

# Vertical Contracting, Product Variety, and Innovation

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The Faculty of Business, Economics and Informatics of the University of Zurich hereby authorizes the printing of this dissertation, without indicating an opinion of the views expressed in the work.

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## Part I

# Dissertation overview



# Dissertation overview

This thesis comprises three chapters. They are concerned with topics of industrial organization, specifically vertical contracting, provision of product variety, and innovative entry.

Chapter 1, written jointly with Armin Schmutzler, considers the effects that the existence of captive consumers of an incumbent has on the entry incentives of a differentiated competitor. An entrant who considers to enter a new market faces an incumbent who has a captive market share. That is, there are consumers who will only ever consider the incumbent's product. This is a potential disadvantage for the entrant, as there is a part of the market that she can never sell to. We show, however, that the existence of captive consumers has more complex effects. Most importantly, an increase in the captive market share can be beneficial for the entrant. In addition, the existence of captive consumers causes the relationship between the intensity of competition (product substitutability) and entry incentives to be non-monotone.

In Chapter 2, I compare two forms of intermediation in a market where several suppliers need to go through a monopolistic intermediary to reach consumers. The modes compared are (i) wholesale and (ii) agency, where the intermediary (i) buys and resells by setting prices to final consumers, or (ii) lets suppliers set prices and takes a fee for a sale. I ask whether the different intermediation modes can result in different market outcomes, in particular whether the monopolistic intermediary has an incentive to limit the product variety available to consumers. In a Salop model with differentiated products, I show that this can in fact be a central outcome that differs between the two modes. The intermediary as a monopolist steers retail pricing in the same way in the wholesale and in the agency mode. But in the wholesale mode, there is an intermediate range for the number of generally available products for which the intermediary wishes to limit the number of products he makes available to consumers. While the literature has so far analyzed differences in prices and profits between the two modes, the possible limitation of product range is another meaningfully differing outcome.

In Chapter 3, I study repeated contracting in vertical chains. I consider an environment where lump-sum transfers from buyers to sellers are not possible. This prohibits franchising contracts, which are commonly used in the literature, but more scarcely in practice. Contracts are restricted to a uniform linear wholesale price and lump-sum transfers only from the manufacturer to the retailers. This prohibits franchise fees and explicit third-degree price discrimination. The manufacturer enters relational contracts to improve vertical coordination and efficiency. With one buyer, the manufacturer pays no transfer but implements self-enforcing quantity fixing. The retailer shares in the efficiency gains and is discouraged from deviating by the threat of reversion to a worse equilibrium. With two retailers in two different markets, the manufacturer will not pay a transfer to at least one. But because now the wholesale price plays a role in coordinating

revenue from different buyers, transfers to buyers may arise as a means to improve vertical coordination. In a simplified setting where the second retailer is short-lived, the manufacturer and the long-lived retailer act like an integrated entity that optimizes quantity internally and sets a wholesale price externally. Downstream transfers such as slotting allowances are thus not a manifestation of buyer bargaining power. In my setting, where they are offered by the seller, they have efficiency-enhancing effects.

## Part II

# Research papers





# 1 Innovative entry in markets with captive consumers<sup>1</sup>

Joint with Armin Schmutzler

## 1.1 Introduction

In many markets innovative firms have successfully challenged established monopolists by offering imperfect substitutes as an alternative. For instance, brick-and-mortar retailers see themselves replaced by online retailers. Taxis face competition from ride-hailing firms. The iPhone successfully entered the market for mobile phones and is subsequently facing competition from newly developed smartphones itself. In several European countries, long-distance bus companies now challenge railway monopolists.<sup>2</sup> Facing this innovative entry, incumbent firms may have advantages over their competitors that stem from their established market position. One such advantage may be that consumers have a bias towards the established product to the advantage that they do not consider an emerging alternative. These captive consumers can emerge for several reasons: technological lock-in, prohibitive switching costs, or cognitive biases such as insufficient awareness all can prevent consumers from considering new offers.<sup>3</sup> In this paper, we analyze under which circumstances entry of new products is likely to arise when the incumbent has a base of captive consumers. Which types of markets are prone to persistent dominance, and which will exhibit product variety? We also ask what the fate of incumbents will be if entry takes place. To which extent will they be able to co-exist in the contested market segment with successful entrants? Captive consumers look like a serious impediment to entry. Their existence reduces the demand potential for the entrant and hence its capability to recover entry costs. Our paper investigates this conjecture. We show that while captive consumers can harm entrants, this conclusion does not hold universally. In some situations, captive consumers can be beneficial to entrant profits and promote innovative entry.

We build a model of competition with differentiated products. One incumbent faces an entrant who produces a differentiated good. Following entry, the firms compete in prices, but a share of the consumers (the captive consumers) will only consider the incumbent's product. Entrant and incumbent compete for the demand of the remaining consumers (the switchers). Crucially,

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<sup>1</sup>Cite as Christian Oertel and Armin Schmutzler (2019): "Innovative entry in markets with captive consumers", Mimeo. A version of this paper has been submitted to the Journal of Economics & Management Strategy.

<sup>2</sup>See Dürr and Hüschelrath (2018) for Germany.

<sup>3</sup>For example, Berger and Dick (2007) document first-mover advantages for retail banks in establishing a captive consumer base, Ho, Hogan, and Scott Morton (2017) show consumer inertia when switching insurance plans, and Murray and Häubl (2007) give support to the theory of consumer lock-in through previous experience with a product.

the incumbent cannot price discriminate between captive and switching consumers. As a result, the pricing incentives from his captive monopoly market will have an impact on the competition with the entrant. We show that the entrant can benefit from an increasing share of captive consumers, because their presence softens the incumbent's pricing policy. This is demonstrated in a general model that shows that, because prices are strategic complements, the presence of a captive monopoly market can have a competition-softening effect on the contested market. To assess the relevant market conditions more carefully, we subsequently analyze a model of competition on the circle à la Salop (1979). This allows us to describe the interplay between competitive conditions, as measured by consumer taste heterogeneity, and the share of captive consumers. In order to focus on the pure effect of captive consumers on the potential success of entry, we abstract from any technological advantages that a product may have. Entrants are not vertically differentiated, have the same variable cost of production as the incumbent, and consumer valuations only reflect horizontal taste differences.

Our first central result is that captive consumers can foster innovative entry. This happens when consumer taste heterogeneity is at an intermediate level. The greater the share of the captive market, the more the incumbent will focus on exploiting his monopoly position. When consumer heterogeneity is not too high, the incumbent can set a relatively high price and still serve all of his captive market. This softens competition, so that the entrant's profits increase in the share of captive consumers. Thus our model sheds light on the conditions under which a captive market can make entry more likely. The joint analysis of the measure of the captive market share and the consumer heterogeneity parameter reveals new interactive effects. An increase in the captive market share reduces the entrant's demand potential, but can be beneficial because of competition softening. Likewise, this happens for intermediate values of consumer heterogeneity, whereas in standard models an increase in heterogeneity reduces competition intensity and thus it is always the higher level that helps entry.

When consumer heterogeneity is high, the incumbent is forced to price aggressively to not lose too much demand. A rising share of captive consumers then does not mean that entry becomes more profitable. When consumer preferences are not too heterogeneous, an increase in captive consumers may even foster *disruptive innovation*, whereby the entrant takes over the entire market for switchers and the incumbent focuses entirely on his captive market. If the share of the captive market is not too big, however, both firms will compete intensely. Finally, when consumer taste heterogeneity becomes low, we start to observe price dispersion for the incumbent's product. Driven by the tension in incentives to charge a high price to exploit the homogeneous captive market, but also to undercut the entrant whose product is now less differentiated, the incumbent follows a mixed strategy.

From a competition policy perspective, it is therefore not obvious how to deal with a dominant firm's attempt to increase its share of captive consumers. However, we need to take into account that the incumbent will only increase his share of captive consumers if this increases his profits. This will usually happen because it softens competition or deters entry. Either way, the effects on consumers will be negative. There is one specific case, however, where the incumbent's interests are aligned with those of consumers. When consumer heterogeneity is sufficiently large, the

incumbent typically wants more captive consumers, even though this reduces prices unless it prevents entry. Because of the favorable demand effect, it is still in the incumbent's interest. This is beneficial to consumers as, due to the decreasing prices, overall consumer surplus increases as the share of captive consumers increases.

We provide a framework for competition and regulation authorities to assess the impact of market conditions on potential entry. Through the interaction with product differentiation, a higher share of existing captive consumers may not always block entry. But we also show that competition authorities should generally be wary of incumbent firms' attempts to create captive consumers. Despite the potential beneficial effect on entry and prices, if the incumbent actively increases the share of captive consumers, it is likely to hurt competition overall.

Our second main point contributes to the large literature on competition and innovation.<sup>4</sup> We find that the existence of captive consumers has important effects on the relation between competition and product innovation. Without captive consumers, softer product competition (induced by greater consumer heterogeneity) unambiguously fosters innovative entry by horizontally differentiated competitors. With captive consumers, this is no longer true. For intermediate levels of consumer heterogeneity, an increase in heterogeneity negatively affect entry profits – so that the overall relation between competition and innovation incentives is no longer monotone.<sup>5</sup> These non-monotonicities reflect the interplay between the standard competition-softening effect of consumer heterogeneity in oligopoly and its effect on monopoly prices. This second effect says that the monopoly price decreases when consumer heterogeneity increases. While the first effect works in favor of entry, the second one works against it. An incumbent who puts sufficient weight on his monopolistic captive market will reduce prices when heterogeneity increases. As a result, prices are typically non-monotone in the competition parameter. These non-monotonicities translate into negative effects of consumer heterogeneity on entry profits in intermediate regions.

In Section 1.2, we introduce a general model, which allows for vertical as well as horizontal product differentiation. The model provides general conditions for incumbent and entrant profits to increase in the share of captive consumers. In order to analyze the effects of market conditions on entry in more detail, we proceed with a specific model of competition with horizontally differentiated products in Section 1.3, which is a variant of Salop's competition on the circle. We adapt this model, so that the transport cost parameter represents pure heterogeneity of consumer valuation for each product.<sup>6</sup> We characterize the equilibrium regions and find that, depending on parameter combinations, different post-entry equilibria arise. Given the entry profits in these equilibria, we are able to say when entry is profitable for a given level of fixed cost of entry. The resulting entry region shows the possible non-monotone effect of captive consumers on entry incentives. We find that post-entry equilibria are in pure strategies until products become sufficiently (but not perfectly) substitutable. At this point, price dispersion will emerge as a result of mixed-strategy equilibria for similar reasons as in the homogeneous

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<sup>4</sup>See Gilbert (2006); Vives (2008); Schmutzler (2010, 2013) for overviews.

<sup>5</sup>Contrary to Aghion et al. (2005), however, the relation is not inverse-U shaped, but U- or N-shaped.

<sup>6</sup>The transportation cost parameter thus captures a demand rotation in the sense of Johnson and Myatt (2006).

goods case analyzed by Varian (1980). In section 1.4 we discuss the relation to the literature. Section 1.5 concludes.

## 1.2 The general framework

We consider a monopolistic industry which has been exposed to a substantial regulatory or technological shock that makes entry potentially feasible. We ask under which circumstances entry actually arises, and what its effects on prices and welfare are.

### 1.2.1 Assumptions

We use a two-stage game with two firms: the incumbent  $i = 0$ , and a potential entrant  $i = 1$ . Variable production costs are assumed to be constant and equal to 0 for both firms. There is a continuum of consumers of different types. Each type corresponds to a different individual preference over products.<sup>7</sup> The resulting monopolistic market demand function is given as  $D_M(p_0, \theta)$ , where  $\theta$  is a demand or policy parameter, taken from a non-degenerate subinterval  $\Theta$  of the real numbers. We assume that the demand function is twice continuously differentiable in both variables almost everywhere and decreasing in the price wherever demand is positive. The entrant's product is differentiated from the one produced by the incumbent. When firms compete for consumers, the demand function is  $D_i(p_i, p_j; \theta)$ . We assume that this demand function is also twice continuously differentiable in the interior (wherever prices and demands are positive). Further, we make the following assumptions:

#### Assumption 1.1.

- (a)  $D_i$  is decreasing in  $p_i$  and increasing in  $p_j$ ,  $j \neq i$ ;
- (b)  $\frac{\partial D_i}{\partial p_j} + p_i \left( \frac{\partial^2 D_i}{\partial p_i \partial p_j} \right) \geq 0 \forall i, j \in \{0, 1\}, j \neq i$ .
- (c)  $D_0(p_0, p_1, \theta) \leq D_M(p_0, \theta)$  for all  $(p_0, p_1, \theta) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \Theta$ ; with strict inequality if  $D_1(p_1, p_0, \theta) > 0$ .

◆

These assumptions mean that the goods are imperfect substitutes, prices are strategic complements and the incumbent's duopoly demand is lower than his monopoly demand. Like the remainder of the set-up, Assumption 1 is compatible with standard models of horizontally and/or vertically differentiated price competition. Note that we allow both for the possibility that  $\theta$  affects both the monopolistic and competitive demand functions (like a shift in income and consumer characteristics) and the possibility that it only affects one of the two demand functions

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<sup>7</sup>In our leading example in Section 1.3, the types will correspond to different locations on the circle, and each consumer will have unit demand.

(like an improvement in the quality of the entrant's good, which has no effect on the monopoly demand).

We suppose that a fraction  $\mu \in [0, 1]$  of each consumer type will not buy from the entrant under any circumstances. These will be called *captive consumers*. As argued in the introduction, they do not consider the entrant product for many possible reasons. We will refer to the remaining consumers as *flexible consumers* or *switchers*. Notice that the distribution of consumer types is the same among captive and flexible consumers. Being captive is not correlated with any other consumer characteristics.

Crucially, we assume that the incumbent cannot price discriminate between captive consumers and switchers. The total demand functions when captive consumers are present are thus not symmetric; they are given by

$$\begin{aligned}\tilde{D}_0(p_0, p_1; \theta, \mu) &= \mu D_M(p_0; \theta) + (1 - \mu) D_0(p_0, p_1; \theta) \\ \tilde{D}_1(p_1, p_0; \theta, \mu) &= (1 - \mu) D_1(p_1, p_0; \theta)\end{aligned}$$

We denote the resulting profit functions for  $i = 0, 1; j \neq i$  as

$$\Pi_i(p_i, p_j; \theta, \mu) = p_i \tilde{D}_i(p_i, p_j; \theta, \mu).$$

The polar cases  $\mu = 1$  and  $\mu = 0$  correspond to the monopoly with demand function  $D_M$  and the duopoly with demand functions  $D_0$  and  $D_1$ , respectively. We refer to the latter case as the *pure duopoly*.

We focus our general analysis in this section on the case that the pricing game has a unique pure-strategy equilibrium.<sup>8</sup> We denote the resulting equilibrium prices as  $p_0^*(\theta, \mu)$  and  $p_1^*(\theta, \mu)$  and the equilibrium profits as  $\Pi_0^*(\theta, \mu)$  and  $\Pi_1^*(\theta, \mu)$ , where we drop  $(\theta, \mu)$  wherever appropriate. In addition to Assumption 1, we assume that the demand functions are such that the functions  $\Pi_i$  are concave and that the stability condition  $\frac{\partial^2 \Pi_0}{\partial p_0^2} \frac{\partial^2 \Pi_1}{\partial p_1^2} > \frac{\partial^2 \Pi_0}{\partial p_0 \partial p_1} \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_0}$  holds.

The firms play the following game. In the first stage, firm 1 decides whether to enter the market at a fixed cost  $F > 0$ . In the second stage, after observing the entry decision, the incumbent sets a price  $p_0$  and, if firm 1 has entered, the latter simultaneously sets a price  $p_1$ . Profits from sales to consumers are then realized.

### 1.2.2 Interpretation of the framework

The central parameter of our model is  $\mu$ , the share of captive consumers. This parameter has to be interpreted cautiously. As we multiply  $\mu$  with the entire demand function  $D_M$  to obtain the incumbent's captive demand, the probability of being captive is independent of the willingness to pay for the incumbent product. This framework fits well with the idea that there are fluctuations in the population of consumers who care about the market, so that, after entry, there are is a subpopulation of old consumers who have previously bought the incumbent's product and an

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<sup>8</sup>In the example in Section 1.3, there will also be parameter regions where only mixed-strategy equilibria exist.

otherwise identical subpopulation of new consumers who have not (see Beggs and Klemperer, 1992). With this interpretation, assuming that switching costs for old consumers are prohibitive and there are no set-up costs for new consumers,  $\mu$  is the share of old consumers.

Other interpretations are consistent with our approach as well. First, some consumers might not realize that the alternative exists because they are inattentive or time constrained. Second, some consumers may erroneously believe that the entrant's product has a quality disadvantage, so that they do not take it into consideration.<sup>9</sup>

### 1.2.3 Results

We now ask how an increase in the share of captive consumers affects prices and profits when the incumbent has entered. We start with price effects. All proofs are in the Appendix.

**Proposition 1.1.**  $p_0^*$  and  $p_1^*$  are both increasing in  $\mu$  if and only if

$$D_M(p_0^*) - D_0(p_0^*, p_1^*) > p_0^*(\mu, \theta) \left( \frac{\partial D_0}{\partial p_0} - \frac{\partial D^M}{\partial p_0} \right), \quad (1.1)$$

or, equivalently,

$$D_M(p_0^*) - D_0(p_0^*, p_1^*) > \varepsilon_M(p_0^*) D_M(p_0^*) - \varepsilon_0(p_0^*, p_1^*) D_0(p_0^*, p_1^*), \quad (1.2)$$

where  $\varepsilon_M$  and  $\varepsilon_0$  are the (absolute values of the) elasticities of  $D_M$  and  $D_0$ , respectively, with respect to the incumbent's price. If (1.1) is violated with strict inequality, both prices are decreasing in  $\mu$ .  $\blacklozenge$

Intuitively,  $\mu$  does not affect the entrant's reaction function directly, as it only scales her profit by a constant. However,  $\mu$  affects the incumbent's reaction function. Thus, by strategic complements, whenever an increase in  $\mu$  shifts the incumbent's reaction curve out, both prices increase, otherwise they both decrease. As the parameter increases, the incumbent pays relatively more attention to the monopoly market. He has incentives to set higher prices after an increase in  $\mu$  if and only if monopoly prices are higher than pure duopoly prices (those set in the absence of captive consumers). Condition (1.1) guarantees that this is true: As  $\mu$  increases, the incumbent has more demand from captive consumers and less demand from flexible consumers; the marginal effect is  $D_M - D_0$ . As he has to share the flexible consumers with the competitor,  $D_M - D_0$  will typically be positive (as captured in Assumption 1 (c)). However, the term  $\left( \frac{\partial D_0}{\partial p_0} - \frac{\partial D^M}{\partial p_0} \right)$  on the right-hand side of (1.1) is usually also positive in parameterized examples: With competition, the demand losses from higher prices will also be lower than in monopoly.<sup>10</sup> If, as required by (1.1), this *elasticity effect* is smaller than the former *demand effect*, then prices are increasing in

<sup>9</sup>This appears to be plausible, for instance, when patented pharmaceutical products face competition from generics.

<sup>10</sup>This is for instance true in the example in Section 1.3.

the share of captive consumers. Condition (1.2) illustrates the elasticity effect. The incumbent price rises when the direct gains from existing demands outweigh the effects of demand changes induced by the monopolist's price change.

This can, but need not be the case. Intuitively, the relative size of the effects determines whether monopoly prices are higher or lower than pure duopoly prices. A stronger focus on captive consumers increases prices only in the former case. As argued by Chen and Riordan (2008), however, the latter case is also possible with differentiated goods.

Next, we consider the effects of  $\mu$  on profits. The results rely crucially on the simple fact that the total effect of a marginal change of  $\mu$  on the profits of firm  $i$  ( $i = 0, 1$ ) is

$$\frac{d\Pi_i^*}{d\mu} = \frac{\partial\Pi_i}{\partial\mu} + \frac{\partial\Pi_i}{\partial p_j} \frac{dp_j^*}{d\mu} \text{ for } j \neq i. \quad (1.3)$$

Thus, the total profit effect  $\frac{d\Pi_i^*}{d\mu}$  of an increase in the share of captives consists of a *direct effect* (which ignores price effects), captured by the partial derivative  $\frac{\partial\Pi_i}{\partial\mu}$ , and an *indirect effect* induced by the adjustments in the rival's price.<sup>11</sup>

**Proposition 1.2.** Let  $p_0^*$  and  $p_1^*$  denote the equilibrium values at  $(\mu, \theta)$ .

(i)  $\Pi_1^*$  is increasing in  $\mu$  at  $(\mu, \theta)$  if and only if

$$\frac{dp_0^*}{d\mu} > \frac{D_1(p_1^*, p_0^*, \theta)}{(1 - \mu) \frac{\partial D_1}{\partial p_0} \big|_{(p, \theta) = (p_1^*, p_0^*, \theta)}} \quad (1.4)$$

.

(ii)  $\Pi_0^*$  is increasing in  $\mu$  if and only if

$$\frac{dp_1^*}{d\mu} > \frac{D_0(p_0^*, p_1^*, \theta) - D_M(p_0^*, \theta)}{(1 - \mu) \frac{\partial D_0}{\partial p_1} \big|_{(p, \theta) = (p_0^*, p_1^*, \theta)}}. \quad (1.5)$$

◆

To understand Part (i), note that  $D_1(p_1^*, p_0^*, \theta)$  captures the entrant's demand loss from a marginal increase in  $\mu$  – the negative direct effect for the entrant. As the denominator  $(1 - \mu) \frac{\partial D_1}{\partial p_0}$  is positive by Assumption 1(a), the entrant can only benefit from an increase in the share of captive consumers if the incumbent responds with a sufficiently high price increase.

Result (ii) is similar to Result (i) in that the critical condition for a positive profit effect is a lower bound on the effect of  $\mu$  on the opponent price. However, contrary to (1.4), the right-hand side of (1.5) is negative because the direct effect on the incumbent's profit, the demand increase  $D_M(p_0^*, \theta) - D_0(p_0^*, p_1^*, \theta)$ , is positive. Thus as long as it does not lead to a substantial drop in the entrant's price (a very negative indirect effect), an increase in the incumbent's share of captive consumers increases the incumbent's profits.

<sup>11</sup>Using the logic of the envelope theorem, there is no indirect effect of  $\mu$  coming from a change in  $p_i^*$ .

To assess the interaction of the parameter for the captive market share with underlying competitive demand conditions in more detail, we introduce a parameterized model in the next section. We will investigate when the incumbent loses profit post-entry when  $\mu$  –his captive base– increases. Further we will determine the conditions under which an increase in the captive market for the incumbent *induces* entry by a competitor.

### 1.3 Competition on the circle

We now specify the demand functions  $D_M$ ,  $D_0$  and  $D_1$  by assuming that they are generated by a model of spatial competition. This not only allows us to investigate the effects of captive consumers on prices and profits and therefore on innovation incentives. It will also show that captive consumers can have strong effects on the relation between a standard measure of competition and product innovation. Without captive consumers, it is necessary the case that a less competitive market always favors entry. With captive consumers, however, there exist parameter ranges for which more competitive pressure favors entry.

#### 1.3.1 Setup and post-entry equilibrium

We first provide the assumptions and some useful auxiliary results. In particular, we identify the post-entry equilibria, which will determine entry profits.

##### 1.3.1.1 Assumptions

We assume that the incumbent  $i = 0$  is located at point 0 on a Salop circle with circumference 1. Transportation costs are linear, with parameter  $t$ . The total mass of consumers is equal to one. They are uniformly distributed along the circle. Each consumer buys at most one unit, either from firm 0 or from firm 1. We normalize valuations so that the average consumers (located at  $1/4$  or  $3/4$ ) have net valuation (after subtracting transport cost) 1. Thus, the net valuations of the consumers for the incumbent good are uniformly distributed on  $[1 - t/4, 1 + t/4]$ , so that an increase in  $t$  increases the taste dispersion without affecting the average consumer valuation. We thus focus on heterogeneity in consumer tastes and shut down effects of  $t$  on the overall desirability of a product.<sup>12</sup> As long as there is no competition, the incumbent is a monopolist with demand function  $D_M(p_0, t) = \min \left\{ 2 \frac{1+t/4-p_0}{t}, 1 \right\}$ . The parameter  $t$  rotates the demand functions around the point with price 1 and quantity  $1/2$ .

We assume that firm 1 can enter at position  $1/2$  on the circle which results in the maximal distance from the incumbent.<sup>13</sup> The fixed cost of entry is  $F > 0$ . We maintain the following assumption:

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<sup>12</sup>The standard Salop model fixes the *maximal* valuation, so that an increase in  $t$  results in a reduction of almost all consumers' valuations.

<sup>13</sup> This assumption simplifies the analysis because it guarantees that demand expressions are symmetric on either side of the firms' locations. It is justified by the principle of maximum differentiation, as an entrant would optimally locate as far away as possible from her competitor after entry.



**Assumption 1.2.**  $t \leq 2$ . ♦

This assumption rules out equilibria in which the market for switchers is not covered after entry, so that both firms would act completely independently.

### 1.3.1.2 Benchmarks

As benchmarks, we consider the pure duopoly without captive consumers ( $\mu = 0$ ) and the monopoly ( $\mu = 1$ ). The following points are useful for future reference:

**Observation:**

- (i) *In a pure duopoly ( $\mu = 0$ ), prices and profits are increasing in  $t$ .*
- (ii) *In a monopoly ( $\mu = 1$ ), prices are increasing in  $t$  only when the market is not fully covered ( $t > 4/3$ ); they are decreasing for  $t < 4/3$ . Profits are decreasing in  $t$ .<sup>14</sup>*
- (iii) *Prices are higher (lower) in the monopoly than in the duopoly if  $t < (>) 4/3$ .*

For  $\mu = 0$ , the standard Salop equilibrium with symmetric prices  $p_i = t/2$  emerges. Entry takes place if and only if  $t \geq 4F$ . Thus an increase in consumer heterogeneity unambiguously favors entry in the pure duopoly as it softens competition and thereby increases entrant profits. In the monopoly ( $\mu=1$ ), the incumbent charges  $p = 1 - \frac{t}{4}$  when  $t \leq 4/3$ , in which case he serves the entire market. For  $t > 4/3$ , the equilibrium price is  $p = \frac{1}{2} + \frac{t}{8}$ , and not all consumers are served. Crucially, consumer heterogeneity therefore only has a positive effect on the monopoly price when the market is not fully covered. As long as the incumbent serves all consumers, prices are decreasing in  $t$  because it becomes harder to keep all consumers on board as heterogeneity increases. Finally, reflecting the logic of the general model, prices can be higher in duopoly than in monopoly, because in the former case the demand elasticity may be lower than in the latter.

### 1.3.1.3 Equilibrium regions

The pure-strategy equilibria in the post-entry pricing game fall into four classes.

**Definition 1.1.**

- (i) *In an **incomplete-coverage equilibrium (ICE)**, the incumbent does not serve all captive consumers. All switchers are served; both firms have positive switcher demand.*

---

<sup>14</sup>The statement on profits relies on Assumption 1.2; beyond  $t > 4$ , monopoly profits are increasing in  $t$ . This non-monotonicity corresponds to Johnson and Myatt (2006) who make the general point that profits are high for very high and very low heterogeneity.

- (ii) In a **full-coverage equilibrium (FCE)**, the incumbent serves all captive consumers at the maximal price ensuring participation. All switchers are served; both firms have positive switcher demand.
- (iii) In a **consumer-friendly equilibrium (CFE)**, the incumbent serves all captive consumers at a price that gives strictly positive surplus to all of them. All switchers are served; both firms have positive switcher demand.
- (iv) In a **market partitioning equilibrium (MPE)**, all switchers are served by the entrant, whereas the monopolist serves all captive consumers.

◆

The following result depicts the existence conditions for each type of equilibrium. It is a corollary of Lemma A.1 in the Appendix which establishes the equilibrium regions and associated prices.

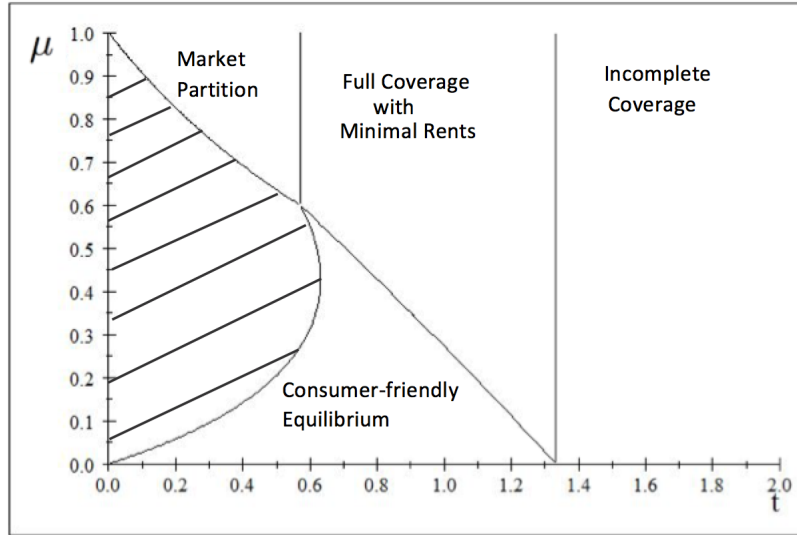
**Corollary 1.1.** With competition for the circle, the following equilibria arise:

- (i) If  $t$  is high ( $t > 4/3$ ), an ICE arises.
- (ii) For intermediate values of  $t$  ( $\frac{4}{7} < t < \frac{4}{3}$ ) and high  $\mu$  ( $\mu > \frac{9t-12}{t-12}$ ), an FCE emerges.
- (iii) For low values of  $t$  and  $\mu$  ( $\frac{36\mu-36\mu^2}{15\mu-8\mu^2+9} < t < \frac{12-12\mu}{9-\mu}$  and  $\mu \leq 0.6$ ), a CFE arises.
- (iv) If  $t$  is relatively low ( $\frac{4-4\mu}{3\mu+1} < t < \frac{4}{7}$ ) and  $\mu$  is high ( $\mu > 0.6$ ), an MPE arises.
- (v) There are no other pure-strategy equilibria than those in (i)-(iv). In particular, for  $\mu \in (0, 1)$  there exists no pure-strategy equilibrium if  $t < \min\left(\frac{36\mu-36\mu^2}{15\mu-8\mu^2+9}, \frac{4-4\mu}{3\mu+1}\right)$ .

◆

Figure 1.1 depicts the equilibrium regions. For sufficiently heterogeneous consumers, the incumbent does not serve all captive consumers, so that an ICE emerges. Conversely, for sufficiently homogeneous consumers, competition for switchers is intense. If there are not too many captive consumers, prices are therefore low enough that there is a CFE. Beyond a certain threshold number of captive consumers and for sufficiently low heterogeneity, an MPE arises. The incumbent focuses on serving the captive consumers at the monopoly price. The entrant sets prices that are just high enough that all switchers buy from her. For a large enough share of captive consumers and intermediate heterogeneity, duopoly competition is sufficiently weak that there is no market separation, but heterogeneity is also sufficiently low so that the captive market is covered; an FCE emerges.

The pure-strategy equilibrium regions described in (i)-(iv) of Corollary 1.1 do not cover the entire



Note: In the shaded regions, no pure-strategy equilibrium exists

Figure 1.1: Equilibrium regions

parameter region  $t \leq 2$ ; for low values of  $t$  as in (v), none of the four pure-strategy equilibria exists. In Section 1.3.4, we deal with mixed-strategy equilibria. Before that, we describe the effects of captive consumers for pure-strategy post-entry equilibria.

### 1.3.2 The effects of captive consumers

We now investigate how the share of captive consumers affects the equilibrium.

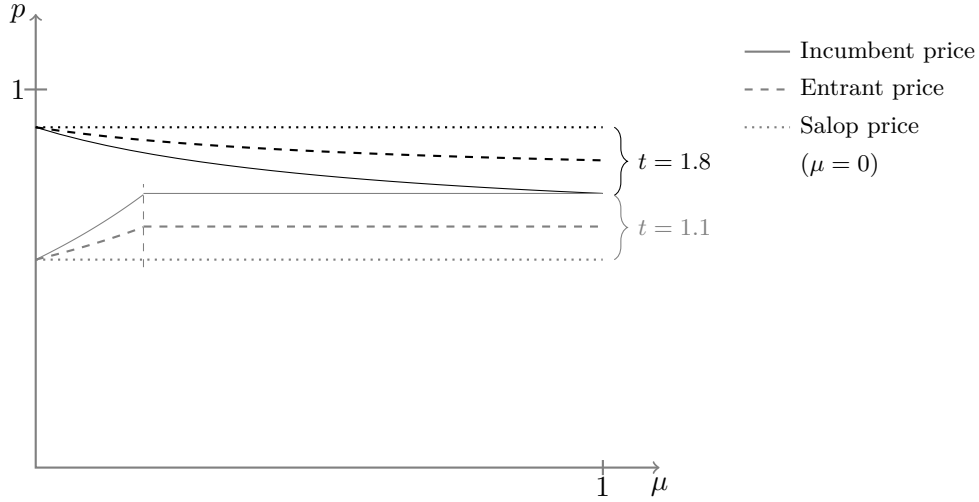
#### 1.3.2.1 Prices

The analysis relies on Lemma A.1 in the Appendix, which specifies the prices in the different equilibrium regions. It immediately implies the following result.

