.1. *We assume that there is at least one scarce good and one non-scarce good and define two qualifications.*

(i) Suppose at prices p the supply for every scarce good i is larger than the demand for every non-scarce good j weighted by the prices, i.e. $S_i = \omega_i^i - \hat{x}_i > Q_j \cdot \frac{p_j}{p_i}$.

(ii) Suppose demand for every non-scarce good j is inelastic or isoelastic, i.e. $|\varepsilon_j| \leq 1$.

Corollary 3.1 (Inequality)**.** *Suppose Assumption 1 holds and suppose that every good h faces positive demand for the fixed price p, i.e. $Q_h > 0$. Then moving from any fixed price equilibrium to the Walrasian equilibrium increases inequality in the following sense: Those with the highest income increase their income, while the income of all others does not increase.*

The result is based on our distinction of scarce and non-scarce goods. Under Assumption 1 (i) suppliers of scarce goods earn more than suppliers of non-scarce goods already under fixed prices. Hence, those with the highest income are the suppliers of scarce goods. When moving to flexible prices, their income increases because both the quantities sold and the prices increase. For suppliers of non-scarce goods on the other hand, Assumption 1 (ii) implies that their income does not increase when moving from fixed to flexible prices because the reduction in prices cannot be compensated by the increase of sold goods. This is due to the inelastic demand. Hence, Corollary 3.1 can also be phrased as "the rich get richer and the poor get poorer" where the "rich" are the suppliers of scarce good and the "poor" the suppliers of non-scarce goods.

This is a genuine increase of inequality. It also links to several common measures and indices of inequality. First, the share of income of the, say, top 25% increases when 25% is the fraction of suppliers of scarce goods. Another simple and common measure takes the ratio of two incomes, comparing a certain percentile, e.g. 10%, with another percentile, e.g. the median. Also this measure of inequality increases when the percentiles are taken such that they compare suppliers of scarce goods with suppliers of non-scarce goods. Several inequality indices are decomposable in a well defined way into inequality within groups and inequality between groups [e.g. Cowell, 2000]. In particular, this is true for the Theil index [Foster, 1983]. Defining groups by suppliers of scarce and non-scarce goods, we get that the inequality between groups increases when moving from FPE to Walrasian equilibrium. However, there is no clear implication for the inequality within groups such that we cannot exclude that inequality within groups falls extremely and dominates the rise of inequality between groups. Similarly for the Gini coefficient.[10] The Gini coefficient is usually defined as the area between the Lorenz curve and the id line. When, however, formalized as a normalized sum of absolute differences, we can see that all differences between groups, say i is supplier of a scarce good and j of a non-scarce good, $|y^i - y^j| < |y^{i,*} - y^{j,*}|$,

[10] For the Gini coefficient this decomposability does not hold in general (there is also an interaction term), but it holds when the groups are non-overlapping [Cowell, 2000], which is indeed true under our Assumption 3.1 because all suppliers of scarce goods are earning more than all suppliers of non-scarce goods.

unambiguously increase.

While Corollary 3.1 has strong implications, it is notably based on a very demanding assumption: Assumption 3.1. In reality, we would expect that both parts of Assumption 3.1 are not fully satisfied. (i) There will not be a perfect separation between suppliers of scarce and non-scarce goods in fixed price equilibrium with all suppliers of scarce goods at the top of the income distribution. (ii) There will be suppliers of non-scarce goods who benefit from flexible prices because their reduction of selling price is over-compensated by the increase in the amount sold. However, the main force that drives the inequality result of Corollary 3.1, will still be at work. Suppliers of scarce goods heavily benefit from the introduction of flexible prices. The boost of their income is due to the combination of larger amounts sold and higher prices, while suppliers of non-scarce goods face lower prices. We consider it as likely that this boost of income increases inequality even if the qualifications of Assumption 3.1 are not met.

Comparisons of income distributions have to be distinguished from welfare comparisons. Whether an agent is better off in the Walrasian equilibrium or in the FPE depends not only on her income, but also on the prices of the goods she demands, and on the quantity constraints she faces at the scarce goods. In general, the Walrasian equilibrium does not Pareto dominate a given FPE; and neither the other way around.

3.4 An Empirical Illustration

3.4.1 The Data Set

Our theoretical investigation provides clear-cut results on how goods are allocated when prices are fixed and how the allocation differs from Walrasian equilibrium. The model applies in particular to time exchange markets. These are the purest real-world examples of exchange economies we can think of. Concretely, these are marketplaces for service exchange, which facilitate decentral trade through a time-based currency. Often, but not always, all prices are fixed and equal, e.g. any hour of service yields one hour on the time account for the supplier and costs one hour for the consumer. Such markets have existed at least since the nineteenth century (see e.g. Warren, 1852), but it was much more recently that many such markets have popped up all around the world.[11] We now set out to describe real transaction patterns of several such platforms in order to check whether these patterns are consistent with our model predictions.

For seven platforms, we obtained data of all transactions made between 2008 and 2016.[12] These 100,000 odd transactions were all managed by the same software and are hence directly comparable. For each platform the recordings of the transactions begin with the introduction of the software. Each platform has a set of rules on how to trade on them. These rules are highly similar to each other on all platforms with one main difference: Prices are fixed to a higher or lower degree. One platform writes [translated from German]: "The exchange rate for performance is 1:1 – one hour of performance entitles to receive one hour of counterperformance." Three other platforms have similar formulations to fix prices.[13] At the other end of the spectrum, there are platforms that only suggest a certain price, but leave the choice to the market participants. One of these platforms writes [translated from German]: "We recommend to charge 100 [currency units] per hour. However, the two exchange partners agree on the price by themselves." Other potentially relevant differences concern restrictions of the budget from below or above; and rules on how much to pay each year as a membership fee, and whether companies are admitted.[14] Some platforms explicitly emphasize exchange of services, while on all platforms both services and goods are admitted. In sum, it is however remarkable how similar the rules on these platforms are.

[11] For instance, already in 2011, 300 registered "time banks" have been counted only in the US, which is just one of 34 countries with such institutions [Cahn, 2011]. There is a broad range of services offered, from ironing clothes, mowing someone's lawn to looking after children, or teaching a certain craft.

