Essays on Risk-Based Capital Standards, Group Regulation, and the Measurement of Model Uncertainty in the Insurance Industry

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The President:

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To my dear parents/Meinen lieben Eltern
Monika & Karl Siegel

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Vorwort

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Introduction

During the last decade, the financial industry has faced several financial crises. Insurance supervisors have reacted by revising the existing regulatory frameworks as well as developing and implementing new solvency models. The economic research of the challenges to the insurance industry arising from these new regulatory systems is therefore an important and contemporary task. This doctoral thesis, which comprises four research papers, seeks to gain new insights into the field of regulation and the solvency assessment of insurance companies.

The first paper "The Impact of Private Equity on a Life Insurer's Capital Charges under Solvency II and the Swiss Solvency Test" is an empirical analysis of the performance of the asset class private equity regarding both its risk-return profile and its impact on an insurer's capital requirements under the Solvency II framework of the European Union as well as Switzerland's Solvency Test. We review the standard market risk models and also propose an approach for an internal model. We show that although the risk-return profile of private equity suggests a solid performance in relation to various other asset classes, the standard approaches of Solvency II and the Swiss Solvency Test overly penalize the asset class in terms of capital requirements.

The following two research papers pertain to the area of solvency assessment for insurance groups. The paper "Solvency Assessment for Insurance Groups in the United States and Europe – a Comparison of Regulatory Frameworks" is an overview and comparison of three innovative group solvency frameworks: the National Association of Insurance Commissioners approach of the United States, the group structure model of Switzerland, and the Solvency II proposal on group solvency assessment. This comparison is based on the recently established criteria for a thorough group solvency approach of the International Association of Insurance Supervisors' Issues Paper on Group-Wide Solvency Assessment and Supervision (see IAIS, 2009b). Our analysis reveals a superiority of the European frameworks over the U.S. approach. In particular, the

Swiss model is able to satisfy most of the reference criteria in full.

The third part of the dissertation contains the paper "Regulating Insurance Groups: a Comparison of Risk-Based Solvency Models". Here, two general classes of group solvency approaches are displayed and compared: the class of legal entity approaches and the class of consolidated approaches. Regarding the challenges of regulatory inconsistency and risk interdependencies, we conduct a theoretical as well as numerical analysis studying shortfall risks and capital requirements under both approaches. Our findings show that a pure consolidated focus is likely to underestimate shortfall risks in times of financial crises, whereas an approach relying on the legal entity viewpoint generally makes it possible to display different group structures but cannot control regulatory arbitrage.

Finally, the last research paper of this dissertation is called "Model Uncertainty and Its Impact on Solvency Measurement in Property-Liability Insurance". It constitutes a study of the model risk immanent in solvency models for property-liability insurers. Based on a basic framework, we analyze the effects of including stochastic jumps, linear, and nonlinear dependencies in a solvency model on shortfall risks as well as the Solvency II capital charges. In addition, we take a regulatory viewpoint, examining the possibility of reducing the deviations in risk measures– that are due to the different model specifications– by requiring interim financial reports. Our simulation results suggest that the sensitivity of capital charges as a risk measure are likely to underestimate the actual model risk to which policyholders are exposed to. Furthermore, we find that mandatory interim reports are able to significantly reduce model uncertainty.

To sum up, the standard approaches of U.S. and European solvency frameworks need additional reforms. Further development of the standard solvency models may be necessary in terms of assessing nonlinear risk dependencies, harmonizing the national capital requirements, and reducing model uncertainty. Although the European frameworks seem,

from an academic perspective, superior to the current U.S. approach, they might need to partially reconsider their implicit incentive scheme. In this context, the thesis uncovers an inappropriate treatment of alternative investments in terms of capital charges under the standard market risk models of the SST and Solvency II. This can have severe economic implications such as an underrepresentation of certain asset classes that could otherwise be well suited for diversifying an insurer's asset portfolio.

Einführung

In den letzten zehn Jahren sah sich die Finanzindustrie mehreren Krisen gegenübergestellt. Die Versicherungsaufsicht hat darauf mit einer Revision der bestehenden Regulierungsrahmenwerke reagiert und neue Solvenzmodelle entwickelt und implementiert. Die ökonomische Untersuchung der sich aus den neuen Regelwerken für die Versicherer ergebenden Herausforderungen ist daher eine wichtige und aktuelle Aufgabe. Diese aus vier eigenständigen Forschungsarbeiten bestehende Dissertation ist bestrebt, neue Einblicke in das Forschungsfeld der Regulierung und Solvenzmessung von Versicherungen zu gewähren.

Die erste Arbeit „The Impact of Private Equity on a Life Insurer's Capital Charges under Solvency II and the Swiss Solvency Test" ist eine empirische Performanceanalyse der Anlageklasse Private Equity, welche das Risiko-Rendite-Profil derselben sowie den Einfluss ihrer Beimischung zum Anlageportfolio des Versicherers auf die Kapitalanforderungen unter dem Solvabilität-II-Rahmenwerk der Europäischen Union und dem Schweizer Solvenztest (SST) untersucht. Hierbei geben wir einen Überblick über die Standard-Marktrisikomodelle und schlagen ausserdem einen Ansatz für ein internes Marktrisikomodell vor. Unsere Ergebnisse weisen darauf hin, dass die Assetklasse Private Equity im Vergleich zu anderen Anlageklassen durch die Standardansätze von Solvabilität II und dem SST im Hinblick auf die Kapitalanforderungen trotz einer soliden Performance stark benachteiligt wird.

Die beiden folgenden Forschungsarbeiten sind in den Themenbereich der Solvenzmessung von Versicherungsgruppen einzuordnen. Das Kapitel „Solvency Assessment for Insurance Groups in the United States and Europe – a Comparison of Regulatory Frameworks" stellt einen Überblick und Vergleich dreier innovativer Gruppensolvenz-Rahmenwerke dar. Es behandelt den US-amerikanischen Gruppenansatz der National Association of Insurance Commissioners (NAIC), das Gruppenmodell der Schweiz und den Vorschlag zur Gruppensolvenzregulierung von Solvabilität II. Der Vergleich basiert auf den erst kürzlich von der International

Association of Insurance Supervisors (IAIS) in ihrem „Issues Paper on Group-Wide Solvency Assessment and Supervision" aufgestellten Kriterien eines ordentlichen Gruppensolvenzansatzes (siehe IAIS, 2009b). Die Analyse zeigt eine deutliche Überlegenheit der europäischen Gruppenansätze gegenüber dem US-amerikanischen Ansatz auf. Das Schweizer Gruppensolvenzmodell ist hierbei durch seine Erfüllung der Anforderungskriterien besonders hervorzuheben.

Der dritte Teil der Dissertation umfasst die Arbeit „Regulating Insurance Groups: a Comparison of Risk-Based Solvency Models". Darin werden zwei verschiedene Kategorien von Gruppensolvenzmodellen betrachtet und verglichen: Die Kategorie der sogenannten Legal-Entity-Ansätze und jene der konsolidierten Ansätze. Im Hinblick auf die Herausforderungen der regulatorischen Inkonsistenz und der Interdependenzen zwischen Risikoklassen untersucht die Forschungsarbeit – auf theoretischer und numerischer Ebene – Ausfallrisiken und Kapitalanforderungen, die sich unter den beiden Kategorien von Gruppenansätzen ergeben. Unsere Resultate zeigen, dass ein Gruppenansatz, der ausschliesslich auf konsolidierter Ebene basiert, unter Umständen die Ausfallrisiken in Krisenzeiten unterschätzt. Dagegen erlaubt es die Kategorie der reinen Legal-Entity-Ansätze generell, die Struktur einer Versicherungsgruppe differenziert darzustellen. Diese Gruppenmodelle sind jedoch oft nicht in der Lage, Regulierungsarbitrage aufzudecken bzw. auszuschliessen.

Die letzte Forschungsarbeit der Dissertation mit dem Titel „Model Uncertainty and Its Impact on Solvency Measurement in Property-Liability Insurance" stellt eine Studie zum immanenten Modellrisiko von Solvenzmodellen für Nichtlebensversicherer dar. Gestützt auf ein Basissolvenzmodell, analysieren wir die sich durch die Einbeziehung von stochastischen Sprungdiffusionsprozessen sowie linearen und nicht-linearen Abhängigkeiten ergebenden Effekte auf Ausfallrisiken und Kapitalanforderungen. Von einem regulatorischen Standpunkt aus untersuchen wir ausserdem die Möglichkeit der Reduktion der Abweichungen in den Risikokennzahlen, welche durch die unterschiedlichen Modellspezifikationen zustande kommen, indem man die Versicherer zu unterjährigen Finanz-

berichten verpflichtet. Die Simulationsergebnisse unserer numerischen Analyse deuten darauf hin, dass die Sensitivität der Kapitalanforderungen als Risikokenngrösse unter Umständen das tatsächliche Modellrisiko, dem letztendlich die Versicherungsnehmer ausgesetzt sind, unterschätzt. Zusätzlich zeigen wir, dass die Einführung verpflichtender Zwischenberichte das immanente Modellrisiko von Solvenzmodellen erheblich reduzieren kann.

Zusammenfassend kann gesagt werden, dass die US-amerikanischen und Europäischen Solvenzrahmenwerke weiterer Reformen bedürfen. Eine Weiterentwicklung der standardisierten Solvenzmodelle ist vor allem in den Bereichen der Berücksichtigung nicht-linearer Abhängigkeiten, der Harmonisierung nationaler Kapitalanforderungen sowie der Reduzierung von Modellrisiken notwendig. Obwohl die Europäischen Rahmenwerke, aus akademischer Sicht, dem aktuellen US-amerikanischen Ansatz überlegen scheinen, sollten diese ihre impliziten Anreizsysteme teilweise überdenken. In diesem Zusammenhang deckt die vorliegende Dissertation bei den Standard-Marktrisikomodellen des Schweizer Solvenztests und Solvabilität II eine unangemessene Behandlung von alternativen Anlageklassen hinsichtlich der Kapitalanforderungen auf. Diese kann ernstzunehmende ökonomische Auswirkungen haben, wie zum Beispiel eine Unterrepräsentation bestimmter Anlageklassen, die ansonsten zur Diversifikation des Anlageportfolios eines Versicherers gut geeignet wären.

Part I

The Impact of Private Equity on a Life Insurer's Capital Charges under Solvency II and the Swiss Solvency Test

Abstract

In this paper, we conduct an in-depth analysis of the impact of private equity investments on the capital requirements faced by a representative life insurance company under Solvency II as well as the Swiss Solvency Test (SST). Our discussion begins with an empirical performance measurement of the asset class over the period from 2001 to 2010, suggesting that limited partnership private equity funds should be well suited for portfolio diversification, especially from the perspective of life insurers with their large bond holdings. Subsequently, we review the standard approaches for market risk set out by both regulatory regimes and outline a potential framework for an internal model. Based on a concrete implementation of these solvency models for a representative life insurance company, it is possible to demonstrate that private equity is overly penalized by the standard approaches. Hence, life insurers aiming to exploit the asset class's potential may expect significantly lower capital charges when applying an economically sound internal model. Finally, we find that, from a regulatory capital perspective, it can even be less costly to increase the exposure to private rather than public equity.[1]

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1   Introduction

Within the last decade, the regulation of the European insurance sector
was subject to fundamental reforms aimed at the introduction of risk-
based solvency standards. One of the first of these initiatives to revise
solvency surveillance came into effect in 2004 in the United Kingdom (see,
e.g., Cummins and Phillips, 2009). Switzerland followed with its Swiss
Solvency Test (SST) in 2006. Beyond these regulatory reforms of indi-
vidual countries, Solvency II, the EU's flagship project to modernize and
harmonize European insurance supervision, has entered its final develop-
ment phase and is expected to come into force in 2013. Solvency II and
the SST in particular are viewed to be the most innovative frameworks
currently available and should thus have a major impact on insurance
regulation in the near future. Despite this fact, design and calibration of
their standard approaches for market risk clearly promote bond holdings.
The prescribed treatment of alternative investments such as private eq-
uity, on the contrary, is at least questionable. Since the attractiveness
of an asset class for insurance companies does not only depend on its
performance characteristics but also on the associated capital charges,
an inappropriate assessment from a solvency perspective may cause an
underrepresentation of asset classes with favorable risk-return profiles,
which would otherwise be well suited for portfolio diversification.

In this paper, we address this issue for the specific case of private
equity. Our contribution is twofold. Firstly, identifying its risk-return
profile within the critical period from 2001 to 2010, we evaluate the at-
tractiveness of the asset class from a performance perspective and com-
pare it with various investment alternatives. Secondly, we shift our focus
to the associated market risk capital requirements for life insurers under
Solvency II and the SST. Our discussion begins with a review of the
standard approaches for market risk set out by both regulatory regimes
as well as an outline of a potential framework for an internal model. Sub-
sequently, we compute and compare the respective capital charges for a
stylized life insurance company based on an empirical calibration and
implementation of these three solvency models. The results are shown

to be robust with regard to the employed benchmark index, the percentage of private equity in the life insurer's portfolio, and the chosen calibration period. Finally, we assess how costly it is, from a regulatory capital perspective, to increase the portfolio weight of private equity in comparison with public equity and hedge funds.

The rest of the paper is organized as follows: Section 2 contains an overview of the literature on private equity and insurance regulation. In Section 3, we briefly discuss the characteristics of private equity investments and conduct an extensive empirical performance analysis, including a comparison with other asset classes. The market risk standard approaches of Solvency II and the SST as well as the outline of an internal model are presented in Section 4. In Section 5, we implement the market risk models and provide an in-depth discussion of the resulting capital charges. Finally, in Section 6 we discuss economic implications and state our conclusion.

2   Literature Review

Since there is a substantial body of extant literature on both private equity and the regulation of financial institutions, we will only review recent research in those areas that are most relevant to our study. We begin with the latest work on the risk-return characteristics as well as the historical performance of the asset class, particularly in comparison with public equity markets. Although industry professionals and general partners regularly stress the attractiveness of private equity investments, existing empirical evidence conveys a rather ambiguous picture. Moskowitz and Vissing-Jorgensen (2002) control for distorted market values and estimate returns for the entire U.S. private equity market, concluding that it did not outperform public equity between 1989 and 1998. Ljungqvist and Richardson (2003), in contrast, conduct a performance analysis of private equity funds over the two decades from 1981 to 2001 and show that their returns exceeded those of the aggregate public equity market by at least five percent per year. Yet, Zhu, Davis, Kinniry, and Wicas (2004) again challenge the allegedly superior risk-return

profile of the asset class. They argue that, due to the long investment
horizon, severe liquidity constraints, as well as high default probabili-
ties of the portfolio companies, private equity funds hide considerable
latent risks, which investors need to take into account. In another em-
pirical paper, Kaserer and Diller (2004) analyze European private equity
funds based on cash flow data, documenting an underperformance rela-
tive to public equity from 1980 to 2003 but an outperformance during
the shorter period from 1989 to 2003. The work of Kaplan and Schoar
(2005) is probably one of the most influential studies of private equity
performance. Examining the capital flows of more than 1,000 limited
partnerships for the period between 1980 and 2001, they demonstrate
that the cross-sectional mean return net of fees did virtually not differ
from that of the S&P 500. Furthermore, Driessen, Lin, and Phalippou
(2008) devise a statistical methodology based on the generalized method
of moments to estimate the risk-return characteristics of nontraded as-
sets from cash flow data. Applying it to a sample of venture capital
and buyout funds between 1980 and 2003, they get a mixed impression
of the investments' performance. Phalippou and Gottschalg (2009) con-
duct a performance analysis for an updated version of the Kaplan and
Schoar (2005) data set, covering the period from 1980 to 2003. From
their results they conclude that the performance figures as disseminated
by industry representatives and earlier research are overstated due to
sample selection issues and artificially inflated net asset values. When
corrected for these biases, the average fund return net of fees falls short
of that of the S&P 500 by as much as three percent per year. Finally,
Franzoni, Nowak, and Phalippou (2011) provide evidence that private
equity returns include a considerable liquidity risk premium. As a conse-
quence, the diversification benefits of the asset class may be lower than
traditionally assumed.

Owing to the absence of objective market values, analyses of private
equity performance need to rely on appraisal-based figures reported by
the limited partnership funds themselves. This, however, causes a vari-
ety of problems that are the subject matter of another extensive strand
of the private equity literature. Kaplan, Sensoy, and Strömberg (2002)

aim to assist academics and practitioners with the interpretation of their empirical results by cross-checking the contract specifications of over 140 venture capital financings and pointing out major biases in the two leading private equity databases. Similarly, Woodward (2004) argues that the returns, as disclosed by general partners, are inaccurate measures of the actual changes in value. Introducing an approach that allows to control for this issue, she is able to show that the risk of private equity is higher than commonly assumed, both in terms of return volatility and beta. Another arguably seminal article has been contributed by Cochrane (2005), who applies maximum likelihood estimation to correct the mean returns, volatilities, alphas, and betas of venture capital investments between 1987 and mid-2000 for selection bias. His results indicate that private equity behaves quite similar to traded securities. Conroy and Harris (2007) find understated risks and exaggerated returns based on data for the period from 1989 to 2005, which they attribute to the prevailing practices of appraisal-based portfolio valuation and information disclosure by private equity funds. Systematic valuation biases are also documented by Cumming and Walz (2010), who analyze a data set of more than 5,000 portfolio companies as well as 221 funds between 1971 and 2003. In addition, Cumming, Hass, and Schweizer (2010a) criticize appraisal-based indices for being subject to return-smoothing and stale pricing, which results in an underestimation of risk. In order to overcome this issue, they suggest correcting limited partnership private equity benchmarks for autocorrelation. Finally, Korteweg and Sorensen (2010) employ a dynamic sample selection methodology to address the issue of selection bias in venture capital data, and come up with considerably higher risk estimates than earlier research.

Other analyses focus on the effects of the asset class in a portfolio context. Ennis and Sebastian (2005), for example, employ equilibrium pricing to derive an expected return for private equity, which they subsequently use in a mean-variance analysis in order to determine the optimal allocation given different portfolio structures. Schneeweis, Gupta, and Szado (2008) consider the diversification benefits of private equity with regard to varying portfolio compositions. Furthermore, Cumming

et al. (2010a) conduct a multi-asset portfolio optimization with a modified appraisal-based and a listed private equity index, employing three different risk measures. Their results illustrate that the portfolio allocation decision is quite sensitive to the selected private equity benchmark. Similarly, Cumming, Hass, and Schweizer (2010b) consider private equity in an optimization framework that incorporates higher central moments of the return distribution and allows them to derive a superior strategic asset allocation compared to the classical Markowitz framework.

With regard to our research objective, we are also interested in previous work on insurance regulation, particularly on Solvency II and the SST. Eling, Schmeiser, and Schmit (2007) as well as Steffen (2008) provide overviews of the state of Solvency II at the time of writing of their articles and point out avenues for future research. Moreover, Doff (2008) assesses the Solvency II framework by means of seven conditions that are required for efficient and complete markets. Pfeifer and Strassburger (2008) point out skewness-related stability problems of the Solvency II standard formula and Eling, Gatzert, and Schmeiser (2008) discuss the main features of the SST as well as its ramifications for the Swiss economy. The implications of Solvency II for the prudential regulation of insurance companies in the U.S. are considered by Vaughan (2009). Cummins and Phillips (2009) additionally conduct an explicit comparison of the U.S. RBC standards, Solvency II, and the SST. Finally, Gisler (2009) compares the insurance risk models embedded in Solvency II and the SST with an emphasis on parameter estimation.

Despite the large number of articles revisited above, research combining the private equity asset class with a regulatory perspective is very scarce. One exception is the work of Cumming and Johan (2007), who draw on a data set of Dutch institutional investors to examine the relationship between regulatory harmonization and portfolio allocations to private equity funds. Besides, Bongaerts and Charlier (2009) analyze regulatory capital requirements for banks' private equity investments under Basel II as well as the Capital Requirements Directive of the European Union, comparing the respective standard approaches with an

internal model. The only studies adopting an explicit insurance perspective are provided by Arias, Arouri, Foulquier, and Gregoir (2010) as well as Studer and Wicki (2010). They argue that the LPX50 listed private equity index, which has been proposed by the Committee of European Insurance and Occupational Pension Supervisors (CEIOPS) for the calibration of Solvency II, should be replaced by a more representative index. In their view, the LPX50 is an inappropriate benchmark since it is biased towards leveraged buyout funds and does not adequately reflect the nature of limited partnership private equity. Consequently, with this paper, we fill a major gap in the literature by investigating the impact of private equity on a life insurer's capital charges under Solvency II and the SST.

3   Private Equity as an Asset Class

In general, "private equity" involves investing in equity securities not listed on a public exchange (see, e.g., Koh, 2009). It provides unlisted firms with long-term share capital in order to nurture their development or restructuring. Investors in this market segment are usually faced with highly illiquid and risky assets and therefore require a compensation by target rates of return that are significantly higher than those for public equity. While there are various reasons for investing in private equity, institutional investors such as banks, insurance companies, or pension funds commonly draw on this asset class in order to diversify their portfolios. Interestingly, recent evidence indicates that a growing fraction of institutional portfolios is invested directly into privately held firms (see, e.g., Dushnitsky, 2012). However, the majority of institutions still prefers an indirect access by becoming a limited partner of a private equity fund (see Nielsen, 2008). The fund itself has a fixed lifetime (usually 10 to 15 years) and is managed by a general partner who calls in money from the limited partners in order to allocate it to target companies (see, e.g., Kaplan and Strömberg, 2009). After the liquidation of an investment, the associated proceeds are distributed to the limited partners.

3.1   Main Investment Styles

Private equity encompasses different investment strategies. According to the age of the firms invested in, the asset class can be subclassified into "venture capital", "growth capital", and "buyouts". Venture capitalists focus on start-up firms that launch and develop the business, whereas growth capital refers to young companies that seek further expansion (see, e.g., Anson, 2002). By contrast, buyouts typically concentrate on the expansion or restructuring of mature companies with stable cash flows (see, e.g., Cendrowski, Martin, Petro, and Wadecki, 2008). Since delisting a company often involves large amounts of debt, such deals are also known as "leveraged buyouts" (LBOs). In addition, a transaction can take the form of a management buyout (MBO) aimed at transferring ownership to the company's executives (see Anson, 2002). Other investment styles are summarized by the term "special situations", which comprises the categories of "mezzanine capital" and "distressed financing". Mezzanine capital, a form of subordinated debt, can be viewed as a hybrid between senior secured debt and equity capital. Therefore, it is used when a company is unwilling to issue additional equity but cannot hope for debt financing beyond the level provided by traditional creditors. Furthermore, distressed financing investors (which are often called "vulture investors", see, e.g., Anson, 2002) acquire capital of companies that are in financial distress or close to declaring bankruptcy. The goal of such transactions is to purchase stakes with a great improvement potential at a small fraction of their face values.

To be able to consistently manage their overall portfolios, it is important for investors that fund managers maintain a clear investment style. If a fund deviates from its proclaimed objectives, investors may end up with an unwanted risk-return profile and fall short of their investment targets. This phenomenon is known as style drift and constitutes a particular concern of institutional investors in limited partnership private equity, since the inherent illiquidity of the underlying assets impedes a prompt withdrawal of the invested capital (see, e.g., Cumming, Fleming, and Schwienbacher, 2009).

3.2   Limited Partnerships vs. Listed Private Equity

The term "private equity" only refers to the target investment itself. It does, however, not require that the investing company is privately held. Hence, alongside limited partnership private equity funds, a number of investment vehicles have evolved that target unlisted firms but are themselves publicly listed on a stock exchange. Such constructs are termed "listed private equity" and can adopt three different organizational forms (see Cumming et al., 2010a). First of all, the majority of the listed private equity universe consists of so-called listed direct private equity companies, which directly acquire target firms. Moreover, listed indirect private equity companies invest via limited partnerships and thus are virtually exchange-traded fund of funds. Finally, there is a smaller number of private equity fund managers that invest neither directly nor indirectly but rather set up and manage a portfolio of funds as general partners, thereby generating fee income. While limited partnership private equity funds usually raise most of their capital from large institutions, the investor base of listed private equity tends to be broader. Through listed private equity firms it is easy to gain exposure to a more or less diversified private equity portfolio. Furthermore, the trading in an organized market largely reduces the illiquidity problem of limited partnership funds (see Bergmann, Christophers, Huss, and Zimmermann, 2009).

3.3   Empirical Performance Measurement

3.3.1   Data and Sample Selection

When searching for an adequate index to assess the risk-return characteristics of the private equity asset class, the previously explained distinction between limited partnership funds and listed private equity is quite relevant. Since the advent of listed private equity, there has been a debate among academics and practitioners as to whether or not publicly listed investment vehicles are a useful proxy for the asset class. Some authors claim that listed private equity is strongly influenced by general stock market dynamics and, due to this noisiness, only partly reflects the behavior of the underlying assets (see, e.g., Arias et al., 2010). More

specifically, due to the fact that listed private equity is known to trade at discounts to net asset value in times of market turbulence and at premiums to net asset value in growth phases, one might expect an overstatement of the asset class's volatility (see, e.g., Studer and Wicki, 2010). In contrast to these views, proponents of listed private equity state that, apart from the prevalent organizational form, there are hardly any real economic differences to unlisted private equity (see, e.g., Bergmann et al., 2009). As a result, the returns of listed vehicles should be a sufficient proxy for the characteristics exhibited by the asset class. Apart from that, market values arising from regular trading activity are viewed to be more reliable than the rather subjective valuations provided by general partners, which offer the possibility to smooth returns (see, e.g., Idzorek, 2007).

An attempt to resolve this discussion is beyond the scope of this paper. Instead, we will aim to capture all facets of the asset class by adopting listed as well as limited partnership private equity indices both for the performance measurement and the calibration of the solvency models in Section 4.[2] In this regard, the listed private equity universe will be represented by the LPX50, the LPX Buyout (LPX BO), and the LPX Venture Capital (LPX VC). Each of these benchmarks is published by the Zurich-based LPX Group on a monthly basis and aims to reflect the value development of representative listed private equity funds and firms. We have deliberately chosen the LPX indices over other available listed private equity benchmarks such as the S&P Listed Private Equity Index, the Global Listed Private Equity Index by Red Rocks Capital, or the Privex by Société Générale, since their data series reach further back in time and the main index LPX50 has been employed by CEIOPS

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[2]In fact, it is not uncommon that institutional investors hold listed and limited partnership private equity at the same time. The decision of how much capital is allocated to each market segment thereby depends on a variety of firm specific factors (see Cumming, Fleming, and Johan, 2011).

for the calibration of the Solvency II standard formula (see CEIOPS, 2010a).[3]

Moreover, the Thomson Reuters Private Equity Performance Index (PEPI), its two subindices for buyout (PEPI BO) and venture capital (PEPI VC), as well as the Cambridge Associates U.S. Private Equity Index (CAPEI) have been selected as benchmarks for limited partnership private equity funds. The PEPI, a common choice for performance measurement purposes among private equity professionals and industry sources such as the U.S. National Venture Capital Association (NVCA), is calculated based on cash flow data and self-reported residual values of over 1,900 limited partnership funds in the Thomson One database. Similarly, the CAPEI incorporates takedowns, distributions, and residual values of 883 limited partnership funds in the U.S., comprising all major investment styles. For all four of these appraisal-based indices so-called pooled end-to-end returns net of fees and carried interest are available on a quarterly basis. Aiming to capture the timing and scale of investments, pooled returns are calculated from a time series of aggregated cash flows, which is derived by summing up the respective figures of all individual limited partnerships that enter the index. Thus, in a sense, all constituents are treated as if they were one single fund. Resulting in investment-weighted instead of average returns, this methodology is supposed to more closely mirror the performance characteristics of typical private equity portfolios.[4]

For comparison purposes we also consider a range of common benchmarks, representing other asset classes. MSCI country indices are employed as measures for the U.S. (MSCI USA), European (MSCI EU), and Swiss (MSCI CH) stock markets. Note that these have been selected because they are also among the major stock market risk factors within the

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[3]Each of the LPX indices dates at least back to January 1998, while none of the other listed private equity benchmarks has a history before January 2004. See www.lpx.ch, www.standardandpoors.com, www.redrockscapital.com, and www.sgindex.com for more information.

[4]A corollary of this method is that larger funds exert more influence on the performance than smaller funds. Refer to the documentation of the Thomson One database for additional details.

SST standard model (see Section 5). In addition, the development of the corresponding government bond markets will be captured through the S&P U.S. Treasury Index (S&P USTI), the S&P Eurozone Government Bond Index (S&P EUGI), as well as the Swiss Government Bond Index (SIX SBI). Finally, the HFRX Global Hedge Fund Index (HFRX) and the HFRI Fund Weighted Composite Index (HFRI) have been chosen as proxies for hedge fund investments. Data for all of these benchmarks has been obtained from Bloomberg. Wherever available, total return indices are used to account for coupons and dividends, which would otherwise not be reflected in prices.

The time horizon for a performance analysis is frequently determined arbitrarily or through restrictions on the available data. Here, we decided to opt for the decade from January 2001 to December 2010, since this period has been chosen by FINMA for the calibration of the 2011 SST standard model for market risk. Thus, monthly and quarterly log-return time series for that period have been collected for each previously mentioned index so as to ensure consistency with the empirical results in Section 4. Furthermore, the average one-month (log) T-Bill rate p.a. (2.02%) and the average three-month (log) T-Bill rate p.a. (2.17%) between January 2001 and December 2010 have been used as risk-free interest rates for the calculation of performance measures based on monthly and quarterly index returns, respectively.

3.3.2   Private Equity in Comparison with Other Asset Classes

With the relevant data at hand, we can now analyze the risk-return profile of private equity in comparison with that of other common asset classes. Tables 1 and 2 summarize the empirical results. In order to preserve the typical characteristics of each asset class, we deliberately refrained from converting the time series of the indices denominated in euros and Swiss francs into U.S. dollars, thus implicitly assuming that the investor is able to sufficiently hedge exchange rate fluctuations at an

immaterial cost.[5] Since hedging foreign investments against exchange rate risk is very common among institutional investors and the trading costs for the required foreign exchange (FX) instruments such as futures and options are relatively small,[6] we believe this to be an acceptable assumption for our purpose.

When comparing the descriptive statistics of Tables 1 and 2, we notice that the PEPI BO (9.80 percent p.a.), the CAPEI (9.60 percent p.a.), and the PEPI (6.97 percent p.a.) exhibited the highest mean returns of all indices under consideration. Only the broad hedge fund index HFRI (6.70 percent p.a.) comes relatively close to these figures, followed by the government bond benchmarks S&P USTI (4.91 percent p.a.), S&P EUGI (4.76 percent p.a.), and SIX SBI (4.20 percent p.a.). And while the HFRX (3.50 percent p.a.) still delivered a positive excess return, stocks, listed private equity, as well as the PEPI VC generated mean returns below the risk-free interest rate. Furthermore, considering the annualized standard deviation of returns of the LPX50 (26.70 percent p.a.), the LPX BO (27.01 percent p.a.), and the LPX VC (27.92 percent p.a.), we find that listed private equity has been by far the most volatile investment, followed by the MSCI EU (20.29 percent p.a.), the MSCI USA (16.73 percent p.a.), and the MSCI CH (15.24 percent p.a.). Consistent with this observation, the maximum and minimum returns are also furthest apart for the three listed private equity and the three stock indices. In contrast to that, the volatilities of the limited partnership private equity benchmarks lie between 11.61 percent p.a. (CAPEI) and 12.48 percent p.a. (PEPI VC), thus being less than half as high as those of their listed counterparts. Besides, the HFRI (6.50 percent p.a.) and the HFRX (5.96 percent p.a.) mark the beginning of the lower end of the range and, according to the S&P USTI (4.29 percent p.a.), the S&P EUGI (3.63 percent p.a.), as well as the SIX SBI (3.83 percent p.a.),

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[5]Consider, for example, an investment in a foreign government bond. If left completely unhedged, this would be an outright speculation on the exchange rate, as the returns in the investor's home currency will be dominated by currency fluctuations, implying that the asset does not exhibit the typical behavior of a government bond.

[6]Flat fees for FX futures traded at the Chicago Mercantile Exchange (CME), the largest regulated FX marketplace worldwide, can be as low as USD 0.11, depending on membership and volume. For more information see www.cmegroup.com.

government bonds were not only the least volatile investment but also generated the highest minimum returns over our examination horizon. In terms of negative months, however, the private equity indices CAPEI (30) and PEPI BO (33) did even better than the three government bond indices.

In addition to these descriptive statistics we have calculated the two widespread performance measures Sharpe Ratio (see Sharpe, 1966) and Calmar Ratio (see Young, 1991) and established the corresponding rank orders for the sample under consideration. The results for each index are displayed in the lower parts of Tables 1 and 2. At first glance, we notice that the three listed private equity indices, the three stock indices, as well as the PEPI VC are not ranked. This is due to the fact that their mean returns fell short of the risk-free interest rate. The Sharpe and Calmar Ratio, however, are not meaningful for negative excess returns since, in that case, a higher value of the risk measure in the denominator would be associated with a better performance (i.e. a less negative ratio). The top three among those indices with positive Sharpe Ratios are the S&P EUGI (0.7529), the HFRI (0.7196), as well as the S&P USTI (0.6732), closely followed by the CAPEI (0.6406) and the PEPI BO (0.6219) on ranks four and five. Thus, two of the four appraisal-based private equity indices end up in the first third of the total sample, outperforming the SIX SBI (0.5686), the PEPI (0.4096), as well as the HFRX (0.2472) on ranks six, seven, and eight.[7] Turning to the Calmar Ratio we notice that, although the HFRX (0.0125) is ranked eighth again, the order for the other indices changes considerably. More specifically, the best performance in the sample is now attributed to the PEPI BO (0.1158) and the CAPEI (0.1111). In addition, the PEPI (0.0747) rises up to rank five, just behind the two government bond benchmarks SIX SBI (0.0985) and S&P EUGI (0.0819). Finally, the S&P USTI (0.0675) as well as the

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[7]One reason for the superior mean-variance profile of the HFRI compared to the HFRX and the resulting outperformance based on the Sharpe Ratio is likely to be its higher degree of diversification. With more than 2,000 constituents, the HFRI is a considerably broader index than the HFRX. See www.hedgefundresearch.com for more information.