**Lemma 1.1.** Both prices are increasing in  $\mu$  in the CFE region and decreasing in the ICE region; elsewhere,  $\mu$  has no effect.

◆

As the general analysis already suggested, price effects can be positive or negative, and prices move in the same direction. The results reflect the interplay of the competition-softening effect of  $\mu$  in duopoly regions and the higher emphasis on the monopolistic market, where demand may be more elastic than in duopoly and prices may therefore be lower (see Observation 1 (iii)). In the consumer-friendly equilibrium,  $t$  is at a moderate level. That means heterogeneity is low enough that relatively high pure monopoly prices can be sustained. At the same time, competitive pressure is noticeable in a duopoly. When  $\mu$  is low, the incumbent focuses more

Figure 1.2: Price effects of  $\mu$ 

strongly on the switcher market. As  $\mu$  increases, the incumbent puts more focus on the captive market and his price rises. In the incomplete-coverage equilibrium,  $t$  is high. The relationship between monopoly and duopoly prices is now inverse. High consumer heterogeneity makes it difficult for a monopolist to serve all consumers. At the same time, a high  $t$  lowers competition and thus causes high duopoly prices. The incumbent thus decreases the price as the captive consumers become more important.

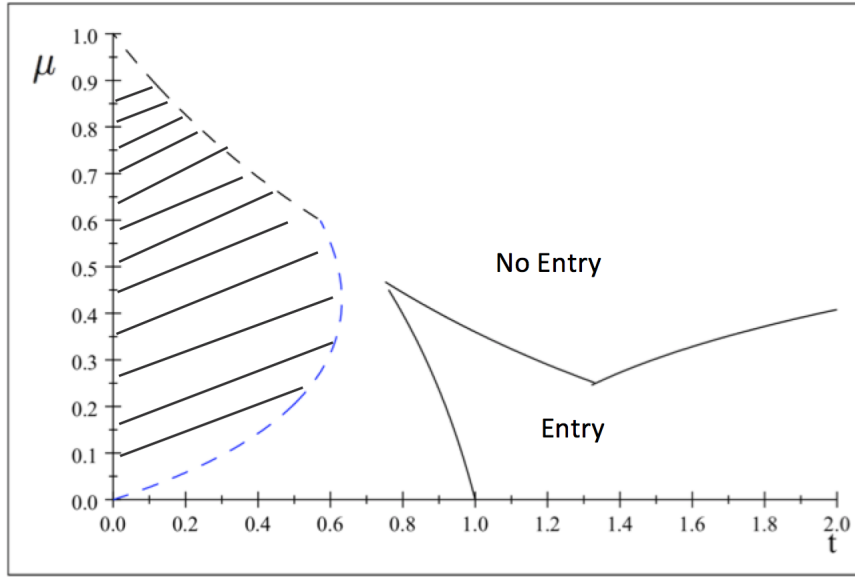
Figure 1.2 illustrates these effects in more detail. For  $t = 1.1$  and sufficiently low shares of captive consumers, the CFE arises, and an increase of  $\mu$  softens competition (increases prices).<sup>15</sup> For  $t = 1.8$ , in the ICE region, duopoly demand is relatively inelastic. Therefore duopoly prices are higher than monopoly prices and, as the monopolist focuses on captive consumers with higher  $\mu$ , prices fall.<sup>16</sup> In line with the discussion of Proposition 1.1, the region with an increasing (decreasing) price corresponds to a situation where the pure monopoly price is above (below) the pure duopoly price. This is also directly manifest in the price level compared to the situation without captive consumers. When heterogeneity is high, consumers benefit from the presence of captive consumers as they force the incumbent to set a lower price and counteract the competition-softening effect of higher heterogeneity.

### 1.3.2.2 Entrant profits

The general analysis of Section 1.2 shows that an increase in  $\mu$  affects the entrant's profit negatively unless it has a strong positive effect on the incumbent price. Lemma 1.1 thus immediately implies that  $\mu$  must have a negative effect on entry profits (and thus on innovation) except possibly in the CFE region. The following central result confirms that the effect is actually positive

<sup>15</sup>Beyond a threshold level of consumers, the game moves into the FCE region. In this region, the incumbent's price is determined by the need to keep the marginal loyal consumer on board – the price is thus completely determined by  $t$ , and it is independent of  $\mu$ .

<sup>16</sup>By strategic complements, the entrant follows the behavior of the incumbent, but reduces its price by a smaller amount.



Note: The equilibrium regions correspond to  $F=0.25$ . In the shaded areas, no pure-strategy equilibrium of the pricing game exists

Figure 1.3: Entry regions

in the CFE region.

**Proposition 1.3.**

- (i) If competition is sufficiently intense ( $t$  and  $\mu$  are sufficiently low), an increase in  $\mu$  fosters innovative entry.
- (ii) If competition for switchers is sufficiently intense ( $t$  is sufficiently low), disruptive innovation takes place (only) if the share of captive consumers is sufficiently high.

◆

In the CFE region, heterogeneity is sufficiently low to make the incumbent care about the captive segment, yet sufficiently high that he does not withdraw from the switcher market. So when an increase in  $\mu$  lets the incumbent put more weight on the captive market, this has a notable competition-softening effect in the switcher market. The second part shows that captive consumers can play an arguably even more fundamental role in fostering disruptive entry: When their share is large and competition for switchers is intense, the incumbent effectively gives up on switchers, exploiting the captive consumers with high prices. Thus, if fixed costs are not too high, innovation will be disruptive if it occurs.

Figure 1.3 illustrates entry behavior for arbitrary values of  $\mu$  and  $t$ , but fixed costs  $F = 0.25$ . For values to the left of  $t = 1$ , increases in  $\mu$  lead from no entry to entry and finally back to entry.

### 1.3.2.3 Incumbent profits

An incumbent can influence the share of captive consumers. For instance, he may use persuasive advertising to make consumers believe that his product is the only solution to their needs. Other strategies to create lock-in might be considered as problematic by competition agencies. For instance, an incumbent may use loyalty rebates and contractual or technological impediments to switching. We now assume that there is an initial stage where the incumbent can influence the share of captive consumers at some cost, given by an increasing function  $C(\mu)$ . The following result shows under which circumstances an incumbent would want to create captive consumers.<sup>17</sup>

**Result:** *An incumbent benefits from an increase in the share of captive consumers, unless this induces entry or there is sufficient heterogeneity and the share of captive consumers is low.*<sup>18</sup>

$\mu$  affects the incumbent's profit positively unless it induces entry or it has a strong negative effect on the entrant's price in post-entry equilibrium (see Proposition 1.2). The former possibility can arise (only) if competition is initially intense (the share of captive consumers is low and consumers are not too heterogeneous). The latter requires that consumers are very heterogeneous, so that the ICE region emerges; in addition, it turns out that the share of captive consumers has to be very small.

### 1.3.2.4 Consumer surplus

The captive consumers always only have the incumbent's product available. Their surplus thus entirely depends on the incumbent's price. Contrary to this, for switchers, the effect of  $\mu$  on consumer surplus is not generally equal to the price effect. Whenever  $\mu$  affects duopoly prices, it also affects the average costs of mismatch incurred by switchers. When prices are identical (for instance, when  $\mu = 0$ ), each switcher buys the product that matches her taste best, thus average mismatch is minimal. For  $\mu > 0$ , firms set different prices, so that not all consumers buy from the nearest firm and average mismatch is not minimal. As the price is a redistribution of surplus between firms and consumers, average mismatch affects total surplus. The more asymmetric market shares are, the larger the loss from mismatch. However, regarding consumer surplus, consumers choose their purchases based on price and distance. Buying a more distant product (which creates asymmetric market shares) thus happens when the distance is compensated by a lower relative price. We obtain the following result.<sup>19</sup>

### Corollary 1.2.

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<sup>17</sup>A full treatment of each strategy would warrant a specific and typically richer model. For instance, for a treatment of rebates for loyal consumers we would have to take price discrimination into account.

<sup>18</sup>For fixed market share, a negative effect arises only in the ICE region if  $t > \frac{72\mu+40\mu^2+48}{24\mu+5\mu^2+27} \cdot \Pi_0$ ; see Corollary A.1.

<sup>19</sup>Lemma A.6 in the Appendix provides the values of the consumer surplus in each region.

- (i) Conditional on the market structure being duopolistic, an increase in  $\mu$  reduces switcher welfare in CFE and increases it in ICE.
- (ii) A marginal increase in  $\mu$  that induces entry increases switcher welfare; a marginal entry-detering increase in  $\mu$  in ICE reduces switcher welfare in CFE (except in a small parameter region near  $(t, \mu) = (2, 0)$ ).



(i) is in line with what one would expect based on price effects: In the CFE region, the surplus of both types of consumers unambiguously falls as  $\mu$  increases: not only do prices go up, but as the effect is more pronounced for the incumbent, market shares become more asymmetric, resulting in a higher total loss from mismatch. In the ICE region, prices are decreasing in  $\mu$  and the surplus of switchers increases (just like the surplus of captive consumers).

Regarding (ii), consider a marginal change in  $\mu$  that affects market structure. From Section 1.3.3.2, we know that in the CFE region an increase in  $\mu$  increases entry profits and can thus induce entry. As entry provides a new differentiated product and hence reduces mismatch, and it also reduces prices in the CFE, it is unambiguously beneficial for consumers in this region (contrary to the effect of  $\mu$  for a given duopoly structure). In the remaining regions, an increase in  $\mu$  is potentially entry-detering, as it reduces entry profits. However, in the ICE region, monopoly prices are lower than duopoly prices, so that it is less obvious that entry is good for consumers. It turns out that nevertheless, except for a very small parameter region near  $(t, \mu) = (2, 0)$ , consumers benefit from entry because the reduction in mismatch dominates the price increase. Therefore, if an increase in  $\mu$  deters entry, consumers usually suffer even though prices fall.

### 1.3.2.5 Policy implications

Should competition policy be concerned about the welfare effects for captive consumers? This is not obvious, given that consumers benefit from an increase in captive consumers in two cases: (i) If the increase induces entry; (ii) If heterogeneity is substantial and the increase does not deter entry (in wide parts of the ICE region). Case (i), which can only arise in the CFE, will not result from an incumbent's strategy, as incumbents would have lower profits after entry. Case (ii) could potentially arise for very large heterogeneity, as the incumbent's profits are typically increasing in  $\mu$  in ICE. With this exception, the incumbent only engages in increasing the share of captive consumers if this softens competition or deters entry, that is, if it is anti-competitive.

### 1.3.3 Competition and innovation: the role of captive consumers for the effects of competition intensity

As we will discuss in more detail in Section 1.4.1, the relation between competition and innovation is one of the central areas in industrial organization and growth theory. The literature has

paid particular attention to the degree of substitution as a competition parameter. According to Observation 1(i), the effect of an increase in the degree of substitution (a reduction in  $t$ ) is trivial when there are no captive consumers. Increasing competition in this sense reduces entry profits, that is, it works against product innovation. When we introduce captive consumers, the competition parameter  $t$  can have different effects on entry profits. In particular, we see that a reduction in  $t$ , and hence more intense competition in a duopoly, may have a positive effect on the post-entry profits of the entrant. This will again happen for intermediate levels of  $t$ .

### 1.3.3.1 Price effects

Again relying on Lemma A.1 in the Appendix, we first state how heterogeneity affects prices.

**Lemma 1.2.** Increasing heterogeneity has positive price effects in the CFE and ICE regions and negative effects in the MPE region. In the FCE region, heterogeneity reduces the incumbent price, but increases the entrant price.  $\blacklozenge$

Broadly speaking, two countervailing forces are at work: Consumer heterogeneity softens competition in a duopoly market, but it may force the monopolist to set lower prices in monopoly markets if he wants to serve all captive consumers (see the discussion in 1.3.1.2). Figure 1.4 shows that, as in the standard Salop model ( $\mu = 0$ ), for  $\mu = 0.1$  both prices are higher when there is much heterogeneity than when consumers are very homogeneous. However, even in this case, the relation is not monotone, as the incumbent price is decreasing in  $t$  in the intermediate FCE region. For a high share of captive consumers ( $\mu = 0.8$ ), the incumbent focuses on this group for low values of  $t$ , setting prices so high that the switchers all buy from the entrant. As  $t$  increases, the incumbent lowers his price to continue selling to all captive consumers. It is this mechanism that generates the downward sloping part of the price curves.<sup>20</sup>

### 1.3.3.2 Entrant profits

Figure 1.3 shows that the effects of heterogeneity on entry may well be non-monotone, with the details depending on the share of captive consumers. For  $F = 0.15$  and intermediate values of  $\mu$ , entry only arises for low and high values of  $t$ , but not for intermediate values.

These patterns reflect non-monotone relations between  $t$  and entry profits. The main insight, which is an immediate implication of Corollary A.1 is that entry profits are decreasing in  $t$  in non-degenerate intervals.

**Proposition 1.4.** For every  $\mu \in (0, 1)$ , there exists a non-degenerate subinterval of  $[0, 4/3]$  on which entry profits are decreasing in  $t$ .  $\blacklozenge$

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<sup>20</sup>For the entrant, the decline comes from the reaction to the incumbent's price reduction.



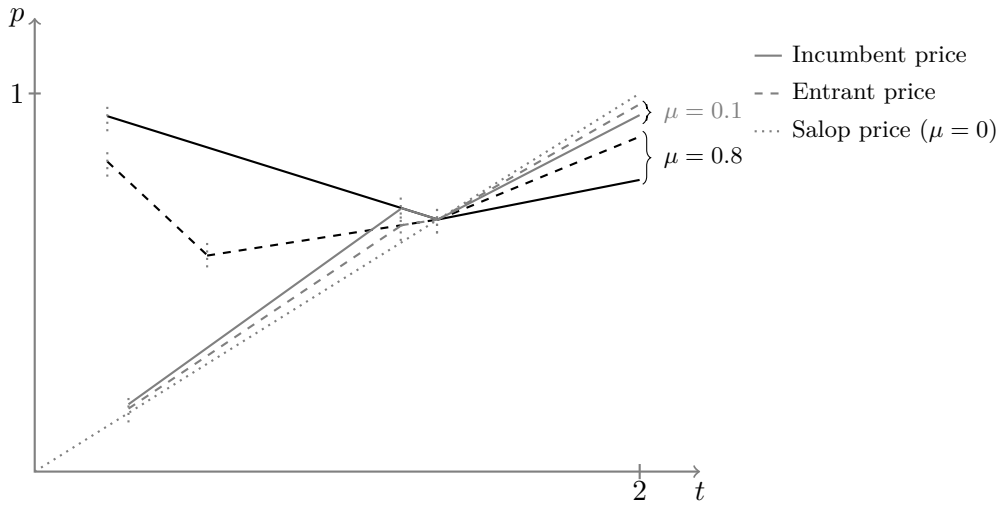
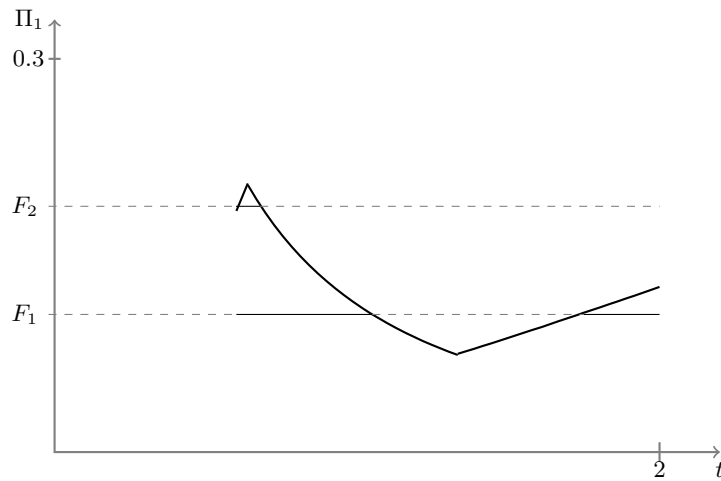
Figure 1.4: Price effects of  $t$ 

Figure 1.5 illustrates the relation between  $t$  and entry profits for  $\mu = 0.55$ . The upward-sloping parts of the graph are unsurprising, as they are consistent with what we know from the case without captive consumers. The negative effect of heterogeneity on entry profits in intermediate parameter regions comes from the incumbent's attempts to keep all captive consumers on board: As higher  $t$  leads to lower monopoly prices if the monopolist covers the switcher market, it can intensify competition when a monopolist pays attention to captive consumers.

Figure 1.5: Nonconvexity of entry regions,  $\mu = 0.55$ 

The negative relation between competition and innovation for some values of  $t$  means that the relation over the entire interval is U-shaped (for  $\mu > 0.6$ ) or N-shaped (for  $\mu < 0.6$ ). As a result, the relation between consumer heterogeneity and entry is generally non-monotone, as depicted in Figure 1.3.<sup>21</sup>

<sup>21</sup>The effect of increasing consumer heterogeneity on the incumbent's profit is similarly complex as for the entrant (see Corollary A.1). Without captive consumers, an increase in  $t$  increases profits. As in the case of the entrant, even with a small share of captive consumers, non-monotonicities occur: There is an intermediate interval of  $t$ -values for which the relation between  $t$  and incumbent profits is negative.

### 1.3.4 Mixed-strategy equilibrium

For sufficiently low values of  $t$ , there is no pure-strategy equilibrium. Figure 1.1 depicts the corresponding parameter region. For intermediate values of  $\mu$ , pure-strategy equilibria fail to exist for larger values of  $t$ , while the region narrows when  $\mu$  moves to its boundaries. The phenomenon of non-existence of pure-strategy equilibria in models with captive consumers has been well known for the case of perfect competition since Varian (1980). As Corollary 1.1 shows, it extends to the case of imperfect competition when products are highly substitutable. Intuitively, competition for switchers is intense while at the same time consumer heterogeneity is low. The incumbent wants to focus on exploiting the captive consumers by setting a high price. This would lead the entrant to respond with a high price. As  $t$  is low, switchers are sensitive to price differences, and the incumbent is tempted to react with a low price to the entrant's high price. Intermediate values of  $\mu$  mean that there is a relevant group of both consumer types, so that the described reasoning has most bite.

We conjecture that in mixed-strategy equilibrium the firms will play a mixture between the bordering equilibria. The market partitioning and consumer-friendly equilibria reflect the two opposing effects described above. We construct an equilibrium in which the incumbent randomizes between the monopoly price and a lower competitive price. The entrant, competing only for the captive consumers, sets one price which is a best reply to the incumbent's expected price. This mirrors the approach of Sinitsyn (2008) who shows that in an asymmetric duopoly where one firm has a captive consumer base, this stronger firm will typically randomize between two prices while the weak firm uses one price. With this approach, we find a mixed-strategy equilibrium in a region that is exactly adjacent to the region of pure-strategy equilibria. This equilibrium covers a part of the region without pure equilibria, but not all of it. This equilibrium is complex to characterize. Prices have analytical solutions. For the left border, we can only establish that it exists, but cannot give an analytical solution.

**Proposition 1.5.** There exists a function  $b(\mu)$  such that a mixed-strategy equilibrium exists in the region  $b(\mu) \leq t < \min\left(\frac{36\mu-36\mu^2}{15\mu-8\mu^2+9}, \frac{4-4\mu}{3\mu+1}\right)$  for all  $\mu \in (0, 1)$ . In this equilibrium, the incumbent randomizes between the monopoly price  $1 - \frac{t}{4}$  and a lower price. The entrant sets one price. ◆

At the border to the pure-strategy equilibrium region, the incumbent's price dispersion vanishes and the prices of both firms converge to the respective prices in the pure-strategy equilibria. The threshold  $b(\mu)$  for the border to the left is uniquely determined by the entrant's non-deviation condition.<sup>22</sup> For  $\mu \in (0, 1)$ , we have  $b(\mu) > 0$ , the region of no pure-strategy equilibrium is thus not entirely covered.

The equilibrium exhibits the realistic feature that the incumbent alternates between two prices. He sets his monopoly price, and sometimes a lower promotion price to compete with his chal-

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<sup>22</sup>At the border, the entrant wants to deviate to the "limit price"  $1 - \frac{3}{4}t$ .

lenger.

Sinitzyn (2009) analyzes equilibria in the differentiated-goods setup with captive consumers and symmetric firms. He shows that the region of mixed-strategy equilibria is split up into several adjacent regions. As the substitutability parameter moves closer to perfect substitutes, the number of prices that the firms randomize over increases. He is unable to find analytical solutions for the more complex equilibria, but he can solve for prices numerically. Sinitzyn (2008) confirms the analytical intractability of the very homogeneous case in an asymmetric duopoly similar to ours.

We conjecture that in our asymmetric setup equilibria with larger price supports will exist in addition to those we found. However, the qualitative implications for entry and welfare are likely to be similar to those we describe in Proposition 1.5. As it is generally complicated to find solutions for these more complex equilibria, we do not endeavor to further solve for them. We conjecture that the mixed equilibria develop from the pure-strategy ones with initially two and one price, adding more prices to the support as  $t$  decreases, to a final randomization over intervals.

Notice that for  $t = 0$ , the problem is easy to solve. Both firms randomize over an interval of prices. That equilibrium is asymmetric, with similar properties as the one in Proposition 1.5. The incumbent's pricing distribution has an atom at the monopoly price, while the entrant's distribution has no atom. The incumbent's expected profits are inverse-U shaped in the share of captive consumers carries over to the mixed-strategy equilibrium at least in the case that  $t = 0$ .

### 1.3.5 General discrete choice models

While the framework of Section 1.2 is quite general, the model used in the current section is more specific. It is possible to understand the equilibrium structure of the pricing game for quite general discrete choice models, with valuation profiles  $(v_0, v_1)$  distributed on  $[\underline{V}, \bar{V}] \times [\underline{V}, \bar{V}]$ . In Appendix A.4, we show what kind of pricing equilibria can arise for such games. We find that, except for a *Doubly Incomplete Coverage Equilibria* where neither the switcher market nor the captive market is covered, no additional types of equilibria can arise. In our application, we excluded double incomplete coverage by Assumption 1.2. The equilibria besides the doubly incomplete one have the structure of one of the equilibria described in Corollary 1.1.

## 1.4 Discussion and relation to the literature

Our paper can be understood as a contribution to several strands of literature. We first discuss the connections to the vast literature on the determinants of innovation. In addition, the paper has some relations to the advertising literature. Finally, the paper is related to the literature on consumer switching costs.

### 1.4.1 Competition and innovation

One of the core topics in industrial organization and growth theory is the relation between competition and innovation.<sup>23</sup> More specifically, we contribute to the literature on product innovation. Within this literature, a small group has focused, like we do, on the incentives of firms to introduce goods that are horizontally differentiated from existing goods.

Most closely related to our paper, Chen and Schwartz (2013) show how the incentives to introduce new goods in a Hotelling setting depend on market structure. One of the constellations in their paper (“duopoly”) considers the incentives of a new entrant to add such a product to the pre-existing product of an incumbent. This corresponds to the degenerate case without captive consumers in our model. However, the authors focus on the comparison between innovation incentives of duopolists and those of a monopolist who decides whether or not to introduce a second good without having to fear entry (a “secure” monopolist).<sup>24</sup> Instead, our paper is concerned with the effects of captive consumers and consumer heterogeneity on innovation, identifying non-monotonicities and interactions between the variables.

Boone (2000) takes a more abstract approach to innovation incentives. He analyzes how changes of different measures of competitive pressure (including substitution parameters like  $t$  in our setting) affect incentives to engage in product and process innovation, respectively. He shows that the relation depends on a firm’s costs relative to the competitor’s. While Boone emphasizes the role of cost asymmetries between firms in a setting with symmetric demand, we focus on demand asymmetries coming from the existence of captive consumers.

Beyond the literature on the introduction of new products, many authors deal with the relation between suitable competition parameters and cost-reducing or demand-enhancing innovations. Very broadly, following Aghion et al. (2005), one part of this literature has converged to the view that an inverse-U relation should at least be the most common, whereas another part of the literature (Gilbert, 2006; Vives, 2008; Schmutzler, 2010, 2013) suggests that even the qualitative nature of the relationship is likely to depend on market-specific details. Our paper confirms the latter view for the case of horizontal product innovations. It shows how a change in the share of captive consumers can introduce non-monotonicities of different types even when there would be a simple monotone relationship between competition and innovation without captive consumers.

### 1.4.2 Advertising

Our paper has some connections to the advertising literature. Authors such as Beladi and Oladi (2006) emphasize the competition-softening effect of advertising, which reflects the “fat-cat” logic of Fudenberg and Tirole (1984) and Bulow, Geanakoplos, and Klemperer (1985). In our

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<sup>23</sup>See, for instance, Gilbert (2006), Vives (2008), Schmutzler (2013) for overviews.

<sup>24</sup>In particular, the authors show that the gain from innovation can be larger for a secure monopolist than for the entrant. Less directly related to our analysis, they also compare the incentives of the secure monopolist with those of Bertrand competitors, showing that Arrows famous result that the monopolist has weaker incentives for drastic innovations need not hold in this case.

setting, it is natural to think of the share of captive consumers in an industry as being influenced by the advertising behavior of incumbents and potential entrants: An incumbent may create captive consumers by suitable persuasive advertising, while informative advertising of entrants may increase awareness for their product and therefore reduce captive consumers. With this in mind, suppose there is an advertising stage before the entry decisions are made. Abstracting from the possibility of entry-deterrence, the incumbent will therefore have strategic incentives for over-investment in advertising (fat cat strategy) whenever an increase in the share of captive consumers increases the entrant's price (in the CFE region). If instead the entrant's price reacts negatively to captive consumers (in the ICE region), the incumbent adopts a puppy dog strategy and advertises less than without strategic considerations. As usual, if entry-deterrence becomes possible, the incumbent has strategic incentives to lower the entrant's profits. This can call for an aggressive strategy (for instance, underinvesting in advertising to avoid the competition-softening effect).

Several authors have emphasized the relation between incentives for advertising and innovation. For instance, Grossmann (2008) and Qi (2019) find complementarities between advertising and quality-improving innovations. Our paper identifies circumstances under which advertising and (horizontal) product innovations are not perfectly complementary: In a sufficiently competitive market, an entrant may want to avoid aggressive informative advertising if this reduces the share of captive consumers: If there are initially not too many captive consumers, reducing their share further would intensify competition.

### 1.4.3 Switching costs, competition, and prices

Our paper also relates to a literature that investigates the effects of (differentiated) competition on prices and a literature on the relation between consumer switching costs and prices with homogeneous goods.

**Differentiated competition and prices** Sinitsyn (2008, 2009) analyzes equilibria in duopoly settings similar to ours. Sinitsyn (2008) considers asymmetric competition where one firm has a captive consumer base and finds similar pricing equilibria to ours. He does not consider entry incentives and crucially, no heterogeneity among the captive consumers. We have shown that this heterogeneity plays an important role for the effects of captive consumers on entry. Sinitsyn (2009) analyzes a similar mode with symmetric bases of captive consumers for each duopolist. He confirms the general structure of the equilibria, and is again not concerned with entry.

Chen and Riordan (2008) show that, starting from a monopoly, the entry of a competitor who produces a differentiated good may well increase prices without captive consumers: Competition reduces an incumbent's demand, which works towards lower prices; however, it also reduces the demand elasticity, which works in favor of higher prices. Similar forces are also present in our paper. As a result, we find that post-entry prices can be lower than pre-entry prices if consumers are sufficiently heterogeneous.

**Captive consumers and prices** The analysis of competition with captive consumers has a long tradition in the literature. In a Bertrand setting with homogeneous products, Varian (1980) explains price dispersion. The existence of (non-informed) captive consumers causes firms to use mixed strategies, which he interprets as price dispersion. In contrast, our game admits pure-strategy equilibria unless consumers/products are too similar and the share of captive consumers is intermediate. We aim to analyze the asymmetric situation where only one firm has a stock of captive consumers. Like Varian, many other papers analyze the effects of captive consumers on price competition. Chioveanu (2008) has ex-ante symmetric firms who invest in advertising to create a stock of captive consumers. She finds that firms will choose asymmetric advertising intensities in equilibrium and some firms will always randomize on prices. Such differences in persuasive advertising may offer an explanation for the existence of asymmetries in captive consumers.

Our paper focuses on the role of captive consumers for entry into a monopolistic industry after an exogenous shock. A related, but distinct question is under which conditions dominance emerges endogenously when firms compete to lock in consumers who face prohibitive switching costs. Beggs and Klemperer (1992) argue that a leading firm's incentive to exploit its captive consumers leads to decreasing dominance. Moreover, in their model, increasing shares of captive consumers always lead to higher profits of all firms. This tends to attract entry because new firms expect future benefits from captive consumers themselves.

## 1.5 Conclusion

This paper proposes a framework to analyze incentives for innovative entry with differentiated goods into a market previously dominated by a monopolist. We identify non-monotone effects of the share of captive consumers and of consumer heterogeneity on potential entry. The paper thereby contributes to the analysis of the relation between competition and product innovation. The results are driven by subtle effects of competition parameters and prices. In particular, consistent with Chen and Riordan (2008) for a setting without captive consumers, we find that increasing competition may increase prices.

In spite of the ambiguous comparative statics conclusions, our analysis has clear competition policy implications. The incumbent usually only benefits from a higher share of captive consumers if this softens competition or deters entry. If an incumbent engages in activities that aim at increasing the share of captive consumers, it is therefore safe to conclude that this has anti-competitive effects.

The conclusions of our analysis are most definite in the special case of competition for the circle. Nevertheless, our general analysis applies more broadly, and it suggests that the lessons from the parameterized model are relevant beyond this case.







## 2 Product variety in the wholesale and agency models of intermediation<sup>1</sup>

### 2.1 Introduction

This paper considers a vertically separated structure where product sales from suppliers to buyers are effected by an intermediary. I ask what consequences follow from the way in which the intermediation is economically organized. In particular, I explore whether a monopolistic intermediary has an incentive to limit the product range available to consumers. I analyze two different modes of intermediation: the wholesale mode and the agency mode. In the wholesale mode the intermediary buys the product from the supplier and resells it to consumers. In the agency mode the intermediary only demands a fee for enabling the trade between supplier and final buyer. When intermediaries carry multiple products, might intermediation have the effect of a gatekeeper who determines which products arrive on the retail market? I analyze the question in a setting of horizontally differentiated products on a Salop circle and find that there is a meaningful difference between the two intermediation modes. In the agency mode, the intermediary always benefits from making a larger number of products available to consumers. When the intermediary is a reseller, however, over a significant range of the number of generally available products, he prefers to limit the number of products he makes available to the buyers. This result follows without stocking costs or any other type of fixed cost. It is purely a consequence of the difference in economic organization of the vertical chain.

More product variety means consumers get a better match of available products to the characteristics they desire. The intermediary as a monopolist can appropriate much of that welfare increase from the consumers. But for an intermediate number of available products, when the intermediary makes more products available to consumers, he has to pass on these welfare gains to the suppliers. When there are few available products overall, not all consumers buy and demand for one product is independent of the other products. Each product is priced individually like in a monopoly (with double marginalization). The profit margin the intermediary makes is independent of the number of products and the intermediary wants to add more products. When the number of products offered by the intermediary rises, the demand of a product becomes influenced by the prices of its neighboring products. Consumers substitute between products if their relative prices change. Because the intermediary sells all the products to consumers, he internalizes the loss in demand that a price cut in one product inflicts on another product. Thus, he reacts less elastically to decreases in the wholesale price. This reduces the competitive

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<sup>1</sup>Cite as Christian Oertel (2019): “Product variety in the wholesale and agency models of intermediation”, Mimeo.

pressure on the suppliers and they charge higher wholesale prices. These prices are such that the wholesale prices rise faster than retail prices when the intermediary carries a larger number of products. The intermediary thus prefers to carry fewer products. Only when the number of available products becomes large the competitive pressure on the suppliers rises and they may start to lower their wholesale prices.

The agency and wholesale modes are most readily compared when the intermediary in the agency mode charges a fixed fee per transaction. The agency model does not exhibit the feature of product range limitation. In this setup of the supply chain, the intermediary always prefers to carry as many products as are available. He benefits from the increase in welfare that comes from increased variety. For a given number of products, consumer prices are then the same under both modes. However, the roles of the intermediary and the supplying industry are reversed. Now, the intermediary sets the access price for each transaction with his fee, a role that was filled by the wholesale price before. But because the intermediary has no competitive pressure, he can keep access prices high also for a large number of products. He thus benefits from larger number of available products and always chooses to carry all available products.