[12] We asked 18 platforms in Austria and Switzerland for their consent to analyze their anonymous transaction data and received a response of 55%, among whom the response was positive in 80% of the cases. One case with positive response was not considered because this data set did not even span one year. When obtaining the data, we agreed not to reveal the identity of these platforms.

[13] Moreover, it is explicitly forbidden to combine transactions with transfers in real currencies, except for costs of material, for which the price of purchase is to be used.

[14] As a test of robustness, we excluded all members that are identifiable as firms. This does not change any of the qualitative results. (In terms of absolute numbers, the trade volume and the inequality on the largest platform, later labelled W3, become more moderate.)

Table 3.1: Peer-to-peer platforms

ID	years	members	TA	currency	price recommendation
F1	5.6	215	1,559	hours	Performance is exchanged 1:1 – one hour of performance entitles to one hour of counter-performance
F2	7.9	330	5,094	hours	An exchange rate of 1:1 is assumed. One hour of performance entitles to obtain one hour of performance for personal use.
F3	8.7	324	4,175	hours	The exchange among those willing to trade is accounted in hours and minutes.
F4	9.6	708	12,513	hours	Concerning the exchange of performance the following holds: Each hour has the same value.
W1	5.6	179	2,804	hour-units	Goods and services are generally traded according to time-units.
W2	6.7	118	2,975	units	The exchange partners determine the performance's value in currency units. As a point of reference, we recommend to value one hour working time by 100 currency units.
W3	10.4	1,037	69,346	units	We recommend to charge 100 [currency units] per hour. But two exchange partners decide on the price themselves.

Note: Members are all participants of a platform who had at least one transaction with another member. TA is the total number of transactions on the platform (excluding system transaction). The price recommendation is a literal translation from German. We categorized platforms into those with fixed prices, labeled "F", and into those with rather flexible prices, labeled "W" for Walrasian, according to the price recommendation and the flexibility of the prices in sample transactions. Platforms ordered first by fixed versus flexible prices and then by time span of data recordings.

Table 3.1 provides some summary statistics about the platforms. According to the formulations on how to set prices, and consistent with sample transactions, we organize the platforms into four with fixed prices labeled F1,...,F4 and three with rather flexible prices labeled W1,..,W3. Within both categories the platforms are ordered and labeled according to the length of our recordings (see column *years*).

Members are defined as participants who engaged in at least one transaction with another participant. We only consider transactions that take place between participants, not system transactions such as the payment of an annual membership fee. In total, we have data on 2,911 members and of 98,527 transactions.

Among our theoretical results, Proposition 3.3 and Corollary 3.1 can be directly taken to the data. They predict that the platforms with rather flexible prices will have more trade and higher inequality. We analyze these two properties in turn.

3.4.2 Amount of Trade

We assess the amount of trade by two complementary measures. The first measure is the number of transactions. The second measure is the trade volume, i.e. the money in the time-based currency spent on trades (converted to hours in the case of W2 and W3).[15] Both measures are normalized by computing the amount per member per year to make the platforms comparable.[16] The amount traded for both measures is illustrated in Figure 3.1. The platforms are still ordered by observed years, but organized into the two categories fixed prices (F1-F4) and flexible prices (W1-W3) for a better comparison.

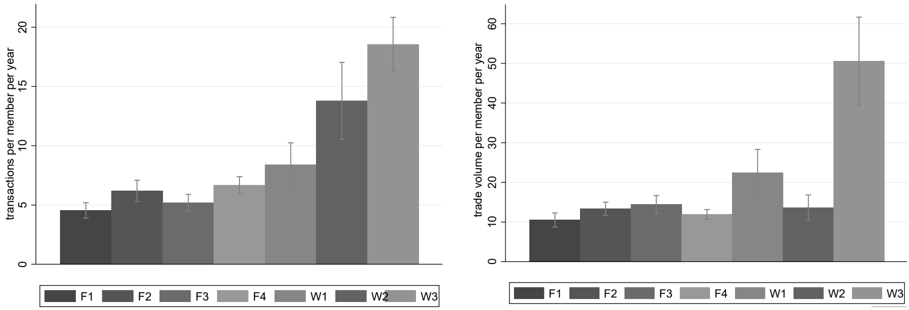
The figure clearly suggests that fixed prices (F1-F4) are associated with less trade than the rather flexible prices (W1-W3), as predicted by our model. On average the platforms with fixed prices only trade 12.6 hours per member and year, while those with flexible prices trade 43.9. On average the platforms with fixed prices only have 6.0 transactions per member and year, while those with flexible prices have 16.9. Concerning the trade volume, platform W2 is an exception to the general pattern since its trade volume is in the range of the platforms with fixed prices, but concerning the number of transactions it is consistent with the pattern.

[15] We do not have the quantities of many transactions, but we always have the price paid.

[16] More precisely, we compute for each member of a platform how much he traded on average per year for all the years that he was active, i.e. had at least one transaction, and average this number over all members. In this way we can account for the fact that individuals can join and leave a platform within the observed years. Another normalization of simply dividing the amount by the age of a platform and the number of members leads to the same qualitative differences between the platforms.

Figure 3.1: Amount of trade

(a) Number of Transactions **(b) Trade Volume**



Note: Amount of trade: Panel (a) shows the average number of transactions per member per year. Panel (b) shows the average trade volume per member per year. Confidence intervals are standard 95% confidence intervals based on the heterogeneity between the members.

3.4.3 Inequality

We investigate inequality of income. Each trader's annual income is the trade volume that he sells in a given year. Inequality typically increases with the length of the observed period because some members are active on the platform for a longer time period than others. We therefore compute inequality measures for each fully observed year separately.