Private Equity:	LPX50	LPX BO	LPX VC	PEPI	PEPI BO	PEPI VC	CAPEI
Bloomberg Ticker	LPX50TR	LPXABOTR	LPXVENTR	-	-	-	-
Currency	EUR	EUR	EUR	USD	USD	USD	USD
Return Interval	Monthly	Monthly	Monthly	Quarterly	Quarterly	Quarterly	Quarterly
Total Return over Period	−21.95%	19.92%	−86.43%	69.73%	97.96%	−18.49%	96.02%
Mean Return	−0.18%	0.17%	−0.72%	1.74%	2.45%	−0.46%	2.40%
− Annualized −	−2.19%	1.99%	−8.64%	6.97%	9.80%	−1.85%	9.60%
Standard Deviation	7.71%	7.80%	8.06%	5.87%	6.13%	6.24%	5.80%
− Annualized −	26.70%	27.01%	27.92%	11.73%	12.27%	12.48%	11.61%
Maximum	28.53%	30.54%	25.30%	10.63%	12.18%	7.93%	14.20%
− Annualized −	342.31%	366.50%	303.58%	42.50%	48.71%	31.71%	56.81%
Minimum	−30.66%	−35.82%	−23.11%	−16.08%	−16.46%	−16.64%	−16.74%
− Annualized −	−367.97%	−429.79%	−277.28%	−64.30%	−65.86%	−66.56%	−66.94%
Number of Negative Months	46	41	61	45	33	54	30
Sharpe Ratio	−0.1580	−0.0012	−0.3821	0.4096	0.6219	−0.3217	0.6406
− Rank −	−	−	−	7	5	−	4
Calmar Ratio	−0.0115	−0.0001	−0.0385	0.0747	0.1158	−0.0603	0.1111
− Rank −	−	−	−	5	1	−	2

Table 1: Descriptive Statistics and Performance Measures (01/01/2001 – 12/31/2010) for the Log-Return Time Series of Seven Widely Recognized Private Equity Indices

Listed private equity: LPX50; LPX Buyout (LPX BO); LPX Venture Capital (LPX VC). Limited partnership private equity funds: Thomson Reuters Private Equity Performance Index (PEPI), including the subindices for buyout (PEPI BO) and venture capital (PEPI VC); Cambridge Associates U.S. Private Equity Index (CAPEI). Note: for the time series with quarterly frequency, the number of negative months has been calculated by simply tripling the number of negative quarters, thus neglecting the possibility of a negative quarter comprising one or two positive months.

Other Asset Classes:	MSCI USA	MSCI EU	MSCI CH	S&P USTI	S&P EUGI	SIX SBI	HFRX	HFRI
Bloomberg Ticker	GDDLUS	GDDLEMU	GDDLSZ	SPBDUSBT	SPBDEGIT	SBIDGT	HFRXGL	HFRIFWI
Currency	USD	EUR	CHF	USD	EUR	CHF	USD	USD
Return Interval	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly
Total Return over Period	14.78%	−7.71%	1.78%	49.11%	47.59%	42.00%	34.99%	66.98%
Mean Return	0.12%	−0.06%	0.01%	0.41%	0.40%	0.35%	0.29%	0.56%
− annualized −	1.48%	−0.77%	0.18%	4.91%	4.76%	4.20%	3.50%	6.70%
Standard Deviation	4.83%	5.86%	4.40%	1.24%	1.05%	1.10%	1.72%	1.88%
− Annualized −	16.73%	20.29%	15.24%	4.29%	3.63%	3.83%	5.96%	6.50%
Maximum	9.17%	14.99%	11.52%	4.41%	3.70%	4.49%	3.10%	5.02%
− Annualized −	110.02%	179.92%	138.18%	52.95%	44.40%	53.83%	37.21%	60.22%
Minimum	−18.76%	−19.27%	−13.24%	−3.56%	−2.78%	−1.84%	−9.81%	−7.09%
− Annualized −	−225.07%	−231.25%	−158.82%	−42.74%	−33.41%	−22.10%	−117.76%	−85.05%
Number of Negative Months	46	53	51	39	42	43	39	37
Sharpe Ratio	−0.0326	−0.1378	−0.1211	0.6732	0.7529	0.5686	0.2472	0.7196
− Rank −	−	−	−	3	1	6	8	2
Calmar Ratio	−0.0024	−0.0121	−0.0116	0.0675	0.0819	0.0985	0.0125	0.0550
− Rank −	−	−	−	6	4	3	8	7

Table 2: Descriptive Statistics and Performance Measures (01/01/2001 – 12/31/2010) for the Log-Return Time Series of Eight Common Indices, Representing Established Asset Classes

Stock markets: MSCI country indices for the U.S. (MSCI USA), Europe (MSCI EU), and Switzerland (MSCI CH). Government bond markets: S&P U.S. Treasury Index (S&P USTI), S&P Eurozone Government Bond Index (S&P EUGI), and Swiss Government Bond Index (SIX SBI). Hedge Funds: HFRX Global Hedge Fund Index (HFRX) and HFRI Fund Weighted Composite Index (HFRI). Risk-free rate: average one-month (log) T-Bill rate p.a. (2.02%) for indices with monthly return interval, and average three-month (log) T-Bill rate p.a. (2.17%) for indices with quarterly return interval. Note: for the time series with quarterly frequency, the number of negative months has been calculated by simply tripling the number of negative quarters, thus neglecting the possibility of a negative quarter comprising one or two positive months.

HFRI (0.0550) rank sixth and seventh.

To complete the analysis, we look at the correlation structure between private equity and the other indices in our sample, which is shown in Table 3. Based on a common t-test we find all Bravais-Pearson correlation coefficients to be significantly different from zero. Interestingly, private equity exhibited unexpectedly high positive correlations of between 0.68 and 0.82 with the three stock indices, regardless of whether we consider limited partnerships or listed vehicles. Thus, it appears as if, during our examination period, the asset class was not quite able to decouple from the developments in the public equity markets. Furthermore, we find positive correlations with the hedge fund indices of between 0.39 and 0.79. The correlation coefficients between private equity and government bond returns, on the contrary, turned out to be moderately negative, ranging between –0.52 and –0.20. Consequently, private equity seems to offer excellent diversification qualities for investors with large bond portfolios such as life insurers.

Clearly, our results need to be interpreted in light of the chosen examination period from January 2001 to December 2010, which has been shaped by several years of major market dislocations due to the burst of the dot-com bubble in 2001 and the global financial crisis of 2007 to 2009. Thus, this is a relatively unfavorable decade for risky asset classes, while it certainly bolsters the observed performance of government bonds, which are heavily sought after in times of market turbulence.[8] Nevertheless, we observe an outperformance of private equity relative to public equity and, depending on the applied performance measure and benchmark index, even hedge funds. Against this background, it is safe to say that, with the exception of the remarkable weakness in the venture capital segment, we get a solid impression of the asset class. As mentioned in Section 2, however, one also needs to bear in mind that an analysis based on self-reported values might, at least to some extent, be

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[8]Unreported results for the period between January 1998 and December 2006 indicate a considerably better performance of both limited partnership funds and listed private equity.

	MSCI USA		MSCI EU		MSCI CH		S&P USTI		S&P EUGI		SIX SBI		HFRX		HFRI	
LPX50	0.82	***	0.82	***	0.79	***	−0.38	***	−0.26	***	−0.27	***	0.60	***	0.77	***
LPX BO	0.76	***	0.75	***	0.72	***	−0.31	***	−0.22	**	−0.24	**	0.57	***	0.69	***
LPX VC	0.74	***	0.77	***	0.75	***	−0.35	***	−0.20	*	−0.21	**	0.50	***	0.72	***
PEPI	0.81	***	0.78	***	0.76	***	−0.50	***	−0.46	***	−0.47	***	0.63	***	0.78	***
PEPI BO	0.78	***	0.74	***	0.72	***	−0.46	***	−0.42	**	−0.44	***	0.64	***	0.75	***
PEPI VC	0.70	***	0.68	***	0.69	***	−0.42	**	−0.44	***	−0.42	**	0.39	**	0.64	***
CAPEI	0.80	***	0.77	***	0.75	***	−0.52	***	−0.48	***	−0.44	***	0.65	***	0.79	***

Table 3: Correlations between Private Equity and Other Asset Classes (01/01/2001 – 12/31/2010)
Significance levels of correlation t-test: ***=1%; **=5%; *=10%. Listed private equity (correlations based on monthly returns): LPX50; LPX Buyout (LPX BO); LPX Venture Capital (LPX VC). Limited partnership private equity funds (correlations based on quarterly returns): Thomson Reuters Private Equity Performance Index (PEPI), including the subindices for buyout (PEPI BO) and venture capital (PEPI VC); Cambridge Associates U.S. Private Equity Index (CAPEI). Other indices: U.S. equities (MSCI USA); European equities (MSCI EU); Swiss equities (MSCI CH); S&P U.S. Treasury Index (S&P USTI); S&P Eurozone Government Bond Index (S&P EUGI); Swiss Government Bond Index (SIX SBI); HFRX Global Hedge Fund Index (HFRX); HFRI Fund Weighted Composite Index (HFRI).

subject to distortions. In particular, the fact that listed private equity has done very poorly during the same decade could be an indication for the presence of return smoothing or a potential sample selection bias in our results for the appraisal-based indices. Yet, at the same time these huge differences in the observed performance raise doubts with regard to the suitability of publicly listed vehicles as a measure for the behavior of the actual underlying assets. In any case, our results illustrate that, within reasonable limits, private equity should be well suited for portfolio diversification. As already mentioned, however, the historical performance of an asset class is not the only key factor when deciding about a potential investment. Instead, financial institutions also need to consider the associated capital charges. Hence, in the remainder of this paper, we will conduct an in-depth analysis of the treatment of private equity under Solvency II and the SST.

4   Solvency Assessment and Market Risk

4.1   Solvency II Market Risk Module

Under Solvency II, the solvency capital requirement (SCR) for insurance companies can be calculated based on a standard approach that has been calibrated by the regulator so as to correspond to a value at risk approach with a confidence level of 99.5 percent and a one-year time horizon (see EC, 2010; CEIOPS, 2010a).[9] In this paper, we focus on the market risk module. A key concept in this regard is the so-called net asset value (NAV), which equals the assets less the liabilities when both magnitudes are valued according to the prices achievable in an arm's-length transaction (see EC, 2010). To calculate the overall SCR for market risk, the changes in the net asset value (ΔNAV) caused by preset shocks to various capital market variables need to be aggregated (see CEIOPS, 2009). For reasons of simplicity and comparability, we restrict ourselves to the capital requirements for interest rate fluctuations (Mkt_{int}), stock market movements (Mkt_{eq}), and shifts in real estate prices (Mkt_{prop}).

According to EC (2010), interest rate risk is defined as the change in the net asset value (ΔNAV) that is caused by movements of the term structure of interest rates (see, e.g., EC, 2010). Since, in general, assets and liabilities of insurance companies are interest rate sensitive, both upward and downward shocks to the yield curve have an effect on NAV. Thus, Mkt_{int} is distinguished in two situations:

$$Mkt_{int}^{up} = \Delta NAV|_{up}, \tag{1}$$

$$Mkt_{int}^{down} = \Delta NAV|_{down}. \tag{2}$$

---

[9]We concentrate on the SCR since it is the key measure of Solvency II. Another magnitude is the so-called "minimum capital requirement (MCR)", which is governed by a linear formula including a certain percentage of the solvency capital requirement as floor and cap (for further information refer, e.g., to EC, 2010).

Mkt_{int}^{up} and Mkt_{int}^{down} capture ΔNAV due to a preset upward and downward shock, respectively. The corresponding stresses for each interest rate r_t in the term structure are applied as follows (see EC, 2010):

$$
\begin{aligned}
&r_t \cdot (1 + s_t^{up}) \quad \forall t, \text{ for the upward shock,} \\
&r_t \cdot (1 + s_t^{down}) \quad \forall t, \text{ for the downward shock,}
\end{aligned}
\tag{3}
$$

where t stands for the maturity and s_t^{up} as well as s_t^{down} equal the shocks for the up and down state.

Mkt_{eq} is based on the change in net asset value due to a drop in equity prices. In the Solvency II proposal, the equity risk category is split into "global equity" and "other equity" (see EC, 2010). The former contains equity investments that are listed on an organized market of an OECD or EEA country, whereas the latter comprises nonlisted equities, emerging market stocks, commodities, hedge funds, and any other investments that are not considered in one of the remaining risk categories. The calculation of Mkt_{eq} is carried out in two steps (see EC, 2010). First of all, the capital requirements for each of the two equity subcategories are determined by the impact of a prespecified shock:

$$
Mkt_{eq,i} = \max\left(\Delta NAV \mid \text{equity shock}_i, 0\right),
\tag{4}
$$

with $i = \{$global equities; other equities$\}$. In a second step, Mkt_{eq} is calculated based on a given correlation structure between global and other equities (see EC, 2010):

$$
Mkt_{eq} = \sqrt{Mkt_{eq,i} \cdot Mkt_{eq,j} \cdot Corr_{eq}},
\tag{5}
$$

where $i, j \in \{$global equities; other equities$\}$ and $Corr_{eq}$ denotes the applicable correlation coefficient.

Similar to the capital requirement for equity risk, Mkt_{prop} is calculated on a prespecified loss in real estate values (see EC, 2010):

$$
Mkt_{prop} = \max\left(\Delta NAV \mid \text{property shock}, 0\right).
\tag{6}
$$

Finally, the overall capital charges for market risk, SCR_{Mkt}, can be determined as (see EC, 2010):

$$SCR_{Mkt} = \max \left\{ \sqrt{\sum Corr_{Mkt}^{up} \cdot Mkt_i^{up} \cdot Mkt_j^{up}}; \right. $$
$$\left. \sqrt{\sum Corr_{Mkt}^{down} \cdot Mkt_i^{down} \cdot Mkt_j^{down}} \right\}, \quad (7)$$

where $i, j \in \{int; eq; prop\}$, the superscripts denote the up and down state for interest rate risk, and $Corr_{Mkt}^{up}$ as well as $Corr_{Mkt}^{down}$ are the preset correlation coefficients, which can be found in Table 8 of the Appendix. Mkt_{eq} and Mkt_{prop}, are independent of the interest rate scenarios.

4.2   SST Standard Model for Market Risk

In this section, we discuss the main features of the SST standard model for market risk as laid down by the Swiss Federal Office of Private Insurance (FOPI) and the Swiss Financial Market Supervisory Authority (FINMA). The SST is based on the market-consistent valuation of an insurance company's assets and liabilities (see FOPI, 2004). A key magnitude for the solvency assessment is the so-called risk-bearing capital at time t (RBC_t), which is defined as the market value of the assets (A_t) minus the best estimate of the liabilities (L_t^b). It represents the firm's available reserve to cope with fluctuations in assets and liabilities over time (see FOPI, 2006):

$$RBC_t = A_t - L_t^b. \quad (8)$$

Since there are no liquid markets for insurance liabilities, the market-consistent value of the liabilities (L_t) in the context of the SST is derived by adding a model-based market value margin (MVM_t) to the best estimate value of the liabilities (see FOPI, 2006):[10]

$$L_t = L_t^b + MVM_t. \quad (9)$$

---

[10]For a discussion of valid approaches to the market-consistent valuation of insurance liabilities, refer to FOPI (2004).

This implies that the market value of the assets is higher than the market value of the liabilities if the risk-bearing capital exceeds the market value margin:

$$RBC_t = A_t - (L_t - MVM_t)$$
$$RBC_t - MVM_t = A_t - L_t.$$

(10)

The market value margin represents the hypothetical capital that would be required for an orderly runoff of the liabilities in case of an insolvency of the insurer, either by closing for new business and exhausting the remaining capital or by transferring the liabilities of the insolvent firm to another institution.[11] In contrast to Solvency II, the SST adopts the tail value at risk (also called conditional value at risk or expected shortfall) as risk measure. The value at risk for the confidence level $1 - \alpha$ (e.g., 99.5%), VaR_α, is generally defined as the loss over a particular period that is only exceeded with probability α. If the random variable X under consideration represents value changes (e.g., returns), the VaR equals the α-quantile (Q_α) of the respective distribution. In line with this definition, the tail value at risk, TVaR_α, is the conditional expected value for those realizations of X that are equal to or lower than the VaR_α: $\text{TVaR}_\alpha = \text{E}[X|X \leq \text{VaR}_\alpha]$. Hence, the TVaR_α represents the size of the average loss in case the VaR_α is exceeded. For the SST, FINMA has set the confidence level to 99 percent, implying an exceedance probability α of one percent (see FOPI, 2006).

Moreover, in line with common practice in the financial services industry, the SST is based on a risk evaluation period of one year. The current values of the assets and liabilities determine their possible realizations at the end of the year and, in turn, the remaining risk-bearing capital. Therefore, the insolvency probability of an insurance company can be controlled by providing for an appropriate level of risk-bearing capital in $t = 0$. Against this background, the SST target capital (TC) is defined as the amount of risk-bearing capital today (RBC_0) for which

---

[11]If the market value margin was unavailable as a compensation for the necessary regulatory capital cost, potential third-party investors would not be willing to acquire the portfolio of insurance policies.

the conditional expected value of the one percent lowest levels of risk-bearing capital at the end of the year ($R\tilde{B}C_1$) equals the market value margin MVM_1 (see FOPI, 2006):

$$E[R\tilde{B}C_{1|RBC_0=TC} \mid R\tilde{B}C_{1|RBC_0=TC} \leq Q_{1\%}] = MVM_1. \qquad (11)$$

To put it differently, the regulator prescribes an amount of RBC_0 which ensures that, even for the most detrimental outcomes, $R\tilde{B}C_1$ is on average sufficient to cover the costs of running off the insurance portfolio, i.e. MVM_1. It can be shown that the following formulation is equivalent to this definition of TC (see FOPI, 2006):

$$\begin{aligned} TC &= -\text{TVaR}_{1\%}\left(\frac{R\tilde{B}C_1}{(1+r_f)} - RBC_0\right) + \frac{MVM_1}{(1+r_f)} \\ &= -\text{TVaR}_{1\%}\left(\Delta R\tilde{B}C\right) + \frac{MVM_1}{(1+r_f)}, \end{aligned} \qquad (12)$$

where r_f denotes the risk-free interest rate and $\Delta R\tilde{B}C$ is the change in risk-bearing capital.

Thus, the target capital (TC) in $t = 0$ is defined as the sum of the (discounted) expected value of the one percent largest declines in risk-bearing capital and the present value of the market value margin. In order to compute TC, the probability distributions for the random variables $R\tilde{B}C_1$ and $\Delta R\tilde{B}C$ are required. For this purpose, the SST includes a standard model, which serves to describe the stochasticity of the change in risk-bearing capital due to changes in a wide range of market risk factors such as stock prices, interest rates, credit spreads, real estate prices, and exchange rates.[12] The k-dimensional random vector of the changes in the risk factors, $\Delta \tilde{RF} = (\Delta \tilde{f}_1, ..., \Delta \tilde{f}_k)'$, is assumed to be multivariate normally distributed with individual means of zero and the variance-covariance matrix Σ (see FOPI, 2004):

$$\Delta \tilde{RF} \sim N_k(\mathbf{0}, \Sigma). \qquad (13)$$

---

[12] A comprehensive list of the current 79 risk factors can be found in FINMA (2010).

Due to the assumption of multivariate normally distributed risk factor changes, the distribution of $\Delta R\tilde{B}C$ is also normal and can be derived based on the mean and variance of $\Delta \tilde{RF}$ (see Appendix A).

However, for some risk factor changes the normality assumption is a more or less strong simplification of reality.[13] In order to mitigate this issue, the analytical model for $\Delta R\tilde{B}C$ is complemented by a number of historical and hypothetical capital market stress scenarios, which account for deviations from normality. The goal is to generate a more sophisticated representation of the highly relevant tail characteristics of $\Delta R\tilde{B}C$. FOPI (2006) describes the scenarios as events that occur with a low frequency but have a severe impact on the risk-bearing capital of an insurance company. Suppose that only one of m possible scenarios can occur once in any year and consider the following distinction of cases:

- S_0 : base case (normal year without a scenario),
- S_j : occurrence of scenario j $\forall j \in \{1, ..., m\}$.

The normal years and the specific scenarios occur with probabilities p_0 and p_j, respectively, and are assumed to be mutually exclusive, implying:

$$p_0 = 1 - (p_1 + p_2 + ... + p_m).$$

For the 2011 SST, a range of $m = 11$ scenarios is preset by the regulator. Apart from these, insurance companies are free to add custom scenarios that are of particular importance to their business situation. In general, scenarios consist of stresses with regard to several risk factors, which can be translated into a deterministic total change in risk-bearing capital, c_j, caused by the occurrence of the respective scenario:

$$c_j = \Delta RBC(S_j) \quad \forall j \in \{1, ..., m\}. \tag{14}$$

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[13]Stock returns, for example, have been repeatedly shown to exhibit skewness and excess kurtosis (see, e.g., Officer, 1972).

Subsequently, this c_j is used to derive a cumulative distribution function (cdf) prevailing under each scenario. To this end, consider the cdf for $\Delta R\tilde{B}C$ in the base case:

$$F_0(x) = P\left(\Delta R\tilde{B}C \leq x|S_0\right). \tag{15}$$

It is now assumed that under each scenario, all possible changes in the risk-bearing capital are lowered by an amount of c_j compared to a normal year. Through this assumption potential scenario-induced deformations of the distribution, such as changes in skewness and kurtosis, are ignored (see FOPI, 2006). Hence, the cdf for scenario S_j is obtained by shifting the cdf of the base case accordingly:

$$F_j(x) = P\left(\Delta R\tilde{B}C \leq x|S_j\right) = F_0(x - c_j) \quad \forall j \in \{1, ..., m\}. \tag{16}$$

Finally, the scenarios and the normal year are consolidated into an aggregate cdf for $\Delta R\tilde{B}C$, which equals the weighted mean of the individual probability distributions(see FOPI, 2006):

$$F(x) = \sum_{j=0}^{m} p_j F_j(x) = \sum_{j=0}^{m} p_j F_0(x - c_j). \tag{17}$$

Based on this mixture of $m + 1$ normal distributions, $F(x)$, it is possible to determine the target capital according to Equation (12).

4.3   Outline of an Internal Model for Market Risk

Apart from the standard approaches under Solvency II and the SST, insurance companies can also rely on internal models to calculate their capital charges for market risk. Hence, in this section we introduce such an alternative framework that will serve us to draw the desired comparisons in the context of the empirical analysis in Section 5. In essence, our approach is a parsimonious structural credit model in which default is characterized as the company's asset value being insufficient

to repay its liabilities.[14] Instead of employing risk factor fluctuations, which are then translated into changes in the firm's risk-bearing capital, our model directly builds upon the stochasticity of the market values of asset classes and liability categories. Consider a one-period evaluation horizon and continuous compounding. In addition, assume that the life insurer has a stable client base and cash flows are exchanged at the beginning of the period. Under this setup, the assets in $t = 1$ can be described as follows:

$$\tilde{A}_1 = A_0 \exp(\tilde{r}_A), \tag{18}$$

where

- $\tilde{A}_1$: stochastic market value of the assets in $t = 1$,
- A_0: deterministic market value of the assets in $t = 0$,
- $\tilde{r}_A$: stochastic return on the assets between $t = 0$ and $t = 1$.

We decide to remain on an abstract level and model $\tilde{r}_A$ based on aggregate asset classes. If deemed necessary, it is straightforward to adopt a more detailed categorization. Thus, the total asset return consists of the individual returns for each asset class in the portfolio of the life insurer:

$$\tilde{r}_A = \sum_{i=1}^{n} w_i \tilde{r}_i, \tag{19}$$

with

- w_i: portfolio weight for asset class i,
- $\tilde{r}_i$: return of asset class i between $t = 0$ and $t = 1$,
- n: number of asset classes in the portfolio.

In general, the value of the life insurance liabilities in $t = 0$ equals the discounted expected future payments to those insured. For each policy, the future cash flow streams need to be estimated based on actuarial assumptions, taking into account the insured's age and mortality profile,

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[14]Structural credit models, which can be traced back to the seminal work of Merton (1974), are well established in the finance literature (see, e.g., Longstaff and Schwartz, 1995; Leland and Toft, 1996).

the obligations arising under the contract, as well as applicable embedded options such as interest rate guarantees. While it is common to employ an actuarial technical discount rate, in a solvency measurement context we are interested in the market value rather than the actuarial value of the liabilities. Hence, the current term structure should be used for discounting, and the resulting market value of the life insurance liabilities is not only sensitive to the life expectancies of those insured, but also reacts to changes in the prevailing interest rate environment. If the market value of the liabilities is assumed to continuously increase (or decrease) throughout the period at a stochastic rate, we obtain the following relationship:

$$\tilde{L}_1 = L_0 \exp(\tilde{g}_\mathrm{L}), \tag{20}$$

where

- $\tilde{L}_1$: stochastic market value of the liabilities in $t = 1$,
- L_0: deterministic market value of the liabilities in $t = 0$,
- $\tilde{g}_\mathrm{L}$: stochastic growth rate of the liabilities between $t = 0$ and $t = 1$.

Similar to the asset side, the aggregated growth rate of the liabilities is determined by individual growth rates for the different categories of liabilities in the life insurance portfolio:

$$\tilde{g}_L = \sum_{i=1}^{l} v_i \tilde{g}_i, \tag{21}$$

with

- v_i: portfolio weight for liability category i,
- $\tilde{g}_i$: growth rate of the liability category i between $t = 0$ and $t = 1$,

- l: number of liability categories in the portfolio.

Furthermore, in order for the company's assets and liabilities to be correlated, let the random variables $\tilde{r}_A$ and $\tilde{g}_L$ adhere to a joint cdf $F(x, y)$:

$$F(x, y) = P(\tilde{r}_A \leq x, \tilde{g}_L \leq y). \tag{22}$$

At this point, we deliberately refrain from making specific distributional assumptions for the asset class returns. Instead, we aim to determine an adequate choice on the basis of distribution fitting in Section 5. Similarly, in contrast to the SST standard model, we do not set the means to zero but explicitly stress the flexibility of describing the return distributions as precisely as possible, including mean, standard deviation, and potentially higher moments.

Based on the stochastic assets and liabilities, it is now possible to derive a distribution for $R\tilde{B}C_1$ and, in turn, the change in risk-bearing capital between $t = 0$ and $t = 1$, which we define according to Equation (12). Analogously to the SST standard approach, this internal model can be extended by scenarios to gain additional flexibility and to capture the tails of the distribution in a more realistic way. Once the aggregate cdf for $\Delta R\tilde{B}C$ has been estimated, the capital requirements can be calculated based on the applicable risk measure, i.e. the $\text{VaR}_{0.5\%}$ (European Union) or the $\text{TVaR}_{1\%}$ (Switzerland).

5 Implementation of the Market Risk Models

5.1 Model Calibration

5.1.1 Stylized Balance Sheet of a Representative Life Insurer

Prior to an implementation of the previously explained solvency models based on real-world data, we need to set out the characteristics of the representative life insurance company whose capital requirements we would like to determine. Table 5 shows the stylized balance sheet underlying our calculations. To enhance the comparability of our results, we abstract from the distinction between Euros (Solvency II) and Swiss Francs (SST) by denominating the balance sheet in currency units (CU). The total asset value and portfolio weights are based on the 2009 financial statements for 21 Swiss life insurers, which are available from the FINMA insurance

Model characteristic	Solvency II	SST	Internal Model
Type	Static stress factor model	Dynamic risk factor model	Structural credit model
Risk measure	value at risk confidence level: 99.5%	tail value at risk confidence level: 99%	Depends on prevailing regulatory regime
Set-up	Solvency capital requirement (SCR) determined by the impact of preset shocks on the net asset value (NAV)	Calculation of target capital (TC) based on the distribution of the change in risk bearing capital (ΔRBC), derived by means of 79 risk factors	Direct modeling of stochastic asset and liability market values
Distributional assumptions	Properties of empirical distributions enter preset stress factor values	Multivariate normally distributed risk factors with $N(0,\Sigma)$	Flexible distributional assumptions, including empirical means, standard deviations, and correlation structure
Calibration	Fully prescribed by CEIOPS based on analysis of time series data	Mainly set by FINMA based on time series data for the latest 10-year period; own calibration for some risk factors possible (e.g., private equity)	Generally flexible; selection of index return time series data and estimation period needs to be accepted by the regulator
Scenarios	n/a	Preset capital market scenarios capture the tail characteristics of ΔRBC	Flexible

Table 4: Comparison of Characteristics of the Three Market Risk Models

report portal (see FINMA, 2009). We have averaged the respective figures over all companies and subsequently aggregated some of the items to stylize the balance sheet. Although annual reports are not consistent with market values as required by the solvency frameworks under consideration, we deem this be the most adequate and reliable proceeding in the absence of market value balance sheets. The firm's equity ratio is also based on figures from the insurance report portal, which range from 5 percent to 12 percent. In general, the difference between market and book values is greater on the asset than on the liability side, implying that a market value balance sheet should exhibit more equity capital. Thus, we decided to employ the upper bound of 12 percent for the equity ratio. Furthermore, investment limits have been retrieved from the applicable regulatory directives (see FOPI, 2008). The maximum percentage of the total assets that insurers are allowed to allocate to the category of alternative investments, for example, is 10 percent.[15] As we do not have any details on the constituent positions within the company's asset categories, we assume that the structure of each subportfolio equals that of a common capital market index. The life insurer's U.S. government bond portfolio, for example, is represented by the S&P U.S. Treasury Index. Modified durations for all three bond portfolios (U.S., EU, and Swiss government bonds) as of December 31, 2010, have been obtained from Bloomberg. The aggregate asset duration equals the weighted average of the bond portfolio durations and amounts to 4.10.[16] Finally, we set the duration of the company's life insurance liabilities to 10.00, implying a duration gap of 5.90. These values are in line with estimations of various practitioner studies for the German life insurance industry (see Finke, 2006, Steinmann, 2006, and Linowski, 2007).

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[15]In addition, no single fund in this category must amount to more than one percent of the insurer's total assets. Similarly, for fund of funds this proportion is 5 percent. The term "alternative investments" comprises the asset classes of private equity, hedge funds, currency overlays, commodities, as well as structured products (see FOPI, 2008).

[16]By employing a single duration figure for the whole asset side, we implicitly assume that interest rates in the USD, EUR, and CHF area move in lockstep. Since, for our calibration horizon, all pairwise correlations of the changes in the respective average interest rates exceed 0.70, we believe that this is an acceptable simplification.

Assets	Index Representing Portfolio	Value (in CU million)	% of Total Assets	Investment Limit (see FOPI, 2008)	Duration (as of 12/31/2010)
Stocks	–	1,690	13%	30%	–
U.S. Stocks	MSCI USA	390	3%	–	–
European Stocks	MSCI EU	650	5%	–	–
Swiss Stocks	MSCI CH	650	5%	–	–
Bonds		8,580	66%	n.a.	–
U.S. Government Bonds	S&P U.S. Treasury Index	1,430	11%	–	4.23
EU Government Bonds	S&P EU Government Bond Index	1,430	22%	–	6.04
Swiss Government Bonds	Swiss Government Bond Index	2,860	33%	–	7.00
Real Estate	Rued Blass Real Estate Index	1,300	10%	25%	–
Alternative Investments		260	2%	10%	–
Hedge Funds	HFRX Global Hedge Fund Index	130	1%		–
Private Equity	LPX50/PEPI/CAPEI	130	1%		
Cash	Swiss Money Market	1,170	9%	n.a.	–
Total		13,000	100%		4.10

Table 5: Stylized Balance Sheet of a Representative Life Insurance Company

Equity and Liabilities	Value	% of Total Assets	Duration
	(in CU million)		(as of 12/31/2010)
Life Insurance Liabilities	11, 440	88%	10.00
Equity	1, 560	12%	–
Total	13, 000	100%	8.80

Table 5: Stylized Balance Sheet of a Representative Life Insurance Company – continued

5.1.2 Solvency II Market Risk Module

Our calibration of the Solvency II standard formula is consistent with the latest CEIOPS directives (see CEIOPS, 2010a; CEIOPS, 2010b; CEIOPS, 2010c), which take the experience from the global financial crisis into account. Table 6 shows the parameter values for the equity, interest rate, and property risk shocks that enter our calculations. In order to derive the stress factor for the market risk category "global equity", CEIOPS employed historical return time series of the MSCI World Developed Equity price and total return indices. In addition, the tails of the empirical distribution were taken into account through extreme value theory. These considerations resulted in a 45 percent stress factor for "global equity", which is supported by a majority of the EU member states (see CEIOPS, 2010c). Furthermore, the preset stress factor for the category "other equities" is based on benchmark indices for the four distinct asset classes private equity, hedge funds, commodities, and emerging market equities. In the case of private equity, for example, CEIOPS relied on the return distribution of the LPX50, from which they derived a $\text{VaR}_{0.5\%}$ of 68.67 percent. In contrast to that, their analysis of the hedge fund index HFRX suggested a stress factor of 23.11 percent. As indicated by this discrepancy, the four subcategories drawn together under "other equities" exhibit a considerable heterogeneity. Hence, it would be highly appropriate to introduce a separate stress factor for each of these four asset classes. Nevertheless, CEIOPS insists on a common stress factor of 55 percent (see CEIOPS, 2010c). For global and other equities they proposed a correlation coefficient of 75 percent.[17]

With regard to interest rate risk, CEIOPS provides an upward and a downward shock for each maturity of the term structure.[18] In order to simplify the analysis, we assume a single interest rate for each of the three currency zones (USD, EUR, and CHF) covered by the insurer's bond portfolios. These flat term structures are calculated by averag-

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[17]The CEIOPS advice for the standard formula also includes a so-called "symmetric adjustment mechanism", which allows to change the equity stress factors in times of financial crises (see CEIOPS, 2010a). We abstract from this feature.