This sheds light on a meaningful difference between the wholesale and the agency mode. With competitive suppliers and a monopolistic intermediary, retail pricing is the same regardless of the intermediation mode for a significant range of the number of available products. For the same number of products, retail prices are the same whether they are set by the intermediary or the suppliers. But there is a region where the intermediary in the wholesale mode wants to limit product availability. The effect of the different intermediation mode thus comes not from pricing, but from the different product offering. The reason for this is the internal organization of the vertical chain of suppliers and intermediaries with a first and a second-moving layer. In the wholesale mode and the agency mode with a per-transaction fee, there is one party which sets a price per transaction within the chain and the other party that sets the retail price. The internal-price setter is the first mover and can appropriate the welfare gains from increased variety. When the supplying industry is the first-moving layer, in the wholesale mode, they benefit from this advantage when the competitive pressure is not too high. The second mover, the intermediary in this case, loses because the first mover set higher prices and thus wishes to limit his product range. The internal organization of the supply chain in the different intermediation modes can thus have real effects beyond pricing.

The intermediary is influencing prices as a monopolist. Prices are thus set to a level such that the consumers just buy. As more products become available, the average net willingness to pay of consumers rises because they find better matches. Retail prices then can rise accordingly. This leads to consumer surplus decreasing in the number of products. In the limit, consumer prices rise to the gross consumer valuation and consumers get no surplus. Interestingly, the restriction on product range that the wholesale mode can bring about is thus beneficial for consumers. This applies also when fixed costs of entry pin down the number of products in the agency mode. For levels of the fixed cost that still makes production viable, the market will always be covered. The number of products is always higher than in the reseller model, but this raises retail prices and lowers consumer welfare. Consumer surplus is thus generally higher in

the wholesale mode.

In addition to the close-knit comparison between the wholesale mode and the agency mode with a fixed per-transaction fee, I also describe results of the agency mode with a proportional fee. The intermediary collects a share of the revenue the seller makes in the transaction by setting the fee as a proportion of the transaction price. As with the per-transaction fee, the intermediary always wishes to carry all the products he can. But prices differ when the number of available products is small. When not all consumers buy and products are priced independently, the prices to consumers are lower than in the other modes. This changes once the market is covered. The intermediary now charges a fee that makes the suppliers set their prices at monopoly levels.

My paper contributes to the literature by fully solving the pricing game for a vertical structure with several suppliers and a multiproduct intermediary. In particular, I characterize the pricing strategy of a multiproduct reseller who sources his products from several suppliers. This leads me to find a region where the reseller does not want to add more products to his offering. The Salop model is a natural vehicle for the analysis of product variety. I will use the number of firms as a description of the extent of variety that is provided in the market. My analysis then focuses on the incentives of the intermediary to limit or expand this number. The Salop model has been used in the past to analyze intermediaries that provide market access for differentiated firms. This includes the reduction of search costs by a marketplace (Bakos, 1997), and more recently the comparison of the agency and wholesale models (Johnson, 2018).

My model is most closely related to Johnson (2018). He compares retail prices under the wholesale and agency models with two competing intermediaries. His model has two periods and his focus lies on intermediary competition when consumers get locked in. For the purpose of that paper, it is not necessary to solve for the full equilibrium of the vertical chain interaction. Its aim is rather to show the effects of a switch of the wholesale to the agency model with wholesale prices and revenue shares, respectively, taken as given. In contrast, my paper is explicitly concerned with the outcome of the vertical contracting under the two models. I analyze the full game when both upstream and downstream firms take decisions on their pricing actions. Johnson's paper also discusses the incentive of the intermediary to limit the number of firms available. He finds that an intermediary using the agency mode may want to limit the number of products. This is in contrast to my finding, where the intermediary in the agency mode always wants to increase the number of suppliers. At the center of the difference lies the fact that unlike in my model, Johnson considers a given revenue share and does not allow the intermediary to choose the share to influence retail prices.

Johnson (2017b) analyzes two situations of monopoly or imperfect competition both upstream and downstream, respectively. He compares the agency and wholesale models in both cases and finds conditions for which the agency model reduces prices. In contrast, I have a model with competition upstream and a monopolistic bottleneck downstream. I ask what effects the different intermediation modes have given this downstream market power.

Foros, Kind, and Shaffer (2017) compare the wholesale mode with the revenue-sharing version of the agency mode. They emphasize the differences in the intensity of upstream and downstream

competition. They consider competing intermediaries and initially take revenue shares in the agency model as given, but later also allow their intermediaries to set them strategically.

The Salop circle model of product differentiation has been used to analyze product variety by considering the effects of vertical separation in the wholesale mode on entry. Entry into the different layers of a vertical structure is analyzed by Reisinger and Schnitzer (2012). Their model uses two successive Salop circles and analyzes the wholesale model under linear wholesale prices and two-part tariffs. They find that the intensity of competition downstream determines the overall profit of the vertical structure, while the upstream conditions determine the sharing of profits. Linear pricing regimes can reduce consumer prices because they induce more entry downstream. Pagnozzi, Piccolo, and Bassi (2016) use the Salop circle to analyze product variety with competing supply chains organized in the wholesale mode. Pairs of suppliers and retailers locate on the circle and compete. Suppliers are imperfectly informed about their retailer's costs. Linear wholesale pricing then induces retailers to raise their prices to reduce their supplier's information rent. This relaxes competition and induces more entry than under two-part tariffs, where retailers reveal their costs.

All the above papers that consider the agency mode use proportional fees. I analyze both this fee structure and a per-transaction payment. My paper is thus also related to the literature that discusses the decision on proportional or per-transaction fees. Shy and Wang (2011) ask why credit card issuers charge proportional fees and find that this always increases their profit. Anderson, de Palma, and Kreider (2001) discuss whether a revenue-maximizing government should set a proportional or ad valorem tax and find that the proportional tax is always more efficient at raising a given revenue. Gaudin and White (2014) show that ad valorem taxes are preferred only in the case that demand is not too convex.

The agency model is commonly used with internet platforms. A big part of the literature focuses on the effects of such platforms on product variety. Casadesus-Masanell and Halaburda (2014) show that intermediaries can create value by restraining choice when there are network effects in product usage. When variety is limited, consumers coordinate more easily on one product and benefit from network effects. The reduction of search costs is another factor increasing product variety on internet platforms. Brynjolfsson, Hu, and Simester (2011) analyze data on online sales and find that internet sellers offer more product variety than traditional offline retailers. The reason is a reduction in search costs, so consumers find a better match more easily.

Hagiu and Wright (2015a) and Hagiu and Wright (2015b) discuss the choice of intermediation mode. They analyze asymmetric information pertaining to the effectiveness of marketing effort or to product quality and determine the optimal decision for the intermediary.

My paper analyzes intermediation under the two different modes of wholesale or agency. Section 2.2 outlines the general setup of the vertical production and sales chain and describes the differences between the two modes. It describes the basic setup of several available suppliers of differentiated goods and one monopolistic intermediary that delivers the products to consumers. The different intermediation modes are then analyzed separately. Section 2.3 starts with the wholesale mode. It describes pricing when product demands are independent and when the in-

intermediary needs to jointly price products with interdependent demand and shows under what circumstances the intermediary wishes to limit product availability. Section 2.4 analyzes the pricing game in the agency mode with a per-transaction fee. The intermediary sets a fixed amount that is payable for each sale that a supplier makes. It turns out that this setup mirrors the wholesale model. Now, the intermediary always prefers to carry all available products. The introduction of fixed costs is then considered to see when product range might be limited in the agency mode. Finally, in Section 2.5, I analyze the agency mode with a fee that is proportional to the transaction value. This revenue-sharing model leads to lower prices when products are independent monopolies, otherwise it exhibits the same retail prices as the other two modes. The intermediary always wants to carry all available products since with his fee he benefits from the higher prices possible through an increase in variety. Section 2.6 concludes.

## 2.2 Model

There are  $K \in \mathbb{N}$  firms (*suppliers*) that each produce one good with marginal cost  $c \geq 0$ . The produced goods are horizontally differentiated. Upon starting production, each firm chooses the location of its product on a circle of circumference 1 as in Salop (1979). There is a continuum of mass one of consumers uniformly distributed along the circle. A consumer buys at most one unit of one product, or nothing. Each consumer is characterized by location  $x$  and has willingness to pay  $v - td_{ix}$  for good  $i$ .  $v > c$  is the valuation of the preferred product (located at  $x$ ),  $t$  is transportation cost, and  $d_{ix}$  is distance of product  $i$  from the consumer at  $x$ . A consumer at location  $x$  who buys product  $i$  at price  $p_i$  thus obtains utility  $U = v - td_{ix} - p_i$  from this purchase. A consumer who does not buy any one of the products realizes an outside option valued at zero.

Trade between firms and consumers has to be enabled by a monopolistic *intermediary*. Firms and consumers cannot bypass the intermediary to trade bilaterally. All consumers can visit the intermediary, but he decides which firms to host. Intermediation is organized according to one of two possible regimes: the *wholesale mode* or the *agency mode*. In the wholesale mode, the intermediary buys the goods from the suppliers and subsequently resells them to consumers. In the agency mode, the intermediary lets suppliers and consumers trade freely, but charges a fee for each transaction. The modes thus differ in who sets the retail price. In the wholesale mode, the intermediary chooses the retail price (for all products), in the agency mode, the retail price for a product is chosen by the respective supplier. I assume that the intermediary in the agency mode charges the respective fee to the suppliers and that consumers pay the price set by the suppliers.<sup>2</sup> The form of the fee in the agency mode characterizes two further submodes. In one, the intermediary charges a fixed amount per transaction, in the other, he charges a fee that is proportional to the transaction price.

In each setup, the intermediary chooses a number  $N \leq K$  of firms to make available to consumers. I assume that the  $N$  firms choose their product characteristics such that the products

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<sup>2</sup>This assumption is consistent with the organization of most real-world intermediaries that charge fees for a transaction, such as Booking.com or Amazon Marketplace.

are spread uniformly around the circle. Each product is located at equal distance from each of its two neighbors. The distance between two products is thus  $1/N$ .<sup>3</sup> The consumers have full information about number, location, and price of the products and have no costs of visiting the intermediary. The suppliers also have full information about the parameters of the game. The solution concept for the pricing games in each intermediation mode is subgame-perfect equilibrium.

The Salop model of horizontal differentiation has the feature that aggregate demand may be inelastic. This is the case when all consumers buy a unit of a good and the consumers all obtain positive surplus. I will say that the market then exhibits complete coverage. Because the consumers have unit demand, when each consumer buys, a further reduction in prices does not increase the total traded quantity. Hence total demand is inelastic at this point. Of course the demand that each product faces individually still depends on relative prices. As these change, consumers will substitute one product for the other and the quantities traded of a product change with its relative price. I will say the market exhibits incomplete coverage when prices are so high that some consumers prefer not to buy anything. In a symmetric equilibrium, each consumer buys the closest product. A fully covered market then means that product demands are interdependent as demand for a given product is affected by a price change of a neighboring product. A market with incomplete coverage means that products have independent demand. A price change in any other product does not affect demand. The products can thus be seen as local monopolies. In the following, the differentiation between covered and uncovered market will play an important role and I will organize the analysis along this difference. This differentiated analysis is warranted because the expressions for product demand change depending on the situation.

## 2.3 Wholesale model

### 2.3.1 Pricing equilibrium

In the wholesale mode, the downstream firm, the intermediary, buys each product  $i$  at wholesale price  $w_i$ ,  $i = 1, \dots, N$ . This price  $w_i$  is set by the upstream firm, the supplier. The intermediary then sets final consumer price  $p_i$  for each product. The timing of the game is as follows. The intermediary chooses  $N$ . The  $N$  firms then simultaneously set their wholesale price. The intermediary sets prices  $(p_i)_{i=1, \dots, N}$  for all products jointly and buys them at  $(w_i)$  to satisfy demand. I am looking for symmetric equilibria of this game where each supplier sets wholesale price  $w$  and the intermediary sets price  $p$  for each product. The demand functions for products differ in the situations of incomplete and complete market coverage. When the market is not fully covered, demand for a product depends on how many consumers obtain nonnegative utility from it at a given price. When the market is covered, all consumers obtain nonnegative utility from some product and compare the utilities they receive from neighboring products. When the

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<sup>3</sup>It is common to look for symmetric equilibria with equally spaced products on a Salop circle. The assumption will be discussed in more detail later.

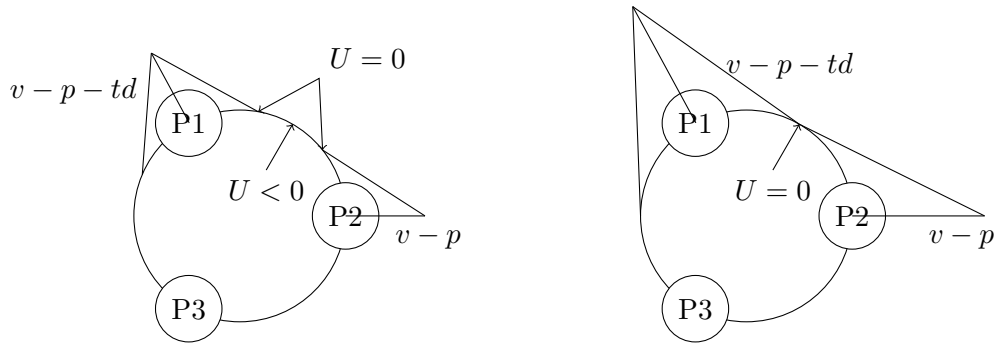


Figure 2.1: Incomplete and complete coverage

number of products offered by the intermediary is low, the average distance from a consumer to a product increases. The market may then not be fully covered at the intermediary's optimal prices. I thus start by analyzing the optimal pricing under incomplete market coverage and determine under what conditions, given the prices chosen by the intermediary, the market is in fact not fully covered.

With symmetric prices, the market is not fully covered when the intermediary chooses prices that are so high that not all consumers on the circle buy. Consumers located sufficiently far away from any product get negative utility from buying and choose the outside option. In this case, the marginal consumer who buys one product is indifferent between buying this product or nothing, not between buying this and another product. The quantity demanded for a product comes from all consumers who get non-negative utility at this price. The marginal consumer to each side of the product is thus defined by the distance  $d_i$  from the product's location such that  $v - p_i - td_i = 0$ . Total demand for the product comes from adding the consumers who buy on each side of its location. Rearranging the preceding expression and taking both sides into account, the product demand under incomplete coverage is found to be  $D^{ic}(p_i) = 2 \frac{v - p_i}{t}$ . The situation is straightforward to analyze. All products are independent monopolies, and thus we will have  $N$  times the result of single-product pricing with double marginalization. The question remains for which parameter range the result applies. As prices are independent of the presence of other products, for a given set of parameters, the retail price will be the same for all  $N$  as long as the market stays uncovered. Call this price  $p^{mW}$ , where the  $mW$  superscript designates the single-product monopoly outcome in the wholesale mode. As  $N$  rises, the products move closer together and for a given consumer price  $p$ , there will be fewer consumers who do not buy at all. This continues until the demands for the products “touch”. That is until the marginal consumer to either side of a product no longer decides between buying the product and not, but buying the product or its neighbor. Until this value of  $N$ , pricing according to the uncovered-market demand function for a product stays in effect. Figure 2.1 illustrates the situation. The lines rising away from the circle line represent the net utility of a consumer as a function of location on the circle (and hence distance from a product). The figure illustrates the situation for a fixed number of products. The right panel features a decrease in the price  $p$  relative to the left panel. The marginal consumers of the products move closer together until they become the same consumer. It is straightforward to see that a similar mechanism applies when the number of products increases for a given price.

Because the optimal retail price  $p^{mW}$  in an uncovered market is independent of  $N$ , so is the demand for each product. The market remains uncovered as long as  $D^{ic}(p^{mW}) \leq \frac{1}{N}$ . As  $N$  rises, this condition will eventually be violated. There is thus a cutoff level of  $N$  until which single-product pricing with demand  $D^{ic}(p)$  is appropriate. The following result shows the outcome of this repeated single-product pricing and the range of parameters under which it applies.

**Proposition 2.1.** For  $N \leq \frac{2t}{v-c}$ , the unique subgame perfect equilibrium is such that the reseller sets single-product monopoly price  $p^{mW}$  and suppliers set monopoly wholesale prices  $w^{mW}$ . The market is not completely covered. Prices are

$$p^{mW} = \frac{3v+c}{4} \text{ and } w^{mW} = \frac{v+c}{2} \text{ for all } i = 1, \dots, N.$$

*Proof.* Suppose the market is not fully covered. The reseller chooses prices to maximize profit  $(p_i - w_i) \cdot 2 \cdot \frac{v-p_i}{t}$  of each product  $i$ . This has solution  $p_i^m(w_i) = \frac{v+w_i}{2}$ . Supplier  $i$  in turn chooses her wholesale price to maximize profit anticipating the retail price:  $(w_i - c_i) \cdot 2 \cdot \frac{v-p_i^m(w_i)}{t}$ . The result of this program is wholesale price  $w_i^m = \frac{v+c}{2}$ . As suppliers are symmetric, the wholesale price for each product is  $w^m = \frac{v+c}{2}$  and the retail price of each product is accordingly  $p^m = \frac{3v+c}{4}$ . Given this, the circle is in fact not fully covered if  $2 \frac{v-p^m}{t} \leq \frac{1}{N} \iff \frac{v-c}{4t} \leq \frac{1}{2N} \iff N \leq \frac{2t}{v-c}$ . ■

The reseller sets monopoly prices as if it sold each product individually. Suppliers are hence monopolists as well and we have the classic double marginalization result with linear prices under monopoly. The reseller makes profit  $\Pi^{mW} = N \frac{(v-c)^2}{8t}$ . Each supplier makes profit  $\pi^{mW} = \frac{(v-c)^2}{4t}$  and total profits of the supplying industry are  $N\pi^{mW} = N \frac{(v-c)^2}{4t}$ . Consumer surplus is  $CS^{mW} = N \frac{(v-c)^2}{16t}$ .

### 2.3.2 Complete market coverage without supplier competition

The derivation of equilibrium under complete coverage is more complex. The difficulty lies in determining the intermediary's reaction to a supplier's deviation from the symmetric wholesale price. Equilibrium is found when, given this reaction, no supplier will deviate. I start by postulating a symmetric equilibrium. Then, I provide a characterization of the multi-product intermediary's reaction to a unilateral deviation of a supplier. If no such deviation is profitable for the supplier, the equilibrium is established. The following result describes the equilibrium that is obtained for an intermediate value of  $N$ . It describes the situation that arises as  $N$  increases and the market starts to be covered. I will prove this result in several steps. In the symmetric equilibrium, each supplier sets the same wholesale price  $w^{oW}$  given the number of firms  $N$ . The  $oW$  superscript designates the oligopoly price in the wholesale mode.

**Proposition 2.2.** For  $N \in \left[2 \frac{t}{v-c}, 4 \frac{t}{v-c}\right]$ , the subgame perfect symmetric equilibrium has full



market coverage. Wholesale prices are

$$w^{oW} = v - \frac{t}{N},$$

and retail prices are

$$p^{oW} = v - \frac{t}{2N},$$

for all  $i = 1, \dots, N$ . ◆

To establish the result, I will first show that the wholesale and retail prices are the highest ones that exactly induce full coverage, i.e. total demand that is inelastic to price decreases. Then, I will establish that under the given conditions, there is no incentive for a unilateral deviation of a supplier. A reduction in the wholesale price is not profitably offset by an expansion in demand. And even though it is possible to push competing products out of the market, this will also be too costly.

We are considering the case that  $N > 2\frac{t}{v-c}$ . In this case, the prices from the single-product pricing equilibrium of the previous section lead the demands to “overlap” on the circle. Once the circle is fully covered, aggregate demand is inelastic to further price decreases. In any situation where symmetric wholesale prices are low enough to support full coverage retail pricing, the intermediary sets the highest possible symmetric retail price, as otherwise he sells the same quantity at a lower price. The reseller’s reaction to symmetric wholesale prices is given in the following result.

**Lemma 2.1.** In the case  $N > 2\frac{t}{v-c}$ , consider symmetric wholesale prices  $w$  and symmetric pricing by the intermediary. The intermediary sets price  $p = \frac{v+w}{2}$  if  $w > v - \frac{t}{N}$  and  $p = v - \frac{t}{2N}$  if  $w \leq v - \frac{t}{N}$ .

*Proof.*  $p = \frac{v+w}{2}$  is the intermediary’s reaction in independent single-product pricing. As long as  $w > v - \frac{t}{N}$  this reaction produces prices such that the market is not fully covered. At  $w = v - \frac{t}{N}$ , the monopoly reaction produces price  $p = v - \frac{t}{2N}$ . This price is such that the indifferent consumer between two adjacent products exactly gets utility zero from both products. The market is just covered and demand is now inelastic to further price decreases. The intermediary thus has no incentive to lower the symmetric price. The intermediary has also no incentive to set asymmetric prices. Raising one price above the others results in local incomplete coverage as it creates a gap of consumers who no longer buy. But under incomplete coverage, the optimal price for each product is a well defined function of the input price and dictates the lower price  $v - \frac{t}{2N}$ . Thus raising the price of one product cannot be profitable. When the intermediary lowers the price of one product, consumers substitute from neighboring products to the cheaper one. Given that suppliers all set the same wholesale price, this cannot be optimal because it moves a part of the constant total demand to a product with a lower profit margin. Thus, for any  $w \leq v - \frac{t}{N}$  the price set by the intermediary is  $p = v - \frac{t}{2N}$ . ■

This establishes the prices in the equilibrium described in Proposition 2.2. At the same time, it characterizes the reseller's optimal reaction to facing symmetric wholesale prices. Notice that the prices are such that given these wholesale prices, the retail prices are on the intermediary's reaction curve as if he would price each product individually. That is, the price is set according to the reaction function  $p(w) = \frac{v+w}{2}$ . When the circle is just covered, we are exactly at the border between individual pricing and pricing that is influenced by the existence of other products. Thus, when convenient, I can still treat each product price in this equilibrium as being set independent of neighboring prices.

To confirm that this is an equilibrium, I now check for unilateral deviations from the suppliers. First, because independent pricing of products would result in overlapping demands and, hence, even lower prices, a deviation to a higher wholesale price is not desirable for suppliers.

**Lemma 2.2.** In the symmetric equilibrium candidate, no supplier has an incentive to unilaterally raise the wholesale price.

*Proof.* A raise of the wholesale price would put the supplier in a situation of single-product pricing, since demand for its product is not affected by the demand for any other product. The optimal price in this case is as above  $w^m = \frac{v+c}{2}$ . However,  $\frac{v+c}{2} \geq v - \frac{t}{N} \iff N \leq 2\frac{t}{v-c}$ . But here we consider the case  $N > 2\frac{t}{v-c}$ . ■

If a supplier lowers her wholesale price, there is a complex reaction by the intermediary. Given symmetric retail prices, a lower wholesale price of one product improves the profit margin of that product relative to the others. The intermediary would thus like to shift demand towards this product by reducing its price. Further, once the price is reduced and demand of that product expanded, the price of the other (neighboring) products could be raised to reclaim rent given up to the indifferent consumer. Some of the consumers pushed away by this increase would further move their demand to the cheaper product. The downside is that some consumers will no longer buy anything.

Label the product by the deviating supplier under consideration generally as product 1. Consider a unilateral decrease in the wholesale price of product 1 to  $\tilde{w}_1 = v - \frac{t}{N} - \varepsilon$ ,  $\varepsilon > 0$ . When this happens, both neighboring products are affected in the same way. Thus, label them both product 2.

In reaction to a decrease of  $w_1$ , the intermediary will not increase  $p_1$ . It is also intuitive that  $p_2$  will not decrease following a decrease of  $w_1$ . This would steal demand from the cheaper product 1 and at the same time give up rent to the indifferent consumer located on the other side of product 2. If  $p_2$  rises, products on the other side of product 2 are not affected, since they are priced on the individual-pricing response curve. Their price was optimal without being influenced by  $p_2$ , and since  $p_2$  is not lowered, there will still not be an interaction.

The reaction of the intermediary to a decrease in  $w_1$  is given by the following result. The proof is in the Appendix.

**Lemma 2.3.** After a small unilateral deviation of supplier 1 to  $\tilde{w}_1 = v - \frac{t}{N} - \varepsilon$ ,  $\varepsilon > 0$ , the reseller's reaction is

$$\tilde{p}_1 = v - \frac{t}{2N} - \frac{\varepsilon}{6} \quad \text{and} \quad \tilde{p}_2 = v - \frac{t}{2N} + \frac{\varepsilon}{6}.$$

Prices of the other products do not change. ◆

As the proof of the lemma shows, the intermediary will lower  $p_1$  and raise  $p_2$  in a way that there is exactly no gap in demand between the two products. Any gap would return product 2 to a situation of independent pricing, where a lower price is warranted. On the other hand, the intermediary still does not want to give up rent to the marginal consumer between the products. The prices then have a mechanical relationship with each other:  $p_1 = 2v - p_2 - \frac{t}{N}$ . Intuitively, the whole tradeoff between low prices and rent given up to consumers can be steered by deciding how many consumers will not be served on the other side of product 2.

With this response, demand for product 1 moves to  $D_1(\varepsilon) = \frac{v - v + \frac{1}{2}\frac{t}{N} + \frac{\varepsilon}{6}}{t} = \frac{1}{2N} + \frac{\varepsilon}{6t}$ , so supplier 1's optimization problem is  $\max_{\varepsilon} (v - \frac{t}{N} - \varepsilon - c) \left( \frac{1}{2N} + \frac{\varepsilon}{6t} \right)$ . The derivative with respect to  $\varepsilon$  is positive as long as  $\varepsilon < \frac{v-c}{2} - 2\frac{t}{N}$ . But the right-hand side of this expression is positive only when

$$v - c > 4\frac{t}{N}. \tag{2.1}$$

Thus, as we consider downward deviations ( $\varepsilon > 0$ ), there is no profitable deviation when  $v - c \leq 4\frac{t}{N}$ .

**Lemma 2.4.** In an exactly fully covered market, a marginal unilateral deviation in the wholesale price is unprofitable when  $v - c \leq 4\frac{t}{N} \iff N \leq 4\frac{t}{v-c}$ . ◆

This is the threshold that appears in Proposition 2.2. As can be seen, the price of the neighboring product is continuously raised. At some point, this price reaches  $v$  which means that there is no demand for product 2 any more, it is pushed out of the market. It needs to be checked that this does not open up the possibility for a large deviation that pushes competing products out of the market. This is not the case: supplier 1 does not benefit from the discontinuous jump in demand when a competing product is shut down. It is too expensive to lower the wholesale price far enough to push other products out. This also holds for full monopolization of the market. The proof is again found in the Appendix.

**Lemma 2.5.** If an initial marginal deviation by a supplier is not profitable, neither is pushing products out of the market. This also holds for full monopolization: lowering the wholesale price to be the only remaining product offered is not profitable. ◆

This establishes Proposition 2.2. In this equilibrium, the profit of a supplier is

$$\pi^{oW} = \left( v - \frac{t}{N} - c \right) \frac{1}{N} = \frac{v - c}{N} - \frac{t}{N^2},$$

the manufacturing industry as a whole earns

$$N\pi^{oW} = v - c - \frac{t}{N},$$

intermediary profit is

$$\Pi^{oW} = N \left( v - \frac{t}{2N} - v + \frac{t}{N} \right) \frac{1}{N} = \frac{t}{2N},$$

and consumer surplus is

$$CS^{oW} = N \cdot 2 \cdot \frac{1}{2} \cdot \left( v - v + \frac{t}{2N} \right) \cdot \frac{1}{2} \frac{1}{N} = \frac{t}{4N}.$$

### 2.3.3 The intermediary's provision of product variety

When the number of products is small, the market is not fully covered. The situation is then the standard one of a vertical chain of monopolies for each product. Each good's pricing upstream and downstream is independent of the presence of the other products and the intermediary makes a constant profit from each product. The situation becomes different once the number of products increases. At the optimal prices, the market is now covered and product prices depend on each other. Adding a new product no longer contributes a constant profit for the intermediary, but its presence affects the profit margin of all products. Remarkably, there is an intermediate range for the value of  $N$  for which the intermediary's profit is declining in the number of products. For  $N \in \left[ 2\frac{t}{v-c}, 4\frac{t}{v-c} \right]$ , the intermediary's profit is  $\frac{t}{2N}$  which is strictly decreasing in  $N$ . Because for  $N \in \left[ 0, 2\frac{t}{v-c} \right]$ , profit  $N\frac{(v-c)^2}{8t}$  is strictly increasing in  $N$ , the highest profit level for any  $N \in \left[ 0, 4\frac{t}{v-c} \right]$  is reached at  $N = 2\frac{t}{v-c}$ . Thus, there is a non-negligible range for  $N$  in which the intermediary limits the number of products he makes available to consumers.

**Proposition 2.3.** Suppose  $K \in \left( 2\frac{t}{v-c}, 4\frac{t}{v-c} \right]$ . The reseller chooses a unique optimal number of products,

$$N^* = 2\frac{t}{v-c} < K.^4$$

◆

The region of decreasing intermediary profit for an increasing number of firms highlights a central feature of the wholesale model on a Salop circle. Because the intermediary internalizes losses from product substitution, he substitutes products less aggressively following a wholesale price decrease. That is, he chooses retail prices such that the change in relative wholesale prices is not

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<sup>4</sup>To be precise, when  $2\frac{t}{v-c}$  is not an integer, he chooses among the two closest ones the one that gives higher profit.

completely passed on to the consumers. The consumers thus react with less substitution towards the cheaper product than if they saw the entire wholesale price decrease in their retail prices. When there are not too many products, competitive pressure is low enough that suppliers will not want to reduce their wholesale prices. The demand increase from a price decrease is too small relative to the loss of the price increase on the existing demand. Even though the suppliers cannot coordinate among themselves, they benefit from the intermediary's coordination motive. And because they are the first movers in the pricing game, the suppliers can extract the welfare increase that comes from increased variety. The intermediary cannot commit to exert higher competitive pressure on the suppliers and thus they set prices to implement intermediated exact coverage and maximal appropriation of consumer surplus in a covered market.

Proposition 2.3 shows that there is a range for the number of potentially available products  $K$  for which the intermediary limits product variety below what is possible. This occurs for  $K$  large enough (larger than  $2\frac{t}{v-c}$ ) until  $K$  reaches  $4\frac{t}{v-c}$ . To get a feeling for the relevance of this range, first notice that it is as wide as  $\left[0, 2\frac{t}{v-c}\right)$ , the range of  $N$  for which the market is not fully covered. This range, in turn, is not irrelevant, as incompletely covered markets, where consumers consume their outside option, are common. Second, consider the factors that affect this range. A high  $t$  or a low  $v - c$  widens the range  $\left[2\frac{t}{v-c}, 4\frac{t}{v-c}\right]$ . This is a consequence of the incentives to lower the wholesale price. When a supplier lowers her wholesale price by  $\varepsilon$ , her loss is  $\varepsilon$  on the existing demand. Her gain is the current profit margin minus  $\varepsilon$  on the new demand. The current profit margin is  $v - c - \frac{t}{N}$ . So, a high  $t$  or a low  $v - c$  lower this margin and make a reduction in wholesale price less attractive. The profit margin is the relevant quantity to consider here because the demand for each supplier in equilibrium is always  $1/N$ , independent of parameters. Both a higher  $t$  or a lower  $v - c$  indicate that the introduction of a product has a smaller welfare contribution. In these cases, the range where the reseller wishes to limit product availability is larger. At the same time, the choice  $N^*$  of the intermediary is larger when  $t$  is higher or  $v - c$  lower. If products are more differentiated or provide less gross value, it is harder to reach all consumers. In order to sustain higher prices, the intermediary thus wishes to carry more products.

## 2.3.4 Complete coverage with supplier competition

### 2.3.4.1 Wholesale prices under competitive pressure

Once  $N > 4\frac{t}{v-c}$ , the suppliers have an incentive to reduce their wholesale price and gain market share. The reaction of the reseller determines how much demand increase is induced by a price reduction. However, the reseller's reaction to a general wholesale-price vector with interdependent demands is hard to assess. Nonetheless, a few statements and conjectures can be made about the structure of such an equilibrium with supplier competition.

First, because the suppliers have an incentive to deviate downwards compared to the previous equilibrium on the range  $N \in \left[2\frac{t}{v-c}, 4\frac{t}{v-c}\right]$ , the symmetric wholesale price is lower than  $v - \frac{t}{N}$ . By Lemma 2.1, the intermediary will still set symmetric retail price  $v - \frac{t}{2N}$  to just cover the

market. That means that consumers do not benefit from any competitive pressure that pushes the suppliers to lower their wholesale prices.