We first describe inequality by the ratio of incomes of different percentiles. The inequality result, Corollary 3.1, has a direct implication for this measure: Given that we compare the income of suppliers of scarce goods with those of non-scarce goods, inequality is larger under flexible prices. Table 3.2 reports the income of different percentiles in relation to the income of the median percentile. The 95 percentile, that is a top 5% earner, earns 8.32 times the earnings of the median in platform F1 and even 10.04 times the median in platform W1. Considering the 95 percentile and the 90 percentile, inequality is larger in platforms W1 and W3 with flexible prices than on the other platforms. The platforms with fixed prices F1-F4 and platform W2 are similar in terms of inequality.

Interestingly, it is only the relative income of the top earners which is higher in W1 and W2. The relative income of lower percentiles is comparable on all platforms. That is in line with Corollary 3.1 if the top 10% earners provide a scarce good, while a fraction of the top 25% earners already provide a non-scarce good. Top earners under flexible prices are therefore likely those who provide the "most" scarce goods.

Table 3.2: Several measures of inequality.

id	95q/50q	90q/50q	75q/50q	25q/50q	Gini
F1	8.32	5.62	3.29	0.27	63.1
F2	7.63	5.50	2.59	0.12	63.5
F3	7.78	5.53	2.56	0.20	64.5
F4	8.76	5.89	2.86	0.27	64.6
W1	10.04	6.52	2.62	0.23	67.2
W2	7.33	5.05	2.83	0.21	60.1
W3	11.44	7.84	3.25	0.19	74.1

Note: Ratio of income quantiles over median (50q); and Gini coefficient average over all fully covered years.

On all platforms relative earnings from the bottom 25% are low. Overall inequality is therefore large. The Gini coefficient is between 63.1 and 68.6. It is again higher for W1 and W3, that is however again driven by the top incomes. Figure 3.3 in the appendix shows the Lorenz curve for each platform for each year. The id line is the benchmark of full equality. The other black lines illustrate inequality of platforms with flexible prices W1-W3 and the gray lines of those with fixed prices. Oftentimes, two black lines – corresponding to W1 and W3 – lie fully below all gray lines, which is known as Lorenz domination. When one distribution Lorenz dominates another one, then the first is more unequal with respect to most inequality measures. Hence, the platforms with flexible prices, apart from W2, lead to greater inequality.[17]

A second look at inequality is possible when analyzing the trade networks that emerged on each platform.

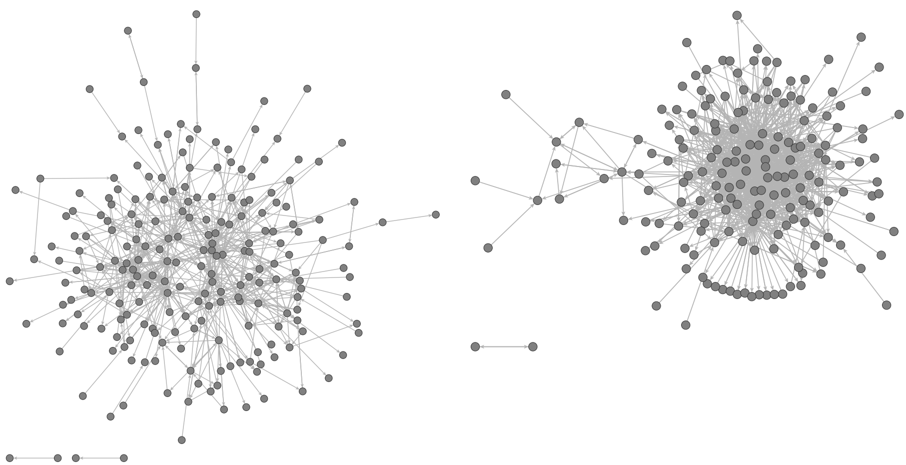
[17]To check whether the differences in inequality are really due to the top earners, we redrew the Lorenz curves for truncated distributions where on every platform the top 10% earners are excluded. Indeed, Lorenz domination is lost by this manipulation.

3.4.4 Trade Networks

We analyze the trade networks that are implied by the transactions on each platform. Each member is a node in the network. An arc from trader i to some trader j indicates that i bought a good from trader j. The network hence illustrates the flow of money. Two such trade networks are visualized by Figure 3.2.

Figure 3.2: Trade Networks

(a) F1 **(b) W1**



Note: Trade network of platform F1 (panel (a)) and of platform W1 (panel (b)). Both networks are of similar age and of similar size (in terms of number of nodes), but the trade network of platform W1, the one with rather flexible prices, is much denser than the trade network of platform F1.

Table 3.3 reports several network statistics for each platform.[18] The platforms are ordered as before. The number of arcs per node is the average number of business partners a member of the platform has. The density is the fraction of present arcs over all possible arcs. The table suggests that more flexible prices are associated with a higher density and more arcs per node. On average the platforms with fixed prices only have 5.1 arcs per node (i.e. business partner per

[18]The network statistics are computed by the package *nwcommands* used in the software *STATA 14*.

member), while those with flexible prices have more than 12.9. This confirms the pattern of more trade for flexible prices and is in line with our theoretical prediction.

Concerning inequality, centralization measures inequality with respect to the number of customers (*indegree*) and to the number of suppliers (*outdegree*). Table 3.3 shows that centralization is substantially higher for flexible prices than for fixed prices, confirming our result on inequality of income. This is reassuring because differences in the inequality of the yearly income, as reported in Table 3.2, were more moderate.

Table 3.3: Network statistics

id	nodes	arcs	arcs/node	density	indegree centralization	outdegree centralization
F1	215	695	3.2	0.015	0.11	0.09
F2	330	1,593	4.8	0.015	0.13	0.12
F3	324	1,756	5.4	0.017	0.10	0.17
F4	708	4,828	6.8	0.010	0.14	0.11
W1	179	1,197	6.7	0.038	0.27	0.33
W2	118	1,150	9.7	0.083	0.48	0.36
W3	1,037	23,071	22.2	0.021	0.41	0.32

Note: Nodes are the members of a platform. Arcs are the trade relationships. Density is the number of present arcs over all potential arcs. Centralization measures inequality with respect to the number of customers (indegree) and of the number of suppliers (outdegree).

3.5 Discussion

Given the theoretical and empirical findings above, what are the advantages of platforms for peer-to-peer exchange in comparison to other market forms?

