Dynamic Elimination Contests with Heterogeneous Participants: Theory and Experimental Evidence

DISSERTATION
of the University of St.Gallen,
School of Management,
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and International Affairs
to obtain the title of
Doctor of Philosophy in Economics and Finance

submitted by

Rudi Stracke

from

Germany

Approved on the application of

Prof. Dr. Uwe Sunde

and

Prof. Dr. Martin Kolmar

Dissertation no. 4130

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The University of St.Gallen, School of Management, Economics, Law, Social Sciences and International Affairs hereby consents to the printing of the present dissertation, without hereby expressing any opinion on the views herein expressed.

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The President:

Prof. Dr. Thomas Bieger

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Abstract

This dissertation uses game-theoretic and experimental methods to analyze the behavior of optimizing agents in different contest environments.

Part 1 contributes to the literature on contest theory and derives the subgame perfect Nash equilibrium solution of a multi-stage pair-wise elimination contest with heterogeneous participants. Subsequently, equilibrium properties of this dynamic contest format are compared to properties of a static one-shot contest. The comparison indicates that the effect of heterogeneity on contest participants is structure specific: While total outlays in both formats are identical in interactions between homogeneous agents, total outlays are strictly higher in the dynamic than in the static contest when agents are heterogeneous.

Part 2 considers a principal who organizes a tournament between heterogeneous employees. Comparing the incentive effect of heterogeneity in different tournament structures, the results indicate that the effect is always negative in static formats, but often positive in dynamic elimination tournaments. Experimental evidence from lab experiments confirms these theoretical predictions. Subsequently, the dissertation investigates to what extent a promotion tournament can accomplish both the selection of the most able employee and the provision of incentives. The results suggest that any tournament with heterogeneous participants provides some incentives for effort and some sorting of types. However, modifications which improve the performance in one will deteriorate the performance in the other dimension, i.e., tournament formats that perform better in terms of incentive provision do worse in terms of selecting the best participant, and vice versa.

Part 3 uses experimental methods to analyze the effect of prize structure variations on optimal behavior. Initially, we consider a single prize treatment, which is supposed to maximize total effort provision, and a treatment with multiple prizes which ensure incentive maintenance across stages. While the experimental design does not introduce any ex-ante heterogeneity between subjects, we observe ex-post heterogeneity – risk attitudes by experimental subjects have a strong effect on their behavior. In a next step, we compare a treatment where agents receive an immediate reward for winning stage 1 with a specification where the reward for winning stage 1 is delayed until the stage-2 interaction is over. The results indicate that stage-1 effort choices by experimental subjects are higher in the delayed than in the immediate reward treatment, while effort provision in stage 2 does not differ between treatments. The finding that differences across treatments are fully explained by risk attitudes suggests that experimental subjects are separately evaluating each stage of the contest.

Zusammenfassung

Die vorliegende Arbeit untersucht die Auswirkungen von Strukturveränderungen auf das Verhalten von Wettkampfteilnehmern.

Im ersten Teil der Arbeit wird das teilspielperfekte Nash-Gleichgewicht in einem mehrstufigen Wettkampfmodell mit paarweiser Elimination und heterogenen Teilnehmern bestimmt. Anschliessend werden die Gleichgewichtsentscheidungen von Spielern in diesem mehrstufigen Wettkampfmodell mit den optimalen Entscheidungen in einem einstufigen Wettkampf verglichen. Unter der Annahme homogener Teilnehmer sind beide Wettkampfmodelle strategisch äquivalent. Im generellen Fall mit heterogenen Teilnehmern zeigt sich jedoch, dass die Investitionsanreize im mehrstufigen Wettkampf höher als im einstufigen Wettkampf sind. Der Effekt von Heterogeneität auf die optimalen Investitionsanreize hängt also von der Wettkampfstruktur ab.