Second, a conjecture about the form of the wholesale price can be made. In response to a difference in wholesale prices, the intermediary will change the relative retail prices of neighboring products to shift demand towards a newly cheaper product or away from a product if its wholesale price is higher than its competitors'. Suppose the intermediary reacts to differences in prices as he does to deviations from the price in the equilibrium of Proposition 2.2. That is, if supplier 1 sets wholesale price  $w_1 = w - \varepsilon$ , where  $w$  is the symmetric equilibrium wholesale price and  $\varepsilon$  can be positive or negative, then the intermediary sets retail price  $p_1 = v - \frac{t}{2N} - \frac{\varepsilon}{6}$ . For the deviating supplier 1, this translates into demand  $\frac{1}{2N} + \frac{\varepsilon}{6t}$  to either side of her product's location. This substitution structure has the same pattern as the substitution by consumers in non-intermediated competition on the Salop circle. In the non-intermediated case, the competitive equilibrium price is  $c + \frac{t}{N}$ . I thus conjecture that the equilibrium wholesale price when suppliers compete has a similar structure. In particular, consider product 1 and its neighbors 2 and 2'. Assume the demand substitution induced by the intermediary is such that for wholesale prices  $(w_i)_{i=1,2,2'}$ , demand for product 1 is given by the expression  $\frac{1}{2N} + \frac{w_2 - w_1}{6t} + \frac{1}{2N} + \frac{w_{2'} - w_1}{6t} = \frac{1}{N} + \frac{w_2 + w_{2'} - 2w_1}{6t}$ . Looking for a symmetric equilibrium yields  $w = c + 3\frac{t}{N}$ . This has the same structure as the non-intermediated Salop equilibrium: marginal cost and a markup that increases in the differentiation parameter and decreases in the number of firms. The price  $c + 3\frac{t}{N}$  is higher than the competitive price  $c + \frac{t}{N}$ . Because the intermediary internalizes the demand losses on another product when lowering a given product's price, he reacts less aggressively with demand shifts to changes in relative prices than consumers do.

With this conjectured wholesale price for  $N > 4\frac{t}{v-c}$  and retail price  $p = v - \frac{t}{2N}$ , the intermediary's profit is  $\Pi = v - c - \frac{7}{2}\frac{t}{N}$ . He would thus again prefer to carry as many products as possible. The suppliers in this case do not benefit from their first-mover position, as they compete with each other. The intermediary can thus play the suppliers off against each other and reap the surplus increases from increased variety. If these conjectured prices hold, the intermediary's profit reaches its level from  $N = 2\frac{t}{v-c}$  again when  $N = \frac{14}{3}\frac{t}{v-c}$ . Proposition 2.3 could thus be amended to say that for each  $K \in (2\frac{t}{v-c}, \frac{14}{3}\frac{t}{v-c})$ , the intermediary in the wholesale mode limits product availability. It could then also be noted that, concerning the relevance of this range, it is longer than  $[0, 2\frac{t}{v-c})$  where some consumers consume their outside option.

### 2.3.4.2 Discussion

Propositions 2.1 and 2.2 provide a characterization of the wholesale mode with products located on a Salop circle. They address pricing when product demands are independent and, crucially, when the intermediary reseller jointly prices product with interdependent demand. Proposition 2.2 shows a way how the reseller substitutes products when he faces different wholesale prices. The proposition also treats the demand discontinuity when a product lowers its price so much so that its neighbor drops out of the market. Unfortunately, the proposition does not characterize the reseller's reaction to a general vector of asymmetric wholesale prices, but only to deviations

from a specific symmetric wholesale price. I can thus only conjecture how an equilibrium looks after the suppliers deviate from the exact-coverage wholesale price  $v - \frac{t}{N}$ . Other papers have analyzed similar problems under additional assumptions. Johnson (2018) takes the symmetric wholesale price as a parameter and considers the intermediaries' reactions under the different intermediation modes. Reisinger and Schnitzer (2012) model upstream and downstream firms both being located on Salop circles. They assume that upstream firms do not know the location of downstream firms and thus have to form expectations about the latter's preferences. This eliminates demand discontinuities for the upstream firms.

I am assuming that with any  $N$  chosen, the products are always equally spaced on the circle. The assumption of equally distributed products along the circle (or Hotelling line) is standard in the literature. It is a consequence of the principle of maximum differentiation introduced by d'Aspremont, Gabszewicz, and Thisse (1979). This principle was shown to hold for more than two firms by Economides (1989). That paper establishes the existence of symmetric subgame-perfect equilibria where products are equally spaced. In the context of my model, a subgame-perfect equilibrium has the intermediary pick  $N$  firms and the suppliers then position their products accordingly.

## 2.4 Agency model with a per-transaction fee

I now move to the investigation of pricing and variety choice under the agency mode. In this mode, the intermediary enables transactions between suppliers and final consumers and charges a fee for it. The price the consumers pay, however, is set by the suppliers. I will consider two subforms of this mode: one with a fixed lump-sum fee per transaction and one with an ad valorem fee. In this section, I analyze the per-transaction fee.

The intermediary sets a fee per transaction with a fixed value  $r > 0$ . As Rochet and Tirole (2006) show, the allocation of the total per-transaction fee between two sides of the market (here: suppliers and consumers) is irrelevant when there are payments from one side to the other. I can thus focus on a per-transaction fee that is levied on the seller. I assume the intermediary does not charge an access fee.<sup>5</sup> The timing is as follows: First, the intermediary chooses to allow  $N$  firms on his platform. Second, he sets the per-transaction fee  $r$ . Third, suppliers set consumer prices ( $p_i$ ). Fourth, consumers decide which product to buy or to consume their outside option.

### 2.4.1 Incomplete coverage

Demand in the case of incomplete coverage again follows the monopolistic single-product expression:  $D^{ic}(p) = \frac{v-p}{t}$ . Supplier  $i$  in this case maximizes

$$\pi(p_i) = (p_i - c - r)2\frac{v - p_i}{t}.$$

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<sup>5</sup>This assumption is made to ensure comparability with the wholesale model where only linear prices, i.e. no two-part tariffs, were allowed.

Taking the first-order condition, the result is  $p_i = \frac{v+c+r}{2}$ . As all suppliers are the same and face the same fee  $r$ , they all set price  $p(r) = \frac{v+c+r}{2}$ . The circle is then not fully covered when  $2\frac{v-p}{t} \leq \frac{1}{N} \iff v - c - r \leq \frac{t}{N}$ .

The intermediary maximizes his profit via the choice of  $r$  by optimizing

$$N2\frac{v-p(r)}{t} \cdot r.$$

The first-order condition with respect to  $r$  gives

$$r^{mA} = \frac{v-c}{2}.$$

The superscript  $mA$  designates individual monopoly pricing in the agency mode.

The resulting symmetric retail price is

$$p = \frac{3v+c}{4}$$

and the condition for incomplete coverage becomes  $v - c - \frac{v-c}{2} \leq \frac{t}{N} \iff N \leq 2\frac{t}{v-c}$ .

**Proposition 2.4.** Consider the case  $N \leq 2\frac{t}{v-c}$ . The unique equilibrium of the agency model has incomplete coverage and prices are

$$p^{mA} = \frac{3v+c}{4}.$$

The fee set by the intermediary is

$$r^{mA} = \frac{v-c}{2}.$$

◆

The intermediary and supplier profits are  $\Pi^{mA} = N\frac{(v-c)^2}{4t}$  and  $\pi^{mA} = \frac{(v-c)^2}{8t}$ . The supplying industry as a whole thus earns  $N\pi^{mA} = N\frac{(v-c)^2}{8t}$ . Consumer surplus is  $CS^{mA} = N\frac{(v-c)^2}{16t}$ .

Notice that when we have a platform intermediary that charges fixed per-transaction fees, the results are equivalent to the equilibrium in the wholesale model, just with switched attribution of profits to suppliers and intermediary. This is because the fixed per-transaction fee works like a wholesale price per unit and in the vertical chain of monopolies, the final prices equilibrate in the same way as in the wholesale model. This finding is mirrored in the approach in Johnson (2017b) who generally considers one first and one second mover who can, in applications, have varying identities.

### 2.4.2 Complete coverage

Suppose now that we are in the case  $N \geq 2\frac{t}{v-c}$ . Isolated pricing for each product would lead to overlapping demand and thus the circle will be covered. This situation between suppliers



is the same as classical competition on a Salop circle with the addition that the intermediary charges fees. The indifferent consumer between two neighboring products  $i$  and  $j$  is given by  $d_{ix} = \frac{p_j - p_i + \frac{t}{N}}{2t}$ . Thus, firm  $i$  with neighbors  $j$  and  $k$  solves  $\max_{p_i} (p_i - c - r) \left( \frac{p_j - p_i + \frac{t}{N}}{2t} + \frac{p_k - p_i + \frac{t}{N}}{2t} \right)$ . Taking first-order conditions and looking for a symmetric equilibrium, we obtain  $p = c + r + \frac{t}{N}$ . This is the standard result from the Salop circle model. The fee acts as an additional cost per unit for the product. This fee, like the production cost, is completely passed through to the consumers. Demand for each product is  $\frac{1}{N}$ .

How can the intermediary set  $r$ ? At  $r = 0$ , a consumer indifferent between two products has utility  $U = v - c - \frac{t}{N} - t \frac{1}{2} \frac{1}{N} = v - c - \frac{3}{2} \frac{t}{N} > 0$ . Under the premises of the Salop competition model, the highest  $r$  which the intermediary can set while still maintaining full coverage is thus  $r = v - c - \frac{3}{2} \frac{t}{N}$ .

However, when there is full coverage, demand is 1 from the intermediary's view. Given this demand level, the platform would like to set the fee as high as possible. Recall that under the assumption of incomplete coverage, the circle is not fully covered when  $r > v - c - \frac{t}{N}$ . Thus, at  $r = v - c - \frac{t}{N}$ , the circle is just covered and lowering  $r$  does not raise demand for the platform. As  $v - c - \frac{t}{N} > v - c - \frac{3}{2} \frac{t}{N}$ , the intermediary sets  $r = v - c - \frac{t}{N}$  and achieves demand level of one. The intermediary does not choose the optimal fee from the incomplete coverage now, because  $\frac{v-c}{2} \leq v - c - \frac{t}{N} \iff N \geq 2 \frac{t}{v-c}$ . Thus, when this condition holds, the platform sets a fee just cover the market under individual pricing of products. To compare with the wholesale mode, I designate the fee and price with superscript  $oA$  to describe an oligopoly in the agency mode.

**Proposition 2.5.** Consider the case  $N \geq 2 \frac{t}{v-c}$ . The intermediary sets

$$r^{oA} = v - c - \frac{t}{N}$$

and resulting prices are

$$p^{oA} = v - \frac{t}{2N}.$$

◆

Each supplier's profit is  $\pi^{oA} = \frac{t}{2N^2}$  and the supplying industry as a whole realizes profit  $N(\pi^{oA} = \frac{t}{2N^2} = \frac{t}{2N})$ . The intermediary's profit is  $\Pi^{oA} = v - c - \frac{t}{N}$ . In the agency model, prices still increase with the number of products. Through the fee, the intermediary appropriates the decreases in total transportation cost. As in the wholesale mode, consumer surplus is  $CS_{Ac} = \frac{t}{4N}$ .

### 2.4.3 Determinants of product range

#### 2.4.3.1 Choice of product range

Although the agency model with a per-transaction fee looks like the wholesale model in terms of outcomes, there is a marked difference for the product range choice of the intermediary. His profit is strictly increasing in  $N$  for all levels of  $N$ . The intermediary is now able to always appropriate the increased surplus from more product variety and thus wishes to carry all available products.

**Proposition 2.6.** In the agency model with a per-transaction fee, the intermediary always chooses  $N^* = K$ . ◆

The intermediary is now the first mover by setting the transaction fee. With this fee, he decides how much of the transaction surplus will be paid to him. He benefits from being the first mover in the same way the suppliers benefitted from it in the wholesale mode for an intermediate range of  $N$ . But because the intermediary is a monopolist, he faces no competitive pressure and will keep the per-transaction price high for all  $N$  such that the market is covered. The discussion leading up to Proposition 2.5 shows that the intermediary prevents the suppliers from setting competitive prices. This allows him to charge a higher per-transaction price to appropriate the welfare gains.

This result illustrates a meaningful difference between the wholesale and the agency mode. It is not about pricing as consumer prices remain the same for a given  $N$ . The monopolistic intermediary, whether he is a reseller or an agent, achieves coordination on the most profitable retail prices for the industry. But the distribution of profits within the supplying vertical chain is affected by the intermediation mode. As Johnson (2017b) shows, the wholesale mode and the agency mode with a per-transaction fee share the feature that one party sets the “access price” for each transaction. It can then be analyzed whether the first mover, that is the party that sets the price, benefits from this position. In the present case, this has implications on the available product range. The first mover can appropriate more of the added surplus from increased product variety and thus benefits from a larger product offer. The second mover loses out. Thus, if the intermediary is the second mover, he will limit product availability. This holds in the setup of the present model at least until the competitive forces for the supplying industry push them to lower prices. Thus, while consumer prices may be different under the different modes, this is not a consequence of different pricing, but comes as a result of the differing product range offered.

#### 2.4.3.2 Fixed cost

Could there be other factors that put different limits on product variety according to the different modes? To assess this, consider costly entry. Suppose that in order to start producing her good,

a manufacturer has to incur a fixed cost  $F > 0$ . Entry of suppliers occurs until the subsequent profits no longer cover the fixed cost. For the agency mode with full coverage, the number of firms is determined through  $\frac{t}{2N^2} = F \iff N_{Ac} = \sqrt{\frac{t}{2F}}$ . The threshold to partial coverage is at  $N = \frac{2t}{v-c}$  which is reached when  $\sqrt{\frac{t}{2F}} = \frac{2t}{v-c} \iff F = \frac{(v-c)^2}{8t}$ . For values of  $F$  bigger than this, there would be partial coverage. But under partial coverage, manufacturers make profit  $\pi_{Aic} = \frac{(v-c)^2}{8t}$ . Thus, when the fixed cost is so high that the market is not fully covered, starting production is actually unprofitable. In the presence of fixed costs, there will be no partially covered market in the agency model with fixed per-transaction fees.

We can now compare the number of firms in the agency model with that in the reseller model. Suppose that  $K$  is in the appropriate range so that  $N^* = \frac{2t}{v-c}$  in the wholesale mode. We have just seen that the number of firms in the agency model is larger than this when  $F \leq \frac{(v-c)^2}{8t}$ . When the fixed cost is higher than this, there are no firms producing in the agency model. Since manufacturer profit in the reseller model is  $\pi_R = \frac{(v-c)^2}{4t}$ , there still are firms producing in the reseller model at this level of fixed cost. Hence, for  $F \leq \frac{(v-c)^2}{8t}$ , when production is active under the agency model, there are more products on the market in the agency model. For  $F \in \left(\frac{(v-c)^2}{8t}, \frac{(v-c)^2}{4t}\right]$ , production is only active under the reseller model. As in both models with full coverage, market demand is one and market price is  $v - \frac{t}{2N}$ , prices are higher under the agency model and consumer surplus is lower.

**Proposition 2.7.** Suppose manufacturers incur a fixed cost to set up production of  $F > 0$ . Further suppose  $K \in (2\frac{t}{v-c}, \frac{14}{3}\frac{t}{v-c})$ . For low levels of  $F$ , the number of firms is higher under the agency mode than under the reseller mode. Market prices are higher under the agency mode and consumer surplus is lower. For high levels of  $F$ , production under the agency model is infeasible while it is still active under the reseller mode.  $\blacklozenge$

## 2.5 Agency model with proportional fee

A feature of many platform markets is that the platform charges a fee that is a proportion of the transaction price. These so-called ad valorem fees are desirable for many reasons. Anderson, de Palma, and Kreider (2001) report that under imperfect competition, ad valorem taxes are welfare superior to per-unit taxes when firms have common costs. Gaudin and White (2014) analyze a similar model and find that prices are lower under ad valorem taxes set by a government for competing firms. In Shy and Wang (2011), ad valorem fees yield higher profits to the payment card provider than per-transaction fees. In this section, I provide some results on the agency model with proportional fees.

Suppose the intermediary leaves a fraction  $r \in (0, 1)$  of the retail price to the supplier and keeps fraction  $1 - r$  of the retail price. The suppliers continue to set the retail price. Thus, first the platform decides on what fraction  $r$  to offer to suppliers and second, suppliers decide on the retail price. Suppose further for the remainder of this section that  $c > 0$ . This causes a wedge between the share of revenue and costs that accrue to the suppliers. While the suppliers

obtain only a fraction of the revenue, they have to incur the entire cost of production. If this wedge was not present, the suppliers would set prices like an integrated firm. The presence of this distortion gives bite to the comparison with the previous modes which involve double marginalization.

### 2.5.1 Incomplete coverage

Under incomplete coverage, each supplier is treated individually. Suppliers choose  $p$  to maximize  $(rp - c)\frac{v-p}{t}$  which leads to price

$$p(r) = \frac{v}{2} + \frac{c}{2r}. \quad (2.2)$$

Foreseeing this, the intermediary chooses  $r$  to maximize  $(1-r)p(r)D(p(r)) = (1-r)\left(\frac{v}{2} + \frac{c}{2r}\right)\frac{v-\frac{v}{2}-\frac{c}{2r}}{t}$ . The optimal  $r^*$  satisfies

$$2c^2 - (r^*)^3v^2 - r^*c^2 = 0.$$

This condition has a unique solution for  $r$  and thus there is a unique  $r^{mP}$  set by the intermediary, where the  $P$  in the superscript designates the proportional-fee mode.

**Lemma 2.6.** There is a unique solution  $r^{mP}(c, v)$  to the maximization problem of the intermediary. The optimal revenue share  $r^{mP}(c, v)$  of the price passed on to suppliers is increasing in  $c$  and decreasing in  $v$ .

*Proof.* The derivative of the firm's profit with respect to  $r$  is  $\frac{2\frac{1-r}{r}c^2 - r^2v^2 + c^2}{4r^2t}$ . Setting this equal to zero yields the condition

$$2c^2 - (r^*)^3v^2 - r^*c^2 = 0. \quad (2.3)$$

At  $r = 0$ , we have  $2c^2 > 0$ . At  $r = 1$ , we have  $c^2 - v^2 = (c+v)(c-v) < 0$ . Because the left-hand side of (2.3) is strictly decreasing in  $r$ , there exists a unique  $r^*(c, v) \in (0, 1)$  that satisfies (2.3).

By the implicit function theorem, if the partial derivative of  $2c^2 - (r^*)^3v^2 - r^*c^2 = 0$  with respect to  $r$  is nonzero, the function  $r^*(c, v)$  is differentiable. The derivative of the the left-hand side of (2.3) is  $-3r^2v^2 - c^2$  which is negative except if either  $(c, v) = (0, 0)$  or  $(c, r) = (0, 0)$ , both of which are not relevant for the analysis. From implicit differentiation,  $\frac{\partial r^*(c, v)}{\partial c} = \frac{4c - 2rc}{3r^2v^2 + c^2} > 0$ , since  $r \in (0, 1)$ , and  $\frac{\partial r^*(c, v)}{\partial v} = -\frac{2r^3v}{3r^2v^2 + c^2} < 0$ . ■

The double-marginalization problem is mitigated. The price consumers face is lower than with a per-transaction fee (or in the wholesale mode). To see this, notice that the value of  $r$  that would lead the suppliers to set the double-marginalization monopoly price of  $\frac{3v+c}{4}$  is  $\frac{2c}{v+c}$ . But entering this value into the profit derivative of the intermediary with respect to  $r$ , it takes the value  $\frac{2c^2(v-c)^2v}{(v+c)^3} > 0$ . Thus, the intermediary raises  $r$  which lowers the retail price.

**Lemma 2.7.** The retail price under incomplete coverage in the agency mode with a proportional fee is strictly lower than  $\frac{3v+c}{4}$ , that is strictly lower than in the wholesale mode and the

agency mode with per-transaction fee. ◆

## 2.5.2 Complete coverage

Complete market coverage is reached when the price set by the suppliers is equal to or lower than  $v - \frac{t}{2N}$ . For any given  $r$  set by the intermediary, this is the case once  $N \geq \frac{t}{v - \frac{c}{r}}$ . Rearranging this condition, we see that at the number of products  $N$  where exact coverage is reached, the revenue share is  $r = \frac{c}{v - \frac{t}{N}}$ . Call the number of products where complete coverage is reached  $\hat{N}$ . Thus, given the optimal  $r^{mP}$  in incomplete coverage, the point where complete coverage is just reached satisfies  $r^{mP} = \frac{c}{v - \frac{t}{\hat{N}}}$ . As  $N$  increases beyond  $\hat{N}$ , and the revenue share is kept at  $r^{mP}$ , demand for each product will decrease and thus the suppliers compete with each other. When suppliers set competitive prices, they solve

$$\max_p (rp - c) \left( \frac{1}{N} + \frac{p_- + p_+ - 2p}{2t} \right)$$

where  $p_-$  and  $p_+$  designate the prices of the products to the left and the right of the supplier in question. Looking for a symmetric equilibrium returns

$$p = \frac{c}{r} + \frac{t}{N}.$$

Now, we have  $\frac{c}{r} + \frac{t}{N} \leq v - \frac{t}{2N} \iff r \geq \frac{c}{v - \frac{3}{2}\frac{t}{N}}$ . If  $r$  is lower than this threshold, we will not be in the situation of complete coverage, and the analysis from the previous section applies. Now, notice that  $\frac{c}{v - \frac{3}{2}\frac{t}{N}} > \frac{c}{v - \frac{t}{N}}$ . That is, if the intermediary allows supplier competition, he retains a lower share of a (weakly) lower price. Because if he sets  $r = \frac{c}{v - \frac{t}{N}}$ , the suppliers price their products according to (2.2) and set price  $v - \frac{t}{2N}$ . This is the highest price consistent with complete coverage, and the intermediary keeps a larger share of it than under supplier competition. Thus, when  $N \geq \hat{N}$ , the intermediary sets revenue share

$$r^{oP} = \frac{c}{v - \frac{t}{N}}. \quad (2.4)$$

Since a price decrease by suppliers makes demands now interdependent, it needs to be checked that no supplier wants to deviate downward. If a supplier deviated downward, demand for her product would be given by the standard Salop expression. To check the profitability of a downward deviation, the supplier's profit with Salop demand thus needs to be considered. Given (2.4) and rival prices of  $v - \frac{t}{2N}$ , the deviating supplier solves

$$\max_p \left( \frac{c}{v - \frac{t}{N}} p - v \right) \left( \frac{1}{N} + \frac{2(v - \frac{t}{2N}) - 2p}{2t} \right)$$

where the expression for demand follows from the Salop model when products are substitutes. The solution to this is  $p = v - \frac{t}{4N} > v - \frac{t}{2N}$ . Thus, there is no profitable downward devia-

tion.<sup>6</sup>

**Proposition 2.8.** In the agency mode with a proportional fee, under complete coverage, the intermediary sets revenue share

$$r^{oP} = \frac{c}{v - \frac{t}{N}}$$

and the suppliers each set price

$$p^{oP} = v - \frac{t}{2N}.$$

◆

### 2.5.3 Profits under incomplete and complete coverage, and product range

Under incomplete coverage, each supplier makes profit

$$\pi^{mP} = 2 \left( r^{mP} \frac{v + \frac{c}{r^{mP}}}{2} - c \right) \frac{v - \frac{c}{r^{mP}}}{2t} = r^{mP} \frac{\left( v - \frac{c}{r^{mP}} \right)^2}{2t}.$$

The intermediary's profit is

$$\Pi^{mP} = N(1 - r^{mP}) 2 \frac{v + \frac{c}{r^{mP}}}{2} \frac{v - \frac{c}{r^{mP}}}{2t} = N(1 - r^*) \frac{\left( v + \frac{c}{r^{mP}} \right) \left( v - \frac{c}{r^{mP}} \right)}{2t}.$$

Obviously, the intermediary's profit is rising in  $N$ , because each additional supplier contributes profit. Both sides' profits decrease in the differentiation parameter  $t$  because an increase in  $t$  decreases the number of consumers who buy at a given price. As both sides obtain a proportion of profits, this hurts both of them. The same holds for the production cost  $c$ , an increase of which reduces monopoly profits.

When  $N$  is large enough such that full coverage is induced, profits of suppliers are

$$\pi^{oP} = \frac{c}{v - \frac{t}{N}} \frac{t}{2N^2}$$

and the intermediary's profit is

$$\Pi^{oP} = \frac{v - \frac{t}{2N}}{v - \frac{t}{N}} \left( v - c - \frac{t}{N} \right).$$

Notice that these profits scale the profits made under a per-transaction fee. For the sellers, the profit is  $r^{oP} \pi^{oA} < \pi^{oA}$ . Profit of the intermediary now is  $\frac{v - \frac{t}{2N}}{v - \frac{t}{N}} \Pi^{oA} > \Pi^{oA}$ . The intermediary thus benefits from the move to a proportional fee. This finding mirrors the results in the literature on proportional versus per-transaction fees like in, for example, Shy and Wang (2011).

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<sup>6</sup>The given demand expression does not apply if the supplier raised her price, because she would then cause a gap between hers and neighboring products. This means the monopoly demand function for her product would be appropriate. The fact that  $v - \frac{t}{4N} > v - \frac{t}{2N}$  thus does not indicate that a price increase is profitable.

With a proportional fee, the intermediary also always chooses to carry all available products. Just like with the per-transaction fee, the intermediary keeps the suppliers from competing in their prices. They set prices according to their independent-product monopoly pricing function so that the market is just covered. The intermediary charges a relatively high fee, so that undercutting is not profitable.

## 2.6 Conclusion

I analyze the effects of intermediation on the available product range in a discrete-choice model of product differentiation. With differentiated products on a Salop circle, I highlight an important difference between the wholesale and the agency mode when there is a monopolistic intermediary and an oligopolistic supplying industry. In the wholesale mode, there is a range of the number of generally available products over which the intermediary limits the number of products he carries. In contrast, in the agency mode, the intermediary would always like to carry all available products.

A lot of the discussion on the agency versus the wholesale model centers on the question of consumer prices. Because in the agency mode, suppliers set the retail prices, different prices might be expected than in the wholesale mode where the intermediary prices all products jointly. This paper shows that there is a different angle that is relevant in the distinction between the two modes. They differ in the split of welfare gains from the offered product range among the suppliers and the intermediary. Whereas, when the market is covered, products are priced in the same way in all modes, differences in the number of offered products may arise (and bring price differences with them). The relevant difference between the two modes that causes these different outcomes is not the authority over the final consumer price, but the move order in the chain. Crucially, the first-mover advantage also applies when the first mover is the oligopolistic supplying industry in the wholesale mode, as long as competitive pressure is not too high. Because the monopolistic intermediary internalizes losses from product substitution, he reduces competitive pressure for the suppliers. The supplying industry then can fully benefit from its first-mover advantage and appropriate the increased surplus. The intermediary, in turn, wishes to reduce the number of products to give away less of the surplus. Interestingly, consumers benefit from this. They have a smaller product offering, but the lower prices overcompensate. The wholesale mode thus presents a case where the monopoly power of the intermediary to extract surplus from consumers is limited.

The Salop model is well suited for this analysis as total demand is constant once the market is covered. Product variety and its welfare consequences can then concisely be analyzed by looking at the number of firms in the market. It has the drawback that the general wholesale equilibrium with interdependent demands is hard to characterize. I have given conjectures on the shape of this equilibrium that hint to the case that eventually the supplying industry loses its first-mover advantage due to competitive pressure. Another question that can be asked concerns competition on the intermediary level. In the present paper, I have analyzed the polar case of a monopolistic intermediary, who deals with a larger number of suppliers. It shows that this

monopolist, who decides over the product variety available to consumers, may have an incentive to act as a bottleneck and limit product variety, but not in order to exploit consumers.







## 3 Vertical coordination through relational contracts and downstream transfers<sup>1</sup>

### 3.1 Introduction

Trade in vertical relationships commonly suffers from the double-marginalization problem. Because each layer of the vertical chain optimizes its own profit, a distortion is added at every step. These accumulated distortions prevent the industry from maximizing its total profit. There are several ways that this problem is usually addressed. Franchising contracts that involve a lump-sum payment from the buyer to the seller eliminate the distortion on the first level. Vertical restraints like the imposition of a specific amount of purchased quantity by the buyer address the distortion from the decisions on the second level. Finally, vertical integration is a direct remedy to the problem and benchmark for the preceding instruments.

In this paper, I will analyze a setup of repeated interaction which supports contracts that improve vertical efficiency. I assume that lump-sum payments from the buyer to the seller are infeasible. The seller, however, can pay lump-sum payments up front to the buyer. These downstream payments, together with the promise of an ongoing relationship, can support outcomes that mitigate double marginalization. I determine the provisions of such a relational contract and in particular, I ask when a lump-sum payment from the seller to the buyer will be made. The seller will be endowed with all the bargaining power and design the terms of the contract. When the seller interacts with one buyer, she will find it optimal to not pay a fixed transfer, but to use the promise of future trade that is more efficient than the static outcome. This promise induces the buyer to buy an amount of the good that is larger than his reply in the one-shot game. I subsequently consider the situation of a single seller who deals with more than one buyer. To give a role to downstream transfers, I consider situations where the seller has to set a common wholesale price to all buyers. The transfers then arise as a mechanism to provide incentives in the relational contract and improve vertical efficiency as well as implement price discrimination between buyers. Downstream payments affect the effective wholesale price that a buyer pays. At the same time, the threat of their withdrawal induces the buyers to order a more efficient quantity.

This paper thus provides an analysis of repeated interaction in vertical relationships. Considering the instruments of a list price and a downstream transfer, I determine how these instruments are used to improve efficiency. The two instruments have different effects on trade in a repeated

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<sup>1</sup>Cite as Christian Oertel (2019): “Vertical coordination through relational contracts and downstream transfers”, Mimeo.

vertical relationship. In order to make a profit, the seller has to set the list price above her marginal cost. This means however, that the buyer has to be compensated if he is to purchase a quantity close to the efficient one. A lower list price moves the one-shot optimal purchasing quantity of the buyer closer to the efficient one and he needs less compensation. How this tradeoff is resolved illustrates the use of different instruments in vertical contracting. When dealing with a single buyer, the seller chooses to minimize deviation incentives by moving the wholesale price towards marginal cost. When dealing with several buyers, the additional instrument of a transfer may be used to differentiate. Crucially, I describe the repeated interaction for all discount factors and show how the instruments permit to move towards efficiency.

While generally considering repeated vertical contracting, my model provides a rationale for the existence of so-called *slotting allowances*. Fixed monetary transfers to downstream buyers are a common occurrence in retail contracts, unlike franchise fees which the theoretical literature commonly invokes to eliminate double marginalization. A large empirical literature analyzes wholesale contracts in vertical chains. Recent papers focus on linear contracts that are augmented by payments to the buyer. For example, Hristakeva (2018) reports that she restricts attention to slotting allowances and linear-price wholesale contracts because franchise fees are not observed in the retail industry. Similarly, Noton and Elberg (2018) report that retail contracts consist of two types of arrangements: one specifying per-unit wholesale prices and one specifying payments to the retailer. In addition, Lafontaine and Shaw (1999), p. 1044, report for industries where franchise fees are observed that “[although] franchise fees receive a lot of attention in theoretical models, in reality, they usually represent a fairly small proportion of the total amount paid by franchisees to franchisors.” They place this proportion at around 8%.

I show that slotting fees can have an efficiency-enhancing motive. They appear in my analysis in a setup where the seller faces two buyers and has to set a common wholesale price. The requirement of a common wholesale price can result from the institutional environment. For example, the European Commission is committed to defending a single market and thus regularly investigates and prohibits differential pricing across regions or the upholding of barriers to arbitrage. Such a barrier that routinely attracts scrutiny from the Commission is a ban on parallel imports. In the United States, the Robinson-Patman Act imposes limits on allowed discriminatory pricing. More generally, the contractual form I analyze has appeal because it is simple. The seller sets a list price and then grants one-time payments to improve trade with specific buyers. In this simple contract, differentiation and improvement of vertical efficiency at the same time are executed via the downstream payment. In many industries, it may also be too burdensome to write contracts with many potential small buyers. I show that a manufacturer can improve efficiency beyond price discrimination by setting a list price and implementing a relational contract with only some of her buyers.