The fact that Walrasian equilibria are Pareto efficient, while fixed price equilibria are not, does not mean that the Walrasian equilibrium Pareto dominates the fixed price equilibrium, as noted before. Pareto efficiency of the Walrasian equilibrium however implies that there is always at least one agent who prefers flexible prices. If that agent would leave the platform with fixed prices, then in the new fixed price equilibrium at least one other agent would prefer flexible prices; and so on. At the end of this hypothetical procedure, only one agent would remain.

It is therefore natural to ask, why such platforms with fixed prices can survive among rational agents. One possibility is that agents would like to commit themselves to buy inside the network. Then the platform-specific currency serves as a local currency in the sense of Mailath, Postlewaite and Samuelson [2016], for which some price stability is considered as necessary. Another reason is transparency. Certain prices are simple and seem focal. If there are high transaction costs for finding mutual agreements on how much to pay for certain services, it can be cheaper to rely on focal prices, which are suggested by a platform operator.

Another possibility is that some participants of these platforms have social preferences. Some prices could be perceived as *fair* such that (a) *procedural fairness* is a motive to engage in these transactions;[19] or it could be that the resulting allocation is considered more fair, than the Walrasian equilibrium allocation such that (b) *distributional fairness* is the motive. If the former motive, (a) procedural fairness, is predominant, the question arises whether there are Pareto superior allocations given the restriction that services are only exchanged according to the fixed prices. Our paper provides an answer to this question by showing that the FPE allocations are constrained inefficient and that Pareto improvements often only necessitate simple trades. Participants motivated by procedural fairness could hence either insist on the decentralized trade and accept the corresponding efficiency loss; or agree to a different allocation mechanism, e.g. a matching algorithm, that keeps the same prices, but leads to Pareto superior outcomes. Concerning (b) distributional fairness, our paper shows that fixed prices tend to induce lower inequality of income. If this motive is predominant, the question arises whether there are alternative (market) mechanisms that lead to Pareto superior outcomes, given the agents' social preferences. For instance, more trade without much higher inequality could be induced by a competitive market combined with some redistribution of income [e.g. Alesina and Angeletos, 2005;

[19] In fact, the origin of time-dependent currencies is the postulate that every hour of work should have the same value [Warren, 1852].

Bénabou and Tirole, 2006].

3.6 Conclusion

We have analyzed platforms for peer-to-peer exchange. These are closed exchange economies, on which price setting is often restricted and markets therefore do not clear. Assuming quasi-linear preferences allowed us to characterize the set of fixed price equilibria. Allocations are typically constrained inefficient, i.e. there are Pareto improvements even within the given price system. Moreover, we can show that trade volume under fixed prices is always lower than under competitive prices. Finally, under more restrictive assumptions, inequality must be lower as well. These findings are corroborated by an empirical illustration of several real platforms with time-based currencies.

Our methodological approach is innovative in that it combines traditional economic theory with a current online phenomenon and also makes use of techniques from network analysis. The main results show that fixed prices come at a high cost (since they lead to a constrained inefficient outcome and to less trade than competitive prices). This finding relates back to known inefficiency results [Herings and Konovalov, 2009; Maskin and Tirole, 1984; Younés, 1975], which seem to become vital and tangible in our setting. By investigating and illustrating how platforms for peer-to-peer exchange are affected by fixed prices, we hope to provide lessons that are not restricted to these markets, but can be addressed in many markets with price restrictions.

3.7 Appendix A: Proofs

3.7.1 Lemma 3.2

Lemma 3.2 (Weak Order). *Property weak order (WO) as defined in Definition 3.1 is equivalent to the following property of Maskin and Tirole [1984]:*

(O") exchange is weakly orderly: for all markets h, there exists no alternative (fully) feasible vector $\tilde{t} \in \prod_i \tau_h^i(t^i)$ such that, for each i, $\tilde{t}^i \succsim^i t^i$ with at least one strict preference.

Proof. Clearly, (O") implies (WO) because if (WO) is violated, then there exists a pair i, j and a trade $(\tilde{t}^i, \tilde{t}^j)$ which is a Pareto improvement. On the other hand suppose (WO) is satisfied. Then there is no such pair as shown below.

Suppose there is a Pareto improvement $\tilde{t}$ concerning market h. Then at least one agent i must be better off: $\tilde{t}^i \succ^i t^i$. Hence, $\tilde{t}_h^i \neq t_h^i$. Assume first that $\tilde{t}_h^i > t_h^i$ (i.e. i would like to buy more of h or sell less of it). By $\sum_i \tilde{t}_h^i = 0$ there must be some $j \neq i$ with $\tilde{t}_h^j < t_h^j$, i.e. who sells more or buys less of h. Since $\tilde{t}$ is a Pareto improvement $\tilde{t}^j \succsim^j t^j$. Thus, either $\tilde{t}^j \succ^j t^j$ and we are done or $\tilde{t}^j \sim^j t^j$. In the latter case, consider $\hat{t} := \frac{t+\tilde{t}}{2}$. Strict convexity implies that $\hat{t}^j \succ^j t^j$. Moreover, $\hat{t}^i \succ^i t^i$. Now, analogously for $\tilde{t}_h^i < t_h^i$ there is a j with $\tilde{t}_h^j > t_h^j$ and $\tilde{t}^j \succsim^j t^j$. Again, we have either $\tilde{t}^j \succ^j t^j$ or $\hat{t} := \frac{t+\tilde{t}}{2}$ has the required properties. □

3.7.2 Proof of Lemma 3.1

In the Walrasian equilibrium x^* for all agents i consuming a positive amount of good h we have $mu_h^i(x_h^{i,*}) = p_h^*$. Now, suppose $p_h^* \geq p_h$. Then $mu_h^i(x_h^{i,*}) \geq p_h$ for every i consuming a positive amount of good h at the price p^*. Since $mu_h^i(\hat{x}_h^i) = p_h$, $x_h^{i,*} \leq \hat{x}_h^i$ by concavity of u_h^i. Moreover, all agents consuming a positive amount of h at price p^* will do so at price $p_h \leq p_h^*$. Thus, $\sum_{i \in N} \hat{x}_h^i \geq \sum_{i \in N} x_h^{i,*} = \omega_h^h$, where the last equality holds because in the Walrasian equilibrium markets clear. Now, suppose $p_h^* < p_h$, then, for the analogous reasons as above, $\sum_{i \in N} \hat{x}_h^i < \sum_{i \in N} x_h^{i,*} = \omega_h^h$. □

3.7.3 Proof of Proposition 3.1

We prove both statements separately.