Der zweite Teil der Arbeit betrachtet Wettkämpfe als Modellrahmen für Bonus- und Beförderungsturniere in Unternehmen. Zunächst wird untersucht, inwieweit die in der Personalökonomie weit verbreitete Ansicht, dass Heterogeneität zwischen Beschäftigten eines Unternehmens die Leistungsanreize in Bonusturnieren reduziert, generell zutrifft. Dabei zeigt sich, dass Heterogenität in dynamischen Turnieren oft zu positiven Anreizeffekten führt, während dieser Effekt in den bisher schon häufig betrachteten statischen Turnieren immer negativ ist. Dieses Muster zeigt sich auch in einer empirischen Untersuchung. Anschliessend werden Beförderungsturniere betrachtet, bei denen es neben absoluten Leistungsanreize auch darauf ankommt Führungspositionen mit den fähigsten Mitarbeiter zu besetzen. Die Untersuchung zeigt, dass diese beiden Ziele inkompatibel sind; wenn Veränderungen der Turnierstruktur die Performance in der Anreizdimension erhöhen, reduzieren sie gleichzeitig die Selektionsfähigkeit des Turniers (und umgekehrt).

Im dritten Teil der Arbeit wird die Anreizwirkung unterschiedlicher Preisstrukturen in mehrstufigen Wettkämpfen mit Hilfe von Laborexperimenten untersucht. In einem ersten Schritt wird die Anzahl der vergebenen Preise variiert. Laut theoretischen Vorhersagen sinken die Investitionsanreize bei gleichbleibender Preissumme mit der Anzahl der vergebenen Preise. Die empirische Untersuchung qualifiziert diese Aussage insofern, als sie nur für risiko-neutrale Entscheider zu gelten scheint; für risiko-averse Entscheider hingegen scheint der Versicherungseffekt eines zweiten Preises den negativen Anreizeffekt zu überwiegen. In einem zweiten Schritt wird der Zeitpunkt der Preisvergabe variiert. Dabei zeigt sich, dass eine spätere Preisvergabe die Anreize erhöht, obwohl der Zeitpunkt laut theoretischen Vorhersagen keinen Einfluss auf das Verhalten der experimentellen Entscheider haben sollte. Genauere Untersuchungen deuten darauf hin, dass dieser Unterschied darauf zurückzuführen ist, dass die Entscheider jede Turnierstufe separat evaluieren anstatt ihren Nutzen über das Gesamtturnier zu maximieren.

Chapter 1

General Introduction

Contest models are prominent in many different areas of economics: Researchers in the field of personnel economics use contests as one possible mechanism to solve incentive and/or informational problems, for example.[1] In addition, contests are used to model election campaigns, rent-seeking games, R&D races, procurement tournaments, the competition for monopolies, litigation, wars, or the competition for titles in sports. Generally speaking,

> *...a contest is a game in which at least two agents compete over at least one prize by making costly and irreversible outlays. Each agent's outlay increases the own probability of success, and reduces the chances of the opponent(s) at the same time.*[2]

The meaning of outlays is application specific and may capture monetary expenses, effort provision, or a combination of these two factors. What is crucial, however, is that outlays are costly for participants of the contest, and that costs are independent of success or failure. As a consequence, each agent faces a trade-off when deciding about his/her outlays; own outlays do not only lead to uncertain gains in the sense that they increase the probability to win, but they also imply certain costs. Assuming that contest participants are risk-neutral and maximize their expected payoff, the resulting decision problem of agent i who chooses his/her outlay x_i in a one-shot interaction with a single prize B formally reads

$$\max_{x_i} \Pi_i(x_i, x_{-i}) = p(x_i, x_{-i}) * B - c(x_i),$$

where $p_{x_i}(\cdot) \geq 0$, $p_{x_{-i}}(\cdot) \leq 0$, and $c_{x_i}(\cdot) > 0$; x_{-i} is the vector of individual outlays chosen by all other agents.

Apart from functional form assumptions about the cost function $c(x_i)$, the so-called contest success function $p(x_i, x_{-i})$, which translates outlays made by participants into

[1] In this field, it is common to use the term tournament rather than contest.
[2] Definition of a contest similar the one given by Clark and Riis (1998b), p.1.