Payments to downstream parties appear clearly in the form of the classic slotting allowance: lump-sum payments from a manufacturer to a retailer in return for the retailer to stock the manufacturer’s product. But also pay-to-stay fees, rewards for specific in-store placements, or other benefits such as direct delivery constitute transfers to retailers that are independent of quantities ordered. In a more general way, vertical policies such as exclusive territories or lump-

sum rebates can generate transfers to downstream firms. Thus, lump-sum payments in models of vertical relationships can stand in for very general agreements to transfer rents to downstream parties.<sup>2</sup> While the discussion on slotting allowances often centers on retailer bargaining power, in my model, it is the manufacturers who have bargaining power. An instrument that looks like an access price set by the retailers, can actually be a means for a manufacturer to implement vertical control.

In the baseline setup of my model, a monopolistic manufacturer trades with a monopolistic retailer. The retailer provides market access for the manufacturer's product. He buys the product from the manufacturer and resells it to final consumers. In the one-shot game, this corresponds to the standard double-marginalization setup as described by Spengler (1950). But parties interact repeatedly. The manufacturer can now design a contract that sets a per-unit price and promises a transfer to the buyer. In turn, the contract designates a quantity that the buyer should order. Because I do not allow lump-sum payments to the seller, such as franchise fees, the wholesale price lies above marginal cost. Hence, the quantity specified by the contract will be distorted from the retailer's point of view. In order to increase efficiency, he will buy more than his one-shot optimum given the specified wholesale price. The incentives provided by the manufacturer are two-fold. One is the threat of a return to the less efficient outcome of the static game, for the second part the manufacturer will lower the wholesale price below the static optimum. In the interaction with one buyer, no transfer is thus paid by the manufacturer. When more markets with monopolistic retailers are added, a price discrimination motive is added to the efficiency improvement in bilateral trade. I show first that the manufacturer can implement and improve upon her outcome under third-degree price discrimination with linear prices by employing lump-sum transfers. The general problem with several markets, however, is very complex. In brief, at least one buyer will still not be paid a transfer, but since the wholesale price cannot be lowered to the optimal level for the remaining buyers, these will be paid transfers in order to lower their effective wholesale prices. To depict the emergence of downstream transfers in a tractable way, I consider the situation of two markets, the buyer in one of which is short lived and hence acts purely myopic. This buyer will then never be paid a transfer because it cannot have an incentive effect. Because the short-lived buyer buys according to his optimal demand given the wholesale price, this price now cannot be too low. If lowering the wholesale price would result in too much of a profit loss for the manufacturer from the trade with the myopic buyer, she resorts to paying the long-lived buyer a lump-sum transfer. The transfer will be withdrawn if the long-lived buyer deviates from buying the assigned quantity. Thus, the lump-sum transfer to a buyer has an efficiency-enhancing effect in the bilateral trade, and also implements price discrimination between buyers. Slotting allowances, or similar instruments, thus emerge from an efficiency-enhancing goal in a setting where discrimination between different buyers is necessary.

The literature on relational contracts has mostly focused on effort provision by an agent, as in

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<sup>2</sup>For example, the *Volvo Trucks v. Reeder-Simco* case from 2006 describes how the upstream firm sets a list price and then selectively grants discounts, taking into consideration the history of trade. See the description in Scott Martin, "Antitrust injury in Robinson-Patman cases: what's left?", at [https://www.competitionpolicyinternational.com/assets/0d358061e11f2708ad9d62634c6c40ad/Martin-Nov08\(2\).pdf](https://www.competitionpolicyinternational.com/assets/0d358061e11f2708ad9d62634c6c40ad/Martin-Nov08(2).pdf).

Levin (2003). Improving pure bilateral trade has been considered by Board (2011). He shows that promises of future payments can mitigate a hold-up problem and implement efficient trade with suppliers. The principal favors to choose an agent she has traded with before, and agents obtain a rent from their hold-up potential. This is similar in my model where the manufacturer cannot appropriate all of the retailers' rents because they can threaten with deviation. Andrews and Barron (2016) consider a similar setup where a principal chooses between different suppliers. They construct an allocation rule that attains first-best when the discount factor is sufficiently high, but rely on a more complex setup of back-and-forth payments between principal and agents. I restrict attention to simpler contracts that are composed of a price per unit paid by the agent (this could be interpreted as a cost of effort) and a lump-sum payment to the agent (often a wage in models of relational contracts). Baker, Gibbons, and Murphy (2002) analyze the role of relational contracts given different forms of vertical integration. I describe the contracts as a (limited) form of integration.

In closely related work to mine, Buehler and Gärtner (2013) analyze repeated trade in a vertically separated structure with incomplete information. They allow only a linear wholesale price and analyze the additional instrument of a recommended downstream price to transmit information and achieve efficiency. They achieve efficiency because they focus on arbitrarily patient parties and use the wholesale price to transfer rents. I analyze the incentive problems for all discount factors, giving the bargaining power to the upstream party. Trade in my model also approaches efficiency in the limit.

Slotting allowances have been rationalized in several ways in the theoretical literature. A strand of the marketing literature explains them as a signaling device for product quality (Chu, 1992; Lariviere and Padmanabhan, 1997). In the industrial organization literature, their role in relaxing downstream competition is highlighted. The arguments are reminiscent of the general argument put forward by Bonanno and Vickers (1988). In Shaffer (1991), slotting allowances paid by manufacturers need to be recovered with higher wholesale prices. This provides a commitment to retailers to soften competition between each other as they face higher input costs. These arguments have been extended to horizontal collusion between downstream firms by Piccolo and Miklós-Thal (2012). Gilo and Yehezkel (2018) use the same approach and show conditions under which collusion can be sustained for all positive discount factors. Importantly, in these models it is the colluding downstream parties that offer contracts including slotting fees. The downstream parties demand transfers to commit themselves to softer competition. In my model the upstream proposes them. I do not have collusion but a dynamic structure that permits the establishment of an efficiency-improving relational contract. My model highlights the role of lump-sum transfers for vertical coordination in a double-marginalization context.

As mentioned previously, the transfers to a downstream firm can be seen as a stand-in for more general transfers of rents downstream. Asker and Bar-Isaac (2014) consider such rent transfers by upstream firms to exclude another upstream rival. They use a dynamic setting, but do not establish a relational contract. Their logic functions perfectly well in a static one-period setting. In my model as in theirs, the upstream firm proposes contracts that include downstream transfers. The upstream firm in my model, however, has no exclusionary motive, but rather an

efficiency-enhancing one. In order to have a chance to improve on the one-shot outcome, the manufacturer in my model needs the dynamic structure to provide intertemporal incentives. The payments offered by an upstream firm can also be a general expression for lump-sum rebates. Ide, Montero, and Figueroa (2016) analyze discount contracts in a static setting. Their focus is on the potential exclusionary effects of discounts. They find that discounts cannot replicate explicit exclusive arrangements because they lack the ex-ante commitment value. Although I am not concerned with exclusion, my model offers a way to think about similar issues. In particular, payments that form a relational contract can be seen as commitment to a particular relationship.

I set up the model with one buyer in Section 3.2 and show that downstream payments will not be used. Next, I introduce additional buyers and show the emergence of downstream transfers in Section 3.3. I briefly consider the situation of several homogeneous retailers in one market in Section 3.4. The manufacturer chooses one retailer to repeatedly trade with. Section 3.5 concludes.

## 3.2 The relational contract with one buyer

### 3.2.1 Model

A manufacturer  $M$  produces a good at constant marginal cost  $c > 0$ . One monopolistic retailer  $R$  provides access to a market of final consumers. The retailer buys the good at per-unit price  $w$  and resells it to consumers. The market is characterized by the inverse demand  $p(q)$ . Assume this demand is well behaved so that the monopoly profit function is strictly concave and admits a unique optimal choice of  $q$  by the retailer for any value of  $w$ .<sup>3</sup> I make an additional assumption about the form of market demand in order to make the later exposition more tractable. In particular, it guarantees that the objective function will be globally concave in its arguments.<sup>4</sup>

**Assumption 3.1.** Inverse demand  $p(q)$  has the following property. For all  $q > 0$ ,

$$2p'(q) + p''(q)q < 0.$$



Denote the profit-maximizing choice of  $q$  for the retailer when he faces input price  $w$  by  $q^*(w)$ :  $q^*(w) = \underset{q}{\operatorname{argmax}} (p(q) - w)q$ . The retailer's second-order condition for a unique global maximum implies that  $q^*(w)$  is strictly decreasing. In the subgame-perfect equilibrium of any one-shot

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<sup>3</sup>The optimal  $q$  can be zero for  $w$  above a certain threshold.

<sup>4</sup>The assumption is for example satisfied for most convex demand functions, including linear, constant elasticity, and linear in logs, but also for concave demand functions such as exponential.

interaction, the manufacturer chooses  $w$  to maximize  $(w - c)q^*(w)$ . Assume that this program has a unique solution  $w^{dm}$ , where the superscript designates that this is the outcome of the canonical double marginalization problem. The outcome  $(w^{dm}, q^*(w^{dm}))$  describes the static benchmark.

Trade between the manufacturer and the retailer takes place repeatedly over infinitely many periods. Time is discrete and periods are denoted by  $t \in \{1, 2, \dots\}$ . Both parties discount payoffs with the same factor  $\delta \in (0, 1)$ . The formal contract has a plain linear structure.  $M$  quotes a price  $w$  per unit of the good.  $R$  can purchase any desired quantity at this price. As a supplement to this formal contract,  $M$  can transfer some rents to  $R$  in the form of a lump-sum payment. This lump-sum payment can stand in for many ways of transferring rents downstream. It is assumed to not be enforceable by an outside court.  $M$  cannot demand any payments or transfers beyond the list price from the buyer. The timing of the stage game then is as follows. (1)  $M$  quotes a list price  $w$ . (2)  $M$  offers a transfer  $F \geq 0$  to  $R$ . If  $R$  accepts, this transfer is paid immediately. (3)  $R$  orders quantity  $q$  and pays the total payment  $wq$ . He then sells the good to final consumers and realizes the revenue from this sale.

### 3.2.2 The relational contract with one buyer

In order to improve on the one-shot outcome with double marginalization, the manufacturer needs to induce the retailer to buy a higher quantity. She could do so by lowering the wholesale price, but if the retailer keeps buying according to  $q^*(w)$ , this alone would push  $M$ 's profits in each period below the static optimum.  $R$  thus needs to be induced to buy a quantity different from  $q^*(w)$ . This can only be done through intertemporal incentives. The manufacturer is endowed with all the bargaining power. In order to maximize her profit in each period, she designs a relational contract in which she sets a wholesale price  $w_e$  and specifies a quantity  $q_e$  that the retailer purchases, subject to the retailer's incentive constraint described below. In addition, she can pay a transfer  $F_e$  to the retailer to support his purchase of  $q_e$ . The relational contract will be implemented in every period of the infinitely repeated game. In order to provide proper incentives, a deviation by either party will result in play of the static double-marginalization outcome forever.

The repeated game is one of perfect monitoring. At each date  $t$ , all players observe the full history  $h^t = (a^1, a^2, \dots, a^{t-1})$ .  $a^t$  is the action profile chosen by the players in period  $t$ :  $a^t = (w^t, F^t, d^t, q^t)$ .  $w^t \in [c, \infty)$  is the wholesale price set by  $M$ ,  $F^t \geq 0$  the transfer offered,  $d^t \in \{0, 1\}$  the retailer's acceptance decision concerning  $F^t$ , and  $q^t \geq 0$  the quantity ordered by  $R$ . I consider strategies that produce a stationary outcome path. As the environment does not change between periods, the actions taken on the equilibrium path will be the same in each period. The strategy of  $M$  is thus in period 1 and for each history that does not include a deviation to set wholesale price  $w_e$  and offer transfer  $F_e$  to  $R$ . In any period where  $h^t$  includes a deviation,  $M$  sets  $w = w^{dm}$  and offers  $F = 0$ .  $R$ 's strategy is in the first period and for each history that does not include a deviation to accept the offer of  $F_e$  and order quantity  $q_e$ . For any history that includes a deviation or if  $F^t \neq F_e$  in the current period,  $R$  accepts  $F^t$  and orders quantity



$q^*(w)$ . The solution concept is subgame perfect equilibrium. No party can have an incentive to deviate after any history given the proposed strategies.

The incentives to play according to the equilibrium strategy are determined by the per-period payoffs for each of the parties. To describe  $R$ 's payoffs along the equilibrium path, we need to know the revenue he makes from the sales of the product to the consumers. The following lemma shows that  $R$  will resell all of  $q_e$  to the final consumers because, essentially, he will behave like a monopolist with a marginal cost of zero.

**Lemma 3.1.** Each quantity  $q_e$  implemented in each period on the equilibrium path of the relational contract will be resold in full by the retailer on the downstream market.

*Proof.* In the equilibrium contract, when the retailer deals with his customers, all payments to purchase his capacity are sunk. The retailer thus maximizes  $p(q)q$  subject to the constraint that  $q \leq q_e$ . Because demand is assumed to be well behaved,  $q > 0$  at the retailer's optimum. From the Kuhn-Tucker constraint with multiplier  $\lambda$ , it then follows that  $\lambda = p'(q)q + p(q)$ . This expression equals zero when  $q$  is at the optimal level of a monopolist with zero cost:  $q^*(0)$ . However, since we assumed the supplier has marginal cost  $c > 0$ , it would not be efficient for her to set such a quantity, hence  $q_e \neq q^*(0)$ . We then have  $\lambda \neq 0$  and thus, from the requirement that  $\lambda(q - q_e) = 0$ , it follows that  $q = q_e$ . ■

Denote the gross profit before transfers for the retailer on the equilibrium path by  $\pi(q_e, w_e) = [p(q_e) - w_e]q_e$ . Along the equilibrium path, the retailer thus has per-period payoff  $\pi(q_e, w_e) + F_e$ . His best deviation is to accept the proposed transfer  $F_e$  and order a quantity  $q^*(w_e)$ . Denote the profit without transfers from ordering the optimal quantity as  $\pi^*(w_e) = [p(q^*(w_e)) - w_e]q^*(w_e)$ . For  $R$  to adhere to the equilibrium strategy, the following incentive constraint needs to hold

$$\pi(q_e, w_e) + F_e \geq (1 - \delta)[\pi^*(w_e) + F_e] + \delta\pi^*(w^{dm}). \quad (3.1)$$

If this condition holds, the retailer will not deviate from the equilibrium strategy on the equilibrium path because the equilibrium payoffs are higher than those from a one-shot deviation and the ensuing punishment phase.

The manufacturer will design the equilibrium contract to maximize her profit. In particular, this will generate a per-period payoff for her that is higher than the one from the one-shot game. Consider possible deviations by the manufacturer. It includes a deviation from  $w_e$  and/or from  $F_e$ . Since in each period the manufacturer moves first and publicly chooses both these actions, the retailer can detect and punish any deviation in the current period. He will then order  $q^*(w)$ .  $M$ 's most profitable deviation is thus to offer  $F = 0$  and  $w = w^{dm}$ . But this gives her the payoff from the static game and is thus not profitable. Finally, both players' strategies after a deviation prescribe infinite play of the unique stage-game Nash equilibrium. Thus, the punishment phase forms an equilibrium and we have established a subgame-perfect equilibrium whenever the retailer's incentive constraint (3.1) is satisfied.

The manufacturer designs the contract that is optimal for her given the retailer's incentive constraint. First, it is clear that for any  $(w_e, q_e)$   $M$  will minimize the transfer to  $R$  and choose  $F_e$  such that (3.1) binds. Rearranging the incentive condition, we obtain an expression for the transfer  $F_e$  in any equilibrium contract that implements  $(w_e, q_e)$ :

$$F_e(q_e, w_e) = \frac{(1 - \delta)\pi^*(w_e) - \pi(q_e, w_e) + \delta\pi^*(w^{dm})}{\delta}. \quad (3.2)$$

This expression can now be inserted into the manufacturer's objective function.  $M$  maximizes her profit from trade with  $R$  minus the fee she has to pay to him. Denote the profit from trade before transfers of the manufacturer as  $\Pi(q_e, w_e) = (w_e - c)q_e$  and the objective function as

$$V(q_e, w_e) = \Pi(q_e, w_e) - F_e(q_e, w_e) = (w_e - c)q_e - \frac{(1 - \delta)\pi^*(w_e) - \pi(q_e, w_e) + \delta\pi^*(w^{dm})}{\delta}.$$

$M$ 's optimization problem is thus

$$\begin{aligned} \max_{w_e, q_e} \left\{ V(q_e, w_e) = (w_e - c)q_e - \frac{(1 - \delta)\pi^*(w_e) - \pi(q_e, w_e) + \delta\pi^*(w^{dm})}{\delta} \right\} \\ \text{s.t. } \frac{(1 - \delta)\pi^*(w_e) - \pi(q_e, w_e) + \delta\pi^*(w^{dm})}{\delta} \geq 0. \end{aligned} \quad (3.3)$$

The solution turns out to be difficult to fully specify in this general setup, but the result can be characterized with respect to the double-marginalization benchmark. The proof of the proposition is illustrated in Figure 3.1.

**Proposition 3.1.** The optimal stationary relational contract for the manufacturer satisfies the following:

- The manufacturer does not pay a lump-sum transfer to the retailer.
- The implemented quantity  $q_e$  satisfies

$$p(q_e) + p'(q_e)q_e - w_e = (w_e - c) \frac{(1 - \delta)q^*(w_e) - q_e}{q_e}. \quad (3.4)$$

It lies strictly between the double-marginalization quantity  $q^*(w^{dm})$  and the efficient quantity  $q^*(c)$ .

- The implemented wholesale price  $w_e$  lies strictly between the double-marginalization price  $w^{dm}$  and the marginal cost  $c$ .

*Proof.* The proposition will be established in several steps.

(i) At any solution,  $w_e > c$  and  $q_e > q^*(w_e)$ . This implies that the objective function  $V(q_e, w_e)$  is strictly decreasing in  $w_e$  at any solution to (3.3). This means that the non-negativity constraint on  $F_e$  is binding:  $F_e(q_e, w_e) = 0$ .

Notice first that the solution to (3.3) cannot contain any  $w_e \leq c$ , because in this case  $\Pi(q_e, c) \leq 0$ . And since  $F_e$  is restricted to be nonnegative,  $M$  cannot make a profit. Consider now the optimization with respect to  $q_e$  and  $w_e$ . For any  $w_e > c$ , the objective function increases in  $q_e$ —holding  $w_e$  fixed—when

$$q_e < q^*((1 - \delta)w_e + \delta c). \quad (3.5)$$

This is because  $\frac{\partial V}{\partial q_e} = w_e - c + \frac{p(q_e) + p'(q_e)q_e - w_e}{\delta}$  and thus  $\frac{\partial V}{\partial q_e} \leq 0 \iff p(q_e) + p'(q_e)q_e - ((1 - \delta)w_e + \delta c) \leq 0$ . This expression is the derivative of the profit with respect to quantity  $q_e$  of a monopolist on the downstream market who faces input cost  $(1 - \delta)w_e + \delta c$  and market demand  $p(q_e)$ . Thus, it is  $> (<)$  0 when  $q_e$  is below (above) this monopolist's optimal quantity choice, given by  $q^*((1 - \delta)w_e + \delta c)$ . Condition (3.5) thus determines a boundary that is determined in  $(q, w)$ -space by the locus  $q = q^*((1 - \delta)w + \delta c)$ . This is a clockwise rotation of  $q = q^*(w)$  in the point  $(q^*(c), c)$ . Thus, to reach this locus,  $M$  will want to increase  $q_e$  above  $q^*(w_e)$  for any  $w_e > c$ . Now,  $\frac{\partial V}{\partial w_e} = \frac{1 - \delta}{\delta}(q^*(w_e) - q_e)$ . Thus, at the solution to (3.3), we need to have  $\frac{\partial V}{\partial w_e} < 0$ . Because  $\frac{\partial F_e}{\partial w_e} > 0$  when  $q_e > q^*(w_e)$ ,  $M$  will reduce  $w_e$  until the non-negativity constraint on  $F_e$  binds:  $F_e(q_e, w_e) = 0$ .

(ii) For any  $q_e$  which is part of a solution  $(q_e, w_e)$ ,  $M$  wants to reduce  $w_e$  until  $F_e \geq 0$  is binding. There exists an implicit function  $w_e = \phi(q_e)$  such that  $F_e(q_e, \phi(q_e)) = 0$ . This function  $\phi$  is decreasing for  $q_e > q^*(w_e)$ .

Since the solution to (3.3) must involve  $F_e(q_e, w_e) = 0$ , it must lie on the locus in  $(q, w)$ -space where this condition is satisfied. To determine properties of this locus, specify a function  $w_e = \phi(q_e)$  such that  $F_e(q_e, \phi(q_e)) = 0$ .  $F_e(q_e, w_e)$  is continuous and its derivative with respect to  $w_e$  is invertible for  $q_e \neq (1 - \delta)q^*(w_e)$ .<sup>5</sup> I will assume that there exists  $w$  such that  $F_e(q, w) = 0$  for each  $q$  in the relevant range of any solution:  $q_e \in [q^*(w^{dm}), q^*(c)]$ . Step (v) will verify that this holds. The derivative of  $\phi$  is given by

$$\phi' \equiv \frac{dw_e}{dq_e} = \frac{p(q_e) + p'(q_e)q_e - w_e}{q_e - (1 - \delta)q^*(w_e)}.$$

The numerator is again the profit derivative with respect to  $q_e$  for a downstream monopolist, now with input price  $w_e$ . It is negative (positive) when  $q_e > (<) q^*(w_e)$ . Thus, when  $\phi$  exists, it is falling when  $q_e > q^*(w_e)$ .

(iii) Any solution to (3.3) satisfies  $(q_e, w_e) = \left( q^*\left(\left[\frac{q^*(w_e)}{q_e}(1 - \delta)\right]w_e + \left[1 - \frac{q^*(w_e)}{q_e}(1 - \delta)\right]c\right), w_e \right)$ . The locus  $q = q^*\left(\left[\frac{q^*(w)}{q}(1 - \delta)\right]w + \left[1 - \frac{q^*(w)}{q}(1 - \delta)\right]c\right)$  is a further clockwise rotation of  $q = q^*((1 - \delta)w + \delta c)$  in  $(q^*(c), c)$ .

Because  $\phi$  is falling for  $q_e > q^*(w_e)$ , each increase in  $q_e$  is associated with a decrease in  $w_e$  to stay on  $\phi$ . While  $q_e < q^*((1 - \delta)w_e + \delta c)$ , both an increase in  $q_e$  and a decrease in  $w_e$  increase the value of  $V$ . To the right of the  $q = q^*((1 - \delta)w + \delta c)$  locus, there is a tradeoff, as now an increase in  $q_e$  decreases  $V$  while a decrease in  $w_e$  still increases  $V$ . This tradeoff is resolved by

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<sup>5</sup>As  $q_e > q^*(w_e)$  and  $\delta < 1$ , this point is not relevant.

checking when

$$-\frac{\partial V}{\partial q_e} = \frac{\partial V}{\partial w_e} \phi'. \quad (3.6)$$

When the left-hand side is smaller than the right-hand side, this condition states that the decrease in  $V$  through an increase in  $q_e$  is smaller than the increase in  $V$  from a decrease in  $w_e$  made possible by the increase in  $q_e$ . The converse reasoning holds when the left-hand side is larger than the right-hand side. Condition (3.6) holds when

$$p(q_e) + p'(q_e)q_e - \left( \left[ \frac{q^*(w_e)}{q_e}(1-\delta) \right] w_e + \left[ 1 - \frac{q^*(w_e)}{q_e}(1-\delta) \right] c \right) = 0. \quad (3.7)$$

Rearranging (3.7) gives an expression to characterize  $q_e$ :

$$p(q_e) + p'(q_e)q_e - w_e = (w_e - c) \frac{(1-\delta)q^*(w_e) - q_e}{q_e}.$$

Again, seeing (3.7) as the first-order condition of a downstream monopolist, the expression means that it is beneficial to change  $q_e$  (with the associated change in  $w_e$  on  $\phi$ ) when

$$q_e \neq q^* \left( \left[ \frac{q^*(w_e)}{q_e}(1-\delta) \right] w_e + \left[ 1 - \frac{q^*(w_e)}{q_e}(1-\delta) \right] c \right). \quad (3.8)$$

Expression 3.8 set to equality defines a function in  $(q, w)$ -space for  $q > q^*(w)$  and  $w > c$ . To see this, notice that, for each  $w$ , when  $q = q^*(w)$  (i.e.,  $\frac{q^*(w)}{q} = 1$ ),  $q < q^*((1-\delta)w + \delta c)$ , because  $(1-\delta)w + \delta c < w$ . Now, set  $q = q^*(c)$  and obtain  $q > q^* \left( \left[ \frac{q^*(w)}{q^*(c)}(1-\delta) \right] w + \left[ 1 - \frac{q^*(w)}{q^*(c)}(1-\delta) \right] c \right)$ . Because  $q^*(w)$  is continuous, there exists a  $q$  for given  $w$  for which (3.8) is satisfied with equality. It can be shown using Rolle's Theorem and Assumption 3.1 that this  $q$  is unique and (3.8) thus defines a function in  $(q, w)$ -space. This function is again  $q = q^*((1-\delta)w + \delta c)$  rotated further clockwise through  $(q^*(c), c)$  since  $\frac{q^*(w_e)}{q_e} < 1$ .

(iv)  $\phi$  crosses  $q = q^*(w)$  only once, hence it must cross any rotation of this locus of the form  $q = q^*((1-x)w_e + xc)$ ,  $0 < x < 1$ , at least once. This happens for  $q^*(w^{dm}) < q_e < q^*(c)$  and  $w_e < w^{dm}$ .

Recall from (3.2) that the condition  $F(q_e, w_e) = 0$  can be written as

$$[p(q^*(w_e)) - w_e]q^*(w_e) - [p(q_e) - w_e]q_e = \delta[p(q^*(w_e)) - w_e]q^*(w_e) - \pi^*(w^{dm}). \quad (3.9)$$

Whenever  $q_e = q^*(w_e)$ , the left-hand side of (3.9) is zero. But because  $[p(q^*(w_e)) - w_e]q^*(w_e)$  is monotonously decreasing in  $w_e$  and  $\pi^*(w^{dm})$  is independent of  $w_e$ , the right-hand side is zero only at  $w_e = w^{dm}$ . Thus,  $\phi$  and  $q^*(w_e)$  cross once at  $(q^*(w^{dm}), w^{dm})$ . Any rotation of  $q^*(w_e)$  of the form  $q^*((1-x)w_e + xc)$ ,  $0 < x < 1$ , generates a decreasing function and that crosses  $q^*(w_e)$  once at  $(q^*(c), c)$ . Hence,  $\phi$  must cross such a rotation at a  $q_e$  such that  $q^*(w^{dm}) < q_e < q^*(c)$ , otherwise it would cross  $q^*(w_e)$  more than once. Since  $\phi$  is strictly decreasing for  $q_e > q^*(w_e)$ , it crosses any rotation at  $w_e < w^{dm}$ .

(v) For each  $q \in [q^*(w^{dm}), q^*(c)]$ , there exists  $w$  such that  $F_e(q, w) = 0$ .

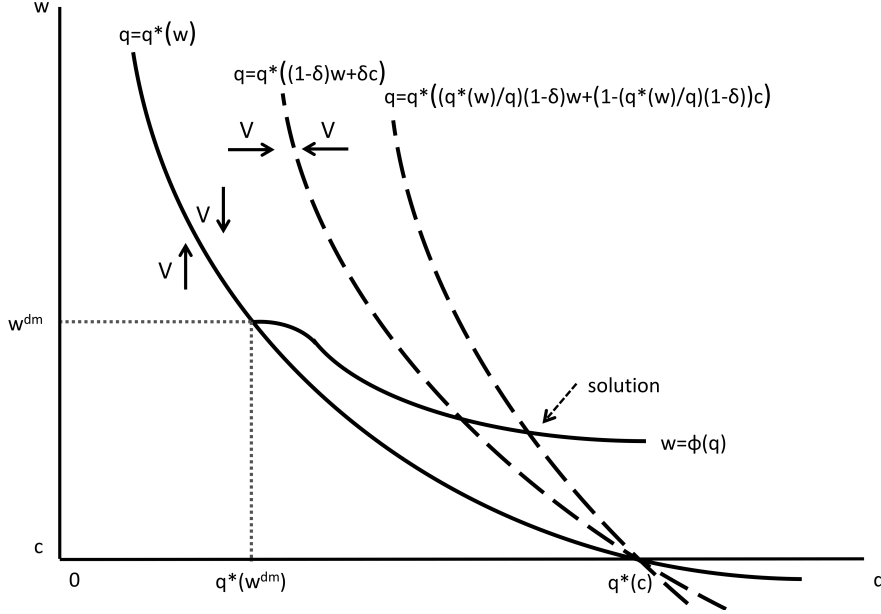


Figure 3.1: Illustration of Proposition 3.1

Hold  $q_e \in [q^*(w^{dm}), q^*(c)]$  fixed. It is immediate from (3.2) that  $F_e > 0$  is attainable by increasing  $w_e$  until  $\pi(q_e, w_e) \leq 0$ . Now, decrease  $w_e$ . Recall that  $q^*(w)$  is continuous and strictly decreasing in  $w$  for  $w \in [c, w^{dm}]$ . Thus, we can decrease  $w_e$  maintaining  $w_e \geq c$  until  $q^*(w_e) = q_e$ . At this point,  $w_e \leq w^{dm}$ , because  $q_e \geq q^*(w^{dm})$ . Then,  $F_e = \pi^*(w^{dm}) - \pi^*(w_e) \leq 0$ . Because (3.2) is continuous, it will cross zero while  $w_e$  is decreased. The conditions for the application of the Implicit Function Theorem are thus satisfied in the range of the solution. ■

Figure 3.1 illustrates Proposition 3.1. It depicts as an example a convex  $q^*(w)$  along with its respective rotations around  $(q^*(c), c)$ . The arrows with the letter  $V$  indicate the direction of increase of the objective function for a movement of  $q_e$  or  $w_e$ , holding the respective other variable fixed.  $V$  increases as the respective variable moves in the direction of the arrow. The relevant locus where the marginal effect of  $q_e$  changes sign is  $q = q^*((1 - \delta)w + \delta c)$  and the marginal effect of  $w_e$  changes sign at  $q = q^*(w)$ . The figure depicts the favorable (but not general, see the discussion below) situation of a unique solution. The curved line  $\phi = w(q)$  crosses the locus  $q = q^*\left(\frac{q^*(w)}{q}(1 - \delta)w + (1 - \frac{q^*(w)}{q}(1 - \delta))c\right)$  only once.

I cannot establish uniqueness of the result in the general setup. Essentially, the question is how often  $\phi$  crosses the locus  $q = q^*\left(\frac{q^*(w)}{q}(1 - \delta)w + (1 - \frac{q^*(w)}{q}(1 - \delta))c\right)$ . The solution is unique if they cross only once, as is depicted in Figure 3.1.  $\phi$  could cross this locus several times, provided it eventually turns convex. If  $\phi$  is globally concave, it can cross only once because it can cross  $q = q^*(w)$  only once. But the curvature of  $\phi$  is in general difficult to assess. In particular,  $\phi$  is globally concave for  $q_e > q^*(w_e)$  if

$$2p'(q_e) + p''(q_e)q_e < 2\frac{p(q_e) + p'(q_e)q_e - w_e}{q_e - (1 - \delta)q^*(w_e)} - (1 - \delta)q^{*'}(w_e) \left( \frac{p(q_e) + p'(q_e)q_e - w_e}{q_e - (1 - \delta)q^*(w_e)} \right)^2.$$

Even under Assumption 3.1, this condition cannot clearly be assessed.

### 3.2.3 Discussion

The proposition describes features of  $M$ 's optimally chosen stationary contract. She is able to improve on the double-marginalization outcome by increasing the traded quantity above its value in the one-shot game. To achieve this, she lowers the wholesale price below  $w^{dm}$ , but pays no transfer to  $R$ . The establishing of a relational contract in itself without any additional instruments improves efficiency in the bilateral trade. Incentives are provided by the threat to return to static double marginalization. From the retailer's binding incentive constraint (3.1), setting  $F_e = 0$ , we can see that  $\pi(q_e, w_e) = \pi^*(w^{dm}) + (1 - \delta)(\pi^*(w_e) - \pi^*(w^{dm}))$ . Because at the solution,  $w_e < w^{dm}$ , the second term on the right-hand side is strictly positive. The retailer shares in the efficiency gains and makes a per-period profit that is greater than in the one-shot game. Over his lifetime, the retailer earns exactly the difference  $\pi^*(w_e) - \pi^*(w^{dm})$  in addition to what he would earn from repeated realization of the one-shot outcome. The retailer thus earns a per-period rent that he loses if he deviates. In order to improve upon the static double-marginalization outcome, the retailer needs to buy a bigger quantity than his one-shot optimum. This gives him an incentive to deviate to the one-shot optimal quantity. The lower (closer to  $c$ ) the wholesale price, the larger is the retailer's one-shot optimal quantity and the smaller are his deviation incentives when a larger quantity is implemented along the equilibrium path of the relational contract. Reducing the wholesale price is the most efficient way for the manufacturer to make the retailer agree to purchase a larger quantity. A downstream transfer would need to be recovered by the manufacturer with a higher wholesale price which would give the retailer greater deviation incentives.