(a) Non-scarce good h: Consider an allocation $\tilde{x}$ that does not satisfy this property. Hence, there is a buyer i and a good $h \neq i$ such that $\tilde{x}^i_h \neq \hat{x}^i_h$.

Suppose first $\tilde{x}^i_h > \hat{x}^i_h$, i.e. i receives more than desired. Then $\underline{Z}^i_h \leq 0 \leq \hat{x}^i_h < \tilde{x}^i_h \leq \bar{Z}^i_h$ (for the canonical constraints, the first and the last inequalities are equalities). Hence, within the constraints and within i's budget set, i could also reduce the amount that he buys from good h to $\tilde{x}^i_h - \epsilon$, and save ϵ of good 0 instead.By concavity $mu^i_h(\tilde{x}^i_h) < mu^i_h(\hat{x}^i_h) \leq p_h$, while the numeraire good has marginal utility of 1.[20] Thus, $MRS^i_{h,0}(\tilde{x}^i) = \frac{mu^i_h(\tilde{x}^i_h)}{mu^i_0(\tilde{x}^i_0)} < \frac{p_h}{1}$ and hence $\tilde{x}$ violates voluntariness (V).

Suppose second $\tilde{x}^i_h < \hat{x}^i_h$, i.e. i receives less than desired. Then he is constrained in market h, $\hat{x}^i_h > \tilde{x}^i_h = \bar{Z}^i_h$ (the last equality follows from feasibility and voluntariness). By concavity $mu^i_h(\tilde{x}^i_h) > mu^i_h(\hat{x}^i_h) \geq p_h$ ($\hat{x}^i_h = 0$ is not possible since $\tilde{x}^i_h < \hat{x}^i_h$), while the numeraire good has marginal utility of 1. Since $\sum_{i \in N} \tilde{x}^i_h = \omega^h_h > \sum_{i \in N} \hat{x}^i_h$ (the inequality is due to the fact that h is a non-scarce good), there must be an agent j with $\tilde{x}^j_h > \hat{x}^j_h$. If $j \neq h$, then $\tilde{x}$ violates voluntariness with respect to agent j as shown above (for agent i). Hence, consider the case that $j = h$. $\tilde{x}^h_h > \hat{x}^h_h$ means that the seller sells less than desired because $mu^h_h(\tilde{x}^h_h) < mu^i_h(\hat{x}^h_h) \leq p_h$ by concavity. Thus, $\hat{x}^h_h - \omega^h_h < \tilde{x}^h_h - \omega^h_h = \underline{Z}^h_h$ (the last equality follows from feasibility and voluntariness), i.e. the seller is constrained from selling more. This is a violation of weak order (WO). Indeed for t such that $t^i_h = \tilde{x}^i_h + \epsilon$ and $t^h_h = \omega^h_h - \tilde{x}^h_h - \epsilon$ and $t^i_0 = \tilde{x}^i_0 - \epsilon$ and $t^h_h = \tilde{x}^h_h + \epsilon$ and otherwise t fully corresponding to $\tilde{x}$, we have $t^i \succ^i t^i$ and $t^h \succ^h t^h$ and $(t^i_h - \bar{Z}^i_h)(t^h_h - \underline{Z}^h_h) = \epsilon \cdot (-\epsilon) < 0$.

(b) Scarce good h: Consider an allocation $\tilde{x}$ that does not satisfy this property. Suppose first that for some $i \neq h$, $\tilde{x}^i_h > \hat{x}^i_h$. This is a violation of voluntari-

[20]Boundary solutions are covered by "$\leq$": $\hat{x}^i_h = 0$ is possible, but $\hat{x}^i_h = \omega^i_h$ not since $\hat{x}^i_h < \tilde{x}^i_h \leq \omega^i_h$.

ness (V) as shown in the proof above.[21] From now on assume that $\forall i \neq h$, $\tilde{x}^i_h \leq \hat{x}^i_h$ and $\tilde{x}^h_h \neq \hat{x}^h_h$.

Suppose first $\tilde{x}^h_h < \hat{x}^h_h$, i.e. h sells more than desired. Then $\underline{Z}^h_h \leq \tilde{x}^h_h - \omega^h_h < \hat{x}^h_h - \omega^h_h \leq 0 \leq \bar{Z}^h_h$. Hence, within the constraints and within h's budget set, h could also reduce the amount that she sells from her good h and consume more herself, $\tilde{x}^h_h + \epsilon$, in exchange for a smaller amount of good h. By concavity $mu^h_h(\tilde{x}^h_h) > mu^i_h(\hat{x}^i_h) \geq p_h$, while the numeraire good has marginal utility of 1. Thus, $\tilde{x}$ violates voluntariness (V).

Suppose second $\tilde{x}^h_h > \hat{x}^h_h$, i.e. h sells less than desired. Then she is constrained in market h, i.e. $\hat{x}^h_h - \omega^h_h < \tilde{x}^h_h - \omega^h_h = \underline{Z}^h_h$ (the last equality follows from feasibility and voluntariness). By concavity $mu^h_h(\tilde{x}^h_h) < mu^h_h(\hat{x}^h_h) \leq p_h$, while the numeraire good has marginal utility of 1. Since $\sum_{i \in N} \tilde{x}^i_h = \omega^h_h \leq \sum_{i \in N} \hat{x}^i_h$ (the inequality is due to the fact that h is a scarce good), there must be an agent i with $\tilde{x}^i_h < \hat{x}^i_h$, i.e. who buys less than desired. By concavity $mu^i_h(\tilde{x}^i_h) > mu^i_h(\hat{x}^i_h) \geq p_h$. Thus, (by feasibility and voluntariness) $\bar{Z}^i_h = \tilde{x}^i_h < \hat{x}^i_h$, i.e. buyer i is constrained from buying more. This is a violation of weak order (WO). Indeed for t such that $t^i_h = \tilde{x}^i_h + \epsilon$ and $t^h_h = \omega^h_h - \tilde{x}^h_h - \epsilon$ and $t^i_0 = \tilde{x}^i_0 - \epsilon$ and $t^h_h = \tilde{x}^h_h + \epsilon$ and otherwise t fully corresponding to $\tilde{x}$, we have $t^i \succ^i t^i$ and $t^h \succ^h t^h$ and $(t^i_h - \bar{Z}^i_h)(t^h_h - \underline{Z}^h_h) = \epsilon \cdot (-\epsilon) < 0$.