[18]The respective figures can be found in the Solvency II calibration paper (see CEIOPS, 2010c).

ing the constituent rates of the respective yield curves on December 31, 2010. In line with this proceeding, we average the CEIOPS stress factors for all maturities. Consequently, we get a single upward and downward interest rate shock of +42 percent and -39 percent, respectively. The firm's bond portfolios and life insurance liabilities are assumed to react to these shocks according to their durations.

Finally, CEIOPS based the calibration of the property risk stress factor on the Investment Property Databank, which provides comprehensive historical total return index data for the U.K. They recommend an overall stress factor for property risk of 25 percent, since the descriptive statistics and the lower percentiles of the empirical return distributions were found to be relatively homogeneous across different property classes (see CEIOPS, 2010b).

5.1.3   SST Standard Model for Market Risk

The market risk standard approach of the 2011 SST comprises a total of 79 risk factors, which have been calibrated by FINMA based on time series data between January 2001 and December 2010.[19] To facilitate our analysis and enhance comparability with the internal model, we decided to reduce the number of risk factors in line with the balance sheet structure of our representative life insurance company. Consequently, from the range preset by the regulator, one appropriate risk factor is adopted for each asset subportfolio. All risk factors underlying our calculations together with the estimated volatilities of their changes (σ_i) are summarized in the central section of Table 6. Since the life insurer's stock portfolios are assumed to resemble the MSCI country indices (MSCI USA, MSCI EU, MSCI CH), those are employed as equity risk factors. With respect to interest rate risk, we again use flat term structures. Fluctuations in the USD, EUR, and CHF interest rates are translated into value changes for the company's bond portfolios and life insurance liabilities by means of the modified durations in Table 5. In addition, the Rued Blass Index (RBREI) as well as the HFRX serve as risk factors for the

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[19]For a complete list of the risk factors and their parameter estimates, refer to the 2011 SST template on the FINMA website.

real estate and hedge fund portfolio, respectively.[20] To prevent an understatement of risk due to index measurement issues, FINMA requires a doubling of the volatility for the hedge fund risk factor (see FINMA, 2010b).

Insurers are generally allowed to select an own risk factor for private equity. However, if the company is incapable of assigning proper volatility and correlation estimates to the asset class, a standard deviation of 37.50 percent needs to be applied and the correlations to all other risk factors are set to one. To avoid this strikingly detrimental calibration prescribed by FINMA, one of the private equity indices introduced in Section 3.3 could be adopted as risk factor. Their suitability for this purpose, however, needs to be assessed on a case-by-case basis. The degree of diversification of an insurer's private equity portfolio as well as the credibility it assigns to valuations provided by its general partners are key aspects to be taken into account for the selection. To get a more complete picture, we will consider the LPX50, the PEPI, and the CAPEI as calibration alternatives. Just as for hedge funds, FINMA doubts the adequacy of risk figures estimated from private equity indices (see FINMA, 2010b). Hence, the volatilities of the appraisal-based PEPI and CAPEI have also been doubled for the analysis.

As explained in Section 4.2, the means of all risk factor changes are set to zero. $\Delta \tilde{f}_i$ represents absolute deviations for interest rates and log-returns for the other risk factors. The corresponding correlation matrix can be found in Table 10 of Appendix B. In order to further simplify the analysis, we have set the market value margin MVM to zero, which implies $L_t = L_t^b$ as well as $RBC_t = A_t - L_t$.[21] Finally, we complement the

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[20]Note that we deliberately choose the HFRX over the HFRI, since it comprises around 40 instead of more than 2,000 funds and its constituents must be open for new investment (see www.hedgefundresearch.com). Swiss insurers are allowed to allocate up to 10 percent of their portfolio to alternative investments. Yet, any single fund must not account for more than one percent of the total assets (see investment limits in Table 5). This implies that portfolios with as little as 10 funds are still admissible. Thus, we believe that the HFRX is more suitable to represent the typical hedge fund portfolio of a Swiss insurance company.

[21]Refer to Equations (9) and (10).

analytical part of the SST market risk model with the eleven capital market stress scenarios provided by FINMA. The occurrence probabilities (p_j) and risk factor shocks that govern the total change in risk-bearing capital (c_j) for each scenario can be found in Table 9 of Appendix B.

5.1.4 Internal Model for Market Risk

To ensure a high level of comparability, we want to align the calibration of our asset-class-based internal model as closely as possible with the risk-factor-based SST standard approach.[22] Therefore, where possible, the indices that represent the life insurer's asset portfolios (see Section 5.1.1) have been chosen so as to correspond to a market risk factor of the SST. Exceptions are the firm's U.S., European, and Swiss government bond holdings, which the internal model captures directly through the historical return time series of the S&P USTI, the S&P EUGI, and the SIX SBI instead of resorting to the underlying interest rates as risk factors.[23] Nevertheless, consistency is ensured, since our calculations for both the Solvency II and SST standard approach are based on the modified durations of these bond index portfolios in combination with the respective interest rate shocks or volatilities. Again, the LPX50, the PEPI, and the CAPEI are utilized as alternative proxies for the private equity portfolio. In addition, we exploit the flexibility of the internal model to explicitly account for the life insurer's cash holdings through the Swiss 3-month Money Market Index (SMMI).

In Section 4.3, we deliberately left the probability distributions for the asset class returns undefined. By means of the Kolmogorov-Smirnov (K-S) and the Anderson-Darling (A-D) goodness-of-fit statistic, we now test whether the normal distribution adequately describes the observed returns of our index portfolios. In line with the calibration horizon of the

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[22] Due to its simplistic design, a reasonable alignment with the Solvency II standard formula hardly seems feasible.

[23] A more sophisticated approach could aim at deriving a return distribution for the bond portfolios based on a stochastic interest rate model such as, e.g., the ones proposed by Vasicek (1977) or Cox, Ingersoll Jr, and Ross (1985). However, in the absence of detailed information (notional, maturities, coupons, etc.) on the bond portfolio constituents, we opt for this alternative.

Solvency II (Shocks)	Down %	Up %	SST (Risk Factors)	$\mathrm{E}\left[\Delta\tilde{f}_i\right]$ % p.a.	σ_i % p.a.	Internal Model (Asset Classes)	$\mathrm{E}\left[\tilde{r}_i\right]$ % p.a.	$\sigma_{\tilde{r}_i}$ % p.a.
Global Equities			**Equity markets**			**Stocks**		
U.S. Stocks	−45.00	−	MSCI USA	0.00	16.73	MSCI USA	1.48	16.73
European Stocks	−45.00	−	MSCI EU	0.00	20.29	MSCI EU**	0.00	20.29
Swiss Stocks	−45.00	−	MSCI CH	0.00	15.24	MSCI CH	0.18	15.24
Interest Rate Risk			**Interest Rates**			**Bonds**		
USD Interest Rate	−39.00	+42.00	USD Interest Rate	0.00	0.95	S&P USTI	4.91	4.29
EUR Interest Rate	−39.00	+42.00	EUR Interest Rate	0.00	0.68	S&P EUGI	4.76	3.63
CHF Interest Rate	−39.00	+42.00	CHF Interest Rate	0.00	0.54	SIX SBI	4.20	3.83
Property Risk			**Real Estate**			**Real Estate**		
Swiss Real Estate	−25.00	−	RBREI	0.00	6.51	RBREI	6.72	6.51
Other Equities			**Hedge Funds**			**Hedge Funds**		
Hedge Funds	−55.00	−	HFRX*	0.00	5.96	HFRX*	3.50	5.96

Table 6: Input Data for the Three Market Risk Models

Solvency II shocks as well as means and standard deviations for the SST risk factors and the asset class return distributions of the internal model. The latter two are based on the period 01/01/2001-12/31/2010. Indices: MSCI country indices for the U.S. (MSCI USA), Europe (MSCI EU), and Switzerland (MSCI CH); S&P U.S. Treasury Index (S&P USTI); S&P Eurozone Government Bond Index (S&P EUGI); Swiss Government Bond Index (SIX SBI); Rued Blass Real Estate Index (RBREI); HFRX Global Hedge Fund Index (HFRX); LPX50 Listed Private Equity Index; Thomson Reuters Private Equity Performance Index (PEPI); Cambridge Associates U.S. Private Equity Index (CAPEI); Swiss three-month Money Market Index (SMMI).

Notes:

* As required by FINMA, the volatilities for the HFRX and the appraisal-based private equity indices are doubled for the SST.

** Negative means have been set to zero.

Solvency II (Shocks)	Down %	Up %	SST (Risk Factors)	$\mathrm{E}\left[\Delta \tilde{f}_i\right]$ % p.a.	σ_i % p.a.	Internal Model (Asset Classes)	$\mathrm{E}\left[\tilde{r}_i\right]$ % p.a.	$\sigma_{\tilde{r}_i}$ % p.a.
			Private Equity			**Private Equity**		
Private Equity	−55.00	–	1) FINMA	0.00	37.50			
			2) LPX50	0.00	26.70	1) LPX50**	0.00	26.70
			3) PEPI*	0.00	11.73	2) PEPI*	6.97	11.73
			4) CAPEI*	0.00	11.61	3) CAPEI*	9.60	11.61
						Cash		
						SMMI	1.21	0.31

Table 6: Input Data for the Three Market Risk Models – continued
Solvency II shocks as well as means and standard deviations for the SST risk factors and the asset class return distributions of the internal model. The latter two are based on the period 01/01/2001-12/31/2010. Indices: MSCI country indices for the U.S. (MSCI USA), Europe (MSCI EU), and Switzerland (MSCI CH); S&P U.S. Treasury Index (S&P USTI); S&P Eurozone Government Bond Index (S&P EUGI); Swiss Government Bond Index (SIX SBI); Rued Blass Real Estate Index (RBREI); HFRX Global Hedge Fund Index (HFRX); LPX50 Listed Private Equity Index; Thomson Reuters Private Equity Performance Index (PEPI); Cambridge Associates U.S. Private Equity Index (CAPEI); Swiss three-month Money Market Index (SMMI).
Notes:
* As required by FINMA, the volatilities for the HFRX and the appraisal-based private equity indices are doubled for the SST.
** Negative means have been set to zero.

P-Values	LPX50	PEPI	CAPEI	MSCI USA	MSCI EU	MSCI CH
K-S Test	0.0430	0.2453	0.5716	0.1033	0.2225	0.1281
A-D Test	0.0276	0.3510	0.5760	0.1593	0.1067	0.1527
P-Values	S&P USTI	S&P EUGI	SIX SBI	HFRX	RBREI	SMMI
K-S Test	0.5946	0.2660	0.9630	0.0122	0.3659	0.0004
A-D Test	0.6249	0.4013	0.9651	0.0156	0.5585	0.0023

Table 7: Goodness-of-Fit Test for the Index Return Time Series (01/01/2001 - 31/12/2010)
P-values for the Kolmogorov-Smirnov (K-S) and the Anderson-Darling (A-D) test of the null hypothesis that the sample has been drawn from a normal distribution. Indices: LPX50 Listed Private Equity Index; Thomson Reuters Private Equity Performance Index (PEPI); Cambridge Associates U.S. Private Equity Index (CAPEI); MSCI country indices for the U.S. (MSCI USA), Europe (MSCI EU), and Switzerland (MSCI CH); S&P U.S. Treasury Index (S&P USTI); S&P Eurozone Government Bond Index (S&P EUGI); Swiss Government Bond Index (SIX SBI); HFRX Global Hedge Fund Index (HFRX); Rued Blass Real Estate Index (RBREI); Swiss three-month Money Market Index (SMMI).

SST market risk standard approach, we employ the return time series for each index from January 2001 to 2010. From the resulting p-values reported in Table 7 only those for the LPX50, the HFRX, and the SMMI are smaller than 0.05. For the other 9 time series, in contrast, we cannot reject the null hypothesis of normality on the five percent significance level. Therefore, to facilitate the analysis and further enhance comparability with the SST standard model, we deem it acceptable to assume normally distributed asset returns. Consequently, $\tilde{A}_1$ is log-normally distributed. Mean $\mu_{\tilde{r}_i} = \mathrm{E}\,[\tilde{r}_i]$ and standard deviation $\sigma_{\tilde{r}_i}$ for the empirical return distribution of each asset class are shown in the right section of Table 6 and the respective correlation matrix can be found in Table 8 of Appendix B. This μ–σ–approach for the asset model is well grounded in the classical portfolio theory (see Markowitz, 1952). Note that the $\mathrm{E}\,[\tilde{r}_i]$ of the MSCI EU as well as the LPX50 have been set to zero, since we obtained negative estimates for the chosen calibration period.

Due to the lack of publicly available time series data, reflecting the behavior of the technical reserves of life insurers, we face a challenge with regard to the calibration of the stochastic liabilities in our internal model. In order to overcome this issue we assume normally distributed liability growth rates, abandon the detailed categorization of the liability side as implied by Equation (21), and resort to approximations on an aggregate level. Since life insurance liabilities are commonly valued using actuarial methodology, we decide to draw on the current maximum technical interest rate i_{tec} in Switzerland which is published on the FINMA website as proxy for the mean growth rate of the liabilities:

$$\mathrm{E}\,[\tilde{g}_L] = i_{\mathrm{tec}} = 0.0175. \tag{23}$$

Furthermore, suppose that the life insurance liabilities exclusively react to fluctuations in the CHF interest rate. Based on this assumption, we estimate the volatility of their growth rate $\sigma_{\tilde{g}_L}$ as follows:

$$\sigma_{\tilde{g}_L} \approx \sigma_{i_{\mathrm{CHF}}} \cdot D_L = 0.0054 \cdot 10.00 = 0.05, \tag{24}$$

where $\sigma_{i_{\text{CHF}}}$ is the volatility of the CHF interest rate (see Table 6) and D_L stands for the modified duration of the life insurance liabilities (see Table 5). Intuitively, this means that we expect the life insurance liabilities to be roughly 10 times as volatile as the underlying CHF interest rate.[24] Owing to these considerations, $\tilde{L}_1$ is log-normally distributed as well.

Having determined the marginal distributions of $\tilde{A}_1$ and $\tilde{L}_1$, we still need to introduce a dependency structure for these two random variables as provided for by Equation (22). In the absence of empirical evidence for a nonlinear relationship, we opt for a linear correlation of $\tilde{r}_A$ and $\tilde{g}_L$ governed by the following bivariate normal distribution:

$$(\tilde{r}_A, \tilde{g}_L)' \sim N_2(\mu, \Sigma_{\tilde{r}_A, \tilde{g}_L}), \tag{25}$$

with the two-dimensional mean vector μ and the variance-covariance matrix $\Sigma_{\tilde{r}_A, \tilde{g}_L}$. To estimate the corresponding correlation $\rho_{\tilde{r}_A, \tilde{g}_L}$, we employ the following approximation:

$$\rho_{\tilde{r}_A, \tilde{g}_L} \approx \begin{cases} D_A/D_L & \text{if} \quad D_A \leq D_L \\ D_L/D_A & \text{otherwise} \end{cases}, \tag{26}$$

where D_A represents the modified duration of the asset side (see Table 5). In our context, D_A and D_L are assumed to be strictly positive. The intuition behind this approach is that the joint variation of assets and life insurance liabilities should arise because they are both sensitive to interest rate fluctuations.[25] Thus, the higher the duration gap between

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[24]Since duration measures are generally based on the assumption of a linear relationship between the value change and the interest rate change, this approximation should hold sufficiently well for small fluctuations. However, it is important to note that, by modeling the liabilities on an aggregate level, we abstract from any diversification effects between different liability categories such as, for example, longevity- and mortality-related risks. Hence, there is a likelihood that we overestimate the volatility to a certain extent.

[25]Again, a corollary of the linear relationship underlying the duration measure is that the accuracy of this approximation deteriorates for larger interest rate movements.

assets and liabilities, the lower their correlation coefficient in the context
of our internal model. Inserting the respective figures, we obtain

$$\rho_{\tilde{r}_A, \tilde{g}_L} \approx \frac{D_A}{D_L} = \frac{4.10}{10.00} = 0.41. \tag{27}$$

Assuming $r_f = 0$ and employing the previously determined distribu-
tional characteristics for $\tilde{A}_1$ and $\tilde{L}_1$ in combination with Equation (12),
we get $\Delta R\tilde{B}C$ as follows:

$$\Delta R\tilde{B}C = R\tilde{B}C_1 - RBC_0 = (\tilde{A}_1 - \tilde{L}_1) - (A_0 - L_0). \tag{28}$$

Since this is the difference of two log-normal random variables mi-
nus a constant, there is no analytical solution for $\text{VaR}_{0.5\%}$ and $\text{TVaR}_{1\%}$.
Thus, we will resort to Monte Carlo simulations (with $1,000,000$ itera-
tions) to derive the capital requirements. Finally, for the SST, we shift
the ΔRBC distribution of the internal model by the c_j associated with
the SST standard scenarios (see Table 9 in Appendix B) and aggregate
the resulting distributions according to Equation (17). In addition, we
double the volatilites of the HFRX and the two appraisal-based private
equity benchmarks as required by FINMA.

5.2 Market Risk Capital Requirements of the Life Insurance Company

In this section we calculate and compare the market risk capital require-
ments (in CU) for the representative life insurance company under the
Solvency II and the SST standard approaches as well as the internal
model. To illustrate the impact of the firm's private equity holdings, we
differentiate between the previously discussed calibration alternatives for
the asset class and vary the associated portfolio weight between zero and
the Swiss legal investment limit of ten percent. Since the weights of all
portfolio constituents must sum to 100 percent, the increasing fraction
of private equity needs to be funded through a reduction in the other
asset classes. In this regard we adopt a procedure suggested by Braun,
Rymaszewski, and Schmeiser (2011). From the basic asset allocation in
Table 5, we calculate the weight of each asset class with regard to the

remaining part of the portfolio if private equity is excluded. The resulting percentages will be called "residual weights". Consider the following example: aside from private equity, the remaining asset classes together make up 99 percent of the portfolio, 3 percent of which are U.S. stocks. Consequently, U.S. stocks are assigned a "residual weight" of $3/99 = 3.03$ percent.[26] For each percentage allocated to private equity, the rest of the asset portfolio is then split according to these residual weights. This implies that any increase in the firm's private equity holdings is associated with an absolute decrease in all other asset classes, while their weights relative to each other remain unchanged. It should be noted that each of the varied portfolio structures necessitates an adjustment of the asset duration, which, in turn, alters the correlation between the firm's assets and liabilities. Similarly, the total change in risk bearing capital for each scenario has to be recalculated. Figure 1 illustrates our results.

Solvency II

We begin our discussion with the capital requirements for the Solvency II standard formula in Figure 1(a) and notice that they generally rise with the private equity portfolio weight.[27] Since changes in the private equity holdings affect the risk-return characteristics of the entire asset portfolio as well as the interaction between assets and liabilities, this phenomenon can be attributed to two distinct effects. Firstly, the rise in the capital charges occurs due to a widening of the duration gap between the firm's assets and liabilities, meaning that the risk-bearing capital is less hedged against market risk. As explained above, an expansion of the private equity portfolio reduces the remaining assets, including the firm's bond positions. Consequently, it implies a reallocation of funds

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[26]In the same fashion, we get 5.05 percent for European stocks as well as Swiss stocks, 11.11 percent for U.S. as well as European government bonds, 22.22 percent for Swiss government bonds, 10.10 percent for real estate, 1.01 percent for hedge funds, and 10.10 percent for cash.

[27]Note that we observe a small capital relief when the private equity allocation rises from nine to ten percent. This occurs due to the fact that the legal investment limit of ten percent holds for both hedge funds and private equity. Thus, to be able to invest ten percent of its portfolio in private equity, the insurer needs to completely dissolve its hedge fund holdings. To account for this issue, we decided to reassign the residual weight of hedge funds to Swiss government bonds, which have a much lesser impact on capital requirements.

from assets with a duration into private equity, which is assumed to be interest rate insensitive. This causes the overall duration of the asset side to decline, while the duration of the life insurance liabilities is unaffected. Secondly, increasing the fraction of private equity in the asset portfolio is equivalent to assigning additional weight to the category "other equities" with its high stress factor of 55 percent at the expense of more favorably treated risk positions.

In Figure 1(c) we have plotted the capital charges that arise when the internal model is run under Solvency II (i.e., based on a $\text{VaR}_{0.5\%}$ and without scenarios).[28] Obviously, for a zero percent weight, the results are independent of the chosen private equity benchmark. Under the LPX50 calibration, more private equity in the portfolio is associated with a clear increase in capital requirements. The previously discussed widening of the duration gap is also a crucial driver here, since a decline in the asset duration leads to a lower correlation between the stochastic assets and liabilities of the life insurer in the internal model (refer to Equation (26)). However, in contrast to the Solvency II standard formula, which exclusively focuses on stress factors, the internal model captures changes in the portfolio structure along two dimensions. Thus, it allows for a second effect that can either counter or reinforce the capital increase through an improvement or deterioration of the overall risk-return-characteristics. This is highlighted in Figure 2, which shows the portfolios from Figure 1 for all three private equity calibration options (LPX50, PEPI, and CAPEI) in the μ–σ– space.

The common point reflects the portfolio without any private equity (zero percent weight in Figure 1). Starting from there, if the asset class is assumed to be represented by the LPX50, the rising proportion of private equity effects a shift further away from the efficient frontier, thereby intensifying the rise in capital requirements. The reason being that, due

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[28]Recall that our estimation period from 2001 to 2010 has been chosen so as to match the requirements of the 2011 SST. Yet, as an additional robustness check we have split this period in two halves: 2001 to 2005 (i.e., excluding the subprime financial crisis) and 2006 to 2010 (i.e., mainly comprising the financial crisis). The corresponding results for the internal model are reported in Figure 4 of Appendix C. Although the observed effects are of a somewhat different magnitude, their direction is stable.

to the low expected return (0 percent p.a.) and the high standard de-
viation (26.50 percent p.a.) of the LPX50, the rebalancing towards pri-
vate equity reduces μ and increases σ of the overall asset portfolio. For
the PEPI and the CAPEI calibrations with their attractive performance
characteristics, in contrast, the additional private equity exposure moves
the portfolio alongside the efficient frontier to the upper right in the μ–
σ–space. As can be observed in Figure 1(c), this notably mitigates the
increase of the capital charges under the PEPI relative to the LPX50
calibration and the standard approach. Furthermore, with the CAPEI
calibration we even document a slight reduction in capital charges of
22 mn between the zero and ten percent private equity weight, which
indicates that the impact of the enlarged duration gap is surmounted
by the transformation of the risk-return profile on the asset side.[29] The
differences in the firm's capital requirements for the three private eq-
uity proxies become larger when the proportion of private equity in the
portfolio increases. At this point it is important to emphasize that the
capital charges associated with the PEPI and the CAPEI should be con-
sidered as a theoretical lower bound, since these indices are not based on
market values and could thus be distorted (refer to the issues raised in
Section 2). With its 50 constituents, the LPX50 is a lot less diversified
than the PEPI and the CAPEI, which comprise several hundred limited
partnership funds. Apart from that, it has performed poorly during the
relevant time period (see Section 3.3). Therefore, we view the capital
requirements arising from a calibration with the LPX50 as an upper
bound.[30]

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[29]Unfortunately, this decline in the capital charges is rather difficult to see in Figure
1(c), since the scale of the y-axis has been chosen so as to ensure comparability with
the other subfigures.

[30]If the portfolio weight for private equity encompassed both listed and limited
partnership private equity, we would generally expect capital charges somewhere in
between those for the pure allocation to the LPX50 and the PEPI/CAPEI.
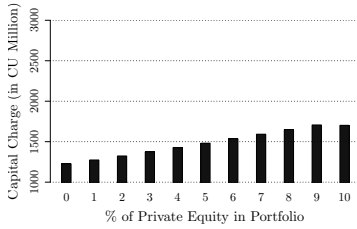
(a) Solvency II Standard Formula
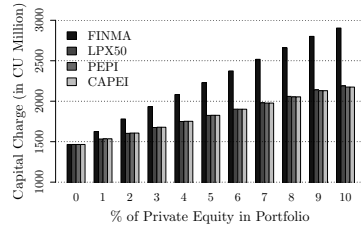
(b) SST Standard Approach
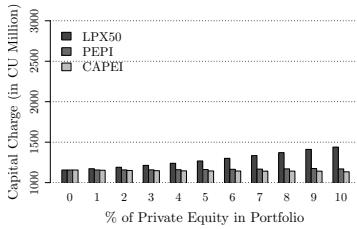
(c) Internal Model (VaR$_{0.5\%}$), no Scenarios
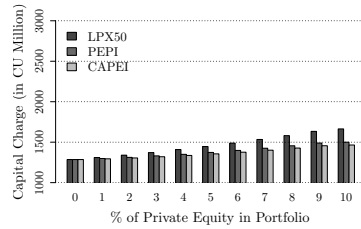
(d) Internal Model (TVaR$_{1\%}$), Scenarios
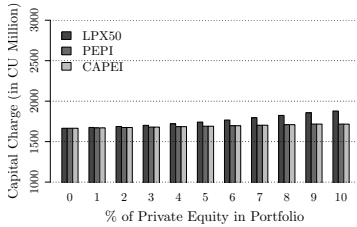
(e) Internal Model (VaR$_{0.5\%}$), no Scenarios (expected returns for all asset classes set to zero)
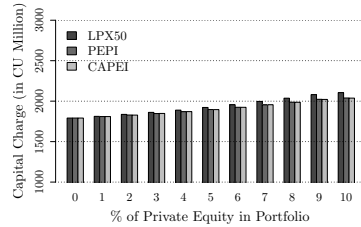
(f) Internal Model (TVaR$_{1\%}$), Scenarios (expected returns for all asset classes set to zero)

Figure 1: Capital Charges for Different Private Equity Portfolio Weights
This figure shows the life insurer's total capital charges for market risk with respect to different proportions of private equity in the asset portfolio: 0% reflects the portfolio without any private equity investments and 10% represents the legal investment limit according to the Swiss Federal Office of Private Insurance (FOPI, 2008). Each subfigure comprises the results for a different model and configuration: (a) Solvency II standard formula; (b) SST standard approach; (c) internal model with VaR$_{0.5\%}$; (d) internal model with TVaR$_{1\%}$ and SST scenarios; (e) internal model with VaR$_{0.5\%}$ and mean returns for all asset classes set to zero; (f) internal model with TVaR$_{1\%}$, SST scenarios, and mean returns for all asset classes set to zero. Alternative calibrations for private equity under the SST standard approach and the internal model: FINMA parameter values; LPX50 Listed Private Equity Index; Thomson Reuters Private Equity Performance Index (PEPI); Cambridge Associates U.S. Private Equity Index (CAPEI).
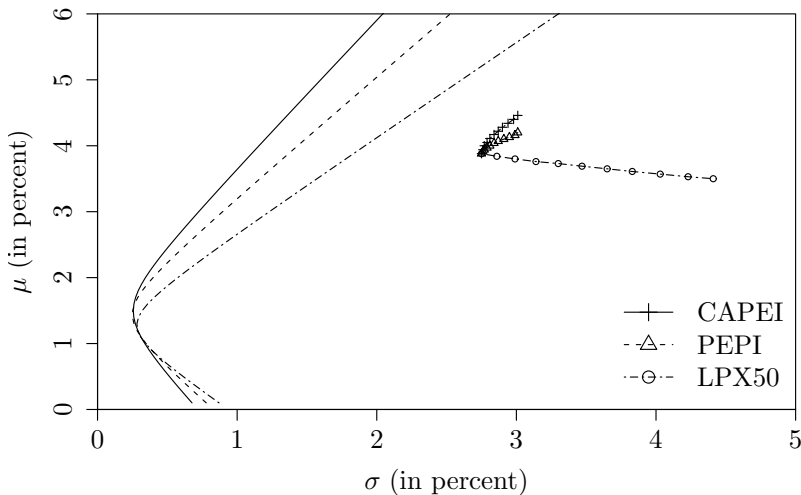
Figure 2: Risk-Return-Profile of the Life Insurer's Asset Portfolio for Different Private Equity Allocations
This figure illustrates the evolution of the life insurer's asset portfolio in the μ–σ–space. The return volatilities σ for each portfolios lie along the x-axis whereas the y-axis represents the corresponding expected returns μ. The crosses, triangles, and circles represent the location of the asset portfolio for different private equity allocations under the three calibration options: LPX50 Listed Private Equity Index; Thomson Reuters Private Equity Performance Index (PEPI); Cambridge Associates U.S. Private Equity Index (CAPEI). The common point marks the portfolio with 0% private equity. Subsequently, the weight is increased in discrete steps up to 10%. The solid, dashed, and dotdashed lines indicate the corresponding efficient frontiers.

Finally, comparing Figures 1(a) and (c) we find that, for any given private equity portfolio weight, the insurer faces a lower solvency capital requirement when employing an internal model. This holds true for each considered calibration option and can be attributed to the fact that the latter takes the investments' expected returns into account. The magnitude of this effect can be assessed through Figure 1(e), which shows the capital charges under the internal model when the expected returns for all asset classes are set to zero. Due to their major impact on the results, neglecting statistical means can evidently cause severe biases in the results. Against this background it should be welcomed that, in the

context of modern solvency frameworks, regulators aim to encourage in-
surers to develop own internal models, which best fit their risk situation
and risk management processes (see, e.g., EC, 2010). As they gener-
ally provide a higher level of detail and sophistication than the standard
approaches, it is possible that these internal models yield lower capital
charges. However, for the accreditation of the regulatory authority, in-
surers will need to demonstrate that the foundations of their models are
built upon sound economic reasoning.

Swiss Solvency Test

Figure 1(b) displays our results for the SST standard approach. First
of all, similar to our findings for Solvency II, the life insurer's capital
charges increase with the percentage of private equity in the portfolio.
Once more, two effects work in the same direction. On the one hand,
reallocating funds to private equity enlarges the asset-liability duration
gap and, on the other hand, it implies a stronger impact of a risk factor
with a high volatility. As could be expected, the supervisory parameter
values of FINMA turned out to be the most expensive calibration op-
tion. To see how inappropriate the 37.50 percent risk factor volatility
combined with a full correlation to all other assets actually is, consider
the following example. When the private equity holdings are expanded
from zero to ten percent, the capital requirements under the standard
approach with FINMA parameter values rise by approximately 1.5 bn.
Taking into account that the firm's balance sheet total equals 13 bn (re-
fer to Table 5), this is more than the absolute value that corresponds
to a private equity weight of ten percent (1.3 bn). In other words, for
each currency unit which is redistributed from the remaining, mostly
more favorably treated asset classes in the portfolio to private equity,
the firm's overall capital charges increase by about 1.15 currency units.
Clearly, under these circumstances it is generally not sensible for insur-
ance companies to invest in private equity at all. Furthermore, we see
that proxying private equity by the LPX50, the PEPI, or the CAPEI
does hardly make a difference under the SST standard approach, since
the associated capital charges move virtually in lockstep. This is in-
duced by the fact that the SST standard approach does not account for

the means of the risk factor changes, implying that the main driver of the capital charges are the respective standard deviations. The doubled estimates for the returns of the PEPI (23.46 percent p.a.) as well as the CAPEI (23.22 percent p.a.) are very close to the volatility of the LPX50 returns (26.70 percent p.a.). As a result, only small discrepancies in the capital requirements can prevail because the three private equity indices exhibit different correlation structures with the remaining positions in the portfolio.

Another phenomenon that is caused by the model set-up of the SST standard approach can be observed in Figure 1(b). Under all four calibration options, the capital charges increase virtually linearly in the private equity portfolio weight. However, the slopes of the corresponding trendlines are not perfectly proportional to the return standard deviations of the indices: the approximately linear slope of the capital charges for the FINMA parameter values equals 3.88 (= 145.43/37.50) times its volatility, which compares to a multiple of 2.79 (= 74.38/26.70), 3.11 (= 72.85/23.46), and 3.14 (= 72.83/23.22) for the LPX50, the PEPI, and the CAPEI, respectively. Again, these slight differences occur due to the respective correlation matrices. Perfect proportionality of the slope to the standard deviation of the risk factor change would require the exact same correlation matrix for all three indices. Intuitively, the FINMA calibration exhibits the highest possible slope for a volatility of 37.50, since, at the same time, all correlations with other asset classes are set to one. If another index with a volatility of 37.50 but lower correlations existed, the slope of its trendline should be flatter.

As shown in Figure 1(d), the increase in capital requirements provoked by an expansion of the firm's private equity investments is less pronounced when the internal model is employed under the SST (i.e., based on a $\text{TVaR}_{1\%}$ and with scenarios). Moreover, the results are now sensitive to the private equity benchmark used for calibration. Both effects arise due to the fact that the internal model captures the full risk-return-characteristics of each asset class. In contrast to Solvency II, however, it is run with doubled volatilities for the appraisal-based indices and the SST scenarios are superimposed on the resulting RBC

distribution. Consequently, the movements in the μ–σ–space have a less dampening impact for the PEPI and are no longer strong enough to exceed the effect of the widening duration gap for the CAPEI calibration. Nevertheless, the relatively high mean returns of the PEPI (6.97 percent p.a.) and the CAPEI (9.60 percent p.a.) provide for lower capital charges compared to the LPX50, which entered the analysis with a mean return of zero (see Table 6). The discrepancies between the different calibration options are amplified by a rising private equity proportion in the portfolio.

Finally, independent of the selected calibration alternative, all feasible allocations to private equity produce lower capital charges under the internal model than under the SST standard approach. Again, this is mainly attributable to the inclusion of expected returns, implying that the internal model establishes a link between the performance characteristics of an asset class and the firm's regulatory capital requirements. Figure 1(f) exhibits the SST market risk capital charges under the internal model in case the means for all return distributions are set to zero. Analogously to our analysis for Solvency II, this illustrates the importance of a μ–σ–approach in the context of solvency measurement.