Because the wholesale price is the only source of revenue for the manufacturer, it will always be set above cost. The implemented quantity approaches the efficient quantity  $q^*(c)$  in the limit. Inspecting (3.4), one can see that the characterization of  $q_e$  approaches the expression  $p(q_e) + p'(q_e)q_e - c = 0$  as  $\delta \rightarrow 1$ . This is the first-order condition of a downstream firm that faces input cost  $c$ . The manufacturer sacrifices some possible efficiency gains by implementing a smaller quantity than the efficient one. That total surplus is not maximized even for large discount factors<sup>6</sup> is surprising. It is a consequence of the fact that no lump-sum transfers are used. Since the manufacturer holds all the bargaining power, she wants to appropriate all the surplus. But transfer of surplus to her can only come from the wholesale price. Since the wholesale price affects the retailer's deviation incentives, there is tension between the goals of raising revenue and reducing the retailer's deviation incentives. Because of the rent that needs to be given up to the retailer, the manufacturer cannot appropriate all the surplus and thus has reduced incentives to maximize total surplus.

This result shares some similarities with Board (2011). In that paper, agents also obtain a rent that cannot be appropriated by the principal because the agents cannot make the associated payment. The agents' lifetime rent is exactly equal to the value of their hold-up threat in a period. Trade with agents, however, is efficient because it is a binary decision. In my model, there is a downward-sloping demand curve of the retailer and trade takes place, but its value

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<sup>6</sup>Large but strictly less than 1.

is not maximized. In Buehler and Gärtner (2013), rent shifting is achieved via the wholesale price in a vertical structure very similar to mine. They are, however, focused on the truthful revelation of information that leads to the implementation of the efficient quantity. They show that this can be achieved for sufficiently large discount factors. In my model, the manufacturer could implement the efficient quantity, but would have to lower the wholesale price and give up profits.

$M$  would, of course, implement the efficient outcome if she could collect the associated rents. In fact, in my setup, if lump-sum transfers to the manufacturer were allowed, the manufacturer would like to set wholesale price equal to cost and collect the profit via the transfer. This is the well-known contract with a franchise fee and wholesale price  $w = c$ . However, deviation incentives of the manufacturer would then need to be considered. By restricting  $F$  to be non-negative, I focus on the case of downstream transfers which is common on retail contracts. I explore the efficiency-enhancing effects of relational contracts that allow at most for transfers to the buyer. As mentioned in the introduction, recent empirical work has reported that franchise fees are not common in retail contracts. Further, even in classical franchising systems, not all profits are collected via franchise fees. Kaufmann and Lafontaine (1994) report that McDonald's franchisees earn rents *ex ante* as well as *ex post*. While the *ex-ante* rents can be explained with financial constraints on the part of the franchisees, even *ex post*, not all rents are collected. Kaufmann and Lafontaine explain this with the incentive effects of leaving surplus to the franchisees. In my setup, I have shown that the downstream buyer obtains a rent when he participates in the relational contract that increases efficiency.

Notice also that in the punishment, the manufacturer does not min-max the retailer. Thus the contract cannot achieve the highest possible payoff for  $M$ . Min-maxing  $R$  would mean granting him a punishment-phase payoff of 0. This could be achieved by refusing to trade, or equivalently by setting the wholesale price  $w_p$  so high that  $q^*(w_p) = 0$ . Since this is not a Nash equilibrium of the stage game, perpetual min-maxing cannot be a punishment in a subgame-perfect equilibrium. Min-max punishments can be implemented by punishment phases of limited length with eventual return to equilibrium play. However, I do not consider these as they are likely to not change the mechanics much, but will allow the manufacturer to appropriate more surplus.<sup>7</sup>

### 3.2.4 Renegotiation proofness and the retailer's rent

The equilibrium path described in Proposition 3.1 relies on a particularly simple punishment. The manufacturer designs the contract to maximize her payoff, and if the retailer deviates from the designated path, the parties return to permanent repetition of the one-shot game. Repeated play of the one-shot Nash equilibrium naturally lends itself as a punishment. Its outcome is the benchmark against which the possibility of improvements is analyzed. Nash reversion is also

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<sup>7</sup>The assumption of a permanent reversal to the one-shot game is also not uncommon in the literature on relational contracts in vertical structures. For example, Baker, Gibbons, and Murphy (2002) assume permanent spot-market trade after a deviation. Buehler and Gärtner (2013) compare the parties' payoffs in their repeated game to the one-shot equilibrium payoffs.

particularly appealing because it is a subgame-perfect equilibrium of the repeated game for all discount factors. Since in Proposition 3.1 I analyze the relational contract for any given discount factor, it is convenient to be able to ignore the discount factor in the punishment.

But this punishment has a drawback. An equilibrium of the repeated game that prescribes it after a deviation is not renegotiation proof. Along the equilibrium path, both the manufacturer and the retailer obtain payoffs in each period in excess of what they earn in the one-shot game. It would thus not seem implausible that they coordinate on continuing on the equilibrium path when in fact the punishment is due. This, however, undermines the threat of the punishment and destroys the incentives on the equilibrium path. A (weakly) renegotiation-proof equilibrium would be such that it specifies a continuation equilibrium after a deviation such that the parties cannot agree to switch to a different continuation equilibrium that would have been followed another history.<sup>8</sup> A punishment from which the parties cannot negotiate away from could thus be constructed as one that gives the same lifetime value to the retailer as the current punishment does, but a higher value to the manufacturer than the equilibrium path.

This second requirement brings a new role for this punishment because I aim to design the contract that is optimal for the manufacturer. If a punishment exists that gives the manufacturer a higher lifetime payoff than the stationary equilibrium path, this punishment path should be played on the equilibrium path to begin with. Consider an equilibrium in the spirit of the punishment in Abreu (1986). In the first period, the retailer faces a wholesale-price-quantity pair that gives him a low payoff. Play then proceeds along the equilibrium path of Proposition 3.1 from the second period onward. The total value of this equilibrium to the retailer is equal to his outside option, taken here to be repeated play of the double-marginalization stage game. In this way, this equilibrium implements the simple optimal stationary path described in Proposition 3.1 in all but one period and minimizes the retailer's lifetime rent. Consider the following equilibrium. In the initial period 1, the contract specifies a wholesale price-quantity combination  $(w_1, q_1)$  different from  $(w_e, q_e)$ , along with a fixed transfer  $F_1 = 0$ , and from the second period onward, the stationary contract  $(w_e, q_e)$  with  $F_e = 0$  is specified as in the previous section. In case of a deviation by the retailer, play starts again in period 1. This equilibrium needs to satisfy three conditions. It needs to set the lifetime value for the retailer equal to his outside option:

$$(1 - \delta)(p(q_1) - w_1)q_1 + \delta\pi(q_e, w_e) = \pi^*(w^{dm}). \quad (3.10)$$

The play prescribed in period 1 needs to be incentive compatible for the retailer:<sup>9</sup>

$$(1 - \delta)(p(q_1) - w_1)q_1 + \delta\pi(q_e, w_e) \geq (1 - \delta)\pi^*(w_1) + \delta\pi^*(w^{dm}). \quad (3.11)$$

And it needs to have a higher value for the manufacturer than the stationary path. This requirement is satisfied when the payoff to the manufacturer in the initial period is larger than

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<sup>8</sup>Weak renegotiation proofness thus means that the players do not want to renegotiate to a continuation equilibrium within their current equilibrium of the repeated game.

<sup>9</sup>The play of  $(q_e, w_e)$  in later periods is incentive compatible for the retailer because a deviation would be punished by re-starting the current path at period 1, which by (3.10) gives lifetime value  $\pi^*(w^{dm})$  to the retailer.



in one of the stationary periods:

$$(w_1 - c)q_1 \geq (w_e - c)q_e. \quad (3.12)$$

If such a contract exists, it is characterized by a high initial wholesale price. The high wholesale price serves to transfer rents from the retailer to the manufacturer. Importantly, it is the determinant of the retailer's deviation payoff in period 1. Conditions (3.10) and (3.11) together imply that  $w_1 \geq w^{dm}$ . By (3.12), the quantity  $q_1$  needs to satisfy  $q_1 \geq \frac{w_e - c}{w_1 - c}q_e$ . Because  $w_1 \geq w^{dm} > w_e$  for  $\delta > 0$ ,  $\frac{w_e - c}{w_1 - c} < 1$ . The resulting combination  $(w_1, q_1)$ , however, may not violate condition (3.10). This condition is difficult to evaluate. For the extreme point  $\delta = 0$ , we need  $\pi(q_1, w_1) = \pi^*(w^{dm})$ , which, for  $w_1 \geq w^{dm}$ , is only possible with  $(w^{dm}, q^*(w^{dm}))$ . All the conditions are trivially satisfied because only the outcome of the one-shot game is attainable. As  $\delta$  rises, more weight is placed on the continuation equilibrium  $(w_e, q_e)$ . The associated per-period payoff for the retailer is  $\pi(q_e, w_e)$ . Recall that from the retailer's incentive constraint along the stationary equilibrium path,  $\pi(q_e, w_e) = \pi^*(w^{dm}) + (1 - \delta)(\pi^*(w_e) - \pi^*(w^{dm})) > \pi^*(w^{dm})$ . Thus,  $\pi(q_1, w_1)$  needs to decrease below  $\pi^*(w^{dm})$  which can be accomplished by raising  $w_1$  and/or raising  $q_1$ . However, as  $\delta$  increases, so does  $q_e$ . It is thus difficult to say whether (3.12) is still satisfied without putting more structure on the model.

What makes rent extraction difficult in my model is the fact that value for the manufacturer can only come from trade with the retailer, but not through payments that are independent of the quantity traded or are executed after trade has taken place. Thus, value for the manufacturer cannot be created independently from the creation of total value. Many papers that analyze relational contracts and renegotiation have the feature that payments can be made to either party after effort has been exerted. For example, Goldlücke and Kranz (2013) avoid surplus-destroying punishments by using side payments that a deviator has to make as punishment. Kostadinov (2019) uses bonus payments after output has been realized in each period.

An interesting avenue regarding renegotiation proofness is opened by the recent work of Miller, Olsen, and Watson (2018). They assume that parties can agree to an externally enforceable contract that prescribes drastic punishments. Each period, following previous compliance, the parties negotiate away from this contract for the current period, but it is kept as a starting point for negotiations for future periods. Renegotiation is thus included in the path of play and the externally enforceable framing contract is put in effect only in a period where negotiation fails. This framing contract destroys surplus. Because it is externally enforceable, it provides commitment for bad outcomes in the case of disagreement in the current period and thus strengthens the power of incentives.

### 3.3 The relational contract with two buyers and downstream transfers

In the equilibrium of Section 3.2, no transfer is paid to the retailer. The manufacturer provided incentives just by threatening to end the mutually advantageous relationship and return to

the standard static outcome. The choice of the manufacturer's instruments,  $w_e$  and  $q_e$ , was determined by the need to balance revenue gains through a higher wholesale price and the efficiency improvement from a higher quantity. When there are two buyers, this balancing effort may change. To illustrate under what conditions a downstream transfer can be paid, I introduce a second market, served by a second monopolistic retailer. I assume that  $M$  cannot set different wholesale prices for the two retailers. She has to set one common list price, and enter relational agreements about the quantity. These agreements will now fulfill a double role, to differentiate between buyers and to improve efficiency in bilateral and overall trade. Transfers to buyers can thus obtain a role in coordinating trade with buyers in separate markets.

### 3.3.1 The role of transfers in differentiation between buyers

There are now two markets, respectively served by monopolistic retailers,  $R_1$  and  $R_2$ . The two markets are independent from another, in that quantities offered on one market have no effect on the price in the other. However, the manufacturer is restricted to setting a common wholesale price to both retailers. Again, denote the optimal quantities demanded by retailers  $R_i$ ,  $i = 1, 2$ , in the one-shot interaction by  $q_i^*(w)$ , and  $R_i$ 's associated profit by  $\pi_i^*(w)$ . The manufacturer sets a unique uniform wholesale price  $w^u$  in the one-shot game. This price maximizes  $(w - c)[q_1^*(w) + q_2^*(w)]$ . Assume that both markets are served under this price:  $q_i^*(w^u) > 0$ , for  $i = 1, 2$ . In the benchmark,  $M$  can set differing wholesale prices that each maximize  $(w_i - c)q_i^*(w_i)$ . Assume the solution for each market is unique and denote it by  $w_i^*$ .

The restriction to a common wholesale price merits some discussion. For instance, it could be that the manufacturer cannot prevent parallel imports. If she sets a different wholesale price in the two markets, the retailer with the higher price would just come to buy in the lower-priced market on the better terms. The European Union, as an example, is very committed to upholding the principle of a single market. The European Commission regularly fines companies that try to prevent parallel imports to support wholesale price differences between markets. One recent such decision is the fine of 200 million euros for brewing company AB InBev.<sup>10</sup> The company restricted parallel imports of beer by retailers and distributors between Belgium and the Netherlands to sustain different wholesale prices in the two countries. This practice was seen as an abuse of dominance by the European Commission. But also competition authorities outside the European Union have imposed fines for prohibiting parallel trade. For example, in 2012, carmaker BMW was fined 157 million Swiss francs by the Swiss competition commission for such practices.<sup>11</sup>

Consider now the repeated interaction between the supplier and the two retailers.  $M$  can use the additional instrument of downstream transfers. These transfers have an effect if they induce the receiving retailer to buy a quantity that is, in his view, distorted relative to  $q_i^*(w)$ .  $M$  has

<sup>10</sup>See the press release of the European Union at [http://europa.eu/rapid/press-release\\_IP-19-2488\\_en.htm](http://europa.eu/rapid/press-release_IP-19-2488_en.htm).

<sup>11</sup>See, for reference, [https://www.swissinfo.ch/eng/import-issue\\_court-confirms-multimillion-dollar-fine-against-bmw/41818596](https://www.swissinfo.ch/eng/import-issue_court-confirms-multimillion-dollar-fine-against-bmw/41818596).

again all the bargaining power and offers contracts and transfers. She wants to implement a quantity choice  $(q_{1,e}, q_{2,e})$  by the retailers. These quantity choices will be supported by transfers to retailers  $F_{i,e} \geq 0$  that supplement the formal contract. The formal contract specifies a uniform wholesale price  $w_e$ . The relational contracts will again be supported by a Nash reversion punishment where  $M$  sets wholesale price  $w^u$ . The equilibrium strategies are the same as in the one-retailer case.  $M$  first sets the public list price, then simultaneously offers transfers to retailers. The retailers simultaneously decide whether to accept and order their desired quantities. The incentive constraint of each retailer  $R_i$  looks like the one of  $R$  in the single-retailer case. It needs to hold for each  $R_i$  individually:

$$\pi_i(q_{i,e}, w_e) + F_{i,e} \geq (1 - \delta)[\pi_i^*(w_e) + F_{i,e}] + \delta\pi_i^*(w^u). \quad (3.13)$$

To assess the payoffs in the game, I note that Lemma 3.1 still applies to all retailers who are party to a relational contract. They resell all the quantity that they purchased from the manufacturer in full on the consumer market.

The restriction to a common wholesale price makes it natural to ask whether price discrimination can be implemented with other means. This designates a possible role for downstream transfers, which did not appear in the single-market case. The following result shows that there is a rationale to use transfers to buyers in order to implement price discrimination in restricted environments. Of course there is only a need for price discrimination if  $w_1^* \neq w_2^*$ .

**Proposition 3.2.** *If  $w_1^* \neq w_2^*$ , the supplier can do strictly better with subsidies in the repeated game than by obtaining repeatedly the outcome of third-degree price discrimination, provided that parties are sufficiently patient.*

*Proof.* Denote the higher of the two  $w_i^*$  by  $w_1^*$ , such that  $w_1^* > w_2^*$ .  $M$  announces  $w_e = w_1^*$  and offers  $F_{1,e} = 0$  every period.  $R_1$  then orders  $q_1^*(w_1^*)$  and  $M$  obtains profit  $(w_1^* - c)q_1^*(w_1^*)$  from trade with market 1.

Given  $w_1^*$ , the lump-sum payment is set so that the effective wholesale price paid by  $R_2$  when he orders quantity  $q_{2,e} = q_2^*(w_2^*)$  is equal to  $w_2^*$ :

$$F_{2,e} = q_2^*(w_2^*)(w_1^* - w_2^*).$$

$F_{2,e}$  is non-negative since by assumption,  $w_1^* > w_2^*$ . With this,  $M$  obtains profit  $(w_1^* - c)q_2^*(w_2^*) - q_2^*(w_2^*)(w_1^* - w_2^*) = (w_2^* - c)q_2^*(w_2^*)$  from trading with  $R_2$ .

At this outcome, the total per-period payoff of  $R_2$  is  $[p_2(q_2^*(w_2^*)) - w_1^*]q_2^*(w_2^*) + q_2^*(w_2^*)(w_1^* - w_2^*) = [p_2(q_2^*(w_2^*)) - w_2^*]q_2^*(w_2^*) = \pi_2^*(w_2^*)$ . Rearranging, we obtain  $\pi_2(q_{2,e}, w_e) = \pi_2^*(w_2^*) - F_{2,e}$ .

Now, for  $R_2$  to cooperate, his incentive constraint needs to be satisfied. Rearrange (3.13) to obtain the threshold for the discount factor as

$$\delta \geq \frac{\pi_i^*(w_e) - \pi_i(q_{i,e}, w_e)}{\pi_i^*(w_e) - \pi_i^*(w^u) + F_{i,e}}. \quad (3.14)$$

For  $R_2$  to take part in the proposed equilibrium, this takes the value:

$$\delta \geq \frac{\pi_2^*(w_1^*) - [\pi_2^*(w_2^*) - F_{2,e}]}{\pi_2^*(w_1^*) - \pi_2^*(w^u) + F_{2,e}} \equiv \Delta_{R_2,e}. \quad (3.15)$$

It is a standard result that in a third-degree price discrimination setup, the optimal uniform price lies between the optimal discriminatory prices when both markets are served under uniform pricing. Thus, when  $w_1^* > w_2^*$ ,  $w^u > w_2^*$ . When  $w^u > w_2^*$ , the value of the threshold in (3.15) is strictly below 1, as  $\pi_2^*(w_2^*) > \pi_2^*(w^u)$ . Furthermore, we see from (3.14) that the numerator must be non-negative, since  $\pi_2^*(w_e) \geq \pi_2(q_{2,e}, w_e)$  by revealed preference. Thus, we have  $\Delta_{R_2,e} \in [0, 1)$ .

Now, it is immediate from (3.14) that the threshold is decreasing in the value of the fixed payment  $F$ . Since  $\Delta_{R_2,e} < 1$ ,  $M$  can decrease  $F_{2,e}$  slightly to  $q_2^*(w_2^*)(w_1^* - w_2^*) - \varepsilon$ , with  $\varepsilon > 0$ . There will still remain a range of discount factors strictly below 1 for which  $q_2^*(w_2^*)$  can be implemented at  $w_1^*$ , but at an effective wholesale price that  $M$  receives now slightly higher than  $w_2^*$ . Thus,  $M$  can strictly increase her profit compared to the third-degree price discrimination outcome. ■

$M$  can use the transfer to discriminate between buyers by entering into different relationships. To recreate the outcome of third-degree price discrimination, she sets the list price to the higher of the two individual prices under price discrimination; the respective retailer buys his optimal quantity.  $M$  subsequently subsidizes the retailer who would otherwise receive a lower price. The subsidy and according quantity agreement can be selected such that the second retailer effectively pays his optimal wholesale price per unit. Since the critical discount factor lies strictly below one in this case,  $M$  could reduce the subsidy and still achieve incentive compatibility for a range of discount factors. She then makes more profit than under third-degree price discrimination. This demonstrates the rationale to use downstream transfers to differentiate between buyers. It implements price discrimination and additionally permits the manufacturer to achieve a better outcome than the third-degree price discrimination benchmark. This is due to the effect from an efficiency-enhancing relational contract as described in the previous section. In the present situation,  $M$  can enforce the price-discrimination quantity while paying a transfer that lets her collect a greater effective payoff.

### 3.3.2 The contract with two buyers and transfer payments

Proposition 3.2 describes the possibility of achieving the outcome from third-degree price discrimination and doing better. The proof constructs a simple combination of wholesale price, transfer, and quantity, that directly mimics third-degree price discrimination. In particular,  $w_e$  is set to  $w_1^*$  and retailer 1 buys according to his optimal demand function. If  $\delta$  is high enough, this construction suffices to generate profits in excess of the third-degree price discrimination benchmark for the manufacturer. The construction in the proof of Proposition 3.2 is not generally optimal for a given  $\delta$ . In this section, I aim at a characterization of the outcome given  $\delta \in (0, 1)$ .

In order to explore what contract with transfer payments the manufacturer chooses when she wants to both price discriminate and improve bilateral trade, I make a simplifying assumption. One market is served by a sequence of short-lived retailers, with whom no relational contract can be implemented. In each period, this market is served by one monopolistic retailer who lives only for that period. After the period, the retailer leaves the market and a new one takes his place. I denote the market and the corresponding short-lived retailer by  $S$ , and the long-lived retailer in the other market by  $R$ .  $S$  will always buy according to  $q_S^*(w)$ . The fact that  $S$  is short lived in model terms could come from him being a small retailer. It is commonly argued that small businesses have low discount factors, due to financing constraints or risk preferences. I make this assumption because it considerably simplifies the analysis of an otherwise very complicated optimization problem. In fact, the following result shows that in any contract with two (long-lived) buyers, at least one of them will not receive a transfer. The assumption of one short-lived retailer who will not receive a transfer to begin with then considerably simplifies the demonstration of the use of downstream transfers.

**Lemma 3.2.** When the manufacturer deals with two long-lived buyers, at least one will not receive a transfer.

*Proof.* In the general problem with two buyers,  $M$  implements  $(w_e, q_{1,e}, q_{2,e})$  to solve

$$\begin{aligned} \max_{w_e, q_{1,e}, q_{2,e}} \left\{ V = (w_e - c)(q_{1,e} + q_{2,e}) - \frac{(1 - \delta)\pi_1^*(w_e) - \pi_1(q_{1,e}, w_e) + \delta\pi_1^*(w^u)}{\delta} \right. \\ \left. - \frac{(1 - \delta)\pi_2^*(w_e) - \pi_2(q_{2,e}, w_e) + \delta\pi_2^*(w^u)}{\delta} \right\} \\ \text{s.t. } \frac{(1 - \delta)\pi_i^*(w_e) - \pi_i(q_{i,e}, w_e) + \delta\pi_i^*(w^u)}{\delta} \geq 0, \quad i = 1, 2. \end{aligned}$$

The derivative with respect to  $w_e$  is

$$\frac{1 - \delta}{\delta} \left( \sum_{i=1}^2 q_i^*(w_e) - \sum_{i=1}^2 q_{i,e} \right).$$

Hence,

$$\frac{\partial V}{\partial w_e} < 0 \iff \sum_{i=1}^2 q_{i,e} > \sum_{i=1}^2 q_i^*(w_e).$$

In any solution, it will be the case that  $\sum_{i=1}^2 q_{i,e} > \sum_{i=1}^2 q_i^*(w_e)$ . Again,  $w_e = c$  cannot be a solution. For  $w_e > c$ , consider the case that  $\sum_{i=1}^2 q_{i,e} = \sum_{i=1}^2 q_i^*(w_e)$ . The cheapest way to implement this is to set  $w_e$  and not pay any transfers. Knowing that the demanded quantity will be  $\sum q_i^*(w_e)$ ,  $M$  ideally sets  $w_e = w^u$  and does not improve on the one-shot game. For the case  $\sum_{i=1}^2 q_i^*(w_e) > \sum_{i=1}^2 q_{i,e}$ ,  $M$  can always raise  $w_e$  to the point where  $\sum_{i=1}^2 q_i^*(w_e) = \sum_{i=1}^2 q_{i,e}$  and sell the same quantity at a higher wholesale price without paying any transfer.  $M$  will also not set  $q_{i,e} < q_i^*(w_e)$  for any individual  $R_i$ , because, in this case, by inducing  $R_i$  to buy  $q_i^*(w_e)$ , she sells a higher quantity at the given price without paying a subsidy, thus increasing her profit. Since  $\frac{\partial F_{i,e}}{\partial w_e} > 0$  for  $q_{i,e} > q_i^*(w_e)$ ,  $M$  reduces  $w_e$  until she hits one of the constraints  $F_{i,e} \geq 0$

and at least one retailer is not paid a transfer; or until for some  $i$ ,  $q_{i,e} = q_i^*(w_e)$ , and again that retailer is not paid a transfer. ■

The lemma provides an outcome justification for the assumption that one buyer will not receive a transfer. A short-lived retailer can take the role of this buyer. The existence of such a player makes the two-buyer problem more tractable. While this is not without loss of generality, it will show an instance where a transfer is paid that serves to discriminate between buyers and improve bilateral trade with one of them. It highlights the different use of the transfer compared to its (non-existent) role in the case with one retailer.

Because the short-lived retailer buys only according to  $q_S^*(w)$ , there is now a more explicit role for  $w_e$ . Contrary to the long-term relationship with the other buyer, for  $S$  the wholesale price has no role in reducing deviation incentives when improving efficiency. Rather, it directly determines the profit made from the trade with  $S$ . There is thus an additional direct optimization role for  $w_e$  independent of relationally implemented quantities. This puts some pressure on  $w_e$  that balances the manufacturer's wish to universally lower  $w_e$  for each quantity level implemented in her long-term relationship. Lowering the wholesale price as far as possible may now give up profit from the trade with the short-lived buyer. Because of this, it is harder to reduce deviation incentives and improve efficiency in the trade with the long-lived buyer. A subsidization of the long-lived retailer may now become necessary to provide incentives. In the following result I provide a characterization of the contract at an internal solution, that is when the non-negativity constraint on the transfer to  $R$  is slack and a transfer is paid.

**Proposition 3.3.** Suppose the manufacturer deals with a short- and a long-lived retailer. At an interior solution with a nonzero lump-sum transfer to the long-lived retailer, she chooses wholesale price and quantities  $(w_e, q_{R,e}, q_{S,e})$  such that

$$\begin{aligned} w_e &= \frac{p_R(q_{R,e}) + q_{R,e}p'_R(q_{R,e}) - \delta c}{1 - \delta}, \\ q_{R,e} &= \frac{(1 - \delta)[\delta q_S^*(w_e) + (1 - \delta)q_R^*(w_e)] + \delta q_S^{*'}(w_e)[p_R(q_{R,e}) - c]}{(1 - \delta)^2 - \delta q_S^{*'}(w_e)p'_R(q_{R,e})}, \\ q_{S,e} &= q_S^*(w_e). \end{aligned}$$

*Proof.* The maximization problem of the manufacturer is

$$\max_{w_e, q_{R,e}} (w_e - c)(q_S^*(w_e) + q_{R,e}) - \frac{(1 - \delta)\pi_R^*(w_e) - \pi_R(q_{R,e}, w_e) + \delta\pi_R^*(w^u)}{\delta}.$$

By Assumption 3.1 and the fact that  $q_i^{*'}(w) < 0$ , this objective is globally concave. Taking first-order conditions with respect to  $w_e$  and  $q_{R,e}$ , we obtain

$$q_S^*(w_e) + q_{R,e} + (w_e - c)q_S^{*'}(w_e) + \frac{1 - \delta}{\delta}q_R^*(w_k) - \frac{q_{R,e}}{\delta} = 0 \quad (3.16)$$

and

$$w_e - c + \frac{p_R(q_{R,e}) - w_e + q_{R,e}p'_R(q_{R,e})}{\delta} = 0. \quad (3.17)$$

Rearrange condition (3.17) to obtain

$$w_e = \frac{p_R(q_{R,e}) + q_{R,e}p'_R(q_{R,e}) - \delta c}{1 - \delta}.$$

This can be inserted into the first condition to yield

$$q_{R,e} = \frac{(1 - \delta)[\delta q_S^*(w_e) + (1 - \delta)q_R^*(w_e)] + \delta q_S^{*'}(w_e)[p_R(q_{R,e}) - c]}{(1 - \delta)^2 - \delta q_S^{*'}(w_e)p'_2(q_{R,e})}.$$

$S$  acts completely myopically and thus his allocation is given by the one-shot optimization response to the list price:  $q_S^*(w_e)$ . ■

The proposition characterizes the internal solution where a downstream fee to buyer  $R$  is in fact paid. The characterization of the wholesale price and the implemented quantity is straightforward. First, the expression for  $w_e$  comes from first-order condition (3.17). Rewriting it gives

$$p_R(q_{R,e}) + q_{R,e}p'_R(q_{R,e}) - w_e = -\delta(w_e - c). \quad (3.18)$$

As usual, compare this to the standard first-order condition of a monopolist on the downstream market  $R$  who chooses quantity facing input cost  $w_e$ . Observe that  $w_e > c$  because the manufacturer could never compensate a negative margin. So, the right-hand side of (3.18) is negative. Compared to a downstream monopolist on market  $R$ , the manufacturer chooses a higher quantity. As parties become less patient, the quantity chosen approaches the one myopically chosen by a downstream firm facing  $w_e$ . As parties become arbitrarily patient, the quantity approaches the efficient quantity. Notice also that (3.18) yields, after rearranging,  $w_e - c = \frac{p_R(q_{R,e}) + q_{R,e}p'_R(q_{R,e}) - c}{1 - \delta}$  so that the margin each period from trading with market  $R$  reflects the social value of continued trade for an integrated vertical structure.

Second, rearranging the first-order condition (3.16) yields:

$$q_S^*(w_e) + (w_e - c)q_S^{*'}(w_e) = \frac{1 - \delta}{\delta}(q_{R,e} - q_R^*(w_e)). \quad (3.19)$$

In comparison to a normal monopoly supplier in a vertical structure, the choice of  $w_e$  is distorted. It takes into account the quantity sold on market  $R$  and the deviation incentives on that market. The left-hand side of (3.19) is the derivative of profit with respect to price of a monopolist who sells to a downstream buyer with demand  $q_S^*(w)$ . If the supplier only sold to market  $S$ , this expression would be equal to zero at the optimum. Now, at  $w_e$ , it is positive. Thus,  $w_e$  is lower than  $w_S^*$ .  $w_e$  affects revenue from both markets  $S$  and  $R$ . But it also affects the deviation incentive of  $R$  each period. The higher the list price, the less profit  $R$  makes from a deviation. As  $\delta \rightarrow 1$ , the wholesale price approaches  $w_S^*$ , the optimal price when a monopolist sells in one-shot interactions to a buyer with demand  $q_S^*(w)$ .

These two observations illustrate the differentiating and integrating effect achieved through the transfer payment. It enables price discrimination between the two markets under the restriction that one uniform wholesale price be set and permits an approximation to vertical integration with the long-lived buyer. Because decreasing the common list price as far as possible sacrifices profit made from the myopic retailer, the value of  $w_e$  is now not pinned down by the binding constraint on the transfer. Rather,  $M$  wants to keep  $w_e$  elevated, even though the incentives to prevent  $R$ 's deviation push it below the one-shot optimal price for market  $S$ . Additional incentives for  $R$  are now provided through the payment of a lump-sum transfer. This transfer has as a goal to move  $M$  and  $R$  together closer towards vertical integration. This integration effect can clearly be demonstrated through the expressions for the equilibrium values of wholesale price and quantity. In (3.18), the optimal quantity is determined in direct reference to the demand in the  $R$  market, taking the wholesale price  $w_e$  into account. This indicates that  $M$  and  $R$  act like an integrated entity when choosing the quantity for the  $R$  market, under the constraint that there is another influence on the wholesale price. The determination of this wholesale price is done through condition (3.19). What enters here is not the market demand on the  $S$  market directly, but mediated through the optimal demand of the  $S$  retailer,  $q_S^*(w_e)$ . Thus, the perspective is one of an upstream monopolist who maximizes profit by setting the wholesale price to a separated downstream retailer. The distortion in the wholesale price is determined by the difference in implemented quantity  $q_{R,e}$  versus one-shot demand  $q_R^*(w_e)$  of retailer  $R$ . This indicates that when setting the wholesale price,  $M$  balances profit from the  $S$  retailer, and efficiency on the  $R$  market. All of this combined paints the image of  $M$  and  $R$  forming a joint entity that tries to achieve efficiency on its “home” market while dealing with an outside third-party retailer  $S$ , who is separated from their structure. Thus, we can see that the relational contract work to improve efficiency in the bilateral trade between its parties. This integration is held together by the relational agreement and hence becomes stronger as  $\delta$  increases. Discrimination between the two retailers is implemented by the fact that one is treated like a member of an integrated structure with  $M$ , while the second retailer faces this structure as a separated outside buyer. The integrated structure of  $M$  and  $R$  is not perfect, however, as in addition to the deviation incentives from impatience, it faces the constraint of a joint price for internal transfers and external sales. The short-lived buyer benefits from an externality that stems from the role of the wholesale price as governing internal transfers between  $M$  and  $R$ . He faces a per-unit price that is lower than under price discrimination. This results from the desire of  $M$  to improve bilateral trade with  $R$ , where a lower wholesale price helps support higher quantities.