□

3.7.4 Proof of Proposition 3.2

Proof. There are two assertions to prove.

1. Pareto efficiency: Suppose good h is non-scarce and $\hat{x}^i_h > 0$ where $i \neq h$. Proposition 3.1 directly implies that in any FPE x: $MRS^h_{h,0}(x^i) < p_h$ and $MRS^i_{h,0}(x^i) = p_h$. Since preferences are continuous, a Pareto improving

[21] Indeed, then $\underline{Z}^i_h \leq 0 \leq \hat{x}^i_h < \tilde{x}^i_h \leq \bar{Z}^i_h$. Hence, within the constraints and within i's budget set, i could also reduce the amount that he buys from good h, $\tilde{x}^i_h - \epsilon$ and save ϵ of good 0 instead. By concavity $mu^i_h(\tilde{x}^i_h) < mu^i_h(\hat{x}^i_h) \leq p_h$, while the numeraire good has marginal utility of 1.

trade, in which h sells some amount to i at a price slightly below p_h, must exist.

2. Constrained efficiency: By weak interiority, the number of non-scarce markets is not equal to one.

   (a) Suppose the number of non-scarce markets is larger than one. Take any supplier i of a non-scarce good i. By Proposition 3.1, in equilibrium $x_i^i > \hat{x}_i^i$ and hence $mu_i^i(x_i^i) < p_i$. By assumption of weak interiority, there exists another non-scarce good h such that $\hat{x}_h^i > 0$, which implies that in equilibrium $mu_h^i(x_h^i) \geq p_h$. Taken together $hi \in R^i$, where the binary relation R^i is defined for a fixed allocation x and fixed prices p_j and p_k as follows: $jk \in R^i \Leftrightarrow x_k^i > 0$ and $MRS_{j,k}^i(x^i) > \frac{p_j}{p_k}$.[22] Denote $i = h_1$ and $h = h_2$. Since h_2 is non-scarce either, a good h_3 exists, such that $h_3 h_2 \in R^{h_2}$. If $h_3 = h_1$, a Pareto improving chain exists. If $h_3 \neq h_1$, a good h_4 must exist such that $h_4 h_3 \in R^{h_3}$. If $h_4 = h_1$ or $h_4 = h_2$, a Pareto improving chain exists. If not, there must be a good h_5, and so on. Eventually at good h_{k+1} it must be that $h_{k+1} = h_1$ or $h_{k+1} = h_2$ or... or $h_{k+1} = h_{k-1}$; and we have found a Pareto improving chain.

   (b) Suppose the number of non-scarce markets is zero. Take any market $h \neq 0$. The assumption $p_h^* \neq p_h$ implies $p_h^* > p_h$ for scarce goods (by Lemma 3.1). Since markets clear in Walrasian equilibrium and Walrasian prices are larger than fixed prices, there is at least one agent who is constrained from buying on this market. Hence, for each good $h \neq 0$, there is some agent i with $mu_h^i(x_h^i) > p_h$, while $mu_i^i(x_i^i) = p_i$ (by Proposition 3.1).

   Now, consider any good h_1. By the argument above, there exists a good h_2 such that $mu_{h_1}^{h_2}(x_{h_1}^{h_2}) > p_{h_1}$, while $mu_{h_2}^{h_2}(x_{h_2}^{h_2}) = p_{h_2}$. Thus, $h_1 h_2 \in R^{h_2}$. Likewise, for good h_2, there is a an agent h_3 and the corresponding good h_3 such that $mu_{h_2}^{h_3}(x_{h_2}^{h_3}) > p_{h_2}$, while $mu_{h_3}^{h_3}(x_{h_3}^{h_3}) =$

[22] The binary relation R^i indicates which trades agent i would accept. $jk \in R^i$ has the interpretation that agent i is willing to give up a small amount of good k to receive $\frac{p_k}{p_j}$ times that amount of good j.

p_{h_3}. Thus, $h_2h_3 \in R^{h_3}$. If $h_1 = h_3$, a Pareto improving chain exists. If $h_1 \neq h_3$, a good h_4 exists $mu_{h_3}^{h_4}(x_{h_3}^{h_4}) > p_{h_3}$, while $mu_{h_4}^{h_4}(x_{h_4}^{h_4}) = p_{h_4}$. Thus, $h_3h_4 \in R^{h_4}$. If $h_4 = h_1$ or $h_4 = h_2$, a Pareto improving chain exists. If not, there must be a good h_5, and so on. Since there are n such goods, eventually at good h_{n+1} it must be that $h_{n+1} = h_1$ or $h_{n+1} = h_2$ or... or $h_{n+1} = h_{n-1}$; and we have found a Pareto improving chain.

□

3.7.5 Proof of Proposition 3.3

There are two assertions to prove.

1. Suppose good h is scarce. By Lemma 3.1, $p_h^* \geq p_h$. Hence, the demand of agent h for her own good is lower under Walrasian prices than under fixed prices. She gets her optimal amount of good h under Walrasian prices, but also under fixed prices since the good is scarce (by Proposition 3.1). Hence, $x_h^{h,*} \leq \hat{x}_h^h = x_h^h$. Thus, $\omega_h^h - \sum_{i \neq h} t_h^{i,*} = x_h^{h,*} \leq x_h^h = \omega_h^h - \sum_{i \neq h} t_h^i$, which yields the result.