Changes in the Capital Requirements: Private Equity vs. Public Equity and Hedge Funds

Our last analysis is based on a specific decision faced by the life insurer introduced in Section 5.1. Imagine the company plans to allocate further funds to an asset class with a higher return potential than government bonds. This could be aimed at increasing the probability that embedded guarantees of life insurance policies can be met. Based on the results of our performance analysis in Section 3, private equity and hedge funds would be natural candidates for this purpose. Moreover, one would generally also consider the stock market. Apart from the performance characteristics of a prospective investment, however, the life insurer needs to take the associated change in capital requirements into account. Hence, we aim to address the question of how costly it is from a regulatory capital perspective to increase the exposure to private equity in comparison to public equity and hedge funds. Figure 3

illustrates the results of this analysis. On the x-axis we have plotted by how many percentage points the respective portfolio weight is increased or decreased from its base-case value shown in Table 5. Meanwhile, the y-axis represents the associated percentage change in the capital charges.

We begin our discussion with Figure 3(a). As could be expected, under the Solvency II standard formula, adding private equity or hedge fund exposure to the portfolio is more expensive than entering further stock investments, since the former belong to the category "other equities" with its unfavorable stress factor. For the analysis with regard to the internal model, we need to additionally determine which one of the three stock portfolios (U.S., EU, Switzerland) is to be expanded. Due to its slightly better performance characteristics we select the life insurer's U.S. stock portfolio, represented by the MSCI USA. Furthermore, since both appraisal-based private equity indices delivered quite similar results in the previous section, we decided to restrict this analysis to the results for the PEPI and the LPX50. Figure 3(b) shows the outcome for the internal model under Solvency II. In case the LPX50 is used for calibration, increasing the proportion of private equity is more expensive than adding the same number of percentage points to the life insurer's U.S. stock or hedge fund portfolio. In contrast, if the private equity portfolio behaved more like the PEPI rather than listed private equity, its extension would be associated with the least increases in capital charges.

Turning to the results for the SST standard approach in Figure 3(c), we see at first glance that the largest increases in capital charges relate to additional investments in private equity. Again, the LPX50 and the PEPI calibration hardly differ due to the zero mean model assumption as well as the doubled volatility for the PEPI. Expanding the U.S. stock market portfolio (MSCI USA), on the other hand, is now associated with the second-smallest rises in capital charges. Finally, adding to the HFRX-like hedge fund holdings turns out to be the least expensive option. Figure 3(d) shows the corresponding results when the firm calculates its SST capital charges based on an internal model. Now it is again more attractive to allocate further funds to a private equity portfolio

represented by the PEPI than to public equity. To sum up, given the appraisal-based indices proxy the true behavior of the insurer's private equity assets sufficiently well, it can be less costly from a regulatory capital perspective to increase the exposure to private rather than public equity. Under Solvency II, this even holds for hedge funds.
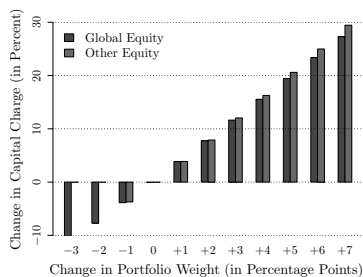
(a) Solvency II Standard Formula
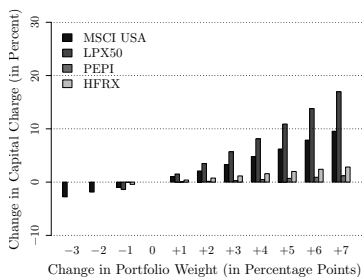
(b) Internal Model (VaR$_{0.5\%}$), no Scenarios
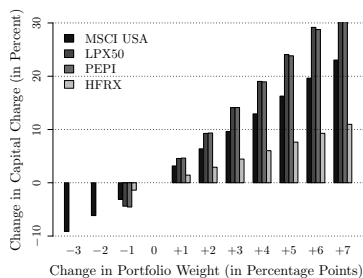
(c) SST Standard Approach
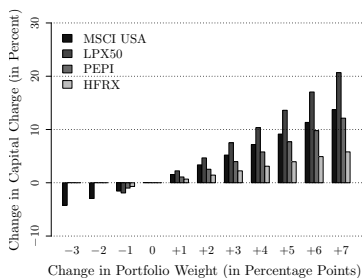
(d) Internal Model (TVaR$_{1\%}$), Scenarios

Figure 3: Percentage Change in Capital Charges for Increasing Portfolio Weights of Risky Asset Classes

Sensitivity analysis of the change in capital charges with respect to increases or decreases of the portfolio weights for private equity, public equity, and hedge funds from their base-case values in Table 5. The x-axis shows by how many percentage points the base case portfolio weight for the considered asset classes is increased or decreased (0 implies no change). The corresponding percentage changes in the capital charges due to the altered portfolio composition are represented by the y-axis. Each subfigure contains the results for a specific model: (a) Solvency II standard formula; (b) internal model with VaR$_{0.5\%}$; (c) SST standard approach; (d) internal model with TVaR$_{1\%}$ and SST scenarios. Indices: MSCI country index for the U.S. (MSCI USA); LPX50 Listed Private Equity Index; Thomson Reuters Private Equity Performance Index (PEPI); Cambridge Associates U.S. Private Equity Index (CAPEI).

6   Economic Implications and Conclusion

In this paper, we conduct an in-depth analysis of the impact of private equity investments on the capital requirements faced by a representative life insurance company under Solvency II as well as the Swiss Solvency Test (SST). Our discussion begins with an empirical performance measurement of the asset class over the period from 2001 to 2010. Subsequently, we review the standard approaches for market risk set out by both regulatory regimes and outline a potential framework for an internal model. Based on an empirical calibration and implementation of these solvency models, it is possible to derive a number of results, which should be of relevance to industry professionals and regulators alike.

Although the chosen examination period is an unfavorable decade for risky asset classes, our empirical performance analysis conveys an overall solid impression of limited partnership private equity funds. Thus, within reasonable investment limits, the asset class should be a fair choice for the purpose of portfolio diversification, especially from the perspective of life insurers with their large bond holdings. Since some uncertainty with regard to the reliability of the employed appraisal-based indices remains, we also believe that relatively large allocations to private equity should only be considered by experienced investors with strong due diligence and manager selection skills. In addition to its performance characteristics, however, the attractiveness of an asset class for insurers also depends on the associated impact on their capital charges. Hence, we turn to the market risk modules of the two most modern solvency frameworks for the insurance industry. By calculating and comparing the capital charges under the Solvency II and the SST standard approach as well as an internal model, we are able to provide evidence that the former disproportionately penalize relatively volatile asset classes such as private equity, which are commonly also associated with higher expected returns. This is mainly attributable to the fact that the Solvency II standard formula relies on a crude stress factor for the category other equities and the SST market risk model solely focuses on volatilities, while setting all risk factor means to zero. Consequently, life insurers

aiming to exploit the potential of private equity may expect significantly lower market risk capital requirements when applying an economically sound internal model. This result is shown to be robust against the indices used for calibration as well as the percentage of private equity in the portfolio. Furthermore, we demonstrate that it can even be less costly to increase the exposure to private rather than public equity. Taking these findings into account, the private equity asset class can be an attractive investment alternative for life insurers both from a performance as well as a regulatory capital perspective.

A final note is due with regard to the examined standard approaches. In our opinion, an inappropriate treatment of assets from a solvency perspective has severe economic implications. Life insurers commonly face the challenge of achieving a sufficient return on investments so as to meet the guarantees that are embedded in their underwritten insurance contracts. However, this goal can virtually not be achieved by solely relying on government bonds, especially in low interest rate environments as they are typical for postcrisis periods. Instead, it is focal to enrich the portfolio with asset classes that exhibit a greater return potential. This is exactly where the problems arise. It is a basic principle in modern finance that, barring exceptional investment skills, higher expected returns are only attainable by assuming higher risks. Thus, investors always need to consider the overall performance of an asset, i.e. the risk-return profile, rather than ignoring one side of the equation. If this fact is not reflected by regulatory frameworks, economic inefficiencies may be the consequence. Inadequate regulatory capital requirements could, for example, lead to an underrepresentation of certain asset classes that are suitable to enhance the risk-return profile of a portfolio and allow life insurers to add value through good asset management skills rather than holding large bond portfolios, which, in many cases, could be simply replicated by the customers themselves. Hence, we deem it crucial that solvency models establish a link between the performance characteristics of an investment and the firm's regulatory capital requirements.

Future research could aim at tackling some of the limitations of our results. Firstly, we ignored certain types of market risk within the analysis. Although our study is in itself consistent, it might be interesting to add FX and credit spreads as additional risk drivers. Secondly, being calibrated based on indices instead of more specific portfolio data, all three solvency models are subject to basis risk. Before inferring from our results to specific cases, we therefore recommend to check with great care whether the employed proxies adequately reflect the situation of the respective life insurance company under consideration. If, for example, the private equity portfolio heavily overweighs venture capital investments, one could rerun our analysis based on a suitable venture capital subindex. Thirdly, some private equity specific issues could be addressed in follow-up work. The phenomenon of style drift may become relevant when the life insurance company invests in a rather small number of funds. In this case one could try and explicitly account for such deviations from a fund's stated investment objectives, although they are difficult to model ex-ante. Apart from that, different methodologies that have been suggested in the literature to adjust appraisal-based private equity indices for certain biases and distortions could be considered as well. However, before such an approach can be employed, it needs to be accepted by the respective regulatory authority. Fourthly, along the way we needed to employ a few assumptions and approximations that could be reassessed once new information becomes available. One example is the linear correlation between the assets and liabilities in our internal model. In case future empirical evidence indicates a nonlinear dependency, the correlation coefficient could be substituted by some sort of copula function. Finally, it might be interesting to extend our research question to other types of institutional investors such as banks and pension funds that face different regulatory environments.

A   SST standard approach: $\Delta \tilde{RF}$ and $\Delta R\tilde{B}C$

$\Delta \tilde{RF}$ can either be measured as returns, i.e. relative changes (e.g. for stock prices) or as absolute deviations (e.g. for interest rates). Let $D = \text{diag}\,(\sigma_1, ..., \sigma_k)$ be a diagonal matrix, carrying the observed standard deviations of the k risk factor changes, and let R be the corresponding correlation matrix:

$$R = \begin{pmatrix} 1 & \rho_{1,2} & \cdots & \cdots & \rho_{1k} \\ \rho_{2,1} & 1 & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \rho_{k-1,k} \\ \rho_{k,1} & \cdots & \cdots & \rho_{k,k-1} & 1 \end{pmatrix}, \qquad (29)$$

where $\rho_{i,j}$ equals Pearson's correlation coefficient between the change in risk factor i and j. Based on these elements, Σ can be derived as follows: $\Sigma = DRD$. The sensitivities of RBC are its partial derivatives with regard to the risk factors (see FOPI, 2006):

$$\delta_n = \frac{\partial RBC}{\partial f_n} \qquad \forall n \in \{1, ..., k\}. \qquad (30)$$

Being denominated in CHF per unit of measurement of the risk factor change, these sensitivities represent the increase or decrease in RBC associated with a one-unit change in the respective risk factor. Based on the assumption of a linear relationship between $\Delta \tilde{RF}$ and $\Delta R\tilde{B}C$, the latter can be derived using the vector of sensitivities $s = (\delta_1, ..., \delta_k)'$:

$$\Delta R\tilde{B}C \approx s' \Delta \tilde{RF}. \qquad (31)$$

As a consequence, mean and variance of the change in risk-bearing capital can be expressed as follows:

$$E(\Delta R\tilde{B}C) = E(s' \Delta \tilde{RF}) = s' E(\Delta \tilde{RF}) = 0, \qquad (32)$$

$$Var(\Delta R\tilde{B}C) = Var(s'\Delta \tilde{R}F) = s'\Sigma s = S'RS, \tag{33}$$

where

$$S = \begin{pmatrix} \delta_1 \\ \vdots \\ \delta_k \end{pmatrix} \circ \begin{pmatrix} \sigma_1 \\ \vdots \\ \sigma_k \end{pmatrix} = \begin{pmatrix} \delta_1\sigma_1 \\ \vdots \\ \delta_k\sigma_k \end{pmatrix}, \tag{34}$$

$\sigma = (\sigma_1, \cdots, \sigma_k)'$ is a column vector containing the volatilities of the risk factor changes, and $\circ$ represents the Hadamard product. By inserting Σ in (33), we find $Var(\Delta RBC) = s'DRDs$ as an equivalent formulation for the variance.

B   Further Input Data

(a) Downward Stress Scenario

$Corr_u^{Mkt}$	Equity	Interest	Property
Equity	1.00	0.50	0.75
Interest	0.50	1.00	0.50
Property	0.75	0.50	1.00

(b) Upward Stress Scenario

$Corr_u^{Mkt}$	Equity	Interest	Property
Equity	1.00	0.00	0.75
Interest	0.00	1.00	0.00
Property	0.75	0.00	1.00

Table 8:   Solvency II: Correlation Matrices for the Aggregation of SCR_{Mkt} (see EC, 2010)

Scenario S_j		S_0	S_1	S_2	S_3	S_4	S_5
Probability p_j		98.90%	0.10%	0.10%	0.10%	0.10%	0.10%
Description		Normal Year	Equity Drop	Property Crash	Black Monday	Nikkei Crash	ERM Crisis
		–	–	–	(1987)	(1990)	(1992)
U.S. Stocks	%	–	−60.00	0.00	−21.20	−13.80	−1.10
European Stocks	%	–	−60.00	0.00	−38.70	−25.60	−0.40
Swiss Stocks	%	–	−60.00	0.00	−23.20	−26.40	−5.80
USD Interest Rate	bp	–	0.00	0.00	−61.70	128.30	40.30
EUR Interest Rate	bp	–	0.00	0.00	−79.20	158.00	39.30
CHF Interest Rate	bp	–	0.00	0.00	−67.40	109.80	−67.50
Real Estate	%	–	0.00	−50.00	-3.10	−2.20	−1.10
Hedge Funds	%	–	−30.00	0.00	−5.00	−0.80	0.50
Private Equity	%	–	−70.00	0.00	−25.10	−28.70	−4.40

Table 9: Risk Factor Shocks Associated with the Scenarios for the SST Standard Approach

Scenario S_j		S_6	S_7	S_8	S_9	S_{10}	S_{11}
Probability p_j		0.10%	0.10%	0.10%	0.10%	0.10%	0.10%
Description		Mexican Crisis	LTCM/ Russia	Dot-Com Bubble	Global Deflation	Global Inflation	Financial crisis
		(1994)	(1998)	(2001)	–	–	(2008)
U.S. Stocks	%	−7.30	−13.90	−32.20	0.00	0.00	−48.40
European Stocks	%	−11.00	−22.50	−42.10	0.00	0.00	−50.30
Swiss Stocks	%	−18.50	−28.40	−35.70	0.00	0.00	−38.80
USD Interest Rate	bp	270.90	−98.00	−123.10	−132.60	136.50	−188.70
EUR Interest Rate	bp	132.30	−57.90	−83.10	−136.50	136.50	−154.30
CHF Interest Rate	bp	151.00	−36.80	−66.30	−127.00	136.50	−109.60
Real Estate	%	−21.50	-3.90	−7.80	0.00	0.00	−10.80
Hedge Funds	%	−3.60	−11.30	−1.90	0.00	0.00	−28.00
Private Equity	%	−11.50	−18.60	−34.10	0.00	0.00	−64.30

Table 9: Risk Factor Shocks Associated with the Scenarios for the SST Standard Approach – continued

SST Standard	MSCI USA	MSCI EU	MSCI CH	USD Rate	EUR Rate	CHF Rate	RBREI	HFRX	FINMA	LPX50	PEPI	CAPEI
MSCI USA	1.00	0.90	0.80	0.40	0.47	0.36	0.29	0.59	1.00	0.82	0.81	0.80
MSCI EU	0.90	1.00	0.88	0.47	0.49	0.40	0.26	0.54	1.00	0.82	0.78	0.77
MSCI CH	0.80	0.88	1.00	0.48	0.47	0.39	0.13	0.46	1.00	0.79	0.76	0.75
USD Rate	0.40	0.47	0.48	1.00	0.82	0.70	0.10	0.27	1.00	0.42	0.52	0.55
EUR Rate	0.47	0.49	0.47	0.82	1.00	0.83	0.09	0.35	1.00	0.40	0.55	0.57
CHF Rate	0.36	0.40	0.39	0.70	0.83	1.00	−0.03	0.25	1.00	0.32	0.53	0.52
RBREI	0.29	0.26	0.13	0.10	0.09	−0.03	1.00	0.34	1.00	0.29	0.31	0.33
HFRX	0.59	0.54	0.46	0.27	0.35	0.25	0.34	1.00	1.00	0.60	0.63	0.65
FINMA	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00			
LPX50	0.82	0.82	0.79	0.42	0.40	0.32	0.29	0.60		1.00		
PEPI	0.81	0.78	0.76	0.52	0.55	0.53	0.31	0.63			1.00	
CAPEI	0.80	0.77	0.75	0.55	0.57	0.52	0.33	0.65				1.00

Internal Model	MSCI USA	MSCI EU	MSCI CH	S&P USTI	S&P EUGI	SIX SBI	RBREI	HFRX	SMMI	LPX50	PEPI	CAPEI
MSCI USA	1.00	0.90	0.80	−0.34	−0.30	−0.32	0.29	0.59	−0.27	0.82	0.81	0.80
MSCI EU	0.90	1.00	0.88	−0.42	−0.31	−0.35	0.26	0.54	−0.26	0.82	0.78	0.77
MSCI CH	0.80	0.88	1.00	−0.43	−0.30	−0.34	0.13	0.46	−0.28	0.79	0.76	0.75
S&P USTI	−0.34	−0.42	−0.43	1.00	0.73	0.66	−0.08	−0.21	0.13	−0.38	−0.50	−0.52
S&P EUGI	−0.30	−0.31	−0.30	0.73	1.00	0.82	−0.04	−0.20	0.02	−0.26	−0.46	−0.48
SIX SBI	−0.32	−0.35	−0.34	0.66	0.82	1.00	0.04	−0.19	−0.02	−0.27	−0.47	−0.44
RBREI	0.29	0.26	0.13	−0.08	−0.04	0.04	1.00	0.34	−0.16	0.29	0.31	0.33
HFRX	0.59	0.54	0.46	−0.21	−0.20	−0.19	0.34	1.00	−0.19	0.60	0.63	0.65
SMMI	−0.27	−0.26	−0.28	0.13	0.02	−0.02	−0.16	−0.19	1.00	−0.32	−0.43	−0.42
LPX50	0.82	0.82	0.79	−0.38	−0.26	−0.27	0.29	0.60	−0.32	1.00		
PEPI	0.81	0.78	0.76	−0.50	−0.46	−0.47	0.31	0.63	−0.43		1.00	
CAPEI	0.80	0.77	0.75	−0.52	−0.48	−0.44	0.33	0.65	−0.42			1.00

Table 10: Correlation Matrices for the SST Standard Approach and the Internal Model

Indices: MSCI country indices for the U.S. (MSCI USA), Europe (MSCI EU), and Switzerland (MSCI CH); S&P U.S. Treasury Index (S&P USTI); S&P Eurozone Government Bond Index (S&P EUGI); Swiss Government Bond Index (SIX SBI); Rued Blass Real Estate Index (RBREI); HFRX Global Hedge Fund Index (HFRX); Swiss three-month Money Market Index (SMMI); LPX50 Listed Private Equity Index; Thomson Reuters Private Equity Performance Index (PEPI); Cambridge Associates U.S. Private Equity Index (CAPEI).

C   Further Results for the Internal Model
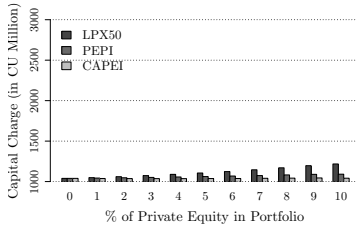
(a) Internal Model (VaR$_{0.5\%}$), no Scenarios (Period 2001–2005)
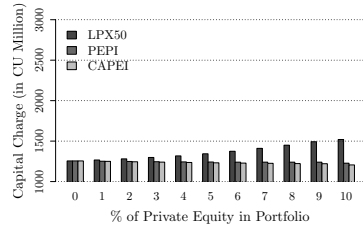
(b) Internal Model (VaR$_{0.5\%}$), no Scenarios (Period 2006–2010)
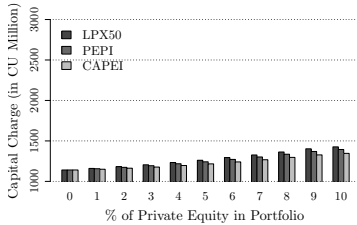
(c) Internal Model (TVaR$_{1\%}$), Scenarios (Period 2001–2005)
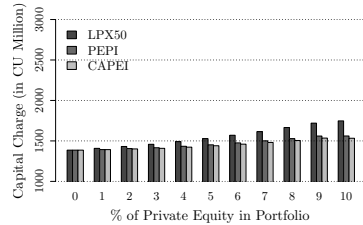
(d) Internal Model (TVaR$_{1\%}$), Scenarios (Period 2006–2010)

Figure 4: Internal Model: Capital Charges for Alternative Calibration Periods (2001–2005, 2006–2010)
This figure shows the life insurer's total capital charges for market risk under the internal model for the alternative calibration periods 2001–2005 (excluding the financial crisis) and 2006–2010 (mainly comprising the financial crisis). 0% reflects the portfolio without any private equity investments and 10% represents the legal investment limit according to the Swiss Federal Office of Private Insurance (FOPI, 2008). Each subfigure contains the results for a specific model: (a) internal model with VaR$_{0.5\%}$ for the period of 2001 to 2005; (b) internal model with VaR$_{0.5\%}$ for the period of 2006-2010; (c) internal model with TVaR$_{1\%}$ and SST scenarios for the period of 2001 to 2005; (d) internal model with TVaR$_{1\%}$ and SST scenarios for the period of 2006-2010. Private equity indices: LPX50 Listed Private Equity Index; Thomson Reuters Private Equity Performance Index (PEPI); Cambridge Associates U.S. Private Equity Index (CAPEI).

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Part II

Solvency Assessment for Insurance Groups in the United States and Europe – a Comparison of Regulatory Frameworks

Abstract

As a reaction to the increasing trend of insurers forming and participating in financial conglomerates and insurance groups, insurance supervisory authorities are currently developing group-wide capital standards. The International Association of Insurance Supervisors (IAIS) recently published an issues paper that discusses the challenges to group supervision and defines criteria for a thorough group solvency framework. Based on these criteria, this article provides an overview and comparison of three important group solvency models – the U.S. solo plus approach of the National Association of Insurance Commissioners, Switzerland's group structure model and the Solvency II proposal on group solvency assessment.

The analysis reveals various deficits within the group capital standard of the United States implying the need for future regulatory work. By contrast, the performance of the European frameworks with regard to the IAIS criteria is good. In particular, the Swiss framework can be seen as a prime example of an innovative and solid group solvency model.[31]

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1   Introduction

Today, most internationally operating insurance companies are organized in financial conglomerates or insurance groups. As a result, over the past decade, many countries have set up additional regulatory frameworks that are applied on the group level. These group-wide capital standards, however, do not replace the solvency assessment of individual legal entities within the group. They are rather meant to supplement the solo supervision, which remains a key tool to provide for policyholder protection (see also IAIS, 2009b).

The interactions of the legal entities within an insurance group may have a substantial impact on the group's solvency as well as the risks to the financial sector as a whole. In order to set incentives for a solid enterprise risk management and a group-wide capital management that complements risk management at the solo level, establishing appropriate group-wide capital standards is of vital importance.

The International Association of Insurance Supervisors (IAIS) has therefore set out principles on group-wide supervision as an internationally applicable guidance for the establishment of consistent and effective group-wide capital standards (see IAIS, 2008b). Based on these principles, the IAIS's issues paper explores different issues and challenges associated with group supervision and provides an analysis of possible approaches (see IAIS, 2009b). This has encouraged us to extend the contribution of the IAIS by conducting a comparison of three current group solvency approaches, based on the different challenges associated with a risk-sensitive group-wide solvency assessment.

Within   the   recent   literature   on   insurance   regulation, Eling and Holzmüller (2008), Cummins and Phillips (2009), as well as Holzmüller (2009) carry out comparisons of different solo capital standards. Eling and Holzmüller (2008) provide an overview and comparison of the solo-level risk-based capital charges of the United States, New Zealand, as well as the European Union and Switzerland, whereas Cummins and Phillips (2009) and Holzmüller (2009) base their analyses on the implications of the U.S. risk-based capital approach, Solvency II, and

the Swiss Solvency Test (SST). However, their work does not focus on the consideration of group solvency issues.

In fact, current literature on insurance group solvency assessment is rather scarce. Within the context of the Swiss Solvency Test, Filipović and Kupper (2007), Keller (2007), Luder (2007) and Filipović and Kupper (2008) present the Swiss group structure model and examine optimal capital and risk transfer and its implications on group diversification. The paper by van Rossum (2005) examines the changes in the insurance industry, such as the emergence of financial groups, and the alignment of its regulatory frameworks to those of the banking industry. Furthermore, Darlap and Mayr (2006) consider important challenges to group supervisors under the Solvency II proposal. The authors argue that there are several risks specific to financial conglomerates and insurance groups that are not covered by modern portfolio theory, such as concentration risks and financial contagion, and recommend the introduction of copula-based solvency models (see Darlap and Mayr, 2006).

Our paper presents an outline and comparison of three current group capital approaches: the group capital approach of the National Association of Insurance Commissioners (NAIC), the group structure model of the Swiss Solvency Test, and the Solvency II proposal on group capital assessment. The U.S. approach and the proposal of the European Union were selected because of their international importance, whereas the group structure model of the SST was included because it is currently regarded as one of the most innovative group capital standards. The comparison is based on five different issues and challenges that are provided by the IAIS's issues paper (see IAIS, 2009b) and that are usually associated within the discussion of a risk-sensitive group-wide solvency assessment of insurance companies.

The paper is structured as follows: Section 2 provides the overview of the three group solvency models. The comparison, the main part of the paper, is conducted in Section 3. Section 4 concludes and evaluates the three group solvency approaches with regard to the results of the previous section.

2 Assessing Group Solvency: an Overview

Typically there are two different approaches according to which a group solvency assessment can take place (see IAIS, 2009b): a legal entity perspective and a consolidated viewpoint of the insurance group. A legal entity approach regards the group as an accumulation of separate legal entities that are interdependent from each other. Here, the capital requirements of each group member are aggregated, taking intra-group transfers into account. By contrast, a consolidated group model regards the insurance group as one single entity and calculates the group capital requirement on the basis of consolidated accounts (see IAIS, 2009b). Ideally, a solid group model should incorporate aspects of both types of approach.

This section presents a description of the three capital standards under consideration, beginning with a general discussion of the framework. Afterwards, an overview of the group solvency model of the respective capital standard is provided.

2.1 NAIC Approach to Group-Wide Capital Standards

The NAIC risk-based capital system was introduced in 1994 and constituted by that time one of the first capital standards to incorporate an insurance company's risk exposition to assess capital requirements. It determines solvency through a two-component approach (for the following paragraph see NAIC, 2009c):

The first component is a factor-based formula specific for each insurance type (life, health, and property/casualty insurance) that calculates the required "risk-based capital" (RBC), a required minimum capital level. The RBC is compared to the "total adjusted capital" (TAC), an insurer's available amount of capital (including surplus). The capital charges depend upon different risk-factor charges, which are multiplied by several financial statement magnitudes of the insurer. Subsequently, a covariance calculation leads to the final adjusted RBC. The second component is a law that identifies five levels of regulatory intervention. This rules-based component defines the level of supervisory action based

on the quotient of the total adjusted capital over the risk-based capital
($\frac{TAC}{RBC}$) (for the following paragraph see NAIC, 2009c):

The first level represents a ratio of $\frac{TAC}{RBC} \geq 200\%$ implying no reg-
ulatory intervention. A solvency quotient between 150% and 200%,
the "company action level", results in the regulatory requirement of
an additional report that comprises a financial plan of how to address
the undercapitalization of the company. The "regulatory action level"
involves a solvency ratio between 100% and 150%. Apart from the
required additional report, this level triggers the intervention of the
insurer's assigned state commissioner. The "authorized control level"
($70\% \leq \frac{TAC}{RBC} < 100\%$) involves the adoption of control over the com-
pany by the regulator. Finally, a solvency ratio of less than 70% triggers
"mandatory control" by the regulator.

The NAIC's approach to group supervision is currently regulated
through the Insurance Holding Company System Regulatory Act (Model
#440) and the Insurance Holding Company System Model Regulation
with Reporting Forms and Instructions (Model #450) (see NAIC, 2010a;
NAIC, 2010b). During the "Solvency Modernization Initiative" (SMI),
which started in June 2008 (see, e.g., NAIC, 2009a), they were modified
and adopted in December 2010. They require disclosure of relevant in-
formation on the change in control of an insurance company, mergers
and acquisitions, material intra-group transactions, as well as informa-
tion on the interrelations between affiliated insurance companies (see,
e.g., NAIC, 2009a). The models apply to "insurance holding company
systems", which are defined as two or more affiliated organizations or
legal persons of which at least one has to be an insurance company (see
NAIC, 2010b).

As pointed out by the NAIC, the current U.S. regulatory system for
insurance groups can be described as a "solo plus" regime that utilizes an
aggregation method for the group adjustments (NAIC, 2010d). That is,
the solvency assessment is based on the single legal entity but is adjusted
for intra-group transactions (see NAIC, 2011d). This means, i.a., that
the regulatory control levels of intervention are left within the single en-
tity solvency assessment (see, e.g., NAIC, 2010d). In addition, insurance

groups are obliged to submit an annual report on the ultimate insurance holding company (see IAIS, 2009b), and regulators are required to consider group capital risks during their annual review process (see NAIC, 2010d).

In response to the financial crisis of 2007, one of the NAIC's declared goal is to enhance U.S. group supervision, i.a., by means of the SMI. The modifications from 2010 to the Insurance Holding Company System Regulatory Act were an important step toward achieving this goal. The most important modifications to Model #440 include (in the following see NAIC, 2011c):

- - The requirement to disclose information on possible operations of the insurer that could potentially give rise to enterprise risk, that is operations or events which might adversely affect the financial condition, liquidity, or reputation of one or more insurers of an insurance holding company system.

- - The expansion of regulators' access to financial information on affiliated companies.

- - The establishment of and participation in supervisory colleges.

Also, the NAIC plans to release Holding Company and Supervisory Best Practices and a study of the financial reporting requirements for insurance holding companies (see NAIC, 2011c).

2.2 Group Structure Model of the Swiss Solvency Test

The Swiss Solvency Test was initiated by the Federal Office of Private Insurance of Switzerland in 2003 and came into effect in 2008. It is a risk-based solvency standard that incorporates both quantitative and qualitative solvency requirements. Concerning the latter, the SST requests an annual report on the overall risk situation of the insurance company. With regard to the quantitative capital charges, the SST is based on an economic capital concept. Here, an insurance company's available economic capital (also called "risk-bearing capital" under the

SST), which constitutes a financial cushion to buffer variations in assets and liabilities throughout the business year, is defined as the company's comprehensive assets minus the discounted best estimate of its liabilities (see, e.g., FOPI, 2006). The SST is based on a market-consistent valuation calculating a lower capital bound, called "minimum solvency", and an upper bound, called "target capital" (see FOPI, 2004). While the former is a statutory magnitude, the latter is calculated consistent with the market and is defined as the tail value at risk of the change in available economic capital plus the capital cost over a one-year time horizon (see, e.g., FOPI, 2006).

An insurance company has to calculate its "SST quotient", the ratio of risk-bearing capital over the target capital (see, e.g., FINMA, 2008b). The three thresholds of supervisory intervention of FINMA are determined according to the value of this ratio (for the following paragraph see FINMA, 2008b).

Threshold 1 is reached with an SST quotient of 100%. Thresholds 2 and 3 are drawn at solvency quotients of 80% and 33%, respectively. An insurer with a SST ratio above threshold 1 is regarded as sufficiently solvent and is not subject to regulatory intervention. A ratio between thresholds 1 and 2, however, triggers an intensified observation of the respective insurance company by FINMA. An insurance company with an SST quotient between 80% and 33% has to submit a restructuring plan within the next two months to FINMA. Furthermore, the authorities can prohibit any risky new business and require an additional liquidity plan. An insurance company falling below threshold 3 is subject to immediate intervention by FINMA, and the insurer is forced to take immediate actions to increase the risk-bearing capital and to decrease the target capital. If FINMA finds the actions to ensure policyholder protection insufficient, it can revoke the insurer's license.

The SST group structure model is a supplement to the individual Swiss Solvency Test for financial conglomerates and in particular for insurance groups. The model is intended to complement the solo SST by applying the same methodology to calculate target capital and available economic capital from a group-level perspective. It is a legal entity

approach in the sense that capital requirements are calculated for each legal entity of the group separately, taking into account group effects such as ownership structure and capital and risk transfer instruments (CRTIs) (see IAIS, 2009b).[32] Consequently, the methodology of the group structure model does not lead to one single SST quotient denoting the solvency of the whole insurance group but calculates separate capital charges for each legal entity of the group. However, an additional solvency assessment on a consolidated basis can be required by the supervisory authority or may be granted upon application of the insurance group (see, e.g., FINMA, 2008b).

The group level SST is based on several general principles and assumptions, which can be summarized as follows (in the following see Filipović and Kupper, 2007 and Keller, 2006):

- An insurance group is considered to be a collection of different legal entities that are connected through a set of legally binding CRTIs and organized as a parent-subsidiary group structure.