winning probabilities, is a central ingredient to any contest model. A contest success function (CSF) can either be *perfectly* or *imperfectly* discriminating. In a contest with a *perfectly* discriminating CSF, the agent whose outlay is higher than the outlay of any other player wins with probability one. Intuitively, it does not matter how big the difference between the outlays is in this specification, it suffices if the own outlay is marginally higher than the outlay of any other agent to win for sure. This situation is commonly referred to as "all-pay auction". Due to its analytical simplicity, the all-pay auction case is probably the most explored setting.[3] At the same time, this approach has two potential flaws: First, equilibria in all-pay auctions are often in mixed strategies, which may constrain empirical and especially experimental testing of theoretically derived results, particularly in small samples.[4] Second, the assumption that a marginal lead by any one agent leads to a deterministic outcome is inappropriate in many real-life situations, where contest outcomes are at least partly determined by chance.[5] *Imperfectly* discriminating CSFs account for these problems; the equilibrium is often in pure strategies, and in addition, the outcome is partly determined by chance. This dissertation employs the standard Tullock (1980) contest model with a linear cost function and an imperfectly discriminating CSF, which formally defines the winning probability of agent i as[6]

$$p_i(x_i, x_{-i}) = \frac{x_i^r}{x_i^r + \sum_{j \neq i} x_j^r}.$$

The parameter r allows for variations of the importance of chance (relative to outlays) for the contest outcome. One says that r measures the discriminatory power, since chance becomes less and less relevant for the contest outcome as r increases; for $r \to \infty$, the ratio CSF by Gordon Tullock approaches the perfectly discriminating all-pay auction CSF.

Many different aspects of the Tullock model have been analyzed in the past. While one strand of the literature investigates application specific research questions, other scholars consider it as equally important to understand general properties of contest games without

[3]Properties of the all-pay auction are discussed by Amman and Leininger (1996), Baye, Kovenock, and de Vries (1996), Clark and Riis (1998a), Moldovanu and Sela (2001), Moldovanu and Sela (2006), as well as Groh, Moldovanu, Sela, and Sunde (2012), for example.

[4]To be precise, the equilibrium is in mixes strategies in settings with complete information, while pure strategy equilibria are usually encountered if player types are private information.

[5]The performance of workers in an evaluation period is usually a composite measure of effort and luck; a sports tournament may be won by the player who provides less effort by chance. Similar arguments hold for most real-life interactions with contest properties.

[6]The second prominent imperfectly discriminating CSF is the additive noise *difference* specification, which assumes that each agent chooses a certain outlay level initially, that some random variable is subsequently added to the chosen outlay, and that the agent whose sum of outlay and random component is highest ultimately wins the contest. In this case, type and bounds of the distribution function from which the random variable(s) are drawn determine the relative importance of chance for the outcome of the contest. For a comparison of these two CSFs, see Hirshleifer (1989); Skaperdas (1996), Clark and Riis (1998b), and Rai and Sarin (2009) present an axiomatization of both contest technologies.

making reference to particular applications: Issues like the number of participating agents, the structure of the competition, the structure of prizes, the type of participants, or informational assumptions, for example, were addressed with respect to their impact on agents' equilibrium behavior. This cumulative dissertation contributes to both strands of the contest literature: Chapters 2 and 3 are rather technical, while Chapters 4 and 5 consider an application, namely bonus and promotion tournaments as an instrument for human resources management on internal labor markets. Finally, the last two chapters are empirical contributions which use experimental data from the lab. Note that each chapter is written as an independent contribution in paper form, which implies that certain aspects, such as the definition of a contest, the description of the experimental implementation, or the experimental instructions, are repeated several times.