2. Suppose h is non-scarce. By Lemma 3.1, $p_h^* \leq p_h$. Hence, the demand of all agents $i \neq h$ is larger under Walrasian prices than under fixed prices. Any agent $i \neq h$ gets her optimal amount of good h under Walrasian prices, but also under fixed prices since the good is non-scarce (by Proposition 3.1). Hence, $x_h^{i,*} \geq \hat{x}_h^i = x_h^i$. Thus, $t_h^i = x_h^i \leq x_h^{i,*} = t_h^{i,*}$, $\forall i \neq h$.

□

3.7.6 Proof of Corollary 3.1

We first show that, under Assumption 3.1 (i), suppliers of scarce goods are earning more than suppliers of non-scarce goods in any FPE. We then show that income increases for suppliers of scarce goods and, under Assumption 3.1 (ii), decreases for suppliers of non-scarce goods. For easier reference, we partition the set of agents into suppliers of scarce goods (SC) and suppliers of non-scarce goods (NSC). Let $i \in SC$ and $j \in NSC$ be two generic suppliers of scarce goods and non-scarce goods, respectively. By Proposition 3.1 the income of each supplier of a non-scarce good in a FPE is $y^j = \sum_{k \neq j} \hat{x}_j^k \cdot p_j$. By Prop. 3.1 the income of each supplier of a scarce good in a FPE is $y^i = (\omega_i^i - \hat{x}_i^i) \cdot p_i$. Assumption 3.1 (i), i.e. $\frac{Q_j}{S_i} < \frac{p_i}{p_j}$, can be written as $(\sum_{k \neq j} \hat{x}_j^k) p_j < (\omega_i^i - \hat{x}_i) p_i$, which then directly implies $y^i > y^j$, i.e. suppliers of scarce goods receive a higher income than suppliers of a non-scarce good.

We now show that $y^{i,*} > y^i$ for $i \in SC$. $y^{i,*} = |t_i^{i,*}| \cdot p_i^* > |t_i^i| \cdot p_i = y^i$ since by Proposition 3.3 $|t_i^{i,*}| > |t_i^i|$ and by Lemma 3.1 $p_i^* > p_i$.

Finally, we use Assumption 3.1 (ii) to show that $y^{j,*} < y^j$ for $j \in NSC$. We first rewrite $\epsilon_j^D = \frac{Q_j^* - Q_j}{p_j^* - p_j} \cdot \frac{p_j}{Q_j}$ to have $Q_j^* = Q_j(1 + \frac{p_j^* - p_j}{p_j} \epsilon_j^D)$, which we plug into the following expression.

$$
\begin{aligned}
y^{j,*} - y^j &< 0 && (3.1)\\
Q_j^* p_j^* - Q_j p_j &< 0 && (3.2)\\
Q_j(1 + \frac{p_j^* - p_j}{p_j} \epsilon_j^D) p_j^* - Q_j p_j &< 0 && (3.3)\\
Q_j \left[p_j^* + \frac{p_j^* - p_j}{p_j} \epsilon_j^D p_j^* - p_j \right] &< 0 && (3.4)\\
Q_j \left[(p_j^* - p_j)(1 + \frac{p_j^*}{p_j} \epsilon_j^D) \right] &< 0 && (3.5)
\end{aligned}
$$

$Q_j > 0$ by assumption. By Lemma 3.1 we have $p_j^* - p_j < 0$. The elasticity ϵ_j^D is negative, but bounded from below by Assumption 1 (ii): $\epsilon_j^D \geq -1$. Since $\frac{p_j^*}{p_j} < 1$ (by Lemma 3.1), we have $\frac{p_j^*}{p_j} \epsilon_j^D > -1$ and $1 + \frac{p_j^*}{p_j} \epsilon_j^D > 0$. Therefore, the inequality holds. □

3.8 Appendix B: Extensions

3.8.1 More General Endowment

We briefly discuss how our results change when we relax the assumption on the endowments, i.e. that every agent is endowed with only one good and that the number of goods m must equal the number of agents n. Hence, there can now be many sellers of a good and an agent can sell many goods. We call every agent who is endowed with more than he desires, i.e. $\omega_h^j > \hat{x}_h^j$, net supplier of this good and all others net demanders. Then the characterization of all FPE becomes:

Proposition 3.4 (General Characterization)**.** *In every FPE, each good $h \neq 0$ is allocated as follows:*

1. *If h is non-scarce, every net demander receives the desired amount, while every net supplier receives at least the desired amount. That is: $\forall i$ with $\omega_h^i \leq \hat{x}_h^i$, we have $x_h^i = \hat{x}_h^i$; and $\forall j$ with $\omega_h^j > \hat{x}_h^j$, we have $x_h^j \geq \hat{x}_h^j$.*

2. *If h is scarce, every net demander receives at most his desired amount, while the net suppliers keep (exactly) the desired amount. That is: $\forall i$ with $\omega_h^i \leq \hat{x}_h^i$, we have $x_h^i \leq \hat{x}_h^i$; and $\forall j$ with $\omega_h^i > \hat{x}_h^i$, we have $x_h^j = \hat{x}_h^j$.*

Proof. The proof is fully analogous to the proof of Proposition 3.1. □

As Proposition 3.4 shows, the characterization of Proposition 3.1 generalizes to the set-up with more general endowments. Only the statement about net suppliers of non-scarce goods becomes weaker. Before, the excess supply was kept by the unique seller. Now, the notion of FPE does not determine how the excess supply is allocated among the net suppliers. The other parts are identical to Proposition 3.1.

For the results on inefficiency (Proposition 3.2) and less trade (Proposition 3.3) this leads to some adaptions but does not change the substance.