- Limited fungibility[33] of capital and limited transferability[34] of assets and risks is assumed, meaning group effects are recognized only by taking into account the web of legally binding capital and risk transfer instruments. In times of financial distress, available economic capital is not transferred between the legal entities unless legally binding CRTIs are in place.

- The available economic capital of subsidiaries is defined as the entities' economic values less a market value margin, the latter being calculated via a cost of capital approach.

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[32]Capital and risk transfer instruments are, for example, dividends, intra-group retrocession, loans, participations, guarantees, and reinsurance agreements (see, e.g., Filipović and Kupper, 2007).

[33]According to Keller (2006), "fungibility" is hereafter defined as the ability to quickly generate cash by converting transferable assets.

[34]The term "transferability" in this context refers to the actual capability of transferring assets and risks from one entity of the group to another, even and especially when the group has to face financial distress in one or more entities (see IAIS, 2009b).

- When determining the SST quotient of the parent company, the economic values of its subsidiaries are taken into account as assets of the parent company.

In addition to the individual capital requirements and the inclusion of all legally binding CRTIs into the calculation of the group solvency, the Swiss framework requires a scenario analysis on the group level. Here, the effects of several possible stress scenarios on all legal entities of the insurance group as well as the expected economic loss to the group as a whole have to be quantified.

The requirements of the group structure model are satisfied when the individual SST ratios of all group members lie above 100% (see FINMA, 2008b).

2.3 Solvency II Proposal on Group Solvency Assessment

Solvency II is the European Commission's showcase to harmonize European insurance regulation across EU countries. From 2013 onward, it will replace the Solvency I framework. The risk-based Solvency II system is based on three main thematic areas, the "pillars" (see EC, 2011). Pillar I determines quantitative capital requirements, which contains, similar to the SST, two key magnitudes that have to be calculated: the "solvency capital requirement" (SCR) and the "minimum capital requirement" (MCR) (see, e.g., EC, 2010). The SCR corresponds to the target capital of the SST and is calibrated on the basis of a value at risk concept with a confidence level of 99.5% (in contrast to the tail value at risk concept with a 99% confidence level under the SST). The second magnitude, the MCR, constitutes a minimum capital level below which the amount of financial resources is not supposed to fall (see EC, 2009), and is comparable to the minimum solvency of the SST approach. It is calculated via a linear formula that is based on the SCR. Additionally, the regulators have defined a fixed minimum absolute floor that is set to € 2.2 million for nonlife insurers and € 3.2 million for life insurers as well as reinsurers (see, e.g., EC, 2010). The focus of Pillar II is on qualitative requirements regarding the risk management policy of insurers, whereas

Pillar III sets out disclosure and transparency rules (see EC, 2009).

The Solvency II proposal on group-wide solvency assessment improves and modernizes the Insurance Group Directive from 1998 (EC, 1998). It assigns the same set of principles and goals that apply to an individual insurer to the insurance group as a whole (see, e.g., CEIOPS, 2009a). To each insurance group a group supervisor is assigned who organizes supervision (see EC, 2011). In order to ensure group solvency, Solvency II requires to determine the group SCR as well as the amount of eligible own funds on the group level (see, e.g., CEIOPS, 2009a). The group SCR is calculated on the same $\text{VaR}_{99.5\%}$ concept as the SCRs for the individual legal entities and equals the amount of economic capital needed to ensure the solvency of the entire group.

The Solvency II proposal for calculating group-wide capital charges tries to combine the two approaches to group solvency assessment mentioned above: the understanding of the insurance group as being a collection of separate legal entities and the integrated view of the group as one consolidated entity (see, e.g., IAIS, 2009b). However, due to the standard formula to assess group-wide capital requirements, and especially in comparison to the group SST and the NAIC approach, it can clearly be categorized under the models with a consolidated focus.

The standard approach to compute group solvency is the "Accounting Consolidation-Based Method" (see, e.g., EC, 2010). It calculates the group SCR on the basis of consolidated balance sheets and can be described as the consolidated solvency capital requirement SCR* of those insurance companies for which the consideration of diversification effects is approved plus the sum of the solo SCRs of the residual group members for which diversification is not approved (see CEIOPS, 2009a). In order to calculate the SCR*, insurance companies may apply the standard formula for solo entities to the consolidated data, as if the group were an integrated entity (see EC, 2010). The insurance group's solvency margin is then defined as the difference between its eligible own funds and the group SCR.

When applying the accounting consolidation-based method, a group capital floor has to be calculated. It is given by the sum of the solo MCRs (determined according to Article 129(1) and Article 129(3) of the Solvency II Framework Directive), of the participating entities, as well as the proportional share of the solo MCRs of the related entities (see EC, 2010).

If the group supervisor comes to the conclusion that the application of the standard method described above is not appropriate for a specific group, Article 220 of the Solvency II Directive 2009/138/EC states that an alternative method should be applied, the "Deduction and Aggregation Method" (see EC, 2009). Under this approach, group solvency is given by the difference between the sum of the aggregated eligible own funds of all group members and the aggregated solo SCRs (see CEIOPS, 2009a).

Apart from the two methods described above, it is also possible for an insurance group to apply for permission to calculate group solvency on the basis of an internal model (see EC, 2009).

3   Comparison

This section sets out a comparison of the three group solvency frameworks displayed above. We aim to contrast the three models with respect to several group solvency issues identified by the Issues Paper on Group-Wide Solvency Assessment and Supervision of the IAIS (in the following see IAIS, 2009b):

1. **Assessment of risk dependencies:** a group solvency approach should be able to appropriately model dependencies between different risk categories.

2. **Fungibility of capital and recognition of diversification effects:** the restriction in the transferability of assets and the fungibility of capital has to be modeled, and diversification effects should be adequately recognized.

3. **Prevention of multiple capital gearing:** a group solvency model needs to prevent any intra-group generation of capital so

that an insurance group's capital resources can be correctly compared with the group capital requirements.

4. **Avoidance of regulatory arbitrage and implementation of supervisory colleges:** in order to harmonize regulatory frameworks, close cooperation between the supervisory authorities of different financial sectors as well as different jurisdictions is needed.

5. **Scope of group supervision and treatment of nonregulated entities:** in order to be able to assess all relevant risks an insurance group is exposed to, a group solvency approach needs to provide adequate mechanisms to deal with nonregulated entities of a group.

3.1   Assessment of Risk Dependencies

The issue of how diversification effects and the pooling of risks within insurance groups should be recognized has gained additional relevance after the subprime crisis of 2007 to 2009. As risk dependencies typically increase in times of financial distress, the modeling of the tail characteristics of a risk category's distribution function becomes particularly important in such situations. As Embrechts, McNeil, and Straumann (2002) point out, the assumptions of multivariate normally distributed returns and linearly correlated risks are especially problematic in the insurance sector, due to the claims data which often exhibits skewness and fat tails.

However, former solvency models often relied on linear correlation measures and were not able to capture heavy tails. Therefore, the IAIS requires in its issues paper the standardized methods of current capital standards to ensure adequate quantification of the underlying risks an insurance group is exposed to and to pay particular attention regarding the modeling of the distribution functions' tails (see IAIS, 2009b).

The NAIC's RBC system uses a standard formula to calculate solo capital charges that aggregates risks on the basis of a covariance adjustment to account for diversification. More precisely, the formula squares the sum of risks that are believed to be not independent of each other,

adds up the squares, and takes the square root of the sum of the squares; the remaining risks that are not believed to be correlated are added to this sum (see, e.g., NAIC, 2009c).

Although most recent changes to the RBC system include scenario analyses for market and interest rate risks within the life insurance formula, the approach still relies on a static formula rather than a full stochastic model (see also Cummins and Phillips, 2009). It assumes linear correlation between risk categories and is therefore not able to take nonlinear tail-dependencies into account. Consequently, the status quo of U.S. capital standards fails to fulfill the first criterion of our comparison.

In order to compute the solo target capital of an insurance company, the SST standard model for market risk uses the change in 79 preset risk factors to measure the change in risk-bearing capital (see, e.g., FINMA, 2010b). The random vector of the changes in risk factors is assumed to be multivariate, normally distributed and linearly correlated. However, the model accounts for the fact that risk factor changes might often exhibit skewness and excess kurtosis by requiring an additional scenario analysis. These scenarios constitute stresses to several risk factors and can be translated into changes in the RBC (see FOPI, 2006). Consequently, a distribution function for each scenario can be calculated, and the scenarios as well as the standard case of a normal year are summed up to an aggregate cumulative distribution function which is no longer normally distributed but exhibits fat tails.

With respect to the model calibration, the risk factors and their pairwise correlation of the standard model for market risk have to be estimated by the insurers according to the latest 10-year data on pre-specified, well-known indices. This ensures a flexible parameter calibration of a model that is grounded on empirical actualities.

Furthermore, the Swiss regulator encourages insurers to develop their own, more sophisticated internal model that might be better able to capture the insurers' specific financial data, for example, by taking other distributional assumptions into account or by defining additional scenarios (see, e.g., FOPI, 2006).

To sum up, the SST's standard model ensures an appropriate assessment of risk dependencies and an adequate recognition of tail dependence by incorporating scenario analysis into an empirically well-grounded and flexible solvency model.

According to the quantitative impact studies 5 technical specifications, the empirically calibrated stress factors that are used to calculate the overall SCR of the standard formula of Solvency II guarantee a solvency level of a 99.5% value at risk (see EC, 2010). Thereby, the European Commission aims to generate capital charges that are stable with regard to different risk dependencies under stressed financial conditions (see EC, 2010).

Although linear correlation techniques are used to aggregate different risks and risk modules, the Committee of European Insurance and Occupational Pension Supervisors points out that the calibration of stress factors is carried out on the basis of extreme value analysis, where necessary, and is therefore able to account for fat tails (see CEIOPS, 2010b).

In contrast to the Swiss Solvency Test, however, the static standard formula of Solvency II does not offer a framework that is able to reflect new market information by readjusting its parameter settings according to the latest available market data. This is a clear disadvantage as new political situations and changing economic structures can affect the dependencies between different risk categories of an insurer in significant ways.

3.2   Fungibility of Capital and Recognition of Diversification Effects

With regard to the appropriate recognition of diversification effects within an insurance group, the extent to which assets and risks are fungible between different group members becomes an important issue. As explained by the IAIS (see IAIS, 2009b), there may be conflicts of interest as well as various legal constraints restricting the transferability of assets and risks and the fungibility of capital.

A pure, consolidated group-wide capital approach implicitly assumes full fungibility of capital and risks leading to a maximum diversification effect on the group level that significantly reduces group capital requirements (see IAIS, 2009b). In practice, however, the transferability of assets and risks is usually restricted, especially when one or more group members experience financial distress (see Keller, 2007). The IAIS therefore points out that, under a consolidated group solvency approach, it is important to consider the impediments to free intra-group capital flows and the transfers of assets and risks by means of stress tests (see IAIS, 2009b).

By contrast, an approach to group supervision with a legal entity focus is generally able to take the actual constraints to transferability of assets and risks and fungibility of capital into account and is therefore likely to reflect the interactions within an insurance group in all of its financial states.

The NAIC solo plus framework (displayed in Section 2.1) fits into the legal entity group solvency approaches that take limited fungibility and transferability into account. As already mentioned, it constitutes a solo approach that focuses on the legal entity, but requires group-level information, as well. Regarding intra-group transactions, the Insurance Holding Company System Regulatory Act forces the insurer to provide the supervisor with information on CRTIs such as intra-group loans, guarantees, reinsurance agreements, management agreements, as well as exchanges of assets (see NAIC, 2010b). Furthermore, the Group Solvency Issues (EX) Working Group, established under the Solvency Modernization Initiative, recently issued a draft on group capital assessment that proposes a risk assessment tool, the "own risk and solvency assessment" (ORSA). The ORSA requires, i.a., all U.S.-based legal entities of an insurance holding company system to conduct an annual qualitative and quantitative analysis of their solvency situation on the group level (see NAIC, 2011a). Within the quantitative group-wide solvency assessment, the draft suggests eliminating intra-group transactions, either by applying a consolidated method in which all CRTIs are canceled out or by adjusting for intra-group holdings when summing up capital

resources and requirements under an aggregation method (see NAIC, 2011a; NAIC, 2011b). However, it does not require the consideration of CRTIs in the sense of imposing minimum capital requirements for the group that result, when violated, in regulatory interventions (see NAIC, 2011b). It needs to be critically noted that, in turn, diversification effects cannot be recognized within the standard RBC formula.

The SST group structure model can also be categorized under the legal entity approaches. Here, only the economically available capital of a subsidiary is considered fungible, that is its economic value less the cost of capital (see, e.g., Keller, 2006). Furthermore, the fungibility is only recognized when legally binding capital transfer contracts are in place. Similarly, the transferability of assets and risks must be ensured by a legal agreement between the group members in order to be taken into consideration for the group solvency test. In contrast to the U.S. solo plus approach, the SST group structure model does not only rely on the declaration of intra-group transfers, but also requires the consideration of these transfers within the quantitative capital requirements of the solvency test. The impact of a CRTI on the transferring and the benefiting company's risk situation has to be assessed, and the change in solvency capital charges, due to the transfer, has to be quantified (see, e.g., Keller, 2007). Therefore, Switzerland's group approach is able to appropriately assign and recognize diversification effects.

As for the Solvency II proposal on group solvency assessment, we already explained in Section 2.3 that it belongs to the consolidated group models. The standard accounting consolidation-based method initially incorporates the problem of implicitly assuming full fungibility of capital and transferability of assets and risks. However, as pointed out by CEIOPS, the current proposal plans to develop requirements under Pillars II and III, demanding scenario analyses on the impact of limited fungibility of capital due to certain stress events for all members of the insurance group as well as a strategic plan on how to deal with financial distress in one or several entities (see CEIOPS, 2009a). Additionally, the

group supervisor is expected to assess the insurance group's management of free capital under stress scenarios such as fungibility constraints.

Under the alternative deduction and aggregation method, diversification effects on the group level are not recognized. Here, intra-group transactions are not implicitly eliminated as within the standard approach. Therefore, capital and risk transfers have to be eliminated in a separate calculatory step.

The assessment of intra-group transactions is placed within the qualitative requirements for governance and risk management under Pillar II. The Solvency II Directive establishes that the (re)insurance company at the head of the group has to report on a regular basis all noteworthy inter-linkages between the group's legal entities (see EC, 2009). Having received the necessary information and after consulting the supervisory authority, the assigned group supervisor has to identify the type of CRTI and its impact on the financial situation of the insurance group.

In summary, we can see that criterion 2 is taken into account under Solvency II only within the qualitative requirements of Pillars II und III. Unless the finalized group approach, which is expected in 2012, considers intra-group transactions and limited fungibility additionally within Pillar I, calculating quantitative group capital requirements, the approach might overestimate the fungibility of capital and therefore the financial health of the insurance group, as its standard approach relies on a consolidated balance sheet.

3.3   Prevention of Multiple Capital Gearing

Group-wide capital standards need to prevent any intra-group generation of capital so that the financial health of the individual companies and the insurance group as a whole is not overestimated (see IAIS, 2009b). This internal capital creation, called "multiple capital gearing", takes place when the same regulatory capital is used to cushion risks in more than one legal entity of the group (see Joint Forum on Financial Conglomerates, 1999).

A group solvency model that is based on consolidated accounts calculates, by definition, consolidated capital resources and capital require-

ments from which intra-group transactions are already subducted. Thus, such an approach ensures that multiple capital gearing cannot occur on the group level. However, as a consolidation method cannot provide information about the distribution of capital between different legal entities of the group, the IAIS requires an additional analysis examining the amount of capital resources of each legal entity (see IAIS, 2009b).

In contrast to the implicit elimination of intra-group transactions under a consolidated approach, group solvency models with a legal entity focus consider the applied capital and risk transfer instruments within an insurance group when determining group capital charges. In order to prevent multiple gearing of capital, legal entity approaches need to take each relevant transaction and participation between group members into account and value each of them consistently with the market (see IAIS, 2009b).

As already mentioned in Section 3.2, the NAIC's solo plus approach so far does not require to consider intra-group transactions within the standard formulas to calculate minimum capital charges. However, ORSA will require a qualitative as well as quantitative group-wide solvency assessment. This implies that the NAIC's group approach will be able to account for multiple capital gearing by eliminating intra-group transactions, as specified in more detail in Section 3.2. Thus, the solo plus approach, once revised and extended, will be able to control capital gearing.

Under the SST group structure model, the mechanism to avoid multiple capital gearing is twofold: Firstly, an insurance group is modeled as a parent-subsidiary constellation in which the market value of the subsidiaries is an asset to the parent company and the risks of the subsidiaries are therefore taken into account within the capital requirement for the parent company (see also IAIS, 2009b). Secondly, by considering every capital and risk transfer between group members quantitatively. Therefore, capital resources and capital requirements are increased/decreased for each legal entity, appropriately. Regarding the qualitative requirements, an insurance group under Swiss regulation has

to semianually prepare an SST report on the group level (see FINMA, 2008b). Consequently, the SST group model fulfills the third criterion.

Articles 222 and 223 of the Solvency II Directive deal with the elimination of multiple capital gearing and intra-group capital creation (for the following paragraph see EC, 2009).

They require the exclusion of the asset values of participating or related companies that simultaneously constitute free capital qualifying for the solvency capital requirement of other legal entities of the group, whenever another calculation method than the consolidation-based method is applied. Furthermore, Article 223 establishes that the calculation of group capital charges is to ignore any eligible own funds for the SCR that are generated through "reciprocal financing"[35] between a participating company and another group member (see EC, 2009). Therefore, the Solvency II group framework is able to anticipate multiple gearing of capital.

3.4 Avoidance of Regulatory Arbitrage and Implementation of Supervisory Colleges

Regulatory arbitrage is the opportunity to exploit differences in regulation between jurisdictions, regulated sectors, or business divisions, to achieve capital or profit goals in the best possible way (see IAIS, 2009b).

The rationale behind the avoidance of regulatory arbitrage, from a supervisory perspective is, on the one hand, that it may entail risks because some countries require significantly lower levels of regulatory capital and the overall quality of supervision is considered insufficient from a European or North American point of view. On the other hand, the principle of regulatory consistency requires that a different regulatory treatment of the legal entities within an insurance group should be based on discrepancies in economic characteristics instead of differences in the legal structure (see IAIS, 2009b). In order to prevent regulatory

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[35]According to the Solvency II Directive, "reciprocal financing" is assumed at least when an insurance company, or a related entity, grants loans to or holds stakes in another entity that, directly or indirectly, holds eligible capital for the SCR of the first company (see EC, 2009).

arbitrage, a successful harmonization of solvency frameworks on an international level is crucial. It is important for the supervisors of the different legal entities within an insurance group to closely cooperate and share information that is relevant for the group's solvency. In this context, the IAIS suggests to designate a group-wide supervisor for each insurance group who is in charge of coordinating the cooperation and assessment of group-wide solvency as well as the group's risk management, risk reporting, and allocation of capital (see IAIS, 2008a). Furthermore, it is argued that an important tool to coordinate regulatory activities and cooperation is the establishment of a college of supervisors (see IAIS, 2009a). These supervisory colleges provide a forum of communication and knowledge transfer for the supervisors involved in the regulation of a particular insurance group and facilitate group supervision.

The first meeting of a supervisory college in the U.S. took place in 2008. It mainly dealt with agreements on information sharing and the assessment of common supervisory goals (see, e.g., NAIC, 2009b). Since that time, the NAIC has continued to develop the regulation tool of supervisory colleges (see NAIC, 2009b). The revised Insurance Holding Company System Regulatory Act of 2010 provides the chief insurance regulatory official with the power to participate in a supervisory college and to cooperate with foreign or other federal or state regulators in order to assess the financial, legal, and regulatory position of any domestic insurance company that is part of an international insurance holding company system (see NAIC, 2010b). However, it prohibits delegation of the supervisory power of the insurance commissioner over the legal entities and affiliates located within its jurisdiction to the supervisory college.

Switzerland's FINMA stays in close contact to foreign supervisory authorities such as the Committee of European Insurance and Occupational Pension Supervisors (CEIOPS), the Committee of European Banking Supervisors (CEBS), the European Commission (EC), as well as the U.K. and U.S. regulatory authorities. Furthermore, it is actively involved in international committees such as the International Associa-

tion of Insurance Supervisors (IAIS) and the Financial Stability Board (FSB). FINMA heads and takes part in a number of supervisory colleges and organizes crisis management for the Swiss banking and insurance industry (see FINMA, 2010a).

In its publication on the lessons learned from the subprime financial crisis, CEIOPS points out that in order to avoid regulatory arbitrage across sectors, to aim to set the stress factors of the different sub-modules of the market risk module such that a cross-sectional consistency with the banking industry is given (see CEIOPS, 2009c).

With respect to arbitrage opportunities across jurisdictions (for this paragraph refer to EC, 2009), Articles 248 to 259 of the Solvency II Directive introduces, similar to the other two group capital standards, the tools of group supervisors and supervisory colleges. It requires that the authorities, involved in the supervision of a particular insurance group, closely cooperate and share information, without bias toward the tasks they have to fulfill with respect to the solo supervision. In case of unsolvable disagreements within the supervisory college of a particular group, it states that any member of the supervisory college is allowed to approach CEIOPS for advice. Furthermore, Article 249 requires the supervisors of the different legal entities of an insurance group to immediately call for a meeting whenever the SCR or MCR of a group member is breached or when the group capital requirements cannot be met in full.

Apart from the cooperation between the insurance supervisors of individual entities of an insurance group, the Solvency II Directive also requests close collaboration between an insurance supervisor and any supervisory authority of a credit institution or an investment firm that is related to or has a common participating company as the insurer (see Article 252 of EC, 2009).

The issue of regulatory arbitrage and the harmonization of different regulatory frameworks is a difficult task. As discussed above, the regulatory authorities of the United States, Switzerland, and the European Union are currently taking steps to enhance international cooperation between insurance supervisors. Notwithstanding these efforts, in the

long run, globally binding minimum capital standards will be needed in order to contribute to the prevention of future global financial crises. Additionally, the regulatory frameworks need to stay flexible enough to concede effective implementation on a national level (see also FINMA, 2010a). To date, this common goal has not yet been achieved.

3.5   Scope of Group Supervision and Treatment of Nonregulated Entities

The rapid development of the financial industry over the past two decades has contributed to an increasing complexity in the structure of financial conglomerates and insurance groups. This has brought forth, i.a., the formation of insurance groups that are made up of a multitude of different legal entities, including "nonregulated entities". According to the definition of the IAIS (see IAIS, 2010), a nonregulated entity is a legal entity of an insurance group that is either a "nonoperating holding company" (NOHC) or an operating entity that is not subject to any form of direct supervisory activities ("nonregulated operating entity" (NROE)).

The existence of nonregulated entities additionally complicates the assessment of capital requirements for insurance groups. For a group solvency approach to ensure transparency and to appropriately measure the nature, scale, and interdependencies of risks faced by the insurance group, it is important to establish mechanisms to provide for an adequate handling of these entities (see IAIS, 2010).

The IAIS guidance paper on the treatment of nonregulated entities in group-wide supervision lists several risks that may be caused by the existence of NOHCs and NROEs (for the following paragraph see IAIS, 2010).

Some of those risks are related to the issue of corporate governance, such as a lack of transparency and inappropriate disclosure policies, as well as conflicts of interest between the different stakeholders of the group. Furthermore, regulatory arbitrage is an issue, as nonregulated entities can be used to avoid capital requirements and to engage in business activities that are not permitted for a regulated group member. Other related risks are financial contagion and reputational risks. NROEs might face

considerable amounts of risks without providing an appropriate capital buffer. These risks might be directly transferred to other entities of the group through CRTIs or might be carried over indirectly by adversely affecting the reputation of the whole insurance group.

In order to effectively deal with nonregulated entities, the IAIS therefore defines certain key characteristics a good group solvency approach needs to entail (in the following see IAIS, 2010):

(a) Supervisors should have a comprehensive understanding of the insurance group's organizational structure, including the activities of nonregulated entities and their influence on other regulated entities' risk exposure.

(b) In order to avoid regulatory arbitrage, enhance the harmonization of regulatory frameworks, and provide enough flexibility to react to new risks, supervisors that are engaged in the same insurance group should cooperate and exchange information across states, countries, and sectors.

(c) Disclosure and transparency rules as well as a possibility to implement risk mitigation measures should certify the timeliness, pertinence, and reliability of information.

(d) The assessment of group capital requirements should take risk exposures from NROEs into account.

There is no explicit mention of how to treat nonregulated entities within insurance groups in the Insurance Holding Company System Regulatory Act as of 2010. However, when interpreted correctly, some of its provisions implicitly exhibit the key characteristics required above. As mentioned before, the U.S. regulatory framework provides for intragroup transactions within Section 5 of the Regulatory Act (see NAIC, 2010b). Furthermore, the powers granted to the group supervisor, especially the permit to engage in supervisory colleges, provides the regulatory framework to react to supranational and group-wide risk exposures.

With regard to disclosure and transparency rules, the Regulatory Act requires to disclose any relevant information on changes in control of an

insurance company, as well as information on any material transactions and interrelations between an insurer and its affiliates, within a pre-specified time period (see NAIC, 2010b).

Although the task force for the SMI suggests considering potential risk sources and contagion effects stemming from nonregulated entities (see NAIC, 2009a), it does not plan to account for such effects within the quantitative capital requirements (see NAIC, 2011d). This holds also true for risks indirectly transferred from nonregulated legal entities that can potentially result in undersized capital requirements.

Therefore, key characteristic (d) is not quantitatively accounted for, under the NAIC group solvency approach.

The organizational structure and transactions of an insurance group that is subject to the Swiss solvency regulation are taken into account, qualitatively and quantitatively, through the granular group solvency model of the SST. The consideration of legally binding risk and transfer contracts between all group members includes interactions with nonregulated entities. Regarding key characteristic (b), the various efforts to enhance the cooperation with other international supervisors have already been referred to in Section 3.4.

Considering the disclosure of relevant and timely information on the solvency situation of a group, FINMA requires semiannual reports on the current group SST results as well as the data from the two previous semesters (see FINMA, 2008a). Apart from relevant information on risks concentrations and the risk management systems of the group members, the reports entail the group's target capital and risk-bearing capital, which are computed on the basis of the solvency margins of all group members, including fictitious solvency margins for nonregulated entities, preset by the Swiss Financial Market Supervisory Authority (see FINMA, 2008a). In addition, Swiss law sets specific criteria for placing NOHCs under supervision insofar as to require adherence to certain corporate governance standards and the existence of appropriate risk management tools (see IAIS, 2010).

The group structure model of the SST, therefore, is fully able to satisfy criterion 5.

According to the Solvency II Directive of November 2009, the supervisory authorities should take all intra-group transactions and relationships between regulated and nonregulated entities of a group into account (see EC, 2009). Similar to the U.S. and Swiss regulatory authorities, the European Commission aims to increase the harmonization of regulatory frameworks across countries and sectors (see Section 3.4).

Furthermore, key characteristic (c) of the IAIS guidance paper can be found in Articles 253 to 256 of the Solvency II Directive. They enforce, i.a., the exchange of relevant and verified information between supervisors and require the disclosure of an annual report on the solvency situation of the insurance group as a whole (see EC, 2009).

Finally, with regard to the group capital charges, nonregulated entities are taken into account by including notional SCRs into the calculation of the group's solvency capital requirement. The notional solvency requirement is the capital requirement an entity would need to fulfill when treated as a regulated entity under the particular sectoral rules (see EC, 2010). Hence, the group solvency approach of Solvency II possesses key characteristic (d) as well.

4   Conclusion

In most jurisdictions, supervision of insurance companies is still based on the solvency assessment of each legal entity. During the past decade, however, group-wide capital requirements have been developed to complement solo supervision so that the risks and chances of a group membership for an insurance company can be quantified. Furthermore, the expansion of financial groups across countries increasingly requires supervisors to internationally cooperate with each other and to converge regulatory frameworks in order to prevent future global financial crises.

This paper gives an overview and a comparison of the group-wide capital standards of the United States, Switzerland, and the European Union on the basis of a criteria catalog that is in line with the group

solvency issues specified by the IAIS's Issues Paper on Group-Wide Solvency Assessment and Supervision (see IAIS, 2009b). Table 11 summarizes the main findings of this comparison. A check mark indicates that the respective criterion is fulfilled, whereas a check mark in brackets indicates that the criterion is only partly fulfilled by the group approach. A cross signifies that the group model is not able to satisfy the criterion at hand.

The main results from our comparison can be summarized and interpreted as follows:

The U.S. RBC approach to group solvency is significantly inferior to the European group models of Switzerland and the European Union. Admittedly, it has been the last of the three approaches under consideration to be revised. Nevertheless, the "solo plus approach" of the United States will need further modernization within the coming years in order to keep up with the regulatory developments in Europe.

Switzerland's group structure model, by contrast, is able to achieve the highest score with regard to the five group criteria. It therefore seems slightly superior to Solvency II in terms of appropriately assessing risk dependencies and with regard to the recognition of group synergies and diversification effects. Nevertheless, the Solvency II proposal on group solvency assessment is a solid group model that incorporates the latest experiences with financial crises and the recent findings in risk management (e.g., the requirement of group-wide capital charges, the assignment of group supervisors to align the regulation of legal entities within an insurance group, as well as the allowance to develop internal group models).

Finally, with regard to the IAIS's goal to avoid regulatory arbitrage and to harmonize the national regulatory frameworks, U.S. and European supervisors are making efforts to cooperate more closely on an international basis. The goal of globally binding minimum capital standards as one possible answer to the increasing internationalization of insurance groups (e.g., as discussed by FINMA, 2010a), however, is still a distant prospect.

Criterion	United States of America	Switzerland	European Union
1. Assessment of risk dependencies	✗	✓	(✓)
2. Fungibility of capital and recognition of diversification effects	(✓)	✓	(✓)
3. Prevention of multiple capital gearing	✓	✓	✓
4. Avoidance of regulatory arbitrage and implementation of supervisory colleges	(✓)	(✓)	(✓)
5. Scope of group supervision and treatment of nonregulated entities	(✓)	✓	✓

Table 11: Summary of the Group Model Comparison

✗ not fulfilled   (✓) partly fulfilled   ✓ completely fulfilled

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Part III

Regulating Insurance Groups: a Comparison of Risk-Based Solvency Models

Abstract

Regulators are currently developing group-wide capital standards that are intended to enable the effective monitoring of insurance groups. Some jurisdictions are taking steps toward models with a focus on the groups' consolidated balance sheets, while other models focus on the interrelations of the groups' legal entities. This paper compares two general approaches to group-wide solvency in light of the regulatory challenges of regulatory inconsistency, risk dependencies and risk aggregation: a consolidated approach and a legal entity approach. In order to contribute to the current discussion on regulating insurance groups, we support our line of reasoning by using a generalized model of Gatzert and Schmeiser (2011). Our findings show that a sole consolidated viewpoint is likely to underestimate shortfall risks in times of financial crises, whereas a sole focus on the interrelated legal entities generally enables to display different group structures but cannot control regulatory arbitrage.[36]

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1   Introduction

The increasing importance of internationally operating financial groups
has given rise to a debate about capital adequacy and appropriate safety
levels within the financial industry. In the past, supervisors and regu-
lators focused primarily on the single legal entity and the protection of
its customers' claims. Consequently, capital requirements were typically
computed on a stand-alone basis (see, e.g., Mälkönen, 2004). However,
more recent risk-based capital standards also aim to consider group ef-
fects by implementing capital requirements at the corporate level. One
group-solvency approach, which treats the insurance group as a set of
interrelated legal entities, calculates capital charges on a legal entity
basis by accounting for capital and risk transfer instruments (CRTIs)
(see IAIS, 2009b). Another approach to group-wide solvency assessment
takes a consolidated point of view by considering the group as one inte-
grated entity and assuming that the legal entities can access each others
cash flows and freely transfer risks (see Keller, 2007 and IAIS, 2009b).
In practice, a variety of models to group-wide solvency assessment are
used, many of which can be regarded intermediate approaches because
they have the characteristics of both a legal entity focus and a con-
solidated viewpoint. Nevertheless, current examples of group solvency
models with a greater emphasis on the legal entity are the NAIC Le-
gal Entity Method of the USA and the Swiss Group Structure Model
(see IAIS, 2009b). Jurisdictions that are moving towards models with
a more consolidated focus, on the other hand, are, e.g., the European
Union, Canada, and Australia (see IAIS, 2009b).

   This paper contributes to the literature by comparing these two ap-
proaches to assessing group-wide solvency in order to determine which
of the two is more appropriate for regulating insurance groups, given
different assumptions and economic circumstances.

   To date, the literature on financial groups can be divided into two
categories: either it explores the issues and practical challenges regula-
tors face when establishing a risk-based capital standard of group-wide

solvency assessment, or it attempts to explore group structures and to quantify the risks and diversification effects within financial groups.

In the latter category, a number of studies examine whether financial groups trade at a discount compared to single line firms. While the majority of articles find evidence of a conglomerate discount in financial groups (see, e.g. Ammann and Verhofen, 2006; Laeven and Levine, 2005; Schmid and Walter, 2009), there is also mixed evidence (see, e.g., van Lelyveld and Knot, 2009) for a sample of European bank-insurance conglomerates. Here, the diversification discount is found to be varying considerably for different conglomerate structures. Furthermore, Gatzert and Schmeiser (2011) simultaneously assess the diversification benefit and conglomerate discount of a two-entity financial conglomerate, given fair pricing for the stakes of equityholders and policyholders. They find that diversification benefits within financial conglomerates are much less considerable when stakeholders obtain risk-adjusted returns.