Slotting allowances as a means for a seller to work together with one buyer while facing a second buyer have also been analyzed by Caprice and von Schlippenbach (2013). In their model, one buyer bargains with two suppliers. One pair potentially acts together and uses slotting allowances to raise the wholesale price in their bilateral trade in order to exploit the other supplier. In my model, the slotting allowance has efficiency-enhancing effects on the trade of the “cooperating” pair and the outside buyer benefits from a lower wholesale price.



*Linear example.*

The preceding result is tractable and can readily be illustrated in a specific example. Consider two markets with linear inverse demand

$$p_i(q) = a_i - bq.$$

In this setup, the enforced equilibrium values are

$$w_e = \frac{a_S + c}{3 - \delta} + \frac{1 - \delta}{3 - \delta}c \quad \text{and} \quad q_{R,e} = \frac{(3 - \delta)a_R - 2c}{2b(3 - \delta)} - \frac{(1 - \delta)a_S}{2b(3 - \delta)}.$$

Now, as  $\delta \rightarrow 1$ ,  $q_{R,e} \rightarrow \frac{a_R - c}{2b}$  which is the quantity an integrated monopolist would sell on market  $R$ . And  $w_e \rightarrow \frac{a_S + c}{2}$  which is equal to the wholesale price that would be set in the simple double marginalization setup (and hence under third-degree price discrimination) for market  $S$ . A corner solution can always be avoided by setting  $b$  high enough. It can be shown that there is a threshold value  $\underline{b}(P)$  for each set  $P$  of parameter values  $((a_i), c, \delta)$  such that  $F_e > 0$  whenever  $b > \underline{b}(P)$ .

The margin for  $R$ ,  $p_R(q_{R,e}) - w_e$ , can be negative in this example. This is the case when  $a_R < \frac{(1+\delta)a_S + 2(1-\delta)c}{3-\delta}$ . Since for  $q_{R,e} > 0$  we need  $a_R > \frac{(1-\delta)a_S + 2c}{3-\delta}$ , the margin for  $R$  is negative when  $a_R \in \left( \frac{(1-\delta)a_S + 2c}{3-\delta}, \frac{(1+\delta)a_S + 2(1-\delta)c}{3-\delta} \right)$ . This sheds light on some occurrences of supposed “loss-leader” pricing where retailers price goods below their input cost. In the context of the present model, this phenomenon is just an effect of the relational contract and compensated by a different payment. Loss leading is commonly seen with suspicion and banned in many jurisdictions.<sup>12</sup> Chen and Rey (2012) show that banning loss leading can increase welfare, while Johnson (2017a), in a model with consumer biases, argues that loss leading can have pro-competitive effects. In my model, there are no behavioral biases. As the relational contract improves efficiency, banning loss leading could have harmful effects.

### 3.3.3 The cost of improving trade with the long-lived retailer

Contrary to the one-retailer case, implementing the relational contract in the presence of short-lived buyers leads the manufacturer to pay a transfer to the long-lived retailer. Under what circumstances will the manufacturer want to enforce the relational contract with this transfer? At some point, paying it might be too costly and  $M$  may wish to just revert to the uniform-price outcome without transfers.

The supplier’s profit under optimal uniform linear pricing is smaller than the one with subsidy as long as

$$(w_e - c)(q_S^*(w_e) + q_{R,e}) - F_{R,e} \geq (w^u - c)(q_S^*(w^u) + q_R^*(w^u)). \quad (3.20)$$

Recall from (3.2) that the transfer  $F_{R,e}$  is determined as  $F_{R,e} = \frac{(1-\delta)\pi_R^*(w_e) - \pi_R(q_{R,e}, w_e) + \delta\pi_R^*(w^u)}{\delta}$ . Rewrite the numerator of this expression as  $[\pi_R^*(w_e) - \pi_R(q_{R,e}, w_e)] + \delta[\pi_R^*(w^u) - \pi_R^*(w_e)]$ . In

<sup>12</sup>See the discussion and references provided in Johnson (2017a).

addition, define as  $\Pi_M(q_{R,e}, w_e) = (w_e - c)(q_S^*(w_e) + q_{R,e})$  the gross profit of the manufacturer when she subsidizes market  $R$  and as  $\Pi_M^u = (w^u - c)(q_S^*(w^u) + q_R^*(w^u))$  her profit under optimal uniform pricing. Then, condition (3.20) can be rearranged to

$$\frac{\pi_R^*(w_e) - \pi_R(q_{R,e}, w_e)}{\delta} + [\pi_R^*(w^u) - \pi_R^*(w_e)] \leq \Pi_M(q_{R,e}, w_e) - \Pi_M^u.$$

The lump-sum subsidy acts as a transfer of profits. It needs to cover the retailer's deviation incentive and is financed through the supplier's gross profit increase. When parties become arbitrarily patient, the deviation incentive vanishes and the quantity is enforceable if the profit gain for the supplier is big enough to compensate the retailer for his gross profit loss. As parties become less patient, more weight is placed on the current deviation incentive. The profit increase of the supplier must be able to compensate the increase in gross profit from current deviation and future punishment outcomes.

Further rearranging and expanding  $\pi_R^*(w^u)$  yields

$$\begin{aligned} [\pi_R^*(w_e) - \pi_R(q_{R,e}, w_e)] + \frac{\delta}{1 - \delta} [(p_R(q_R^*(w^u)) - c)q_R^*(w^u) + (w^u - c)q_S^*(w^u)] \\ \leq \frac{\delta}{1 - \delta} [(p_R(q_{R,e}) - c)q_{R,e} + (w_e - c)q_S^*(w_e)] \end{aligned}$$

The value of future efficiency gains in market  $R$  minus the cost to the supplier from the distortion in market  $S$  must cover deviation incentives in each period.

This result is similar in spirit to the work of Asker and Bar-Isaac (2014) who analyze conditions under which a manufacturer may want to pay a downstream transfer. They consider exclusionary motives and find that the manufacturer is willing to pay a transfer when he has enough rents to share to compensate retailers for not accommodating his rival. Their model has homogeneous retailers and thus no double marginalization. Here, I show how a subsidy can be paid to improve bilateral trade and raise efficiency. This, as in their model, also transfers some rents to the retailers. Asker and Bar-Isaac (2014) provide a dynamic formulation of their model that shows that the profit increase has to cover foregone losses by the retailers in all future periods. In my model, the dynamic setup permits an improvement over the double-marginalization outcome. The profit increase for the manufacturer through the relational contract needs to cover the deviation incentives, the importance of which are determined by the patience parameter  $\delta$ .

### *Linear example.*

Consider again the linear example with market demand  $p_i(q) = a_i - bq$ ,  $i = R, S$ . Enforcement is profitable when

$$a_R \leq \frac{1}{3} \left( a_S + 2c + \frac{\delta 4(a_S - c)^2}{\sqrt{(a_S - c)^2 \delta (3 - \delta) (3b - 4)}} \right).$$

When  $b < \frac{4}{3}$ , enforcement is always profitable. When  $b > \frac{4}{3}$ , enforcement is profitable when  $a_R$  is not too big relative to  $a_S$ . For  $\delta \geq \frac{3b-4}{b}$ , this would for instance be satisfied by the requirement

that  $a_R < a_S$ . But even for low  $\delta$  enforcement can be profitable. For instance, if  $b = 1.5$ ,  $c = 0.5$ ,  $\delta = 0.2$ , and  $a_S = 5$ , we need  $a_R \leq 4.27$ .

### 3.4 Multiple long-lived retailers and homogeneous downstream competition

Suppose there are  $n \geq 2$  identical long-lived retailers in market  $R$ . If there is no subsidy, competition between the retailers causes the market price to be equal to the list price  $w$ . The total quantity sold in market  $R$  is therefore  $q_R(w)$ , where  $q_R(p)$  is the direct demand in market  $R$  for price  $p$ . If the manufacturer wants to sell a higher quantity on that market, she has to pay a subsidy to at least one retailer. Assume the remaining retailer(s) will keep setting their price equal to  $w$ . Any subsidized retailer has the possibility to take the subsidy  $F$ , set price equal to  $w$ , and thus obtain a one-time payoff of  $F$ . In order to provide incentives for adherence to the relational contract, the supplier has to promise a lifetime payoff strictly greater than zero to the subsidized retailer. The incentives can be most efficiently provided when the manufacturer promises to subsidize the same retailer again as long as he did not deviate. For any total rent the manufacturer pays out, the continuation payoff is largest and hence non-deviation incentives are strongest for any retailer when he receives all of this rent. Because the retailers are identical, the manufacturer does not want to split up the rent given out to downstream firms.

**Lemma 3.3.** If retailers are identical and the manufacturer wants to subsidize market  $R$ , she always trades with the same retailer. ◆

This result is reminiscent of Board (2011) where the principal prefers trade with agents she has traded with before (“insiders”). It opens up a new scope for punishment for the manufacturer. As the market is contested, she can now exclude a deviating retailer from trade and thus easily min-max him with a permanent-punishment strategy. This exclusion strategy is also used by Calzolari and Spagnolo (2017) to provide incentives.

### 3.5 Conclusion

The literature on retail contracts commonly uses two-part tariffs set by an upstream firm to analyze wholesale contracting. In actual retail contracts, however, such tariffs are rare. Instead, contracts with lump-sum payments from a the upstream seller to the downstream buyer are common. I describe the role these transfers from seller to buyer can play when the parties interact repeatedly in a full-information setting. The analysis is embedded in a broader discussion of repeated contracting in a vertical chain when lump-sum transfers from the buyer to the seller are impossible. In the repeated game with one buyer, the seller does not pay a transfer. It is cheaper for her to provide incentives by lowering the per-unit price. Implemented wholesale price and

quantity purchased by the buyer are then determined by the condition that the transfer will not be negative. The instrument of a transfer could easily compensate the buyer for any quantity he buys. But it has the drawback that the buyer can take the money without adhering to the prescribed quantity order. Reducing the wholesale price is a move toward an imperfect vertical integration. It is fragile because of the deviation incentives of the buyer. As the buyer becomes arbitrarily patient, the parties implement the efficient quantity.

An explicit role for the downstream transfer arises when the seller deals with several buyers in different markets. If the seller is constrained to set a common wholesale price for all buyers, she can use the downstream transfer to differentiate between them. At the same time, a step towards efficiency-enhancing integration is undertaken with the subsidized buyer.

My paper contributes to the literature on downstream transfers. In the main treatment, I assume they are offered by the seller and paid to non-competing buyers. This is in contrast to a large part of the literature that shows how such transfers can be set by the buyers to facilitate collusion. While in these cases such transfers are anti-competitive, I show an efficiency-enhancing effect they can have in repeated contracting. The seller uses simple contracts to coordinate between several markets and mitigate double marginalization. It is thus insightful to carefully consider the market structure in practice when analyzing the effects of such contractual provisions. An interesting case to consider for future work would be upstream competition and to ask whether the effect of transfers of approaching integration could also have exclusionary effects. Finally, my paper also provides an example of a model of repeated interaction with limited possibilities for side payments between parties. Analyzing the consequences of such setups for incentives, rent extraction, and renegotiation, can be promising future work.

## Part III

# Appendices



# A Appendix: Chapter 1

## A.1 General model

### A.1.1 Proof of Proposition 1.1

Total differentiation of the first-order conditions implies that, for  $i = 0, 1$  and  $j \neq i$ ,

$$\frac{dp_i}{d\mu} = \frac{\frac{\partial^2 \Pi_i}{\partial p_i \partial p_j} \frac{\partial^2 \Pi_j}{\partial p_j \partial \mu} - \frac{\partial^2 \Pi_i}{\partial p_i \partial \mu} \frac{\partial^2 \Pi_j}{\partial p_j^2}}{\frac{\partial^2 \Pi_i}{\partial p_i^2} \frac{\partial^2 \Pi_j}{\partial p_j^2} - \frac{\partial^2 \Pi_i}{\partial p_i \partial p_j} \frac{\partial^2 \Pi_j}{\partial p_j \partial p_i}},$$

where all expressions are evaluated at the equilibrium. By the stability condition, the denominator is positive. Using the entrant's first-order condition,  $\frac{\partial^2 \Pi_1}{\partial p_1 \partial \mu} = 0$  at the equilibrium. Hence,  $dp_0/d\mu > 0$  if and only if  $-\frac{\partial^2 \Pi_0}{\partial p_0 \partial \mu} \frac{\partial^2 \Pi_1}{\partial p_1^2} > 0$ , that is, using concavity of  $\Pi_1$ ,  $\frac{\partial^2 \Pi_0}{\partial p_0 \partial \mu} > 0$ . Similarly,  $dp_1/d\mu > 0$  if and only if  $\frac{\partial^2 \Pi_1}{\partial p_0 \partial p_1} \frac{\partial^2 \Pi_0}{\partial p_0 \partial \mu} > 0$ , that is, using strategic complements (A1(b)),  $\frac{\partial^2 \Pi_0}{\partial p_0 \partial \mu} > 0$ . Finally,  $\frac{\partial^2 \Pi_0}{\partial p_0 \partial \mu} > 0$  if and only if

$$D_M - D_0 + p_0 \left( \frac{\partial D_M}{\partial p_0} - \frac{\partial D_0}{\partial p_0} \right) > 0.$$

Simple rearrangements show that this condition is equivalent to conditions (1.1) and (1.2) in the proposition.

### A.1.2 Proof of Proposition 1.2

(i) Using (1.3), it suffices to show that (1.4) is equivalent to  $\frac{\partial \Pi_1}{\partial \mu} + \frac{\partial \Pi_1}{\partial p_0} \frac{\partial p_0^*}{\partial \mu} > 0$ . This equivalence follows from  $\frac{\partial \Pi_1}{\partial \mu} = -p_1 D_1$  and  $\frac{\partial \Pi_1}{\partial p_0} = p_1 (1 - \mu) \frac{\partial D_1}{\partial p_0}$  after simple rearrangements, using  $\frac{\partial D_1}{\partial p_0} > 0$  (by A1(i)).

(ii) Using (1.3), it suffices to show that (1.5) is equivalent to  $\frac{\partial \Pi_0}{\partial \mu} + \frac{\partial \Pi_0}{\partial p_1} \frac{\partial p_1^*}{\partial \mu} > 0$ . This equivalence follows from  $\frac{\partial \Pi_0}{\partial \mu} = p_0 (D_M - D_0)$  and  $\frac{\partial \Pi_0}{\partial p_1} = p_0 (1 - \mu) \frac{\partial D_0}{\partial p_1}$  after simple rearrangements, using  $\frac{\partial D_0}{\partial p_1} > 0$  (implied by A1(i)).

## A.2 Salop model

The following lemma establishes the equilibrium regions and their according prices. It establishes the equilibria referred to in Corollary 1.1.

**Lemma A.1.** With competition for the circle, the following equilibrium regions arise:

- (i) If  $t$  is high ( $t > 4/3$ ), an ICE arises. Prices are

$$p_0^* = \frac{(3-\mu)t + 8\mu}{2(5\mu+3)}, p_1^* = \frac{(3+2\mu)t + 4\mu}{2(5\mu+3)}.$$

- (ii) For intermediate values of  $t$  ( $\frac{4}{7} < t < \frac{4}{3}$ ) and high values of  $\mu$  ( $\mu > \frac{9t-12}{t-12}$ ), an FCE emerges. Prices are

$$p_0^* = 1 - \frac{t}{4}, p_1^* = \frac{1}{2} + \frac{t}{8}.$$

- (iii) If  $t$  takes on relatively low values and  $\mu$  is low ( $\frac{36\mu-36\mu^2}{15\mu-8\mu^2+9} < t < \frac{12-12\mu}{9-\mu}$  and  $\mu \leq 0.6$ ), a CFE arises. Prices are

$$p_0^* = \frac{(6+2\mu)t}{(3-3\mu)4}, p_1^* = \frac{(6-2\mu)t}{(3-3\mu)4}.$$

- (iv) If  $t$  is relatively low ( $\frac{4-4\mu}{3\mu+1} < t < \frac{4}{7}$ ) and  $\mu$  is high ( $\mu > 0.6$ ), an MPE arises. Prices are

$$p_0^* = 1 - \frac{t}{4}, p_1^* = 1 - \frac{3t}{4}.$$

- (v) If Assumption 1.2 holds, there are no other PSE than those mentioned in (i)-(iv). In particular, there exists no pure-strategy equilibrium with  $\mu \in (0, 1)$  and  $t < \min\left(\frac{36\mu-36\mu^2}{15\mu-8\mu^2+9}, \frac{4-4\mu}{3\mu+1}\right)$ .

◆

### A.2.1 Proof of Lemma A.1

(i) In the proposed equilibrium, the incumbent does not serve all captive consumers. His demand from captive consumers is thus  $D_0^M = 2\frac{1+\frac{t}{4}-p_0}{t}$ . The demand from switchers is derived from the indifferent consumer, who is located at  $x = \frac{\frac{t}{2}-p_0+p_1}{2t}$ . The profit of the incumbent as  $\Pi_0 = p_0 \left[ \mu \left( 2\frac{1+\frac{t}{4}-p_0}{t} \right) + (1-\mu) \left( \frac{\frac{t}{2}-p_0+p_1}{t} \right) \right]$ . The entrant's demand comes only from switchers. Its profit is thus  $\Pi_1 = (1-\mu)p_1 \frac{\frac{t}{2}+p_0-p_1}{t}$ . The entrant's interior best-response condition is thus:  $p_1 = \frac{t+2p_0}{4}$ . First-order conditions give equilibrium price candidates  $p_0 = \frac{(3t+8\mu-t\mu)}{2\mu+8\mu+6}$  and  $p_1 = \frac{1}{2} \frac{3t+4\mu+2t\mu}{5\mu+3}$ .

For the incumbent's demand functions posited above to be valid, demand from switchers must satisfy  $D_0^M < \mu$ . This is the case when  $t > \frac{4}{3}$ . Further, the indifferent switcher needs to have nonnegative valuation and be located between positions 0 and  $\frac{1}{2}$ . For  $t \leq 2$ , we can verify that this holds.

(ii) We posit that the captive market is exactly covered in this region. The price that achieves this sets the net utility of the captive consumer located at  $\frac{1}{2}$  equal to zero. We have thus  $p_0 = 1 - \frac{t}{4}$ . The entrant reacts to this price according to the same reaction function as in case (i) and thus sets  $p_1 = \frac{1}{2} + \frac{t}{8}$ . We now need to check for possible profitable deviations of the



incumbent. For an upward deviation, the elastic demand from case (i) applies, for a downward deviation, loyalists' demand will remain fixed at one.

Considering this, the incumbent has no profitable upward deviation when  $t \leq \frac{4}{3}$ . It has no profitable downward deviation when  $t > 12\frac{1-\mu}{9-\mu}$ . The indifferent switcher has non-negative utility because all consumers would be prepared to buy from the incumbent. Finally, we need to check that the indifferent consumer sits between 0 and  $\frac{1}{2}$ . This is assured when  $t > \frac{4}{7}$ . To determine which lower bound for  $t$  applies, we observe that  $12\frac{1-\mu}{9-\mu} \geq (<) \frac{4}{7}$  when  $\mu \leq (>) 0.6$ .

(iii) In the proposed equilibrium, the captive market is covered and the captive consumer at  $\frac{1}{2}$  obtains a positive surplus. The incumbent thus has an inelastic captive demand of  $\mu$ . Its profit is  $\Pi_0 = p_0\mu + (1-\mu)\left(\frac{1}{2}\frac{t-2p_0+2p_1}{t}\right)$ . The entrant's profit function remains the same as in the previous cases ( $\Pi_1 = p_1(1-\mu)\left(\frac{1}{2}\frac{t-2p_1+2p_0}{t}\right)$ ). First-order conditions yield equilibrium candidate prices  $p_0 = \frac{1}{6}t\frac{\mu+3}{1-\mu}$  and  $p_1 = \frac{1}{6}t\frac{3-\mu}{1-\mu}$ .  $t < 12\frac{1-\mu}{9-\mu}$  guarantees that all captive consumers get a positive surplus. The indifferent switcher is located between 0 and  $\frac{1}{2}$  when  $\mu < 0.6$ . We further need to check deviations by the incumbent to  $\hat{p}_0 = 1 - \frac{t}{4}$  where he only serves the captive market. When  $t > \frac{36\mu-36\mu^2}{15\mu-8\mu^2+9}$ , no profitable deviation of this type exists. First note that for  $t > \frac{12\mu-12}{11\mu-15}$ , the price  $1 - \frac{t}{4}$  is so low that the incumbent would still serve switchers: To see this, compare the net utility of the consumer located at 0, which is  $1 + \frac{t}{4} - 1 + \frac{t}{4} = \frac{t}{2}$  when buying from the deviating incumbent, whereas at the entrant it is  $1 - \frac{t}{4} - \frac{1}{6}t\frac{3-\mu}{1-\mu} = \frac{1}{12}\frac{-9t-12\mu+5t\mu+12}{1-\mu}$ . The claim follows after some rearrangements. Thus suppose  $t < \frac{12\mu-12}{11\mu-15}$ , so that the deviation is feasible. As the profit before deviation is  $t\frac{\mu+3}{6-6\mu}\left(\mu + (1-\mu)\left(\frac{\frac{t}{2}-t\frac{\mu+3}{6-6\mu}+t\frac{3-\mu}{6-6\mu}}{t}\right)\right) = \frac{1}{36}t\frac{(\mu+3)^2}{1-\mu}$ , the net benefit from deviation is  $\mu\left(1 - \frac{t}{4}\right) - \frac{1}{36}t\frac{(\mu+3)^2}{1-\mu} = -\frac{1}{36}\frac{-9t+36\mu-15t\mu-36\mu^2+8t\mu^2}{\mu-1}$ . This expression remains negative as long as  $-9t+36\mu-15t\mu-36\mu^2+8t\mu^2 < 0$ , or equivalently,  $t > \frac{36\mu-36\mu^2}{15\mu-8\mu^2+9}$ . Finally note that  $t > \frac{12\mu-12}{11\mu-15}$  implies  $t > \frac{36\mu-36\mu^2}{15\mu-8\mu^2+9}$ .

(iv) In case (iii), if  $\mu \geq 0.6$ , the indifference condition for switchers gives a value  $\leq 0$ . In this case, the incumbent gives up serving the switchers and sets price  $p_0 = 1 - \frac{t}{4}$  to serve only the captive consumers at maximum profit. The entrant sets the highest possible price such that all switchers buy from it: the switcher at position zero is indifferent between buying from the entrant and buying from the incumbent. This is achieved with price  $p_1 = 1 - \frac{3t}{4}$ . The entrant has no incentive to lower its price. An upward deviation is not profitable when  $t \leq \frac{4}{7}$ . The incumbent has no incentive to deviate downward and get switchers to buy from it when  $t \geq 4\frac{1-\mu}{3\mu+1}$ .

(v) First, consider equilibria where the switcher market is not completely covered (and thus the captive market is not covered either). A firm that marginally increases prices would thus not lose any demand to the competitor. Hence, its optimal price must correspond to the price of a monopolist with demand  $2\frac{1+t/4-p}{t}$ . Under Assumption 1.2, maximization of this function would lead to a price that is so low that demand would be greater than 1, a contradiction. Thus, in the following, we can rule out equilibria with incomplete coverage in the switcher market. Second, consider an equilibrium with incomplete coverage on the captive market. Contrary to the ICE in (i), first assume that there is market partition. Then the price of the incumbent

must correspond to the optimum on the monopolistic captive market. As  $t < 2$  by Assumption 1.2, this price is inconsistent with incomplete coverage. Thus any equilibrium with incomplete coverage in the captive market must have both firms competing actively in the switcher market. Then the first-order conditions imply that it must have the form of the equilibrium in (i). Thus, we can rule out additional incomplete coverage equilibria. Third, we rule out additional equilibria with full coverage in both markets. There can be no such equilibrium where market partition takes place and, unlike in (iii), the marginal consumer earns positive surplus. In this case, the monopolist could profitably raise prices. Thus, any equilibrium with full coverage and market partition must give zero surplus to the marginal captive consumers, as in (iii). This fixes both prices as in (iii). As to equilibria with full coverage, but without market partition, they either give zero surplus to marginal captive consumers as in (ii) or a positive surplus as in (iv). In the former case, the zero surplus requirement immediately determines both prices as in (ii). In the latter case, the equilibrium conditions must necessarily hold for both firms. Finally, it is straightforward to see that the region where none of the conditions (i)-(iv) holds is given by  $\mu \in (0, 1)$  and  $t < \min\left(\frac{36\mu-36\mu^2}{15\mu-8\mu^2+9}, \frac{4-4\mu}{3\mu+1}\right)$ . Thus, by the previous analysis, there can be no pure-strategy equilibrium in this region.

### A.2.2 Profits

As an immediate corollary of Lemma A.1, we obtain the following characterization of equilibrium profits.

**Lemma A.2.** The profits in the equilibrium regions are given as follows:

- (i) ICE:  $\Pi_0 = \frac{1+\mu}{4t} \frac{(3t+8\mu-t\mu)^2}{(5\mu+3)^2}$  and  $\Pi_1 = \frac{1}{4} (1-\mu) \frac{(3t+4\mu+2t\mu)^2}{t(5\mu+3)^2}$
- (ii) FCE:  $\Pi_0 = \frac{7t+4\mu+t\mu-4}{8t} \left(1 - \frac{t}{4}\right)$  and  $\Pi_1 = \frac{1}{64} (1-\mu) \frac{(t+4)^2}{t}$
- (iii) MPE:  $\Pi_0 = \mu \left(1 - \frac{t}{4}\right)$  and  $\Pi_1 = (1-\mu) \left(1 - \frac{3t}{4}\right)$
- (iv) CFE:  $\Pi_0 = (1-\mu) \left(\frac{3+\mu}{3-3\mu}\right)^2 \frac{t}{4}$  and  $\Pi_1 = (1-\mu) \left(\frac{3-\mu}{3-3\mu}\right)^2 \frac{t}{4}$

◆

Lemma A.2 is useful to understand how profits depend on parameters.

### Corollary A.1.

- (i)  $\Pi_0$  is increasing in  $\mu$  everywhere except in the ICE region for  $t > \frac{72\mu+40\mu^2+48}{24\mu+5\mu^2+27}$ .  $\Pi_0$  is increasing in  $t$  in the CFE region and decreasing in the MPE region. It is increasing in  $t$  in the ICE region if and only if  $\mu < \frac{3t}{t+8}$ . It is increasing in the FCE region if and only if  $\mu < \frac{16-7t^2}{t^2+16}$

- (ii)  $\Pi_1$  is increasing in  $\mu$  in CFE, decreasing in the remaining regions.  $\Pi_1$  is increasing in  $t$  in ICE and CFE, decreasing in the remaining regions.

*Proof.* (i) From Lemma A.1, it follows that  $p_1$  is non-decreasing in  $\mu$  except in the ICE region. Hence  $\Pi_0$  is increasing in  $\mu$  everywhere except possibly in the ICE region. There,  $\Pi_0 = \frac{1+\mu}{4t} \frac{(3t+8\mu-t\mu)^2}{(5\mu+3)^2}$ . Hence

$$\frac{\partial \Pi_0}{\partial \mu} = \frac{1}{4} (-3t - 8\mu + t\mu) \frac{27t - 72\mu + 24t\mu - 40\mu^2 + 5t\mu^2 - 48}{t(5\mu + 3)^3}.$$

This expression is positive if and only if the denominator is positive or  $t < \frac{72\mu+40\mu^2+48}{24\mu+5\mu^2+27}$ .

Next, we consider the relation between  $t$  and  $\Pi_0$ . In the CFE region,  $\Pi_0 = (1 - \mu) \left( \frac{3+\mu}{3-3\mu} \right)^2 \frac{t}{4}$ , which is increasing in  $t$ . In the MPE region  $\Pi_0 = \mu \left( 1 - \frac{t}{4} \right)$ , which is decreasing in  $t$ . In the ICE region,  $\frac{\partial \Pi_0}{\partial t} = \frac{1}{4} (\mu + 1) (-3t - 8\mu + t\mu) \frac{-3t+8\mu+t\mu}{t^2(5\mu+3)^2}$ . This is positive only if  $\mu < 3\frac{t}{t+8}$ . In the FCE region,  $\Pi_0 = \frac{7t+4\mu+t\mu-4}{8t} \left( 1 - \frac{t}{4} \right)$ . Thus  $\frac{\partial}{\partial t} \frac{7t+4\mu+t\mu-4}{8t} \left( 1 - \frac{t}{4} \right) = -\frac{1}{32t^2} (16\mu + t^2\mu + 7t^2 - 16)$ . This is positive only if  $\mu < \frac{16-7t^2}{t^2+16}$ .

- (ii) From Lemma A.1, it follows that  $p_0$  is non-increasing in  $\mu$  except in the CFE region. Hence,  $\Pi_1$  is decreasing in  $\mu$  everywhere except possibly in the CFE region. There  $\Pi_1 = (1 - \mu) \left( \frac{3-\mu}{3-3\mu} \right)^2 \frac{t}{4}$ . Hence,  $\frac{\partial \Pi_1}{\partial \mu} = \frac{1}{36} \frac{t}{(\mu-1)^2} (-\mu^2 + 2\mu + 3) > 0$ . Now consider the relation between  $t$  and  $\Pi_1$ . In the ICE region,  $\Pi_1 = \frac{1-\mu}{4t} \frac{(3t+4\mu+2t\mu)^2}{(5\mu+3)^2}$ . Hence  $\frac{\partial \Pi_1}{\partial t} = \frac{1-\mu}{4t} \frac{(3t+4\mu+2t\mu)^2}{(5\mu+3)^2} > 0$ . In the FCE region  $\frac{\partial \Pi_1}{\partial t} = \frac{1}{64t^2} (1 - \mu) (t^2 - 16) < 0$ . In the CFE region,  $\Pi_1 = (1 - \mu) \left( \frac{3-\mu}{3-3\mu} \right)^2 \frac{t}{4}$ , which is increasing in  $t$ . In the MPE region,  $\Pi_1 = (1 - \mu) \left( 1 - \frac{3}{4}t \right)$ , which is decreasing in  $t$ . ■

### A.2.3 Entry

**The Effect of Captive Consumers:** To prove Proposition 1.3, we first rephrase it using concrete parametric conditions.

#### Proposition A.1.

- (i) Suppose  $0 < t < 4/7$ . Then entry takes place only at intermediate levels of  $\mu$  if and only if  $t/4 < F < t\frac{4-3t}{3t+4}$ . This entry will occur near the lower boundary of the MPE region and possibly near the upper boundary of the CFE region. If  $F > t\frac{4-3t}{3t+4}$ , there is no entry for any value of  $\mu$ ; if  $F < t/4$ , entry occurs for all  $\mu$  below a cut-off value.
- (ii) Suppose  $4/7 < t < 4/3$ . Then entry take place only at intermediate levels of  $\mu$  if and only if  $t/4 < F < \frac{1}{8} \frac{(t+4)^2}{12-t}$ . This entry will occur on both sides of the boundary between the MPE region and the CFE region. If  $F > \frac{1}{8} \frac{(t+4)^2}{12-t}$ , there is no entry for any value of  $\mu$ ; if  $F < t/4$ , entry occurs for all  $\mu$  below a cut-off value.
- (iii) Suppose  $4/3 < t$ . Then entry will only occur below a threshold level of  $\mu$ .

*Proof.* (i,ii) For  $0 < t < 4/3$ , the CFE arises at  $\mu = 0$ . In the CFE  $\Pi_1 = (1 - \mu) \left( \frac{3-\mu}{3-3\mu} \right)^2 \frac{t}{4}$ . If  $t/4 < F$ ,  $\Pi_1 < F$  for  $\mu = 0$ . As  $\Pi_1$  is increasing in  $\mu$ , it takes its maximum in the CFE region at the upper boundary of this region. Suppose  $t > 4/7$ . Then this maximum satisfies  $t = \frac{12-12\mu}{9-\mu}$  or equivalently  $\mu = \frac{9t-12}{t-12}$ . Inserting this into the profit expression (which is also the lowest profit in the FCE region) gives  $\frac{1}{8} \frac{(t+4)^2}{12-t}$ . Thus, we require  $\frac{1}{8} \frac{(t+4)^2}{12-t} > F$ .