3.8.2 More General Preferences

In this section, we extend the model by relaxing the assumption that the utility function is quasi-linear. The more general utility function has the following form:

$$U^i(x^i) = u_0^i(x_0^i) + u_1^i(x_1^i) + ... + u_h^i(x_h^i) + ... + u_n^i(x_n^i),$$

with marginal utility $mu_h^i(x_h^i) > 0$ and $\frac{\partial mu_h^i(x_h^i)}{\partial x_h^i} \leq 0$ for all i, h and x_h^i; the inequality $\frac{\partial mu_h^i(x_h^i)}{\partial x_h^i} \leq 0$ is strict for all $h \neq 0$. A simple characterization as in Proposition 3.1 is then no longer possible because demand and supply on each market may now depend on the allocation on all other markets. It is even possible that a scarce good "becomes non-scarce" in the sense that there is excess supply in the fixed price equilibrium; and vice versa. Since Proposition 3.1 was key to show inefficiency (Proposition 3.2), the question arises whether this result can be reestablished. The short answer is: yes, partially.

We can show first that for each scarce good i there must exist an agent j who would be willing to trade good i in exchange for his own good j (but not necessarily for good 0).

Lemma 3.3. *If good h is strictly scarce, i.e. $\sum_{i \in N} \hat{x}_h^i > \omega_h^h$, then in any FPE x there is an agent j who would like to trade h in exchange for his own good, i.e. $MRS_{h,j}^j(x^j) > \frac{p_h}{p_j}$.*

Proof. We first show that the seller of the scarce good h, receives at least the desired amount, i.e. $x_h^h \geq \hat{x}_h^h$. Assume not such that $x_h^h < \hat{x}_h^h$. By voluntariness (V), we then have $x_0^h < \hat{x}_0^h$. Again by voluntariness (V), this implies $x_k^h < \hat{x}_k^h$ for any good k. Thus, $x_h^h < \hat{x}_h^h$ implies $x_k^h < \hat{x}_k^h$ for any good k (including the numeraire). But then $px < p\omega$. Hence, x cannot be an equilibrium allocation. Second, if h is strictly scarce, there must be an agent j such that $mu_h^j(\hat{x}_h^j) < mu_h^j(x_h^j)$. Together, we therefore have $p_h mu_j^j(x_j^j) \leq p_h mu_j^j(\hat{x}_j^j) = p_j mu_h^j(\hat{x}_h^j) < p_j mu_h^j(x_h^j)$. $\square$

Lemma 3.3 can be interpreted as follows: every (initially) scarce good remains "somewhat scarce." The main reason is that quantity constraints on the demand side can never increase supply. Hence, if there are two agents i and j, who both have a larger demand for the other's good than the other's (unconstrained) supply is, then they could improve in each FPE by mutual trade at the given price scheme. This leads to one kind of inefficiency that we establish in the following extension of Proposition 3.2.

Proposition 3.5. *If there is a set of agents S such that their demand for their own goods exceeds the endowment, i.e. $\forall i, h \in S$, $\sum_{i \in S} \hat{x}^i_h > \omega^h_h$, then no FPE is constrained efficient.*

Proof. From Lemma 3.3 we know that $\forall h \in S$, $x^h_h \geq \hat{x}^h_h$. Thus, for some $i \in S$, $x^i_h < \hat{x}^i_h$. This directly implies $hi \in R^i$ because $mu^i_h(\hat{x}^i_h) = \frac{p_h}{p_i} mu^i_i(\hat{x}^i_i)$ and $x^i_i \geq \hat{x}^i_i$ (again from Lemma 3.3). At the same time there must exist an agent $j \neq i \in S$ such that $x^j_i < \hat{x}^j_i$. For the same reason as above $ij \in R^j$. We can continue as in the proof for Proposition 3.2 until we have found a Pareto improving chain. □

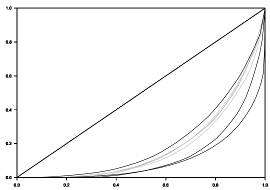
Hence, fixed prices often lead to constrained inefficient allocations even with more general preferences. We have shown this for one type of inefficiency (scarce goods, prices are "too low"), while for another (non-scarce goods, prices are "too high") the analogous result cannot be established. The reasons is that quantity constraints on the demand side can easily increase demand for other goods. Hence, our inefficiency result, Proposition 3.2 partially extends to more general preferences.

Importantly, the effects isolated in the special case of quasi-linear preferences are still at work, they are in general simply accompanied by other potential effects.

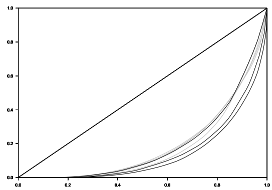
3.9 Appendix C: Additional Figures

Figure 3.3: Lorenz curves

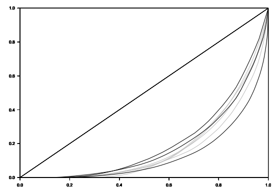
(a) 2016



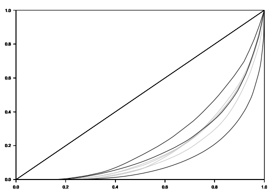
(b) 2015



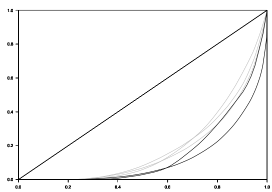
(c) 2014



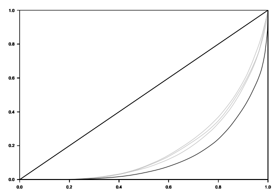
(d) 2013



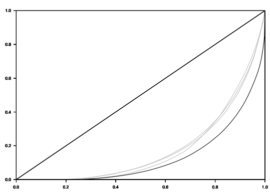
(e) 2012



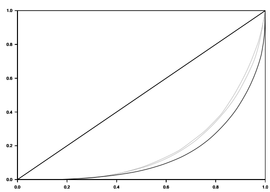
(f) 2011



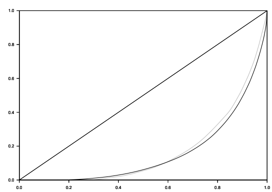
(g) 2010



(h) 2009



(i) 2008



Note: Lorenz curves of income distribution for each platform by year. The id line is the benchmark of full equality. The other black lines illustrate inequality of platforms with flexible prices W1-W3 and the gray lines of those with fixed prices F1-F4. Lorenz domination is visible when one line fully lies below another line.

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