Freixas, Loranth, and Morrison (2007) compare the risk-taking appetite of single firms and financial conglomerates and find that, in comparison to stand-alone financial institutions, the diversification in conglomerates can increase risk-taking incentives. Analyzing moral hazard within financial groups, Kahn and Winton (2004) propose a model framework to explain the "bipartite" subsidiary structure often found within banking conglomerates. With regard to the group-level Swiss Solvency Test, Keller (2007) and Luder (2007) model risks and diversification effects and calculate capital charges when capital and risk transfers between the legal entities of the insurance group take place. Within the same context, Filipović and Kupper (2007) and Filipović and Kupper (2008) derive optimal capital and risk transfer instruments in order to explore group diversification under convex risk measures.

Another segment of the literature deals with the group effects of financial conglomerates and their impact on systemic risk. In light of the subprime financial crisis, Harrington (2009) discusses, from a theoretical perspective, the question of whether insurance generally exhibits systemic risk. Other studies take an empirical approach: As an indicator of the systemic risk potential in the United States and Europe, De Nicolo and Kwast (2002) and Schüler (2002) examine the interdependen-

cies among banks proxied by the correlations of the banks' stock returns. Both empirical studies find evidence that consolidation contributes to the interdependencies between firms and, thus, to an increase in systemic risk. Allen and Jagtiani (2000) create "synthetic universal banks" in order to analyze the effect of investments and insurance activities on the banks' total risks and conclude that conglomeration leads to an increase in both systematic market risk and systemic risk.

Most of the literature that deals with the issues and practical challenges of establishing group-wide solvency standards, takes a nonquantitative perspective. Diereck (2004) discusses the legal structures of financial conglomerates and the conglomerates' relevant risks and benefits from a supervisory perspective. Mälkönen (2004), Morrison (2003), and Schilder and van Lelyveld (2003), derive the possible causes of the establishment of financial groups, set out justifications for their regulation, and address the issues and challenges with which supervisors of financial conglomerates are confronted. In addition, Mälkönen (2004) examines limitations to solvency regulation by comparing a silo approach with a consolidated view. Along the lines of these studies, the issues paper on group-wide solvency assessment and supervision prepared by the International Association of Insurance Supervisors discusses the regulatory issues of the solvency assessment for insurance groups and identifies four main challenges to group supervision (in the following see IAIS, 2009b):

1. Regulatory inconsistency, that is "capital gearing" and "regulatory arbitrage",

2. "Fungibility[37] of capital and transferability of assets",

3. "Measurement of risk dependencies and aggregation of risks", and

4. "Treatment of nonregulated entities".

The paper also provides a qualitative overview of current approaches to group-wide solvency regulation.

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[37]In the following we will define fungibility as the ability to transfer capital easily and freely within the insurance group (see also Filipović and Kupper, 2007).

Our paper makes both a theoretical and a numerical comparison between the different approaches to group-wide solvency assessment by quantifying risks and capital requirements. It determines which approach is more appropriate in which situation when dealing with different regulatory challenges.

Our analyses are based on the model framework proposed by Gatzert and Schmeiser (2011). Their study simultaneously assesses the diversification benefit and conglomerate discount with respect to the capital charges and shortfall risks of a two-entity financial conglomerate with and without accounting for the altered shareholder value. The authors derive capital requirements in the context of the tail value at risk concept of the Swiss Solvency Test.

Generalizing the model framework by Gatzert and Schmeiser (2011) to $N + 1$ legal entities (one parent company and N subsidiaries), we aim to compare the two approaches to assess the solvency of insurance groups in light of different regulatory issues under the real world measure $\mathbb{P}$. Within a one-year solvency horizon, we compare results from a legal entity approach, which takes different capital and risk transfer instruments (CRTIs) into account, and a consolidated approach. Keeping the capital structure fixed, we study shortfall risk and capital charges under different parameter assumptions. In order to derive the capital requirements we apply the value at risk measure of the proposed Solvency II regulatory framework. Since, in general, no closed-form solutions can be derived, numerical results are generated by means of a Monte Carlo simulation. We interpret our findings in light of two main challenges to group-wide solvency regulation: regulatory inconsistency and risk interdependencies, with a special focus on the latter.

The remainder of the paper is organized as follows. Section 2 introduces the model framework describing in detail the solvency approaches under consideration. Section 3 contains a numerical analysis and the simulation results for an insurance group comprised of three legal entities as well as an interpretation of those results in light of the regulatory challenges to group-wide solvency assessment. Section 4 undertakes an overall comparison of the approaches and Section 5 summarizes our findings.

2 Model Framework

2.1 Basic Setting

Generalizing the model framework proposed by Gatzert and Schmeiser (2011), we consider a set $\mathcal{F} = \{0, ..., N\}$ of firms denoted by $i = 0, ..., N$ within an insurance group. The index $i = 0$ denotes the parent company; $i = 1, ..., N$ stand for the subsidiaries. The market value of liabilities and the market value of assets of the ith entity are given by $L_{t,i}$ and $A_{t,i}$, respectively, with the time index $t = 0, 1$. $A_{0,i}$ is defined as the sum of the initial payments of equityholders $E_{0,i}$ and policyholders $D_{0,i}$ to firm i: $A_{0,i} = E_{0,i} + D_{0,i}$.

The development of assets and liabilities is modeled by means of geometric Brownian motions. For a one-year time horizon ($t = 0, 1$), the stochastic processes are given by:

$$dA_t = \mu_A A_t dt + \sigma_A A_t dW_t^A \tag{35}$$

$$dL_t = \mu_L L_t dt + \sigma_L L_t dW_t^L, \tag{36}$$

with constant means μ_A and μ_L, and volatilities σ_A and σ_L, over time. W^A and W^L are correlated standard $\mathbb{P}$-Brownian motions, with a Pearson's correlation coefficient of $\rho_{A,L} dt = dW^A dW^L$.

At $t = 1$ two scenarios are possible: In the first, company $i \in \mathcal{F}$ is able to cover its liabilities, so policyholders and other debtholders obtain the value of the liabilities and equityholders receive the difference between the market value of assets and the market value of liabilities. In the second scenario, the liabilities cannot be met in full, therefore policyholders receive the total value of assets and equityholders leave empty handed. The payoff to policyholders can be expressed by the value of liabilities less the payoff of the default put option at time $t = 1$ (see Doherty and Garven, 1986):

$$D_{1,i} = L_{1,i} - \max(L_{1,i} - A_{1,i}, 0), \tag{37}$$

where $\max(L_{1,i} - A_{1,i}, 0) = DPO_{1,i}$ constitutes the default put option value of firm i at time t (see Doherty and Garven, 1986).

The payoff to equityholders can be expressed as a call option on the firm's assets, while the liabilities represent the strike price. Thus, for the equityholders of entity i at time 1, one obtains (cf. Doherty and Garven, 1986):

$$E_{1,i} = A_{1,i} - D_{1,i} = \max(A_{1,i} - L_{1,i}, 0). \tag{38}$$

2.1.1 Economic Capital

We derive available and necessary economic capital based on fixed a-mounts of initial debt and equity payments (cf. Gatzert and Schmeiser, 2011). In insurance regulation, available economic capital (AEC) is often called risk-bearing capital, as in the Swiss Solvency Test (see FOPI, 2006) or risk-based capital as in the U.S. NAIC method (see NAIC, 2009c). Following Keller (2007), and Filipović and Kupper (2007), we define the AEC of company i at time t as the market value of assets less the market value of liabilities:

$$AEC_{t,i} = A_{t,i} - L_{t,i}. \tag{39}$$

The necessary economic capital (NEC), also called solvency capital requirement (Solvency II) or target capital (Swiss Solvency Test), is the economic capital needed at $t = 0$ to limit the probability of default to a pre-specified confidence level α (see, e.g., FOPI, 2004). The NEC^α depends on the underlying stochastic model, the input parameters and the risk measure chosen. For the latter, value at risk (VaR) is applied, in line with Solvency II (see, e.g., EC, 2009). The value at risk for a given confidence level $1 - \alpha$ is given by the quantile of the distribution $\mathrm{F}^{-1}(\alpha)$ such that $\mathrm{VaR}^\alpha(X_i) = \inf\{x : \mathrm{F}_X(x) \geq \alpha\}$. For the ith firm, we set X_i to:

$$X_i = AEC_{1,i} \cdot e^{-r_f} - AEC_{0,i}. \tag{40}$$

That is, we define $\mathrm{VaR}^\alpha(X_i)$ as the value at risk of the change of available economic capital of firm i during one time period (see, e.g., FOPI, 2006). Therefore, the necessary economic capital for $i \in \mathcal{F}$ is given by:

$$NEC_i^\alpha = -\mathrm{VaR}^\alpha(X_i). \tag{41}$$

We set the minimum level of economic capital (ML) below which financial resources are not supposed to fall (see EC, 2009), the so-called minimum capital requirement under EU solvency regulations for nonlife insurers, to the maximum of the premium basis (PB_i) and the claims basis (CB_i) of an insurance company i (EC, 2002a):[38]

$$ML_i = \max(PB_i, CB_i). \tag{42}$$

The premium basis and the claims basis for firm i are calculated as follows (EC, 2002a):

$$PB_i = 0.18 \cdot (\min(P_i; 50\,\text{€ million})) + \\ 0.16 \cdot (\max(P_i - 50\,\text{€ million}; 0)), \tag{43}$$

$$CB_i = 0.26 \cdot (\min(C_i; 35\,\text{€ million})) + \\ 0.23 \cdot (\max(C_i - 35\,\text{€ million}; 0)), \tag{44}$$

where P_i stands for the net premium income of insurer i at $t = 0$ and C_i denotes the average net claims of company i - in general based on the last three years.

2.1.2 Individual and Joint Shortfall

In line with Gatzert and Schmeiser (2011), we assume that shortfall can occur in two cases (in the following see Gatzert and Schmeiser, 2011):

1. Either the available economic capital in $t = 1$ falls below zero, so the insurer is insolvent, or

2. $AEC_{1,i}$ falls below the minimum level ML_i, meaning that firm i is not insolvent, but cannot continue in business, unless it raises additional capital.

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[38]For the sake of simplification we ignore reinsurance coverage.

Thus, the individual shortfall probabilities for the ith entity can be calculated by (see Gatzert and Schmeiser, 2011):

$$P_i^{ind} = \mathbb{P}(AEC_{1,i} < 0) \tag{45}$$

and

$$P_i^{ind,ML} = \mathbb{P}(AEC_1, i < ML_i). \tag{46}$$

The probability of a joint shortfall of exactly $m = 1, ..., N+1$ legal entities can be expressed by:

$$P_m^{joint} = \sum_{\substack{\mathcal{F}^* \subset \mathcal{F} \\ |\mathcal{F}^*| = m}} \mathbb{P}\left[(\wedge_{i \in \mathcal{F}^*} AEC_{1,i} < 0) \wedge (\wedge_{i \in \mathcal{F} \setminus \mathcal{F}^*} AEC_{1,i} > 0)\right].$$

The sum runs over all subsets $\mathcal{F}^*$ of $\mathcal{F}$ counting exactly m elements.[39] The first term inside the square brackets describes the joint shortfall of all legal entities within the subset $\mathcal{F}^*$, given that the residual firms of $\mathcal{F}$ (the second term inside the square brackets) are solvent at $t = 1$.

Similarly, the probability that the available capital of $m = 1, ..., N+1$ legal entities simultaneously falls below the minimum level, is:

$$P_m^{joint} = \sum_{\substack{\mathcal{F}^* \subset \mathcal{F} \\ |\mathcal{F}^*| = m}} \mathbb{P}\left[(\wedge_{i \in \mathcal{F}^*} AEC_{1,i} < ML_i) \wedge (\wedge_{i \in \mathcal{F} \setminus \mathcal{F}^*} AEC_{1,i} > ML_i)\right].$$

2.2   Legal Entity Approach

A group-wide solvency assessment approach with a legal entity focus treats the insurance group as a collection of interdependent legal entities (see IAIS, 2009b). Capital requirements and risks are determined for each legal entity taking into account intra-group transactions.

In this section, we extend the model framework provided by Gatzert and Schmeiser (2011) to the general case of $N + 1$ legal entities which are separately capitalized. Within this framework, firm $i = 0$, the parent

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[39]$|\mathcal{F}^*|$ denotes the cardinality of the subset $\mathcal{F}^*$. (m is the number of legal entities insolvent at $t = 1$).

company, covers its subsidiaries' liabilities only in the presence of legally binding transfer contracts (see Keller, 2007). This approach therefore relies on different assumptions regarding the capital and risk transfer between entities (for the following paragraph see Gatzert and Schmeiser, 2011):

The first assumption is that the parent company $i = 0$ can access its subsidiaries' surplus capital. Furthermore, a going concern assumption for the subsidiaries after $t = 1$ is included, which requires that the subsidiaries $i = 1, ..., N$ must at least be endowed with the minimum level of economic capital at time 1. Thus, the available economic capital of a subsidiary $i = 1, ...N$ in $t = 1$ can be expressed by $\min(A_{1,i} - L_{1,i}, ML_i)$. Taken together, these assumptions imply that the parent can sell its subsidiaries for the value of $\sum_{i=1}^{N} \max(A_{1,i} - L_{1,i} - ML_i, 0)$.

In line with Gatzert and Schmeiser (2011), our analysis examines two different CRTIs: a guarantee and a quota-share retrocession when each is transferred from the parent company to one subsidiary and when each is transferred from one subsidiary to another.

Under the guarantee, we assume that the transferring company, denoted by i_{tr}, covers the shortfall $DPO_{1,i_{bfc}} = \max(L_{1,i_{bfc}} - A_{1,i_{bfc}}, 0)$ of the beneficiary i_{bfc} with $i_{bfc} \neq i_{tr}$ only, when the transferor's available economic capital at time 1 is above the minimum level. Thus, the transfer T to the benefiting firm is restricted to $\max(A_{1,i_{tr}} - L_{1,i_{tr}} - ML_{i_{tr}}, 0)$.

Therefore, the value of the guarantee T^G can be expressed by:

$$T^G = \min\left(DPO_{1,i_{bfc}}, \max(A_{1,i_{tr}} - L_{1,i_{tr}} - ML_{i_{tr}}, 0)\right). \qquad (47)$$

The second type of CRTI considered, is a quota-share retrocession in which q denotes the quota. When the transferring company is legally obligated to assume the share q of the beneficiary's liabilities, the quota-share retrocession's value is given by:

$$T^R = \min\left(q \cdot L_{1,i_{bfc}}, \max(A_{1,i_{tr}} - L_{1,i_{tr}} - ML_{i_{tr}}, 0)\right). \qquad (48)$$

Considering the case of a transfer from the parent company $i = 0$ to the benefiting subsidiary, i_{bfc}, we can express available economic capital in $t = 0$ for all $i = 0, ..., N$, by: $AEC_{0,i} = A_{0,i} - L_{0,i}$. At time $t = 1$ the AEC of the beneficiary is:

$$AEC_{1,i_{bfc}} = \min(A_{1,i_{bfc}} - L_{1,i_{bfc}}, ML_{i_{bfc}}) + T. \qquad (49)$$

For the parent company $i = 0$, we obtain at $t = 1$:

$$AEC_{1,0} = A_{1,0} - L_{1,0} + \max(A_{1,i_{bfc}} - L_{1,i_{bfc}} - ML_{i_{bfc}}, 0) +$$
$$\sum_{\substack{i=1 \\ i \neq i_{bfc}}}^{N} (\max(A_{1,i} - L_{1,i} - ML_i, 0)) - T. \qquad (50)$$

For all other subsidiaries $i = 1, ..., N, i \neq i_{bfc}$, we receive:

$$AEC_{1,i} = \min(A_{1,i} - L_{1,i}, ML_i). \qquad (51)$$

For the case in which a transfer is made from one subsidiary i_{tr} to another subsidiary i_{bfc}, with i_{tr}, given $i_{bfc} \neq 0$, the available economic capital in $t = 0$ is again defined by $AEC_{0,i} = A_{0,i} - L_{0,i}$ for all $i = 0, ..., N$. The AEC of the transferor and the beneficiary in $t = 1$ can be expressed by:

$$AEC_{1,i_{tr}} = \min(A_{1,i_{tr}} - L_{1,i_{tr}} - T, ML_{i_{tr}}) \qquad (52)$$

and

$$AEC_{1,i_{bfc}} = \min(A_{1,i_{bfc}} - L_{1,i_{bfc}}, ML_{i_{bfc}}) + T. \qquad (53)$$

Finally, we receive for the available economic capital of the parent company in $t = 1$:

$$
\begin{aligned}
AEC_{1,0} = {} & A_{1,0} - L_{0,1} + \max(A_{1,i_{tr}} - L_{1,i_{tr}} - ML_{i_{tr}} - T, 0) + \\
& \max(A_{1,i_{bfc}} - L_{1,i_{bfc}} - ML_{i_{bfc}} + T, 0) + \\
& \sum_{\substack{i=1; \\ i \neq i_{bfc}; \\ i \neq i_{tr}}}^{N} \max(A_{1,i} - L_{1,i} - ML_i, 0).
\end{aligned}
\tag{54}
$$

2.3 Consolidated Approach

Following Gatzert and Schmeiser (2011), we define available economic capital under the consolidated approach as the difference between the sum of the legal entities' assets and the sum of the liabilities:

$$
AEC_t^{cons} = \sum_{i=0}^{N} A_{t,i} - \sum_{i=0}^{N} L_{t,i},
\tag{55}
$$

for $t = 0, 1$.

Under this approach, individual and joint shortfall probabilities coincide such that $P^{ind} = P_m^{joint}$ and $P^{ind,ML} = P_m^{joint,ML}$, for any $m = 1, ..., N + 1$.

3 Numerical Analysis and Implications

In our numerical analyses, we present results for a stylized example. For the sake of simplicity, we consider an insurance group that is comprised of three legal entities: two subsidiaries and their parent company. Our analysis examines the introduction of a guarantee and a quota-share retrocession transferred either from the parent company 0 to subsidiary 1 or from subsidiary 2 to subsidiary 1. The numerical example is conducted via a $100,000$-run Monte-Carlo simulation, each run employing the same set of random numbers (see, e.g., Glasserman, 2004).

The remainder of this section is structured as follows: After Section 3.1 provides the input parameters, Section 3.2 sets out the calculations

of the available economic capital for each legal entity under the two solvency approaches. Section 3.3 presents the simulation results and interprets the model framework with respect to the two regulatory challenges so as to compare the group structure models. To this end, we first provide the working definitions of risk dependencies as well as regulatory inconsistency on which the comparison in Section 4 is based.

3.1 Parameter Settings

In the following, we assume that the three firms (i.e., the two subsidiaries and their parent) have the same asset-liability structure but that the parent company is twice as large as its subsidiaries. We set the nominal value of the liabilities of subsidiaries 1 and 2 to $L_{0,1} = L_{0,2} = 50$ mn currency units (CU) and the market value of the liabilities of the parent company 0 to $L_{0,0} = $ CU 100 mn. The equity capital of the two subsidiaries $E_{0,1}$ and $E_{0,2}$ is fixed at CU 15 mn and for the parent $E_{0,0}$ at CU 30 mn. The initial values of the default put option are fixed at CU 100,000 for company 0 and at CU 50,000 for the subsidiaries, so the value of the debt capital of subsidiaries 1 and 2 is given by $D_{0,1} = D_{0,2} = $ CU 49.95 mn, and the value of the debt capital of the parent company is given by $D_{0,0} = $ CU 99.9 mn. Thus, the market value of the assets of the two subsidiaries $A_{0,1}$ and $A_{0,2}$ is CU 65 mn and for the parent company it amounts to $A_{0,0} = $ CU 130 mn. The net premium income of subsidiaries 1 and 2 is set to $P_1 = P_2 = $ CU 7.5 mn and that of the parent is set to $P_0 = $ CU 15 mn. We assume the average net claims over the last three years to be $C_1 = C_2 = $ CU 4.5 mn for the subsidiaries and $C_0 = $ CU 9 mn for the parent company. Drift and standard deviation of the assets and liabilities are given by $\mu^A = 5\%$, $\sigma^A = 10\%$ (for assets) and $\mu^L = 3\%$, $\sigma^L = 0.5\%$ (for liabilities). The risk-free rate of return is set to $r_f = 2\%$, and the quota of the quota-share retrocession is assumed to be $q = 5\%$.

The correlation coefficients between pairs of assets and liabilities are fixed at: $\rho(A_i, L_i) = 0.2$ and $\rho(A_i, L_j) = \rho(A_j, L_i) = 0.0$, with $i \neq j$ and $i, j = 0, 1, 2$. For a more profound comparison of the two solvency models, we compare results for different values of $\rho = \rho(A_i, A_j) = \rho(L_i, L_j)$, with

	$AEC_{1,0}$	$AEC_{1,1}$	$AEC_{1,2}$
Legal Entity	$A_{1,0} - L_{1,0}$ $+ \max(A_{1,1} - L_{1,1} - ML_1, 0)$ $+ \max(A_{1,2} - L_{1,2} - ML_2, 0)$	$\min(A_{1,1} - L_{1,1}, ML_1)$	$\min(A_{1,2} - L_{1,2}, ML_2)$
Legal Entity $G_{0/1}$	$A_{1,0} - L_{1,0}$ $+ \max(A_{1,1} - L_{1,1} - ML_1, 0)$ $+ \max(A_{1,2} - L_{1,2} - ML_2, 0) - T^G$	$\min(A_{1,1} - L_{1,1}, ML_1) + T^G$	$\min(A_{1,2} - L_{1,2}, ML_2)$
Legal Entity $G_{2/1}$	$A_{1,0} - L_{1,0}$ $+ \max(A_{1,1} - L_{1,1} - ML_1 - T^G, 0)$ $+ \max(A_{1,2} - L_{1,2} - ML_2 + T^G, 0)$	$\min(A_{1,1} - L_{1,1}, ML_1) + T^G$	$\min(A_{1,2} - L_{1,2} - T^G, ML_2)$
Legal Entity $R_{0/1}$	$A_{1,0} - L_{1,0}$ $+ \max(A_{1,1} - L_{1,1} - ML_1, 0)$ $+ \max(A_{1,2} - L_{1,2} - ML_2, 0) - T^R$	$\min(A_{1,1} - L_{1,1}, ML_1) + T^R$	$\min(A_{1,2} - L_{1,2}, ML_2)$
Legal Entity $R_{2/1}$	$A_{1,0} - L_{1,0}$ $+ \max(A_{1,1} - L_{1,1} - ML_1 - T^R, 0)$ $+ \max(A_{1,2} - L_{1,2} - ML_2 + T^R, 0)$	$\min(A_{1,1} - L_{1,1}, ML_1) + T^R$	$\min(A_{1,2} - L_{1,2} - T^R, ML_2)$
Cons	$A_{1,0} + A_{1,1} + A_{1,2} - L_{1,0} - L_{1,1} - L_{1,2}$	-	-

Table 12: Available Economic Capital at $t = 1$ for the Two Approaches of Group Solvency Assessment

Legal Entity = legal entity approach without CRTIs; Legal Entity $G_{0/1}$ = legal entity approach with a guarantee from company 0 to subsidiary 1; Legal Entity $G_{2/1}$ = legal entity approach with a guarantee from subsidiary 2 to subsidiary 1; Legal Entity $R_{0/1}$ = legal entity approach with a retrocession from company 0 to subsidiary 1; Legal Entity $R_{2/1}$ = legal entity approach with a retrocession from subsidiary 2 to subsidiary 1; Cons = consolidated approach

$i \neq j$ and $i, j = 0, 1, 2$. We show outcomes for the uncorrelated case ($\rho = 0.0$), for a case of moderate correlation ($\rho = 0.4$), and for a case of relatively high correlation ($\rho = 0.8$).

3.2 Available Economic Capital

This section sets forth calculations of the available economic capital for the group solvency approaches in both $t = 0$ and $t = 1$ for the insurance group that is comprised of three legal entities.

For the legal entity approach, the available economic capital at $t = 0$ for the ith legal entity is determined by $AEC_{0,i} = A_{0,i} - L_{0,i}$, whereas it is given by $AEC_0^{cons} = A_{0,0} + A_{0,1} + A_{0,2} - L_{0,0} - L_{0,1} - L_{0,2}$ for the consolidated approach. The AEC for the different transfer cases under the legal entity approach as well as the consolidated available economic capital at $t = 1$ are summarized in Table 12 (see also Gatzert and Schmeiser, 2011).

3.3 Numerical Results and Interpretation

3.3.1 Risk Dependencies

We follow the working definition of risk dependencies by the International Association of Insurance Supervisors (for the following paragraph see IAIS, 2009b).

Our discussion therefore focuses on two main drivers of risk dependencies: Risk concentration and risk diversification. According to the IAIS, risk concentration refers to common risk factors that are able to threaten the financial soundness of the entire insurance group, while diversification effects cause the aggregated risks of the entire group to be in general lower than the sum of the individual companies' risks.

We take two perspectives in comparing the different solvency approaches with regard to how they assess shortfall risks and concentration as well as diversification effects within the insurance group. In the first step, we assess the riskiness of each financial institution by considering individual shortfall probabilities (see Figure 5) and the necessary economic capital (see Figure 6). In the second step, we focus on the insti-

tutions' exposure to common risk factors and their interconnectedness, measured by joint shortfall probabilities (see Figure 7).

The Riskiness of the Individual Financial Institution Our simulation results, shown in Figure 5 and Figure 6, are based on the fixed capital structure given in Section 3.1. Figure 5 shows individual shortfall probabilities (left column) as well as the probabilities that the available economic capital in time 1 will fall below the minimum level (ML) (right column) for different specifications of the correlation coefficient ρ.

The left column of Figure 5 ($\rho = 0.0$) shows that under our legal entity approach, the parent company's shortfall probability is practically reduced to zero due to the group diversification (see also Gatzert and Schmeiser, 2011).[40] The subsidiaries' shortfall probabilities, on the other hand, depend on the transfer case, considered. With no CRTIs in place, the subsidiaries do not participate in the diversification effects. By contrast, the introduction of a CRTI leads to a considerable reduction in the shortfall probability of the subsidiary that benefits from the transfer, although the extent of the reduction depends on the type of CRTI and on the transferring company's solvency. A guarantee reduces the beneficiary's shortfall probability to practically zero, regardless of whether the transferor is the parent company or another subsidiary. By contrast, the introduction of a quota-share retrocession reduces the benefiting company's shortfall probability to a lesser degree, particularly when it is the other subsidiary that is making the transfer. The parent company's shortfall probability is unchanged and close to zero in all cases, since the transfer from the parent is undertaken only when the company is solvent.

Only one bar is shown for the consolidated approach, because the insurance group is treated as one consolidated entity. As a consequence, individual and joint shortfall probabilities are indistinguishable in this framework. Due to a maximum realization of diversification effects and synergies under this solvency approach, the probability of shortfall is close to zero for $\rho = 0.0$.

---

[40]Here, diversification effects can arise, because assets and liabilities of the three companies are not fully correlated (see Gatzert and Schmeiser, 2011)

The right column of Figure 5 shows the probability that the available economic capital at time 1 will fall below the minimum level of economic capital, meaning that the firms will not be able to continue in business, unless they raise additional capital. Thus, $P^{ind,ML}$ includes P^{ind}. Under the legal-entity approach, the benefiting subsidiary's $P^{ind,ML}$ remains stable both with and without a guarantee, but it is reduced in case of a quota-share retrocession. The parent's individual shortfall probability and the consolidated model's P^{ML} are, again, close to zero.

Turning to the second and third row of Figure 5, we find that the higher the correlation coefficient ρ, the more diversification effects are reduced in both group solvency approaches and consequently the individual shortfall probabilities are increased in all cases.
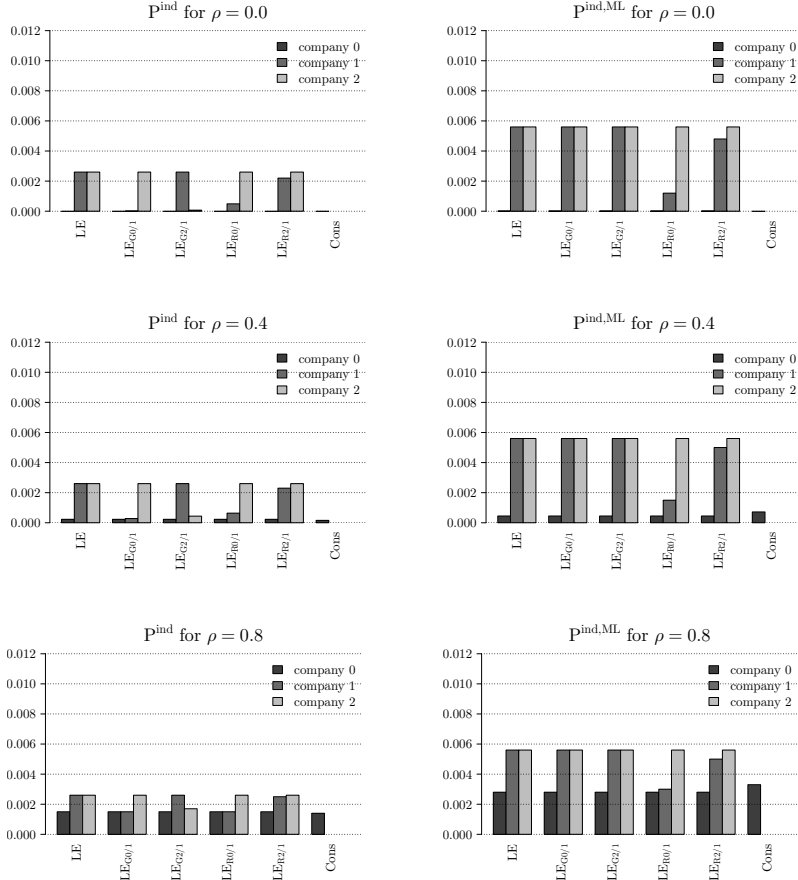
Figure 5: Individual Shortfall Probabilities for $\rho = 0.0, 0.4$ and 0.8
LE = legal entity approach without CRTIs; $LE_{G0/1}$ = legal entity approach with a guarantee from company 0 to subsidiary 1; $LE_{G2/1}$ = legal entity approach with a guarantee from subsidiary 2 to subsidiary 1; $LE_{R0/1}$ = legal entity approach with a retrocession from company 0 to subsidiary 1; $LE_{R2/1}$ = legal entity approach with a retrocession from subsidiary 2 to subsidiary 1; Cons = consolidated approach
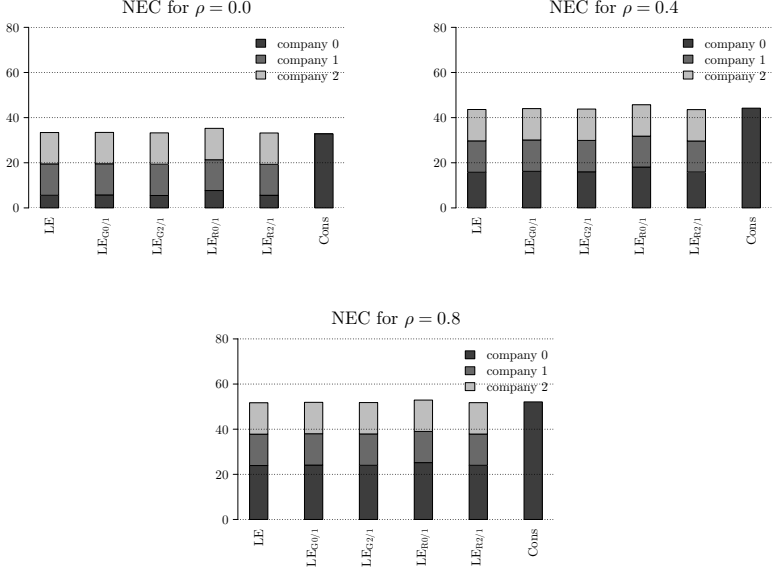
Figure 6: Necessary Economic Capital for $\rho = 0.0, 0.4$ and 0.8
LE = legal entity approach without CRTIs; $LE_{G0/1}$ = legal entity approach with a guarantee from company 0 to subsidiary 1; $LE_{G2/1}$ = legal entity approach with a guarantee from subsidiary 2 to subsidiary 1; $LE_{R0/1}$ = legal entity approach with a retrocession from company 0 to subsidiary 1; $LE_{R2/1}$ = legal entity approach with a retrocession from subsidiary 2 to subsidiary 1; Cons = consolidated approach

Figure 6 shows the capital requirements for the entire insurance group under both approaches. The different shades of gray in the cases of the legal entity approach indicate the entities' individual contribution to the group capital charge. Considering the uncorrelated case in the first row, we see that under the legal entity approach, the parent's necessary economic capital is substantially lower compared to the NECs of the two subsidiaries. The introduction of a guarantee leads to a slight decrease in the NEC of the benefiting subsidiary, but to a slight increase in the capital requirement of the parent company. Therefore, the group NEC remains relatively constant. By contrast, when a quota-share retrocession is in place, the increase in the parent's NEC is substantial, so that the group capital requirement is higher than in the case without any CRTIs.
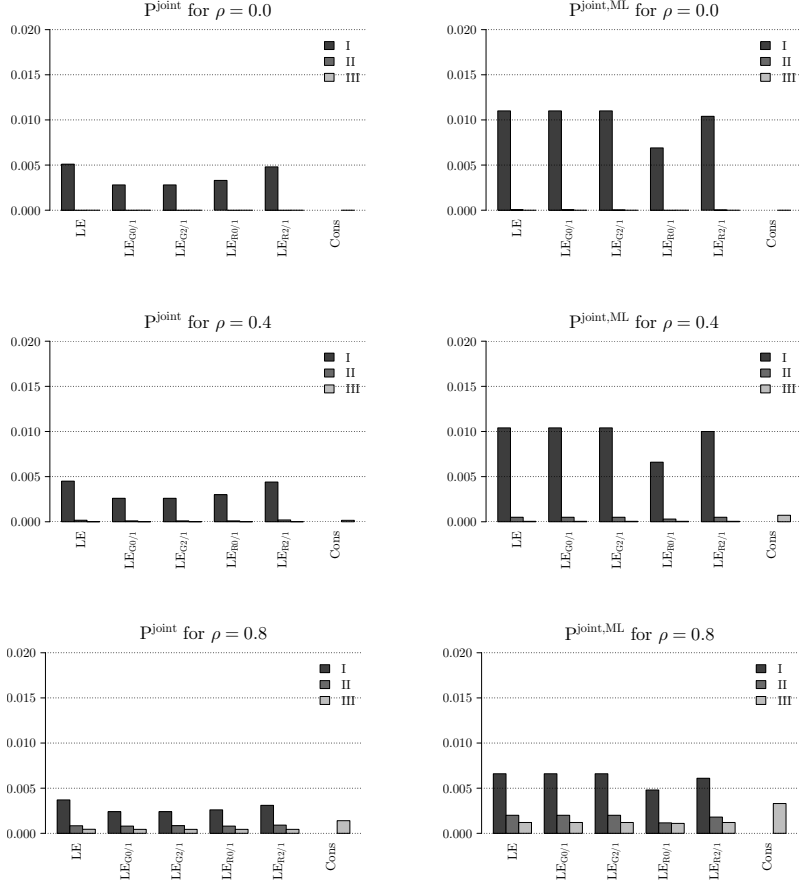
Figure 7: Joint Shortfall Probabilities for $\rho = 0.0, 0.4$ and 0.8

LE = legal entity approach without CRTIs; $LE_{G0/1}$ = legal entity approach with a guarantee from company 0 to subsidiary 1; $LE_{G2/1}$ = legal entity approach with a guarantee from subsidiary 2 to subsidiary 1; $LE_{R0/1}$ = legal entity approach with a retrocession from company 0 to subsidiary 1; $LE_{R2/1}$ = legal entity approach with a retrocession from subsidiary 2 to subsidiary 1; Cons = consolidated approach

Turning to the consolidated approach where the insurance group is considered on the basis of its consolidated balance sheet, the NEC shown is already the capital requirement for the entire insurance group. Comparing the necessary economic capital of the two solvency models, we find that they are very similar to each other.