If  $t < 4/7$ , the highest value of  $\Pi_1$  arises at the lower bound of the MPE region. The entry condition becomes  $(1 - \mu) \left( 1 - \frac{3}{4}t \right) > F$  for the critical value of  $\mu$  satisfying  $\frac{4-4\mu}{3\mu+1} = t$ , which is  $\mu = \frac{4-t}{3t+4}$ ; thus  $\left( 1 - \frac{4-t}{3t+4} \right) \left( 1 - \frac{3}{4}t \right) = t \frac{4-3t}{3t+4} > F$  must hold. Thus, if  $t \frac{4-3t}{3t+4} - \frac{t}{4} > 0$  or equivalently  $t < 0.8$ , there exists intermediate fixed cost levels for which entry takes place; this holds because  $t < 4/7 < 0.8$ . The condition for entry to arise at the upper boundary of the CSE region as well is more stringent, but will hold if  $F$  is sufficiently close to  $t/4$ .

(iii) follows because profits are decreasing in  $\mu$  in the ICE region ( $t > 4/3$ ). ■

**The Effect of Consumer Heterogeneity:** Here, we substantiate the claims concerning the relation between consumer heterogeneity  $t$  and product innovation. For  $\mu > 0.6$ , we require the following auxiliary result.

**Lemma A.3.** Suppose  $\mu > 0.6$ . There is a U-shaped relation between  $t$  and entry profits. The minimum is obtained at  $t = \frac{4}{3}$ .

*Proof.* Profits are decreasing in the MPE and FCE regimes, increasing in the ICE regime. ■

Intuitively, this result immediately implies that entry tends to take place for extreme values of  $t$  if  $\mu > 0.6$ . We formalize this intuition as follows.

**Proposition A.2.** Suppose  $\mu > 0.6$ .

- (i) Entry will take place for arbitrary values of  $t$  if and only if  $F < \frac{1}{3} - \frac{1}{3}\mu$ .
- (ii) If  $F > \frac{1}{3} - \frac{1}{3}\mu$ , then one of the following cases arises:
  - (a) If  $F < \frac{1}{2} (4\mu + 3)^2 \frac{1-\mu}{(5\mu+3)^2}$ , then entry will take place for low and high, but not for intermediate levels of  $t$ .
  - (b) If  $\frac{1}{2} (4\mu + 3)^2 \frac{1-\mu}{(5\mu+3)^2} < F < 2(3\mu - 1) \frac{1-\mu}{3\mu+1}$ , then entry will take place only for low values of  $t$ .
  - (c) If  $2(3\mu - 1) \frac{1-\mu}{3\mu+1} > F$ , there is no entry for any value of  $t$ .

*Proof.* By Lemma A.3, profits are U-shaped, with minimum at  $t = 4/3$ . There, profits are  $\frac{1-\mu}{4t} \frac{(3t+4\mu+2t\mu)^2}{(5\mu+3)^2} = \frac{1}{3} - \frac{1}{3}\mu$ . This implies (i) and it also implies that in case (ii) entry never arises for intermediate levels (in a neighborhood of the minimum). The profits of the entrant at the left and right maximum are  $2(3\mu - 1) \frac{1-\mu}{3\mu+1}$ , and  $\frac{1}{2} (4\mu + 3)^2 \frac{1-\mu}{(5\mu+3)^2}$ , respectively. As

$2(3\mu - 1) \frac{1-\mu}{3\mu+1} > \frac{1}{2}(4\mu + 3)^2 \frac{1-\mu}{(5\mu+3)^2}$ , then one of the three cases (a)-(c) arises, depending on  $F$ . ■

For  $\mu > 0.6$ , we summarize the relation between consumer heterogeneity and entry profit as follows.

**Lemma A.4.** Suppose  $\mu < 0.6$ .

- (i) Entry profits have four extreme values  $t_1(\mu) < t_2(\mu) < t_3(\mu) < t_4(\mu)$ , where  $t_1$  and  $t_3$  are local minima, whereas  $t_2$  and  $t_4$  are local maxima:  $t_1$  is the leftmost point in the CFE regime, at  $t_2$  the FCE starts, at  $t_3$  the ICE starts and  $t_4 = 2$ .
- (ii) There exist values  $\mu_1 \sim 0.3749$ ,  $\mu_2 \sim 0.40531$ ,  $\mu_3 \sim 0.48561$  and  $\mu_4 = 0.6$  such that  $\Pi_1^*(\mu, t_2) < \Pi_1^*(\mu, t_4)$  if and only if  $\mu < \mu_1$ ;  $\Pi_1^*(\mu, t_1) < \Pi_1^*(\mu, t_3)$  if and only if  $\mu < \mu_2$ ;  $\Pi_1^*(\mu, t_1) < \Pi_1^*(\mu, t_4)$  if and only if  $\mu < \mu_3$

*Proof.* (i) follows immediately from Lemma A.2, with  $t_1(\mu) = \frac{36\mu-36\mu^2}{15\mu-8\mu^2+9}$  and  $t_2(\mu) = \frac{12-12\mu}{9-\mu}$  as the left and right boundaries of the CFE region and  $t_3(\mu) = \frac{4}{3}$  and  $t_4(\mu) = 2$  as the left and right boundaries of the ICE region.

(ii) By Lemma A.2 and the definitions of  $t_1(\mu), \dots, t_4(\mu)$  in (i),  $\Pi_1^*(\mu, t_1) = \frac{\mu(\mu-3)^2}{15\mu-8\mu^2+9}$ ,  $\Pi_1^*(\mu, t_2) = \frac{1}{3} \frac{(\mu-3)^2}{9-\mu}$ ,  $\Pi_1^*(\mu, t_3) = \frac{1}{3}(1-\mu)$  and  $\Pi_1^*(\mu, t_4) = \frac{1}{2}(4\mu+3)^2 \frac{1-\mu}{(5\mu+3)^2}$ . The claim now follows from simple calculations. ■

Intuitively, Part (i) of this result shows that entry profits are N-shaped in  $t$  if  $\mu < 0.6$ . Part (ii) helps to understand at which values of  $t$  entry takes place.

To understand the relation between consumer heterogeneity and entry, we need the following comparison of entry profits in the extreme points identified in Lemma A.4.

**Lemma A.5.**

- (i) If  $\mu < \mu_1$ , then  $\Pi_1^*(\mu, t_1) < \Pi_1^*(\mu, t_3)$  and  $\Pi_1^*(\mu, t_2) < \Pi_1^*(\mu, t_4)$ ;
- (ii) If  $\mu_1 < \mu < \mu_2$ , then  $\Pi_1^*(\mu, t_1) < \Pi_1^*(\mu, t_3)$  and  $\Pi_1^*(\mu, t_2) > \Pi_1^*(\mu, t_4)$ ;
- (iii) If  $\mu_2 < \mu < \mu_3$ , then  $\Pi_1^*(\mu, t_1) > \Pi_1^*(\mu, t_3)$  and  $\Pi_1^*(\mu, t_2) > \Pi_1^*(\mu, t_4)$  and  $\Pi_1^*(\mu, t_1) > \Pi_1^*(\mu, t_4)$ ;
- (iv) If  $\mu_3 < \mu < \mu_4$ , then  $\Pi_1^*(\mu, t_1) > \Pi_1^*(\mu, t_3)$  and  $\Pi_1^*(\mu, t_2) > \Pi_1^*(\mu, t_4)$  and  $\Pi_1^*(\mu, t_1) < \Pi_1^*(\mu, t_4)$

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The proof follows directly from Lemma A.2.

### A.2.4 Consumer surplus

In this section, we derive general expressions for the consumer surplus. We use these expressions to justify the claims made in Section 1.3.2.4. Defining  $I \in [0, 1/2]$  as the location of the consumer who is indifferent between both firms,<sup>1</sup> the average switcher surplus is

$$S^F = 2 \left( \int_0^I \left(1 + \frac{t}{4} - tx - p_0\right) dx + \int_I^{1/2} \left(1 + \frac{t}{4} - t\left(\frac{1}{2} - x\right) - p_1\right) dx \right) \text{ and hence}$$

$$S^F = (tI - p_1 - 2Ip_0 + 2Ip_1 - 2tI^2 + 1).$$

Defining  $C \in [0, 1/2]$  as the location of the critical consumer with the highest value of  $x$  who still buys from the incumbent, the average surplus of captive consumers is  $S^C = 2 \left( \int_0^C \left(1 + \frac{t}{4} - tx - p_0\right) dx \right) = C \left(2 + \frac{t}{2} - tC - 2p_0\right)$ ; similarly, if there is no entry the average surplus of all (captive and flexible) consumers is  $S^M = C \left(2 + \frac{t}{2} - tC - 2p_0\right)$ .

Using the expressions just derived, we can calculate the surplus in the ICE region in the monopoly and duopoly case.

**Lemma A.6.** In the ICE region,

$$(a) \text{ The average switcher surplus is } S^F = \frac{1}{8} \frac{72t + 24t^2\mu^2 + 168t\mu + 16\mu^2 + 56t\mu^2 - 36t^2\mu - 27t^2}{t(5\mu+3)^2}.$$

$$(b) \text{ The average consumer surplus in a monopoly is } S^M = \frac{1}{64} \frac{(t+4)^2}{t}$$

*Proof.* (a) In the ICE,  $p_0 = \frac{(3-\mu)t+8\mu}{2(5\mu+3)}$  and  $p_1 = \frac{(3+2\mu)t+4\mu}{2(5\mu+3)}$ , respectively. Thus indifference requires  $tI + \frac{(3-\mu)t+8\mu}{2(5\mu+3)} - t\left(\frac{1}{2} - I\right) - \frac{(3+2\mu)t+4\mu}{2(5\mu+3)} = 0$ , so that  $I = \frac{3t-4\mu+8t\mu}{12t+20t\mu}$ . As  $\frac{\partial}{\partial \mu} \frac{3t-4\mu+8t\mu}{12t+20t\mu} = \frac{3}{4t} \frac{3t-4}{(5\mu+3)^2} > 0$  for  $t > 4/3$ , the market share becomes more asymmetric as  $\mu$  increases, tilted towards the incumbent. Thus,  $S^F = \frac{1}{8} \frac{72t + 24t^2\mu^2 + 168t\mu + 16\mu^2 + 56t\mu^2 - 36t^2\mu - 27t^2}{t(5\mu+3)^2}$ . This expression is increasing in  $\mu$ , reflecting lower prices.

(b) A monopolist maximizes  $p_0 \left( \min \left\{ 2\frac{1+\frac{t}{4}-p}{t}, 1 \right\} \right)$ . He thus sets  $p = \left(\frac{1}{2} + \frac{t}{8}\right)$  if this does not induce full-coverage, that is, if  $2\frac{1+\frac{t}{4}-\left(\frac{1}{2}+\frac{t}{8}\right)}{t} < 1$ , or equivalently  $t > 4/3$ . The resulting average consumer surplus in the ICE is thus  $C \left(2 + \frac{t}{2} - tC - 2p_0\right) = \frac{1}{64} \frac{(t+4)^2}{t}$ . ■

Using Part (a) of Lemma A.6, the derivative of  $S^F$  in the ICE region is  $= \frac{3}{4t} \frac{3t-4}{(5\mu+3)^3} (9t - 4\mu + 18t\mu)$ , which is positive in the ICE region, as claimed in the text.

Further, comparing the surplus under duopoly and monopoly, we find that there indeed is a small region near  $(t, \mu) = (2, 0)$  where the monopoly surplus is higher than the duopoly surplus

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<sup>1</sup>In the boundary cases that all consumers prefer firm 0 (1), we set  $I = 1/2$  ( $I = 0$ ).

(reflecting the lower prices in the monopoly).

### A.3 Proof of Proposition 1.5

The incumbent chooses high price  $p_0^h$  and low price  $p_0^l$  that satisfy two conditions. (a) Both prices must promise the incumbent the same expected profit. (b) The low price must be a best reply to the entrant's price.

$p_0^h$  will optimally be the monopoly price  $1 - \frac{t}{4}$ . With this price, the incumbent obtains profit of at least  $\mu(1 - \frac{t}{4})$ . We know from the market partitioning equilibrium that we are in a parameter region where the entrant's optimal reaction to  $1 - \frac{t}{4}$  results in zero switcher demand for the incumbent. As the entrant now reacts to an expected price of the incumbent that will be strictly lower than the monopoly price, we deduce that the incumbent's profit at the monopoly price is  $\mu(1 - \frac{t}{4})$ .

The entrant's price must be a best reply to the incumbent's pricing strategy.

Suppose that the incumbent chooses the monopoly price with probability  $\alpha$ . The entrant chooses  $p_1$  to solve

$$\max_{p_1} p_1 \left( \alpha + (1 - \alpha) \frac{p_0^l - p_1 + \frac{t}{2}}{t} \right).$$

This yields a reaction function  $\tilde{p}_1(p_0^l)$ . The incumbent's indifference condition is

$$p_0^l \left( \mu + (1 - \mu) \frac{p_1 - p_0^l + \frac{t}{2}}{t} \right) = \mu \left( 1 - \frac{t}{4} \right).$$

Solving this yields a first condition for the low price  $\bar{p}_0^l(p_1)$ . Further,  $p_0^l$  needs to solve

$$\max_{p_0^l} p_0^l \left( \mu + (1 - \mu) \frac{p_1 - p_0^l + \frac{t}{2}}{t} \right).$$

This yields the best-reply function  $\tilde{p}_0^l(p_1)$ . Substituting  $\tilde{p}_0^l(p_1)$  into  $\tilde{p}_1(p_0)$  gives the entrant's equilibrium price  $p_1(\alpha)$  depending on  $\alpha$ . We then obtain  $\alpha$  by equating  $\bar{p}_0^l(p_1(\alpha))$  and  $\tilde{p}_0^l(p_1(\alpha))$  and solving for  $\alpha$ . Finally, this value is inserted into  $p_1(\alpha)$  to obtain the equilibrium value  $p_1^e$ . This is then substituted into  $\tilde{p}_0^l(p_1)$  to obtain the equilibrium value  $p_0^{l,e}$ .

This procedure results in the following values.

$$p_0^{h,e} = 1 - \frac{t}{4},$$

$$p_0^{l,e} = \frac{t \left( 3\mu^3(4 - t) + 6\mu^2(t - 4) + 3\mu \left( \sqrt{(1 - \mu)^3 \mu(4 - t)t} + 4 - t \right) + \sqrt{(1 - \mu)^3 \mu(4 - t)t} \right)}{2(1 - \mu) \left( 2\mu t - 3\mu^2 t + 3\sqrt{(1 - \mu)^3 \mu(4 - t)t} + t \right)},$$

$$p_1^e = \frac{t \left( 48\mu^2 - 24\mu^3 + (3\mu^3 - 13\mu^2 + 9\mu + 1)t - 3\mu \left( \sqrt{(1-\mu)^3\mu(4-t)t} + 8 \right) + \sqrt{(1-\mu)^3\mu(4-t)t} \right)}{2(1-\mu) \left( 3\mu^2 t - 2\mu t - 3\sqrt{(\mu-1)^3\mu(t-4)t} + t \right)},$$

and

$$\alpha = \frac{(19\mu - 6\mu^2 + 3)t + 6 \left( 6\mu^2 - 6\mu + \sqrt{(1-\mu)^3\mu(4-t)t} \right)}{15\mu t + t - 36(1-\mu)\mu}.$$

It can be verified that  $p_1^e < 1 - \frac{3}{4}t$  and that  $d_0(p_0^l, p_1^e), d_1(p_1^e, p_0^l) > 0$ , so the demand expressions in the incumbent's and entrant's objectives are indeed the correct ones.

It needs to be checked that no firm wants to deviate. The optimal deviation for the incumbent is to always set the monopoly price. The optimal deviation for the entrant is to rely on the situation where he gets the entire switcher demand by setting the limit price  $1 - \frac{3}{4}t$ . It can be verified that, as  $t$  decreases, the entrant is the first one to have a profitable deviation. The point at which the entrant is indifferent between deviating and playing the equilibrium pins down a function  $t = b(\mu) > 0$  for  $\mu \in (0, 1)$  such that the entrant will deviate when  $t < b(\mu)$ . It can be verified numerically that this function is unique and continuous.

## A.4 Equilibrium structure with general demand

In this section, we analyze the equilibrium structure in a general discrete choice model that contains the Salop example as a special case, but is itself a special case of the general model of Section 1.2. We replace the assumptions of Section 1.3 as follows.

**Assumption A.1.** 1. *There is continuum of consumers with mass 1.*

2. *Each consumer has unit demand.*

3. *There is a joint valuation distribution  $F(v_0, v_1)$  on  $\mathcal{V} = [\underline{V}, \bar{V}] \times [\underline{V}, \bar{V}]$  with joint density  $f(v_0, v_1)$ .  $F$  has positive mass in every neighborhood of  $(\underline{V}, \bar{V})$ ; similarly for  $(\bar{V}, \underline{V})$ .*

4. *The distribution of valuation differences  $v_1 - v_0$  has a density that is everywhere positive.*

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Note that we do not assume that all valuation profiles in  $\mathcal{V}$  have positive density. The positivity requirements are sufficiently weak that the model of Section 1.3 fulfills Assumption A.1 with  $(\underline{V}, \bar{V}) = (1 - \frac{t}{4}, 1 + \frac{t}{4})$ . We write  $\Delta = \bar{V} - \underline{V}$  to denote the maximal valuation advantages.

Our first result restricts the possible equilibrium price vectors. All conceivable equilibria have the same structure as in Corollary 1.1, except that we cannot rule out a *Doubly Incomplete Coverage Equilibrium (DICE)*, in which neither the captive market nor the switcher market is covered.



- Lemma A.7.** Suppose  $t > 0$ . (i) There can be no PSE with  $p_i^* = 0$  or  $p_i^* \geq \bar{V}$ . ( $i = 0, 1$ ).
- (ii) In any equilibrium with  $0 < p_0^* < \underline{V}$ , both firms must be active in the switchers' market; the entrant's price must satisfy  $p_1^* \in (p_0^* - \Delta, \min\{p_0^* + \Delta, \bar{V}\})$ . Thus, the equilibrium must be a CFE.
- (iii) In any equilibrium with  $p_0^* = \underline{V}$ , one of two possibilities arises:
- (a)  $p_1^* = p_0^* - \Delta = \underline{V} - \Delta$ , which corresponds to an MPE;<sup>2</sup>
- (b)  $p_1^* \in (p_0^* - \Delta, p_0^* + \Delta) = (\underline{V} - \Delta, \bar{V})$ , which results in an FCE.
- (iv) In any equilibrium with  $\bar{V} > p_0^* > \underline{V}$ , the captive market is incompletely covered. One of the following cases can arise:
- (a)  $p_1^* \in (p_0^* - \Delta, \underline{V})$ : In this case, the switcher market would be jointly covered by the two firms. Thus, there would be an ICE.
- (b) If  $p_1^* \in (\underline{V}, \min\{p_0^* + \Delta, \bar{V}\})$ , the switcher market would also be incompletely covered; thus a DICE arises.<sup>3</sup>

*Proof.* (i) As there is a positive measure of consumers with  $v_0 < v_1$  and  $v_0 > v_1$ , there can be no equilibrium with  $p_0^* = 0$ . If  $p_i^* \geq \bar{V}$ , then the firm earns zero profits. It can avoid this, for instance by setting prices sufficiently close to zero.

(ii) By contradiction. If  $p_1^* \leq p_0^* - \Delta$ , then firm 1 would get the entire switcher demand.<sup>4</sup> Firm 0 could therefore deviate by increasing the price, thereby increasing profits in the loyal market without any losses in the switcher market (where profits are zero anyway). If  $p_1^* \geq p_0^* + \Delta$ , then the entrant would have no demand. A deviation to any positive price below  $p_0^* + \Delta$  would be profitable.

(iii) By contradiction.  $p_1^* \geq p_0^* + \Delta = \bar{V}$  is inconsistent with (i). If  $p_1^* < p_0^* - \Delta = \underline{V} - \Delta$ , then market partition arises, but firm 1 could increase the price without losing demand. For  $p_1^* \in (p_0^* - \Delta, p_0^* + \Delta)$ , both firms have positive demand.

(iv)  $p_0^* > \underline{V}$  implies incomplete coverage. As we already know  $p_0^* < \bar{V}$ , if  $p_1^* < p_0^* - \Delta$ , the entrant could profitably increase prices without losing demand. If  $p_1^* \geq p_0^* - \Delta$ , the entrant would have zero demand. Thus  $p_1^* \in (p_0^* - \Delta, p_0^* + \Delta)$ . Clearly, either (a) or (b) must hold. ■

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<sup>2</sup>Here and elsewhere, we stay agnostic about the behavior of consumers who are indifferent between the two firms: As they have mass zero, we can neglect them.

<sup>3</sup>If there is no mass near zero.

<sup>4</sup>As there are no mass points by assumption, we ignore the possibility of ties.



## B Appendix: Chapter 2

### B.1 Proof of Lemma 2.3

In order to solve this problem, I first need to establish the need for a further assumption because the unrestricted optimization of the reseller's problem does not have a solution.

**Lemma B.1.** Assume that after a decrease of  $w_1$  from  $v - \frac{t}{N}$  to  $v - \frac{t}{N} - \varepsilon$ ,  $\varepsilon > 0$ , the intermediary wants to (weakly) lower  $p_1$  and (weakly) raise  $p_2$ . Without further conditions, there is no inner solution to the reseller's optimization problem.

*Proof.* The important aspect is the correct characterization of the demand function. It is a Hotelling-style demand between products 1 and 2, since demand there touches at least, and monopoly demand for product 2 on the other side. I can consider only one side of product 1, as the situation on the other side is completely symmetric. The reseller's optimization problem is then

$$\max_{p_1, p_2} \left\{ \Pi = (p_1 - w_1) \frac{p_2 - p_1 + \frac{t}{N}}{2t} + (p_2 - w_2) \left( \frac{p_1 - p_2 + \frac{t}{N}}{2t} + \frac{v - p_2}{t} \right) \right\}.$$

Notice that the demand expressions in this objective function are accurate only under the following constraints:

$$p_2 \geq v - \frac{t}{2N} \tag{B.1}$$

and

$$p_1 \leq 2v - p_2 - \frac{t}{N}. \tag{B.2}$$

These conditions express that  $p_2$  rises weakly and that there is no gap of non-buying consumers between products 1 and 2. Taking FOCs with respect to prices 1 and 2 gives

$$\begin{aligned} \frac{\partial \Pi}{\partial p_1} = 0 &\iff \frac{p_2 - p_1 + \frac{t}{N}}{2t} - \frac{p_1 - w_1}{2t} + \frac{p_2 - w_2}{2t} = 0 \\ &\iff 2p_1 = 2p_2 + \frac{t}{N} + w_1 - w_2. \\ \frac{\partial \Pi}{\partial p_2} = 0 &\iff \frac{p_1 - w_1}{2t} + \frac{p_1 - p_2 + \frac{t}{N}}{2t} + \frac{v - p_2}{t} - \frac{p_2 - w_2}{2t} - \frac{p_2 - w_2}{t} = 0 \\ &\iff 6p_2 = 2p_1 + 2v + \frac{t}{N} - w_1 + 3w_2. \end{aligned}$$

Now, combining these two conditions and substituting the candidate values for the wholesale

prices,  $w_1 = v - \frac{t}{N} - \varepsilon$  and  $w_2 = v - \frac{t}{N}$ , gives

$$p_1 = v + \frac{t}{2N} - \frac{\varepsilon}{2} \text{ and } p_2 = v.$$

This is not an inner solution. For small  $\varepsilon$ , condition (B.2) is violated. ■

Condition (B.2) is binding. This pins down  $p_1$  in terms of  $p_2$ :  $p_1 = 2v - p_2 - \frac{t}{N}$ . The whole optimization problem of the reseller can then be formulated in terms of  $p_2$ .

If this constraint is imposed, the demands of the two products just touch and do not overlap. I can thus use the independent-product monopoly demand expressions and obtain the following problem:

$$\max_{p_2} \left( 2v - p_2 - \frac{t}{N} - w_1 \right) \frac{v - 2v + p_2 + \frac{t}{N}}{t} + (p_2 - w_2) \cdot 2 \cdot \frac{v - p_2}{t}.$$

The FOC with respect to  $p_2$  is

$$\frac{1}{t} \left( 2(v - p_2) - 2(p_2 - w_2) - \left( p_2 - v + \frac{t}{N} \right) + \left( 2v - p_2 - \frac{t}{N} - w_1 \right) \right) = 0.$$

Substituting the proposed equilibrium values  $w_1 = v - \frac{t}{N} - \varepsilon$  and  $w_2 = v - \frac{t}{N}$ , solving, and using binding (B.2) yields

$$p_1 = v - \frac{t}{2N} - \frac{\varepsilon}{6} \tag{B.3}$$

and

$$p_2 = v - \frac{t}{2N} + \frac{\varepsilon}{6}. \tag{B.4}$$

## B.2 Proof of Lemma 2.5

The intermediary's response established in Lemma 2.3 holds until product 2 becomes so expensive that it is shut down ( $p_2 = v$ ). This happens when  $\varepsilon = 3\frac{t}{N}$ . At this point, the price of product 1 calculated with the given response from the reseller is  $\tilde{p}_1 = v - \frac{t}{N}$ .

When product 2 is shut down, however, product 1 is a local monopoly again and individual pricing applies. The monopoly price at  $\varepsilon = 3\frac{t}{N}$  is  $p^m|_{\varepsilon=3\frac{t}{N}} = v - 2\frac{t}{N} < v - \frac{t}{N}$ . Thus, at  $\varepsilon = 3\frac{t}{N}$ , there is a discrete downward jump. This downward jump will affect the next product on the circle. To see this, notice that the next product (call it product 3) will be affected if demand for product 1 is more than  $\frac{3}{2}\frac{1}{N}$  to either side. With the wholesale price at  $w_1 = v - 4\frac{t}{N}$ , we have  $D_1^m = 2\frac{1}{N}$ . Thus, we cannot actually apply unrestrained monopoly pricing as this interferes with the demand of product 3. Applying the same logic as before, I will determine pricing using the condition that the indifferent consumer between products 1 and 3 gets utility exactly zero.

More generally, consider the  $M^{\text{th}}$  neighbor to either side of product 1 and label that neighbor with slight abuse of notation as product  $M$ . This neighbor is affected once the prices of the  $M - 1$  previous neighbors have been set to  $v$ . The indifferent consumer between product 1 and

its  $M^{\text{th}}$  neighbor is located at  $d_{1,M} = \frac{p_M - p_1 + M \frac{t}{N}}{2t}$  from the position of product 1. Taking this together with the condition of zero utility for this marginal consumer, we can substitute

$$p_1 = 2v - p_M - M \frac{t}{N}. \quad (\text{B.5})$$

The reseller's optimal reaction solves

$$\max_{p_M} \left( 2v - p_M - M \frac{t}{N} - w_1 \right) \frac{p_M - v + M \frac{t}{N}}{t} + 2(p_M - w_M) \frac{v - p_M}{t}.$$

The solution yields

$$6p_M = 5v - 2M \frac{t}{N} - w_1 + 2w_M. \quad (\text{B.6})$$

Now, the current level of  $w_1$  depends on which  $M$  is being considered. ( $w_M$  is always at the candidate equilibrium level  $v - \frac{t}{N}$ .) The first neighbor's price is set to  $v$  when  $w_1 = v - 4\frac{t}{N}$ . So, at the point where we consider  $M = 2$ ,  $w_1 = v - 4\frac{t}{N} - \varepsilon_2$ , where  $\varepsilon_2 \geq 0$  is indexed to indicate it is different from (and is applied on top of) the  $\varepsilon$  considered until now. The price reaction of the intermediary for product  $M = 2$  is thus  $p_{M=2} = v - \frac{t}{3N} + \frac{\varepsilon_2}{6}$ . This price is  $v$  when  $\varepsilon_2 = 2\frac{t}{N}$ .

Notice that at  $M = 2$ ,  $w_1 = v - 2M \frac{t}{N} - \varepsilon_M$ . As price  $v$  is reached when  $\varepsilon_M = 2\frac{t}{N}$ , this formula stays valid for  $M + 1$ . Inserting this general formula in the pricing formula (B.6), we observe that the term  $2M \frac{t}{N}$  will always cancel out. As  $w_M$  is the same at each  $M$ , the number  $M$  no longer appears in the expression. The resulting price reaction of the intermediary to  $\varepsilon_M$  for  $p_M$  is thus the same for all  $M \geq 2$ :

$$p_M = v - \frac{t}{3N} + \frac{\varepsilon_M}{6}.$$

Using this together with (B.5) yields  $p_1 = v - \frac{3M-1}{3} \frac{t}{N} - \frac{\varepsilon_M}{6}$  and  $D_1 = \frac{3M-1}{3} \frac{1}{N} + \frac{\varepsilon_M}{6}$ .

For the optimization problem of supplier 1, we then have for each  $M$ , after  $w_1$  has been lowered so much that  $M - 1$  neighbors have been pushed out of the market,

$$\max_{\varepsilon_M} \left( v - 2M \frac{t}{N} - \varepsilon_M - c \right) \left( \frac{3M-1}{3} \frac{1}{N} + \frac{\varepsilon_M}{6} \right).$$

The solution to this is  $\varepsilon_M^* = \frac{v-c}{2} - (4M-1) \frac{t}{N}$  and

$$\varepsilon_M^* > 0 \iff v - c > (8M-2) \frac{t}{N}.$$

The right-hand side of this expression is increasing in  $M$  and the condition at  $M = 2$  is  $v - c > 14\frac{t}{N}$ . The condition that makes a deviation profitable does not become less restrictive than condition (2.1), which was needed for a marginal deviation to be profitable.

Is exactly pushing competitors to price  $v$  beneficial? The profit immediately after having pushed out a product is given by the above formulas. Profit after having pushed out  $M - 1$  neighbors is  $\pi_M = (v - 2M \frac{t}{N} - c) \frac{3M-1}{3} \frac{1}{N}$ . This profit is increasing in  $M$  when  $v - c > 4M \frac{t}{N} - \frac{2}{3} \frac{t}{N}$ . This condition becomes tighter as  $M$  increases. At  $M = 2$ , the condition is  $v - c > \frac{22}{3} \frac{t}{N}$ . But we know from (2.1) that an initial deviation is unprofitable when  $v - c < 4\frac{t}{N}$ . Because  $4\frac{t}{N} < \frac{22}{3} \frac{t}{N}$ , when an initial downward deviation by a supplier is unprofitable, so is pushing neighboring products

out of the market.

It remains to check that monopolization of the market is not profitable. In order to monopolize the market, supplier 1 needs to push out  $N - 1$  competitors. Now it is relevant whether the total number of products on the circle is even or uneven. Up to now, we have considered affecting products symmetrically to the left and right of product 1, thus always regarding an uneven total number of products.

For  $N$  uneven, the formulas used before continue to be appropriate. To monopolize the market,  $M = \frac{N-1}{2}$  neighbors need to be pushed out. This, as we have seen, is not profitable.

For  $N$  even, the condition for pushing out the last product is slightly different. Before the last product is affected,  $\frac{N-2}{2}$  neighbors of product 1 have to be shut down. Thus,  $w_1$  will be at  $v - \left(\frac{N-2}{2} + 1\right) \frac{t}{N} = v - \frac{t}{2}$ . The indifferent consumer between products 1 and  $N$  is  $d_{1,N} = \frac{p_N - p_1 + \frac{N}{2} \frac{t}{N}}{2t} = \frac{p_N - p_1 + \frac{t}{2}}{2t}$  and thus price 1 can be expressed as  $p_1 = 2v - p_N - \frac{t}{2}$ . The intermediary maximizes  $(2v - p_N - \frac{t}{2} - w_1) \frac{p_N - v + \frac{t}{2}}{t} + (p_N - w_N) \frac{v - p_N}{t}$ . The first-order condition yields  $4p_N = 4v - t - w_1 + w_N$ . Substituting the wholesale prices results in  $p_N = v - \frac{t}{8} - \frac{1}{4} \frac{t}{N} + \frac{\varepsilon_N}{4}$ .  $p_N = v$  when  $\varepsilon_N = \frac{t}{N} + \frac{t}{2}$ , hence  $w_1 = v - t - \frac{t}{N}$ . Supplier 1's profit is then  $\pi = (v - t - \frac{t}{N} - c) \cdot \frac{1}{2}$ . This is bigger than the initial equilibrium profit if  $\frac{v - t - \frac{t}{N} - c}{2} > \frac{v - \frac{t}{N} - c}{2N} \iff v - c > \frac{N^2 + N - 1}{N(N-1)} t$ . To compare with our parametric environment, we analyze  $\frac{N^2 + N - 1}{N(N-1)} t > \frac{4}{N} t \iff N^2 - 3N + 3 > 0$ . This is the case for all  $N$ . Thus, the condition for monopolization to be profitable is stricter than that for a marginal deviation to be profitable.

## Part IV

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## Part V

# Curriculum vitae





# Curriculum vitae

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