With an increase in ρ, the necessary economic capital for company 0 increases substantially within the legal entity approach due to group diversification effects. This is particularly evident when looking at the entities' contribution to the overall group capital charge: While the subsidiaries' necessary economic capital remains approximately the same, the capital requirement of the parent company increases considerably. The necessary economic capital under the consolidated approach increases to a similar extent for higher correlation coefficients.

Interconnectedness within the Insurance Group   In the next step, joint shortfall probabilities are calculated based on the capital structure of the numerical example. Results are presented in Figure 7. Again, we consider three different values for ρ.

In the uncorrelated case (left column of Figure 7), we find that the probability that all three entities, or two out of three of them will default at the same time ((P_{III}^{joint} and P_{II}^{joint})) is close to zero for both approaches. Since under the consolidated approach joint shortfall probabilities correspond to the individual ones, P_{I}^{joint} and P_{II}^{joint} are not defined (see also Gatzert and Schmeiser, 2011). The probability that exactly one firm defaults (P_{I}^{joint}) is lowest for the case of a guarantee under the legal entity approach. In the case of a quota-share retrocession, P_{I}^{joint} is significantly higher when the transfer is made from one subsidiary to another than when the transfer is made from the parent company to one of the subsidiaries.

Similar results can be observed in the right column of Figure 7. However, the legal entity approach results in the lowest probabilities that the available economic capital of exactly one firm will fall below the minimum level of economic capital in the presence of a quota-share retrocession.

The second and last row of Figure 7 depict the results for higher correlations. While the probability that only one entity of the insurance group will default is reduced significantly when assets and liabilities of the different entities are highly correlated, the joint shortfall probabilities II and III are significantly increased in all cases.

Comparing the two solvency models, the probability of all three firms defaulting at the same time is approximately three times higher in the consolidated framework than in all cases of the legal entity approach.

3.3.2   Regulatory Inconsistency

According to Mälkönen (2004), regulatory inconsistency occurs in the presence of regulatory arbitrage and double/multiple gearing of capital.

Regulatory arbitrage is the process of taking advantage of the discrepancies between different regulatory regimes and is sometimes referred to as "capital arbitrage" or "jurisdictional arbitrage" (see, e.g., Freixas et al., 2007). In the context of financial conglomerates and insurance groups, regulatory arbitrage can be defined as the possibility of separately capitalized legal entities to transfer assets to the divisions that are subject to the lowest capital charges.

According to the Joint Forum on Financial Conglomerates (1998), double gearing of capital occurs if one legal entity of a financial group holds solvency capital issued by another legal entity, and the issuing company counts the capital in its own balance sheet (for this paragraph refer to Joint Forum on Financial Conglomerates, 1998). Thus, external capital of the group is counted twice, so it may serve to fulfill capital adequacy requirements in both entities. Multiple capital gearing occurs when the externally generated capital is geared up multiple times, such as when a company that holds regulatory capital issued by another legal entity downstreams this capital to a third-tier legal entity.

With regard to the legal entity approach, intra-group transfers are properly assessed because this approach models the web of CRTIs. However, regulatory arbitrage between countries and financial sectors is generally possible whenever capital charges are calculated differently in different jurisdictions (see Table 12). On the other hand, this approach models the market value of the subsidiaries as an asset of the parent com-

pany, so double/multiple gearing is avoided by splitting up the value of a subsidiary $i = 1, ..., N$ into two parts: the transferable value to the parent $(\max(A_{1,i} - L_{1,i} - ML_i, 0))$, and the subsidiary's available economic capital $(\min(A_{1,i} - L_{1,i}, ML_i))$, which at least equals the minimum level (see Gatzert and Schmeiser, 2011).

Finally, considering our consolidated approach, we find that due to the implicit assumption of full fungibility and transferability of capital and risks and the fact that capital adequacy requirements are based on one consolidated balance sheet, regulatory arbitrage and double/multiple gearing of capital are not possible (see also Freixas et al., 2007).

4   Comparison

This section compares the two approaches to group-wide solvency assessment presented in detail in Section 2 in order to determine which of the two is more appropriate under which circumstances.

The consolidated approach treats the insurance group as one integrated entity, so all intra-group transactions cancel out. Thus, the approach implicitly controls for regulatory inconsistency. Yet, Keller (2007) points out that it is a valid group solvency approach only when its assumption of full mobility of capital and risks between members of the insurance group holds, allowing for a maximum realization of synergies and diversification. These effects are reflected in our simulation results for the individual shortfall probabilities, as the consolidated approach produces the lowest probabilities, regardless of the value of ρ (see Figure 5).

In line with Keller's reasoning, the Committee of European Insurance and Occupational Pensions Supervisors points out, that such an assumption is particularly problematic during financial crises because diversification benefits tend to diminish or at least do not operate the same way they do in normal times (see CEIOPS, 2009c).

In addition to the problematic assumption of full transferability, the consolidated approach does not provide any information about the individual entity or its risk contribution to the total risk faced by the insurance group as it is based on a consolidated balance sheet.

On the other hand, the analysis in Section 3.3.1 suggests that the consolidated approach is the more conservative approach when it comes to computing the probability that all legal entities within an insurance group will default at the same time (see Figure 7).

By contrast, the legal entity approach to group solvency provides for the shortfall risk of each institution and its individual capital endowment by taking into account risk and capital transfer instruments. As it is based on the individual entities' balance sheets, and therefore does not need to assume full transferability of capital and risks within the insurance group, Keller (2007) argues that it is a group solvency approach directly compatible with a solo assessment of the solvency of an individual entity.

Despite the problem of not being able to account for regulatory arbitrage, in our model framework the legal entity approach is more conservative with regard to the risk assessment of the individual members of the insurance group (refer to Figure 5). It is also able to control for capital gearing. However, it is likely to be the most complex to implement in practice and therefore probably the more expensive group-wide solvency approach (see also IAIS, 2009b).

Nevertheless, if the web of CRTIs is modeled accurately, the legal entity approach can model all kinds of group structures, including the extreme case of no intra-group transactions at all, as well as the case when capital and risks are freely transferable among the legal entities. Therefore, it is the more generally applicable framework.

5   Summary

This paper compares two approaches to group-wide solvency assessment of insurance groups in light of the regulatory challenges of regulatory inconsistency and risk dependencies: a legal entity approach and a consolidated approach. Generalizing the model framework by Gatzert and Schmeiser (2011), we examine capital charges, individual shortfall risks as well as joint shortfall for an insurance group of $N + 1$ legal entities - one parent company and N subsidiaries - and interpret the results with

respect to the supervisory challenges of regulatory inconsistency and risk dependencies, with a special focus on the latter one.

Our findings contribute to the current discussion of solvency regulation of large financial groups, especially insurance groups. Firstly, we present the two group solvency approaches emphasizing the different implicit and explicit assumptions made in each framework since these are of special relevance from a regulatory perspective. The results of our numerical analyses reveal that the choice of a particular group solvency approach has a substantial influence on capital charges and shortfall risks. Individual shortfall risks decrease considerably with the level of consolidation assumed by each of the different solvency approaches, although this effect diminishes as the correlation between the entities' returns on assets and liabilities increases.

Secondly, the two solvency approaches are compared in terms of their advantages and shortcomings and it is determined under which circumstances each approach is more appropriate. The assumptions of a consolidated framework are particularly problematic when asset and liability returns become highly correlated as the effects of diversification diminish. On the other hand, our numerical analyses show that the consolidated approach is more conservative than the legal entity framework with respect to the calculation of joint shortfall probabilities. In addition, the legal entity approach provides for each entity's individual shortfall risk and capital endowment by taking into account the web of CRTIs, whereas a consolidated approach provides no information about the individual entity or its contribution to total risk (see also Gatzert and Schmeiser, 2011). Finally, the legal entity framework is more complex to implement and cannot control regulatory arbitrage.

We conclude from the analyses that a legal entity approach is more generally applicable, as it is able to take different group structures into account and find it therefore, despite its shortcomings, superior to an approach that is solely based on a consolidated viewpoint.

Although the models used to assess group-wide solvency in practice are intermediate models with characteristics of both the legal entity and the consolidated approach, a comparison of these two extremes on a theoretical and numerical basis in light of regulatory challenges is es-

pecially important as regulators and supervisors work toward designing and implementing a sound system of group-wide solvency regulation in the insurance sector.

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Part IV

Model Uncertainty and Its Impact on Solvency Measurement in Property-Liability Insurance

Abstract

The aim of this paper is to study the model risk of solvency models for property-liability insurers. From a basic model framework, we examine the effects of introducing stochastic jumps and linear, or nonlinear dependencies into the model on capital requirements and shortfall risk measures. Additionally, we take a regulatory view and consider the degree to which the deviations in risk measures due to different model specifications can be diminished, by means of requiring interim financial reports. The simulation results suggest that the sensitivity of capital requirements as a risk measure may underestimate the actual model risk that policyholders are exposed to. We also find that mandatory interim reports can significantly reduce the model uncertainty faced by a regulator. This has important implications for the design of risk-based capital standards and the implementation of internal solvency models.[41]

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1   Introduction

Since the early 1990s, most insurance regulators have introduced a system of risk-based capital standards (for the following paragraph see, e.g., Cummins and Phillips, 2009). Some of the first countries to do this were Canada with its Minimum Continuing Capital and Surplus Requirements in 1992 and the United States with the NAIC risk-based capital approach in 1994. In 1996, Japan followed with its Solvency Margin Standard and Australia passed the General Insurance Reform Act in 2001. In Europe, the recent developments in solvency assessment include new capital requirements, which ensure that the insurance companies' eligible own funds suffice to fulfill the solvency capital requirements.

Although well-designed capital standards can reduce the insolvency risk of an insurance company (see also Holzmüller, 2009), the recent financial crisis has shown that quantitative models especially can give insurers, regulators and above all policyholders a false sense of security. Academics, practitioners, and regulators have recently pointed out that, while some models might do a good job in normal times, their performance during times of financial distress or in times of a financial crisis can turn out to be rather poor (see, e.g., CEIOPS, 2009c). Therefore, the risk associated with working with misspecified models, must not be underestimated.

The risk of model misspecification can be categorized under the term of "model risk" (see, e.g., McNeil, Frey, and Embrechts, 2005 and Sandström, 2006). When considered in the literature, model risk is often analyzed alongside the so called parameter risk: the risk of errors in the parameter values within a specific framework (see, e.g., Cairns, 2000). Previous studies have (mostly) focused on financial derivative products when considering those two types of risk. For example, Hull and Suo (2002) investigate the model risk associated with illiquid exotic options based on an implied-volatility model. The works by Cont (2006), Giannetti, Clark, and Anderson (2004), Green and Figlewski (1999), and Kato and Yoshiba (2000) also discuss model risk within a financial derivative context. Kerkhof, Melenberg, and Schumacher (2002) quantify model

risk due to model misspecifications and estimation errors and apply their approach to stock portfolios as well as derivative products. Interestingly, they find that model risk is much more important than parameter risk. Furthermore, a study on model risk in interest rate markets, which concentrates on the risk of using incorrect parameter values is conducted by Gibson, Lhabitant, Pistre, and Talay (1999). The study by Cairns (2000) considers the process of parameter and model risk in an insurance context. The author presents a methodology to coherently assess the risks with regard to some particular factors of interest (see Cairns, 2000).

Finally, with respect to the sensitivity of capital requirements from a solvency perspective, one can find scenario based analyses within European Solvency Frameworks such as the Swiss Solvency Test and the Solvency II framework of the European Union (see, e.g., FOPI, 2006 and EC, 2010). A study by Olivieri and Pitacco (2009) proposes a partial internal model to quantify the impact of mortality risks on risk management actions. Based on a given portfolio of life annuities, it investigates the resulting capital requirements when considering different solvency targets. The authors compare the results of their internal model to the capital requirements according to solvency frameworks such as Solvency II.

In contrast to most of the previously mentioned papers, our analyses are simulation based and consider model misspecifications immanent in the solvency assessment of insurance companies. Based on a rather simplistic and general solvency framework, that models the development of insurance assets and liabilities as independent geometric Brownian motions, we examine the impact of different model specifications on capital requirements, shortfall probabilities and expected policyholder deficits of an insurer. In order to do so, a jump component is introduced into the stochastic process of liabilities and linear or nonlinear dependencies are considered under the basic solvency model.

To measure the deviations in solvency capital requirements, shortfall probabilities and expected policyholder deficits, the ratio of the risk mea-

sure value calculated from the basic setting is examined, to the respective value that results from one of the modifications to the framework.

This risk measure ratio reflects the explanatory power of the basic setting when considering the introduction of different modifications into the solvency framework (jumps or dependencies). Therefore, its complementary value $(1 - $ risk measure ratio$)$ can be interpreted as a measure for the risk of model misspecification. Additionally, we take a regulatory view and consider the degree to which the deviations in capital requirements and shortfall risks due to different model specifications can be diminished, by requiring interim financial reports in addition to the annually required information on the solvency and financial condition of an insurance company. In particular, we contrast the situation of only annual reports to a situation of semiannual, quarterly and monthly updates of the financial information, disclosed by the insurer.

The results from our Monte Carlo simulation show that changes in the specification of a solvency model have a much greater impact on shortfall probabilities and expected policyholder deficits than they have on capital requirements. The shortfall risk measures react much more sensitively to small changes in the model assumptions, than the capital requirements. This leads us to the conclusion that regulators should not solely rely on capital requirements to monitor the solvency situation of an insurer, but should additionally consider shortfall risk measures. More precisely, an analysis of model risk focusing on the sensitivity of capital requirements will typically underestimate the relevant risk of model misspecification from a policyholder's perspective. Finally, the simulation results suggest that mandatory interim reports on the solvency and financial situation of an insurance company are a powerful tool in order to reduce the model uncertainty faced by regulators.

The remainder of the paper is organized as follows: Section 2.1 describes the basic solvency model assumed to be used to assess the solvency situation of insurers. The different model modifications are introduced and explained in Section 2.2. In Section 3 we display the numerical analyses. Section 4 concludes.

2   Model Framework

2.1   Basic Setting

In the basic setting, we consider an insurance company with a market value of liabilities L_t and a market value of assets A_t within a one-year solvency horizon $t = [0, T]$. Assets are divided into high-risk investments $A_{1,t}$ and low-risk investments $A_{2,t}$, so that the market value of total assets at time t is given by:

$$A_t = A_{1,t} + A_{2,t}. \tag{56}$$

In the basic setting it is assumed that the market value of both asset classes $A_{i,t}$ with $i = 1, 2$ and the market value of liabilities L_t evolve according to independent geometric Brownian motions under the objective probability measure $\mathbb{P}$ (cf. also Cummins and Sommer, 1996):

$$dA_{i,t} = \mu_{A_i} A_{i,t} dt + \sigma_{A_i} A_{i,t} dW^{\mathbb{P}}_{A_i,t}, \tag{57}$$

$$dL_t = \mu_L L_t dt + \sigma_L L_t dW^{\mathbb{P}}_{L,t}, \tag{58}$$

with $t = 0, ..., 1, ...T$ and $i = 1, 2$. Drift and volatility of the stochastic processes are denoted by μ_{A_i}, μ_L and σ_{A_i}, σ_L. $W^{\mathbb{P}}_{A_i,t}$ and $W^{\mathbb{P}}_{L,t}$ are standard $\mathbb{P}$-Brownian motions.

2.2   Modifications to the Basic Setting

2.2.1   Introduction of Stochastic Jumps

Since many types of insurance expose the insurer to large jumps in its liabilities,[42] we contrast capital requirements and shortfall risks of the basic setting above to those of a framework including a jump component in the liability process. We hereby use a jump-diffusion process as suggested by Merton (1976) and applied to insurance liabilities by, for

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[42]For example, when property-liability insurers offer catastrophe coverage or when life insurers are faced with pandemic risks and longevity risks (see, e.g., Cummins, 1988).

example, Cummins (1988) and Gatzert and Schmeiser (2008).

Retaining the assumption of a geometric Brownian motion for the development of assets, we now assume that the market value of liabilities evolves according to (in the following see Gatzert and Schmeiser, 2008):

$$\frac{dL_t}{L_{t-}} = \mu_L dt + \sigma_L dW_{L,t}^{\mathbb{P}}(t) + dJ_t, \tag{59}$$

with drift μ_L and volatility σ_L and $L_{t-} = \lim_{v \uparrow t} L_v$.

Thereby, the variable J_t is independent of $W_{L,t}^{\mathbb{P}}$ and N_t and can be expressed by (see Gatzert and Schmeiser, 2008):

$$J_t = \sum_{j=1}^{N_t} (Y_j - 1). \tag{60}$$

with N_t, denoting a Poisson process with intensity λ and the size of the jump $Y_j - 1$. λ will be interpreted in the following as the average number of jumps per period.

We assume that consecutive values of Y_j are independent and identically distributed and that they follow a log-normal distribution with $\ln(Y_j) \sim \Phi(a, b^2)$.

The solution to the stochastic differential equation (59) can be expressed, for example, by (see Gatzert and Schmeiser, 2008):

$$L_t = L_0 \cdot \exp\left(\left(\mu_L - \sigma_L^2/2\right)t + \sigma_L W_{L,t}^{\mathbb{P}}(t)\right) \cdot \prod_{j=1}^{N_t} Y_j. \tag{61}$$

2.2.2 Modeling Dependencies

Another modification to the basic setting that we consider is the introduction of linear and nonlinear dependencies. In a first step, we examine the deviations in capital requirements and shortfall risks that originate from the introduction of linear dependencies into our basic setting.

Linear dependencies can be displayed via Pearson's linear correlation coefficient ρ. We assume a pairwise linear correlation between the stan-

dard Brownian motions $W^{\mathbb{P}}_{A_1,t}$, $W^{\mathbb{P}}_{A_2,t}$ and $W^{\mathbb{P}}_{L,t}$ of Equations (57) and (58):

$$\begin{aligned}
dW^{\mathbb{P}}_{A_1,t}dW^{\mathbb{P}}_{A_2,t} &= \rho(A_{1,t}, A_{2,t})dt, \\
dW^{\mathbb{P}}_{A_1,t}dW^{\mathbb{P}}_{L,t} &= \rho(A_{1,t}, L_t)dt, \text{ and} \\
dW^{\mathbb{P}}_{A_2,t}dW^{\mathbb{P}}_{L,t} &= \rho(A_{2,t}, L_t)dt.
\end{aligned} \tag{62}$$

Since the literature implies that the sole inclusion of linear correlation in modeling dependencies is often not sufficient when risks are heavy-tailed and skewed (see Eling and Toplek, 2009; Embrechts, McNeil, and Straumann, 2002), we subsequently examine the introduction of nonlinear dependencies via a copula function. The copula concept enables us to separate the marginal distributions from the multivariate dependence structure for continuous multivariate distribution functions.[43]

We investigate the modeling of nonlinear dependencies between high-risk investments, low-risk investments and the liabilities of the insurer using the Clayton copula, a copula function with a closed-form solution belonging to the family of the so-called Archimedean copulas. This copula has been studied extensively in the literature (see, e.g., Blum, Dias, and Embrechts, 2002; Embrechts, Lindskog, and McNeil, 2001; Embrechts et al., 2002; Embrechts, Höing, and Juri, 2003; Frees and Valdez, 1998; Kole, 2007; Malevergne and Sornette, 2003) and is often used in practice, since it is easy to calibrate (see Eling and Toplek, 2009; Sun, Frees, and Rosenberg, 2008; SCOR Switzerland AG, 2008). A key characteristic of Archimedean copulas is their construction via generator functions (see Nelsen, 2006).

We select the Clayton copula as it exhibits lower tail dependence.[44] This desirable copula property enables the Clayton copula to visualize typical adverse scenarios of the insurance industry, e.g., situations in which an insurer is simultaneously exposed to high losses in the insurance business and low asset returns from the capital markets (see also Eling and Toplek, 2009).

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[43]For an introduction to copula functions, see, e.g., Nelsen (2006).

[44]For an introduction to tail dependence see, e.g. Nelsen (2006) or Juri and Wüthrich (2002).

If $\phi : [0,1] \to [0,\infty]$ is a strict Archimedean generating function, an N-dimensional Archimedean copula is given by (McNeil et al., 2005):

$$C(u_1, ..., u_N) = \phi^{-1}\left(\phi(u_1) + \cdots + \phi(u_N)\right), \tag{63}$$

if and only if the generator inverse $\phi^{-1} : [0,\infty] \to [0,1]$ is completely monotonic.[45]

The generating function ϕ of the Clayton copula and its inverse ϕ^{-1} are given by (Wu, Valdez, and Sherris, 2006):

$$\phi(u) = \frac{u^{-\theta} - 1}{\theta} \tag{64}$$

and

$$\phi^{-1}(u) = (\theta \cdot u + 1)^{-1/\theta}. \tag{65}$$

Thus, the Clayton copula is represented by (Gatzert, Schmeiser, and Schuckmann, 2008):

$$C_{\theta,N}^{Cl}(u_1, ..., u_N) = \left(\sum_{i=1}^{N} u_i^{-\theta} - N + 1\right)^{-1/\theta}, \tag{66}$$

with $0 \leq \theta < \infty$.

The family of Archimedean copulas contains both exchangeable copulas and nonexchangeable copulas. As multivariate exchangeable Archimedean copulas produce a dependence structure not always applicable (see McNeil et al., 2005), we are going to use a three-dimensional nonexchangeable copula construction, as described by McNeil et al. (2005):

$$C(u_1, u_2, u_3) = \phi_2^{-1}\left(\phi_2 \circ \phi_1^{-1}\left(\phi_1(u_1) + \phi_1(u_2)\right) + \phi_2(u_3)\right), \tag{67}$$

consisting of the two strict Archimedean generators ϕ_1 and ϕ_2, with generator inverses ϕ_1^{-1} and ϕ_2^{-1} that are completely monotonic decreasing functions and the composite function $\phi_2 \circ \phi_1^{-1}$, which is a completely monotonic increasing function (see McNeil et al., 2005).

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[45]A decreasing function $g(t)$ is completely monotonic over an interval $[c,d]$ if it satisfies (McNeil et al., 2005): $(-1)^k \frac{d^k}{dt^k} g(t) \geq 0, k \in \mathbb{N}, t \in (c,d)$.

This construction makes it possible to combine different copula functions. Nevertheless, we will focus on the Clayton copula for both generators, so that ϕ_1 and ϕ_2 differ only in their parameter values (see also Eling and Toplek, 2009).

The generating function ϕ_1 and its corresponding parameter θ_1 model the dependence between high-risk and low-risk investments, and the generator ϕ_2 and its corresponding parameter θ_2 model the dependence between assets and liabilities.

2.3 Risk Measurement

2.3.1 Capital Requirements Based on Value at Risk

Most recent solvency frameworks, such as the Basel Accords, the Solvency II Proposal, as well as the Swiss Solvency Test are based on an economic capital concept. Following this concept, capital requirements for an insurance company calculate available economic capital AEC and solvency capital SC. An insurer's available economic capital is often called risk-bearing capital or risk-based capital (see, e.g. Basel II, Swiss Solvency Test, U.S.-NAIC Standards). It can be defined as the difference of the market value of assets and the market value of liabilities at time $t = [0, T]$ (see, e.g., FOPI, 2006):

$$AEC_t = A_t - L_t. \tag{68}$$

Solvency capital, on the other hand, is the amount of capital required at time 0, given a certain confidence level $1 - \alpha$, to be able to meet obligations over a particular future time period (see, e.g., FOPI, 2004). It is also called target capital (e.g., under the Swiss Solvency Test). Generally, regulators require that the AEC of the previous period $t - 1$ to be larger or equal to the SC in t (see Gatzert and Schmeiser, 2008):

$$AEC_{t-1} = A_{t-1} - L_{t-1} \geq SC_t. \tag{69}$$

The amount of SC_t depends on the stochastic model applied, the risk measure considered and the parameter setting. In the following,

we calculate solvency capital using the value at risk measure on the stochastic variable X_t ($\mathrm{VaR}_\alpha(X_t)$) at a confidence level of 99.5%, as is planned in the Solvency II framework (see EC, 2009). We hereby define the random variable X_t as the change in available economic capital within one year, discounting AEC_t with the risk-free interest rate r_f (see FOPI, 2006):

$$X_t = AEC_t/(1 + r_f) - AEC_{t-1}. \tag{70}$$

The amount of solvency capital is then calculated by:[46]

$$SC = -\mathrm{VaR}_{0.5\%}(X_t).^{47} \tag{71}$$

2.3.2 Shortfall Probability and Expected Policyholder Deficit

In addition to the capital requirements of Solvency II, we examine two shortfall risk measures. The shortfall probability at time t, $SP_t = P(A_t < L_t)$, defined as the event of the available economic capital becoming negative and the expected policyholder deficit at time t, $EPD_t = \mathrm{E}(\max(L_t - A_t, 0))/(1 + r_f)$, in order to capture the severity of insolvency via the expected insolvency cost (see, e.g. Barth, 2000).

2.4 Reducing the Risk of Model Misspecification by Means of Interim Financial Reports

The idea of requiring mid-year updates in order to certify solvency has gained new relevance in Europe, especially after the global financial crisis of 2007 to 2009. Within the context of its principles on integrated insurance supervision, the Swiss Financial Market Supervisory Authority (FINMA), for example, has conducted several interim reviews of the solvency situation and tied assets of Swiss insurers throughout 2008 (see FINMA, 2011). Moreover, the Insurance Supervision Act of Germany

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[46]The analyses were also carried out using tail value at risk as a risk measure with a 99% confidence level as required by the Swiss Solvency Test. However, since our basic setting is based on geometric Brownian motions and does therefore not incorporate heavy tails, the results for tail value at risk are very similar to the outcomes under value at risk. Thus, we eliminated tail value at risk from our analyses.

[47]Value at risk for a given confidence level is given by the quantile of the distribution $F^{-1}(\alpha)$ so that $\mathrm{VaR}_\alpha(X_t) = \inf\{x : F_X(x) \geq \alpha\}$.

requires quarterly financial reports on the latest accounting data and information on the insurer's portfolio composition (see, e.g., BaFin, 1992).

In this section, we aim to determine the extent to which the deviations in capital requirements and shortfall risks due to different model specifications can be diminished by mandatory interim financial reports in addition to the annual financial statements. The status quo of annual reports only (as required, for example, by the Solvency II Directive, EC, 2009), is compared to theoretical situations where the regulatory authority requires semiannual, quarterly or monthly updates of the financial information. In order to run this analysis we need the assumption that the basic setting (see Section 2.1) corresponds to the solvency model a regulatory authority uses to calculate capital requirements and shortfall risks and that the modifications to the framework displayed in Sections 2.2.1 and 2.2.2 are able to model the actual asset and liability processes in a more realistic way than the basic setting.[48]

The stylized graphs in Figure 8 illustrate this procedure for the case of the risk of model misspecification caused by introducing a jump component into the liability process of the insurance company. The analysis of the reduction in the model uncertainty caused by linear and nonlinear dependencies is conducted analogously.

The graph in Figure 8(a) depicts the view on liabilities of the regulatory framework, i.e., the basic setting, which will be denoted by the superscript BS in the sequel. Figure 8(b) depicts the evolution of the liabilities including a jump component in the insurer's liability process (model framework from Section 2.2.1). The corresponding values of the liability process are superscripted with JP.

In the absence of interim reports, a regulatory authority can only compare the values of assets and liabilities produced by the model framework it uses to calculate capital requirements and shortfall risks with the actual values of assets and liabilities disclosed in the annual reports at the end of each year. In the setting of Figure 8(a), the regulator there-

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[48]This assumption is needed in order to select the basic setting as the benchmark model so that the impact of mid-year financial updates on the deviations in the risk measures that are due to different model specifications can be measured.
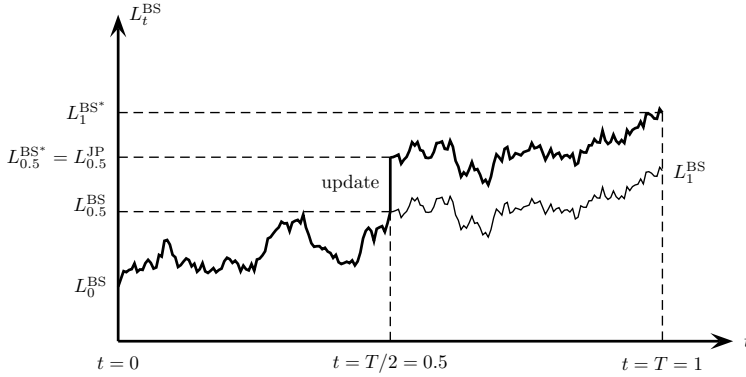
(a) Stylized illustration of a semiannual update of the liability process from the basic setting (BS) with values from the framework including stochastic jumps (JP), see Figure 8(b). The starred values correspond to the values obtained after updating at time t=0.5.
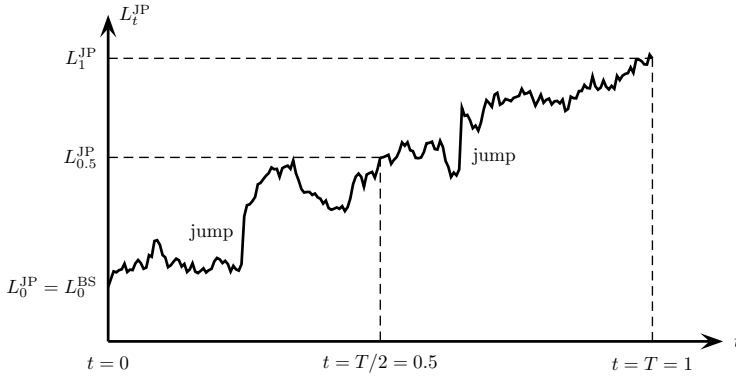
(b) Stylized illustration of the development of liabilities in the model framework including a jump component (JP), displayed by two jumps in the considered period $(T = 1)$.

Figure 8: Illustration of the Update Mechanism when Requiring Semiannual Reports in Addition to the Annually Submitted Financial Information

fore compares the end-of-year value of liabilities of the basic setting L_1^{BS} to the end-of-year value of liabilities of the modified framework with a jump component L_1^{JP}.

In contrast, in the presence of semiannual interim reports, the regulatory authority is able to update its information on the values of assets and liabilities of the insurer after the first half of the financial year. That is, it can compare the value of liabilities of the basic setting $L_{0.5}^{\text{BS}}$ to the value of liabilities of the jump component model framework $L_{0.5}^{\text{JP}}$ and in case of a deviation can set $L_{0.5}^{\text{BS}}$ to $L_{0.5}^{\text{JP}}$ for further calculations of capital requirements and shortfall risks throughout the rest of the business year. The updated value of $L_{0.5}^{\text{BS}}$ is denoted by $L_{0.5}^{\text{BS}^*}$ in Figure 8(a). By doing so, the regulatory authority implicitly takes into account the jump in liabilities that occurred during the first half of the business year, but it cannot account for the jump occurring during the second half of the business year. Therefore, there is still a deviation between $L_1^{\text{BS}^*}$ and L_1^{JP} due to the evolution between $t = 0.5$ and $t = 1$, even though it is smaller than the deviation when looking at annual reports only.

3   Numerical Analyses

This section contains numerical analyses for examining the risk of model misspecification and the ability to reduce this risk as described in the last section. Results are based on a reference case, so as to focus directly on the research question. Additionally, a sensitivity analysis is conducted. Our numerical examples are based on Monte Carlo simulations.[49] Since some results are displayed as the ratio of partly very low values of risk measures (e.g., close-to-zero shortfall probabilities), a high number of simulations is needed in order to keep reasonable accuracy. Therefore, all numerical examples are calculated with a minimum of 5'000'000 iterations. Simulations involving nonlinear dependencies or combining geometric Brownian motions with a jump process are evaluated using 10'000'000 runs.

After providing the input parameters in Section 3.1, we present our simulation results regarding the uncertainties of different model specifica-

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[49]For an introduction see, e.g., Glasserman (2004).

tions included into the basic setting in Section 3.2. Furthermore, Section 3.3 contains the numerical results of the reduction of model uncertainty that is due to the requirement of interim reports. Finally, we conduct a sensitivity analysis for the deviations in the three risk measures with respect to the liability-to-asset ratio and an asset-to-asset ratio in Section 3.4.

3.1 Input Parameters

In the following, we present the parameter configuration for our reference case. Table 13 summarizes the model parameters, their definitions, as well as their initial values for our numerical analyses.

In the simulation study we consider a one-year time horizon ($T = 1$). To this end, we assume asset class 1 to consist of different stocks and asset class 2 to be a portfolio of government bonds. As a representative for the mean μ_{A_1} of asset class 1, we therefore take the average rate of return on the Swiss Market Index of 8% between 1988 and 2009, and for σ_{A_1} the volatility of the SMI of approximately 20%. In order to proxy asset class 2, we calculate the average return on the SBI Domestic Government for the period of 1997 to 2009 to get $\mu_{A_2} = 4\%$. Taking the same proxy for the volatility, we get a σ_{A_2} of approximately 4%. We set the market values of the high-risk assets to 0.5 billion currency units (CU), and the market value of low-risk assets (asset class 2) to 9.5 billion currency units, according to values found among Swiss market participants. The market value of liabilities is set to CU 8 billion.

Since the analyses are conducted for the insurance sector, it is reasonable to assume that correlation coefficients between assets and liabilities are zero.[50] The linear correlation coefficient for asset class 1 and 2, $\rho(A_1, A_2)$, is examined within the interval of [0,1]. The copula parameter θ_1 of the Clayton copula is examined in the interval $[0, 8]$ and the second copula parameter θ_2 is set to 0.

Another important set of input parameters concerns the jump component within the liability process of Section 2.2.1. Since there is no readily available data on magnitudes of catastrophes (see Cummins, 1988), we

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[50]This is also in line with the assumptions of the Solvency II standard approach.

Parameter	Symbol	Initial value at $t = 0$
Time horizon	T	1
Size of time steps within T	dT	$\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{12}\}$
Market value of asset class 1 (high-risk investments)	$A_{1,0}$	CU 0.5 billion
Market value of asset class 2 (low-risk investments)	$A_{2,0}$	CU 9.5 billion
Market value of liabilities	L_0	CU 8 billion
Risk-free rate of return	r_f	0.02
Drift of the geometric Brownian motion of asset class 1	μ_{A_1}	0.08
Drift of the geometric Brownian motion of asset class 2	μ_{A_2}	0.04
Drift of the geometric Brownian motion of liabilities	μ_L	0.03
Volatility of the geometric Brownian motion of asset class 1	σ_{A_1}	0.2
Volatility of the geometric Brownian motion of asset class 2	σ_{A_2}	0.08
Volatility of the geometric Brownian motion of liabilities	σ_L	0.05
Pearson's correlation coefficient between asset class 1 and asset class 2	$\rho(A_1, A_2)$	$[0, 1]$
Pearson's correlation coefficient between asset class 1 and liabilities	$\rho(A_1, L)$	0
Pearson's correlation coefficient between asset class 2 and liabilities	$\rho(A_2, L)$	0

Table 13: Input Parameters for the Reference Case

Parameter	Symbol	Initial value at $t = 0$
Copula parameter modeling the dependence between asset class 1 and 2	θ_1	$[0, 8]$
Copula parameter modeling the dependence between assets and liabilities	θ_2	0
Expected value of Y_j	$E(Y_j)$	1.05
Volatility of Y_j	$\sigma(Y_j)$	0.05
Average number of jumps over time horizon (intensity)	λ	$[0, 0.5]$
Parameter for the log-normal distribution of Y_j	a	0.048
Parameter for the log-normal distribution of Y_j	b^2	0.023
Exceedance probability of value at risk	α	0.005

Table 13: Input Parameters for the Reference Case – continued

set $\lambda = 0.2$ and $E(Y) = 1.05$, which implies a jump of magnitude 5% every five years, on average. The dispersion parameter $\sigma(Y)$ is fixed at 5%. Parameters a and b^2 for the log-normal distribution of the jump process are derived as 0.048 and 0.0023, respectively.

3.2 Numerical Illustration

This section numerically illustrates the impact of different model specifications on capital requirements and shortfall risks.

Table 14 shows the numerical results from the basic setting of Section 2.1 and the results from introducing the different model modifications, as described in Section 2.2.1 and 2.2.2. The values in brackets denote the percentage increase, compared to the value of the basic setting due to different model specifications.

Considering the first column of the Table, we find that without a jump component in the liability process and no dependencies, the expected policyholder deficit (EPD) and the shortfall probability (SP) are relatively low in the basic setting. With respect to the solvency capital it is noticeable that within the model specification including stochastic jumps (second column), the requirement of regulators that the available economic capital of the previous year should always exceed the solvency capital requirement of the current year (see Equation 69) is violated (cf. $AEC_0 = A_0 - L_0 = 20 \cdot 10^{-8} < SC_1 = 23.3 \cdot 10^{-8}$).

When looking at the percentages depicted in brackets, one can see that the solvency capital reacts much less sensitive to changes in the specification of the solvency model than the shortfall measures (SP and EPD). For example, a change from no jumps in the assumed liability process of the insurer to jumps that occur every five years increases capital requirements by approximately 74%. In contrast, the shortfall probability is increased by 10'500% and the expected policyholder deficit by approximately 31'000%. The observation that solvency capital requirements can be relatively insensitive toward changes in model specifications while shortfall probabilities and expected policyholder deficits are subject to considerable change can also be made in case of introducing linear and nonlinear dependence into the basic setting (see columns three and

four of Table 14). This finding is in line with previous studies, such as Butsic (1994) and Barth (2000). For example, Barth (2000) finds that insurers with different shortfall probabilities and vastly different severities may still have the same EPD ratio.

In a next step, we examine the deviations in each of the three risk measures due to the inclusion of stochastic jumps or dependencies. Figures 9, 10, and 11 illustrate our simulation results that represent the impact of different model specifications on capital requirements and shortfall risks (shortfall probability and expected policyholder deficit). In particular, they display the ratio of the respective risk measures in %:

$$\text{risk measure ratio} = \frac{\text{risk measure}^{\text{BS}}}{\text{risk measure}^{*}}, \tag{72}$$

where the superscript BS refers to the basic setting, while the superscript * represents one of the modifications to the solvency model described in Section 2.2 that will be denoted in the following by JP for the jump component, LD for linear dependence, and NLD for the Clayton copula case.

Equation (72) can be interpreted as the fraction of the respective risk measure's value, calculated on the basis of the modifications to the framework that can be "explained" by this risk measure's value computed from the basic setting (BS). It is therefore a measure of the explanatory power, the basic setting possesses, when considering the introduction of a jump component into the liability process of the insurance company, linear dependence between asset classes, or nonlinear dependence between asset classes. Consequently, the complementary value $(1 - \text{risk measure ratio})$ can be seen as a measure of the model misspecification risk.

Figure 9 shows the percentage deviations (the risk measure ratio in %) in the three risk measures that are due to the inclusion of a jump component in the insurer's liability process of the basic setting. Here, the average number of jumps per period (λ) is considered. In order to compare the two model frameworks the parameter λ is examined within an interval of $[0, 0.5]$. That is, we allow for up to one jump every two

Risk measure	Basic Model	Model with jumps $(\lambda = 0.5)$	Model with linear correlation $(\rho = 1)$	Model with Clayton copula $(\theta_1 = 8)$
SC	$13.4 \cdot 10^8$	$23.3 \cdot 10^8$	$14.8 \cdot 10^8$	$14.7 \cdot 10^8$
	(0.0%)	(73.9%)	(10.4%)	(9.7%)
SP	0.0001	0.0106	0.0003	0.0003
	(0.0%)	$(10'500.0\%)$	(200.0%)	(200.0%)
EPD	$15.3 \cdot 10^3$	$47.6 \cdot 10^5$	$53.4 \cdot 10^3$	$49.7 \cdot 10^3$
	(0.0%)	$(31'011.1\%)$	(249.0%)	(224.8%)

Table 14: Risk Measure Values

Values of the three risk measures in the basic setting, the jump component model modification (for $\lambda = 0.5$), the specification of the model including linear correlation between $A_{1,t}$ and $A_{2,t}$ (with $\rho = 1$) and the modification that introduces nonlinear dependence between $A_{1,t}$ and $A_{2,t}$ via a Clayton copula (with $\theta = 8$). Values in brackets denote the percentage increase compared to the value of the basic setting.

years. Obviously, the two model frameworks (the basic setting and the modification including a jump component) lead to the same values of the risk measures, when λ in the jump component model is set to 0. Therefore, at $\lambda = 0$, the basic setting is able to "explain" 100% of the model framework of Section 2.2.1. However, as λ increases (allowing for a higher jump intensity), the deviations in the risk measures increase and therefore the basic setting "explains" less and less of the other model specification. Thus, all three curves are downward sloping.

When looking at Figure 9, one notices the similar development of the deviations in the shortfall probability (SP) and the expected policyholder deficit (EPD), although the slope of the EPD curve is more steep and the EPD curve lies always below the SP curve. Another important finding is that the deviations in solvency capital and shortfall risk measures evolve very differently. While the basic setting is still able to "explain" more than 50% of the development of value at risk when $\lambda = 0.5$, the deviations in shortfall probability and expected policyholder deficit of the two model frameworks are close to 100% when $\lambda = 0.5$, so that the basic setting "explains" less than 5% considering the values of the deviations in EPD and SP.

Figure 10 illustrates the deviations in risk measures when comparing the basic setting with the model framework in Section 2.2.2 including the positive linear correlation between the two asset classes A_1 and A_2. One can see that the three curves in Figure 10 are less steep and stay at a higher overall level than those of Figure 9. This is due to the parameter settings that involve a liability-to-asset ratio of 80% so that stochastic jumps in liabilities affect the solvency of the firm to a large extent.

Turning to Figure 11, we again find three downward sloping curves displaying the respective deviations in the risk measures by comparing the basic setting with the Clayton copula model of Section 2.2.2. In this case, the copula parameter θ is considered within an interval of $[0, 8]$. Contrasting the copula case to the case of linear correlation, we can observe that the three curves displaying the deviations in solvency capital and in the shortfall risks evolve in a similar way and at a similarly high
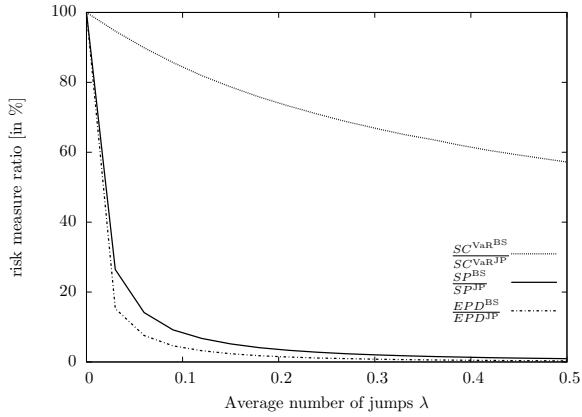
Figure 9: Deviations in Risk Measures when Including a Jump Component into the Basic Setting with Different Values for the Average Number of Jumps per Year $\lambda \in [0, 0.5]$
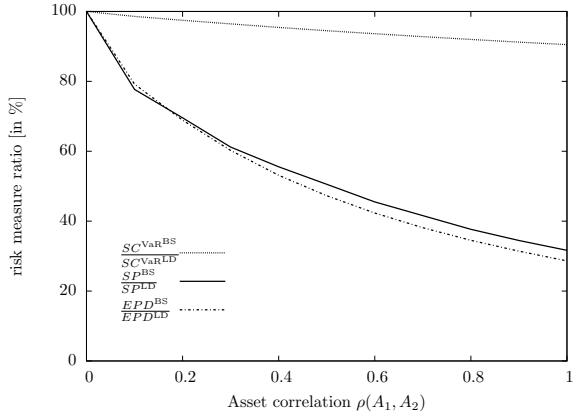

Figure 10: Deviations in Risk Measures when Including Linear Correlation Between the Two Asset Classes, A_1 and A_2, for $\rho(A_1, A_2) \in [0, 1]$
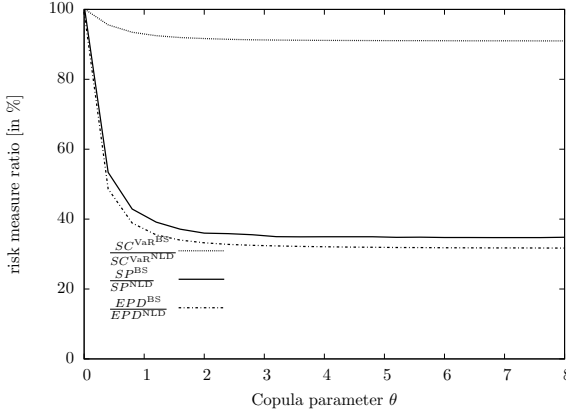
Figure 11: Deviations in Risk Measures when Including Nonlinear Dependence through a Clayton Copula with Parameter $\theta \in [0, 8]$

level. The difference relative to Figure 10 lies in the steeper slopes of the curves. That is, the explanatory power of the basic setting is restricted a lot faster when including nonlinear dependence instead of linear correlation.

When comparing Figure 11 with Figures 10 and 9, we find that the resulting values for the risk measure ratio calculated from the highest considered value of $\rho(A_1, A_2)$ and θ (that is $\rho(A_1, A_2) = 1$ and $\theta = 8$) are approximately the same for all three risk measures. In contrast, the risk measure ratios in the model specification including jumps with $\lambda = 0.5$ are different from those of Figures 10 and 11. As mentioned before, this is due to the liability-to-asset ratio which causes manipulations to the liability process having a greater impact than manipulations to the asset processes.

3.3 Numerical Results of the Reduction in Misspecification Risk by Means of Interim Financial Reports

In the following, we present the simulation results for Section 2.4. Table 15 displays the deviations in the risk measures (according to Equation (72)) depending on the number of updates of the financial information from a regulatory view. Four different frequencies in the updating of financial information are considered: A yearly update which corresponds to the disclosure of annual financial reports of insurance companies, semi-annual, quarterly, and monthly updates on the financial information. The three sub-tables refer to the impact of different numbers of updates on the risk measure ratios when comparing the basic setting with the modifications described in Section 2.2, respectively.

Table 15(a) shows the impact of different numbers of updates on the respective risk measure ratios when comparing the basic setting with the model specification including a jump component (λ set to 0.2). As already seen in Section 14, we again find that the deviations in solvency capital and the two shortfall risk measures have very different values.

A new important finding here is that the explanatory power of the basic model framework can already be improved substantially, when the regulatory authorities require semiannual reports instead of only annual reports. Here, the explanatory power of the basic setting with respect to SC is increased from approximately 74% to approximately 88% and regarding the deviations in the shortfall risk measures (SP and EPD) the explanatory power of the basic setting is more than ten times greater in case of semiannual reports rather than annual reports.

Obviously, the additional improvement in the explanatory power of the basic setting is decreasing in the number of updates as in case of continuous updates the basic setting would exactly reproduce the risk measure values of the other model specifications of Sections 2.2.1 and 2.2.2.

Monthly updates of the values of assets and liabilities of the insurer already imply an explanatory power of the basic setting of approximately 98% with respect to capital requirements and 88.8% (87.1%) when con-

sidering the shortfall probability (the expected policyholder deficit) of our insurance company.

Table 15(b) displays the impact of the number of updates, n, on the risk measure ratio when comparing the basic setting to the case including a linear correlation coefficient of $\rho = 0.2$. One can see that the values of the respective risk measure ratio for $n = 1$ are already relatively high for all risk measures considered.[51] Therefore, the improvements in the risk measure ratios when switching from annual financial reports to semiannual financial reports are not as large as in Table 15(a). Nevertheless, the semiannual values of the deviations in SP and EPD are increased by 16.8% and 14.2% respectively, and the values of the deviations in SC are increased by more than 1%. With monthly updates, the basic setting receives an explanatory power of more than 99% for all three risk measures.

Finally, Table 15(c) comprises the values for the risk measure ratios in case of a comparison between the basic setting and the Clayton copula case with $\theta = 0.5$. Here, the values for $n = 1$ are lower compared to Table 15(b), so the improvements in the explanatory power of the basic model framework switching from $n = 1$ to $n = 2$ are larger, particularly in case of the two shortfall risk measures. Again, requiring monthly interim reports implies an almost complete reduction of misspecification risk.

Although the risk measure ratios in case of annual updates are different for the modifications to the solvency model compared to the basic setting (see Tables 15(a), 15(b), and 15(c)), we find similar developments of the risk measure ratios when increasing n in all three tables. Overall, monthly updates are able to reduce misspecification risk (defined as $1 -$ risk measure ratio in Section 3.2) faced by regulators to a large extent. Even though a frequency of $n = 12$ for disclosing financial information of an insurance company might not be a realistic and manageable requirement, our analysis shows that semiannually or quarterly reports would already reduce the misspecification risk significantly.

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[51] This is due to the choice of a relatively low correlation coefficient of $\rho = 0.2$.

(a) Impact of n on risk measure ratios when including stochastic jumps with $\lambda = 0.2$.

n	$\frac{SC^{BS}}{SC^{JF}}$	$\frac{SP^{BS}}{SP^{JF}}$	$\frac{EPD^{BS}}{EPD^{JF}}$
1	74.0%	3.7%	1.5%
2	87.7%	42.5%	37.2%
4	94.4%	70.1%	64.8%
12	98.0%	88.8%	87.1%

(b) Impact of n on risk measure ratios when including linear correlation with $\rho = 0.2$.

n	$\frac{SC^{BS}}{SC^{LD}}$	$\frac{SP^{BS}}{SP^{LD}}$	$\frac{EPD^{BS}}{EPD^{LD}}$
1	97.5%	69.6%	68.9%
2	98.7%	86.4%	83.1%
4	99.3%	89.8%	87.4%
12	99.8%	99.6%	99.1%

(c) Impact of n on risk measure ratios when including a Clayton copula with $\theta = 0.5$.

n	$\frac{SC^{BS}}{SC^{NLD}}$	$\frac{SP^{BS}}{SP^{NLD}}$	$\frac{EPD^{BS}}{EPD^{NLD}}$
1	94.6%	48.4%	44.1%
2	97.7%	78.0%	73.7%
4	98.9%	88.2%	89.3%
12	99.8%	98.4%	98.9%

Table 15: The Impact of the Number of Updates (n) on the Financial Information per Year on Risk Measure Ratios

Number of updates (n) on financial information per year and its impact on risk measure ratios comparing the basic setting to the model specifications including stochastic jumps, linear, or nonlinear dependencies. (Values are rounded to one decimal place.)

3.4   Sensitivity Analysis

In this section, a sensitivity analysis for the deviations in the different risk measures is conducted with respect to the liability-to-asset ratio and the ratio of high risk assets (asset class 1) to total assets.

Figure 12 shows the comparison of the basic setting to its modification that includes a jump component in the insurer's liability process with $\lambda = 0.2$.

In Figure 12(a) the risk measure ratios for the quotient of high-risk assets to total assets ($A_{1,0}/A_0 \in [0,1]$) is illustrated. The first thing one notices is that the course of the SP and EPD curve is similar. All three curves exhibit a minimum at an asset-to-asset ratio of approximately 0.05 and are increasing in $A_{1,0}/A_0$. The parameter setting of our numerical examples above with an asset-to-asset ratio of $A_{1,0}/A_0 = 0.05$ therefore considers a worst case scenario in which the explanatory power of the basic setting is lowest with respect to all risk measures. While the deviations in the shortfall risk measures vary a lot according to the asset-to-asset ratio, the explanatory power of the basic setting stays at a relatively high overall level in case of the solvency capital.

Turning to Figure 12(b), the risk measure ratios for the quotient of liabilities to assets (L_0/A_0) is illustrated. We find that the values of the deviation in SC increase in the liability-to-asset ratio, so that the curve is downwards sloping. Considering the SP and EPD curves, one can see that they exhibit minimum values of 0 at an L_0/A_0 ratio of approximately 0.65. However, the values for the deviations in SP and EPD are displayable only from a value for the liability-to-asset ratio of 0.65 onwards as SP^* and EPD^* are close to zero for values $L_0/A_0 < 0.65$. As a consequence, those values are not available. The same reasoning applies to the missing values for the deviations in SP and EPD in Figures 13(b) and 14(b). What concerns our reference case, Figure 12(b) shows that a liability-to-asset ratio of 0.8 (see Table 13) implies a relatively low explanatory power of the basic setting with respect to the jump diffusion model.
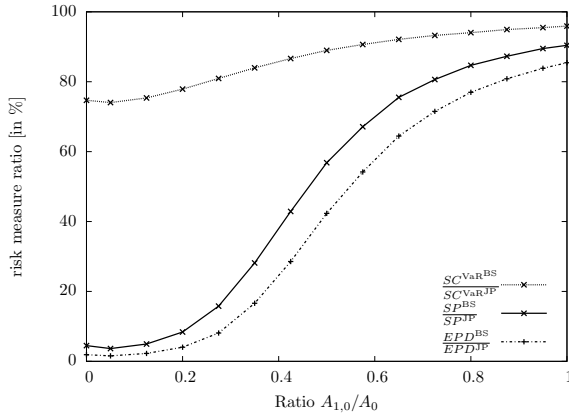
(a) Risk measure deviations depending on the $A_{1,0}/A_0$ ratio when comparing the basic setting with the modification of Section 2.2.1 with $\lambda = 0.2$.
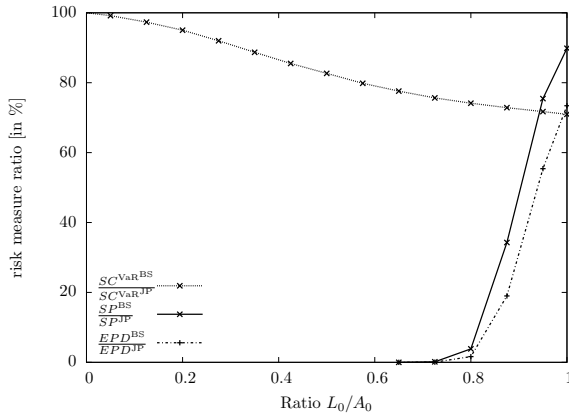

(b) Risk measure deviations depending on the L_0/A_0 ratio when comparing the basic setting with the modification of Section 2.2.1 with $\lambda = 0.2$.

Figure 12: Illustration of the Risk Measure Deviations with Respect to Variations of the Investment Riskiness in Graph (a) (Ratio of High Risk Assets $A_{1,0}$ to Total Assets A_0) and the Liability-to-Asset Ratio in Graph (b) (Liabilities L_0 to Total Assets A_0) in the Model Specification with Stochastic Jumps for $\lambda = 0.2$
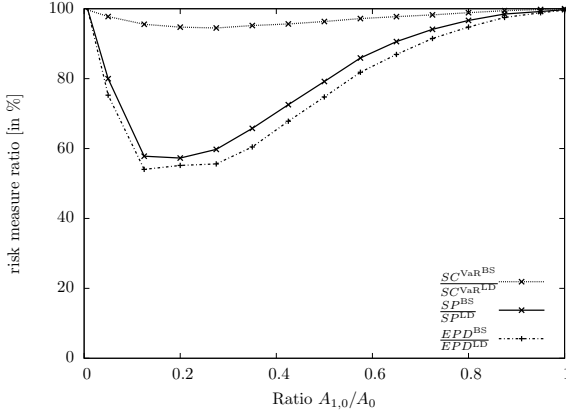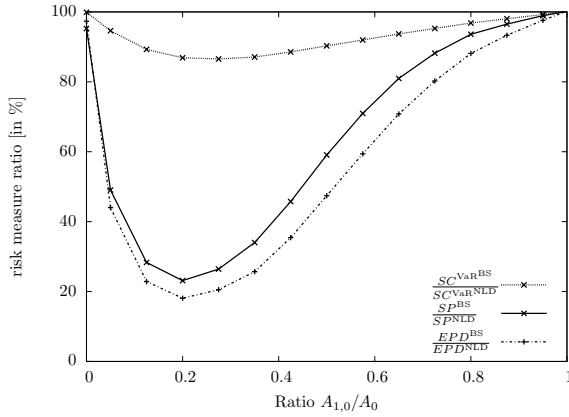
(a) Risk measure deviations for different $A_{1,0}/A_0$ ratios in model specification with linear correlation $\rho(A_1, A_2) = 0.2$.
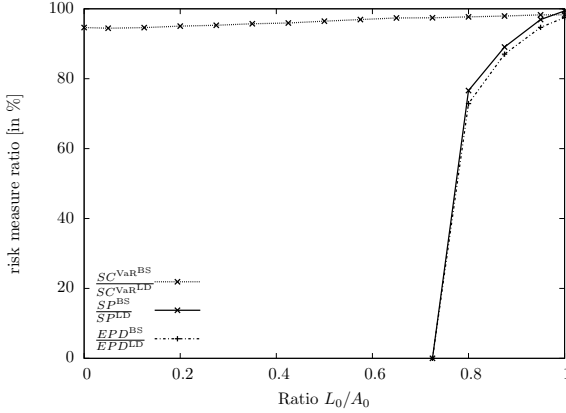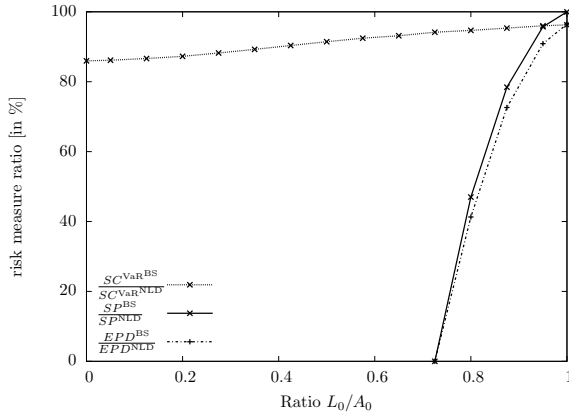

(b) Risk measure deviations for different L_0/A_0 ratios in model specification with linear correlation $\rho(A_1, A_2) = 0.2$.

Figure 13: Illustration of the Risk Measure Deviations with Respect to Variations of the Investment Riskiness in Graph (a) (Ratio of High Risk Assets $A_{1,0}$ to Total Assets A_0) and the Liability-to-Asset Ratio in Graph (b) (Liabilities L_0 to Total Assets A_0) in the Model Specification Including Linear Dependence between the Two Asset Classes

(a) Risk measure deviations for different $A_{1,0}/A_0$ ratios in model specification with nonlinear correlation $\theta = 0.5$.

(b) Risk measure deviations for different L_0/A_0 ratios in model specification with nonlinear correlation $\theta = 0.5$.

Figure 14: Illustration of the Risk Measure Deviations with Respect to Variations of the Investment Riskiness in Graph (a) (Ratio of High Risk Assets $A_{1,0}$ to Total Assets A_0) and the Liability-to-Asset Ratio in Graph (b) (Liabilities L_0 to Total Assets A_0) in the Model Specification Including Nonlinear Dependence between Assets via a Clayton Copula

Figure 13 shows the sensitivity analysis comparing the basic setting with the model specification that includes linear dependence between the two asset classes (assuming $\rho(A_1, A_2) = 0.2$).

Figure 13(a) depicts the risk measure deviations for different ratios of $A_{1,0}/A_0$. It is very similar to Figure 12(a), except that all curves are located at a higher overall level. This is due to the parameter settings chosen (especially, the relatively moderate linear correlation of $\rho = 0.2$). Also the minimum values of the curves are shifted to the right compared to the specification of the model with a jump component.

With respect to Figure 13(b), one finds a very flat SC-curve when increasing the liability-to-asset ratio. In contrast to Figure 12(b), the values here are a lot higher and slightly increasing in L_0/A_0. Turning to the deviations in the shortfall risk measures of Figure 13(b), we see a great similarity to the developments of the deviations in SP and EPD in Figure 12(b). However, the curves in Figure 13(b) are steeper and concavely shaped, so that the volatility of the deviations in SP and EPD is even higher.[52]

In Figure 14, we finally consider the Clayton copula case of our analysis varying again both $A_{1,0}/A_0$ and L_0/A_0 within the interval $[0,1]$. In these calculations, parameter θ is set to a relatively low value of 0.5. Figure 14 is similar to the last figure considered. However, the variations in the risk measure ratios for all measures of risk are larger so that the minimum values of the curves displayed in 14(a) and 14(b) lie below the corresponding values of Figures 13(a) and 13(b). This implies that the introduction of nonlinear dependence via a Clayton copula has in principle the same effect on the risk measure ratios as positive linear correlation when considering the quotient of asset class 1 to total assets and the quotient of liabilities-to-assets, only the impact of positive nonlinear dependence in form of a Clayton copula is greater.

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[52]The values for a liability-to-asset ratio within the interval of $[0,0.725]$ are missing as the denominators (SP^* and EPD^*, respectively) in Equation (72) are close to zero.

3.5   Implications

In Section 3.2, we have shown that the deviations in shortfall risk measures (SP and EPD) develop in a similar manner when comparing the basic setting to both a jump component and different dependencies between assets as described in Sections 2.2.1 and 2.2.2.

Section 3.2 also displays the different impact of the three model specifications on the explanatory power of the basic setting. The introduction of nonlinear dependencies by means of a Clayton copula and the implementation of a stochastic jump component seem to restrict the explanatory power of the basic setting the fastest. This suggest that if the different asset classes of insurance companies really exhibit nonlinear dependencies or if the insurers are exposed to jumps in their liabilities, then the lack of including these characteristics of assets and liabilities into a solvency model results in considerable risk of model misspecification.

An important finding that is illustrated by Table 14 is that the two shortfall risk measures considered in this paper react much more sensitive to changes in the modeling of an insurer's solvency than the value at risk measure specified by the Solvency II Directive. The same result can be found when considering Figures 9, 10 and 11. Furthermore, the convergence values of almost zero for the shortfall risk measures, and approximately 75% for the solvency capital lead to the conclusion that it might not suffice to consider just one class of risk measures within a solvency framework or capital standard. The concentration on capital requirements might be misleading in the sense that insurers and regulators could conclude that model misspecifications did not lead to considerable changes in the solvency situation of the insurer as they do not lead to considerable changes in capital requirements. In contrast, the two shortfall risk measures of our analyses deliver a completely different picture of the model uncertainty arising from the change in model specifications. Only slight increases in λ, ρ, and especially θ suffice to change the risk measure ratio significantly and therefore introduce considerable misspecification risk. A regulator aiming to protect policyholders and shareholders should therefore be taking the sensitivity of the expected policyholder deficit to changes in the model specifications additionally into account when developing a risk-based solvency framework.

With respect to the reduction of misspecification risk through interim financial reports, Table 13 shows that monthly updates on the financial information, available to regulators, lead (in the simulation context of this paper) to high risk measure ratios and therefore reduce the risk to a large extent. But as already pointed out in Section 3.3, semiannually disclosed financial information can already reduce misspecification risk in a significant way. This suggests that increasing the number of required interim financial reports might be a powerful tool to reduce the risk that is due to an inappropriate model choice in the context of solvency assessment of insurance companies.

4   Conclusion

This paper analyzes the risk of model misspecification within the context of solvency models for insurance companies. Misspecification risk, as a significant part of model risk, is not only important to be considered by financial institutions applying internal models of solvency but also by regulatory authorities aiming to design and implement sound risk-based capital standards and standard solvency models.

From a general solvency framework which models the market value of assets and liabilities as independent geometric Brownian motions, our paper examines the effects of changes to this basic setting on three risk measures: solvency capital, shortfall probability and expected policy-holder deficit. In particular, the deviations in risk measures is considered when including a jump component into the stochastic process of liabilities, or when introducing linear correlation or nonlinear dependencies (via a Clayton copula function) between asset classes. In order to do so, we calculate the ratio of the respective risk measure calculated from the basic model framework to the same risk measure calculated from the modified framework including either stochastic jumps into the liability process or dependencies between asset classes. Additionally, we take a regulatory view and consider to which degree the deviations in capital requirements and shortfall risks due to different model specifications can be diminished, by means of requiring interim financial reports in addi-

tion to the annually required information on the solvency and financial condition of an insurance company.

We have three main findings, each with important insights into the misspecification risk immanent in solvency frameworks for insurance companies.

Firstly, our numerical results show that the three modifications to the solvency model (jumps, linear correlation, and nonlinear dependence) affect the deviations in capital requirement and shortfall risks to different extents. The greatest impact on the risk measure values can be found, when introducing nonlinear dependence between asset classes by means of a Clayton copula function and when allowing for jumps in the market value of liabilities. This implies that if the dependence between the different asset classes of insurance companies can really be best approximated by the copula concept or if insurers are exposed to jumps in their liabilities, then the lack of including these features into a solvency model results in considerable risk of model misspecification.

Secondly, the shortfall risk measures react much more sensitive than the solvency capital when modifying the basic setting. We therefore conclude that analyzing the sensitivity of capital requirements within a solvency model might underestimate the actual misspecification risk that an insurance company is exposed to. The sensitivity of shortfall risk measures such as the expected policyholder deficit is particularly relevant from a policyholder's point of view.

Thirdly, the numerical results show that mandatory interim reports on the solvency and financial situation of an insurer might be a useful way to reduce the risk of model misspecification faced by a regulatory authority.

Current regulatory frameworks in Europe include linear dependence, but often ignore nonlinear dependence and stochastic jumps, which is also true for many internal models used by insurers (see also Eling and Toplek, 2009). The subprime financial crisis of 2007 to 2009 has illustrated the relevance of such scenarios and the importance of model risk in a solvency context. In addition to a theoretical underpinning of these

insights, our paper shows that the sensitivity of risk measures plays an important role for the misspecification risk immanent in solvency models. We therefore conclude by recommending to include sensitivity analyses for shortfall risk measures, such as the expected policyholder deficit, in addition to the capital requirements into the quantitative models of solvency regulation.

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