The next chapter (Chapter 2) presents the subgame perfect Nash equilibrium solution of a multi-stage pair-wise elimination contest. While previous analyses of this contest format restricted attention to the case where all agents are homogeneous, I consider the general case with heterogeneous contest participants. Chapter 3 uses the solution from Chapter 2 to compare the dynamic multi-stage to a static one-shot contest. The comparison indicates that the effect of heterogeneity on contest participants is structure specific: While total outlays in both formats are identical in interactions between homogeneous agents, I find that total outlays are usually higher in the dynamic than in the static contest when agents are heterogeneous. In contrast to Chapters 2 and 3, the focus of Chapters 4 is on a particular application. Together with my co-author Uwe Sunde, I consider a principal who uses a tournament compensation scheme to incentivize workers. We find that the overall effect of heterogeneity on total effort provision by all workers depends on the tournament format. While the effect is always negative in static formats, we find that the incentive effect of heterogeneity can be strictly positive in dynamic elimination tournaments. Experimental evidence from lab experiments confirms these theoretical predictions. Chapter 5 is joint work with Wolfgang Höchtl, Rudolf Kerschbamer, and Uwe Sunde. As in Chapter 4, we consider the personnel economics application of a principal who uses tournaments as a means of human resource management. In particular, we investigate to what extent a promotion tournament can accomplish both the selection of the most able employee and the provision of incentives. The results suggest that any tournament with heterogeneous participants provides some incentives for effort and some sorting of types. However, modifications which improve the performance in one will deteriorate the performance in the other dimension, i.e., tournament formats that perform better in terms of incentive provision do worse in terms of selecting the best participant, and vice versa. From a policy perspective, this suggests that multiple instruments should be used whenever both goals are equally important. The last two chapters of this dissertation use experimental methods to analyze prize structure variations in multi-stage pair-wise elimination contests. Chapter 6 (joint work with Wolfgang Höchtl, Rudolf Kerschbamer, and Uwe Sunde) considers a single prize treatment, which is supposed to maximize total effort provision, and a treatment with multiple prizes which ensure incentive maintenance across stages. While the experimental design does not introduce any ex-ante heterogeneity between subjects, we observe ex-post heterogeneity – risk attitudes by experimental subjects have a strong effect on their behavior. In particular, we find that total effort is maximized in the single prize treatment for risk-neutral, but not for risk-averse subjects. Independent of risk attitudes, we observe incentive maintenance across stages in the multiple prizes setting. The last chapter (joint work with Rudolf Kerschbamer and Uwe Sunde) analyzes which effect the timing of rewards has on the behavior of agents in two-stage pair-wise elimination contests. We compare a treatment where agents receive an immediate reward for winning stage 1 with a specification where the reward for winning stage 1 is delayed until the stage-2 interaction is over. Theory predicts that the two treatments are strategically identical in both stages if agents are risk neutral, or if agents jointly evaluate the payoff of both interactions. Yet, we find that stage-1 effort choices by experimental subjects are higher in the delayed than in the immediate reward treatment, while effort provision in stage 2 does not differ between treatments. In particular, average differences of stage-1 effort choices between treatments are fully explained by choices of risk averse subjects: While their stage-1 effort choices in the delayed are much higher than in the immediate reward treatment, there is no difference across treatments for risk neutral subjects. This pattern is consistent with theoretical predictions only if experimental subjects separately evaluate the payoff of each stage. In this case, delayed rewards provide an insurance for risk averse decision makers, such that stage-1 effort choices should indeed differ across treatments for risk averse, but not for risk neutral decision makers.

Chapter 2

Multi-Stage Elimination Contests with Heterogeneous Agents

2.1 Introduction

Contest models are used to describe strategic interactions between agents in many different settings, including diverse areas such as war, rent-seeking or R&D competitions, and sport tournaments. Due to the impressive variety of possible applications, many different contest structures have been considered in the literature already. One of the most prominent structures is the multi-stage pair-wise elimination format, which is sometimes also referred to as knock-out contest, since the loser of each interaction is eliminated from any future competition, while the winner moves on to the next stage. This contest structure is probably best known from sports: Disciplines like baseball, boxing, hockey, soccer, tennis, or even chess make use of this structure at least in later stages of the competition, in the so-called "playoff" stage. However, the structural feature of subsequent elimination is relevant in many other fields as well: In personnel economics, for example, where promotion tournaments within firms are usually modeled as elimination contests, or in political sciences, where multi-stage election campaigns like the one for US presidency have this structural feature.

In this paper, I analyze multi-stage pair-wise elimination contests with heterogeneous agents, assuming that types are common-knowledge among participants of the contest. I show under which conditions a subgame perfect Nash equilibrium exists when a general Tullock contest success function (CSF) is used. Moreover, the equilibrium solution is derived analytically for the special case of a lottery CSF, and characterized for the remaining cases. Note that the main difficulty which arises in multi-stage contests once agents are allowed to be heterogeneous is that continuation values in early stages become endogenous due to feedback effects across different branches of the game. Therefore, I devote special attention to the analysis of these feedback effects. At the end of the paper, I investigate several properties of the model, such as comparative static results, and effort maximizing prize structures. In particular, I also compare "Seeding" properties in my Tullock CSF model with properties previously established by Groh, Moldovanu, Sela, and Sunde (2012) for the perfectly discriminating all-pay auction CSF.

Given the wide variety of potential applications, it is almost surprising that the existing literature has (almost) exclusively concentrated on the most simple case of multi-stage elimination contests in which all participating agents are identical. Even a very recent paper on the optimal design of multi-stage contests by Fu and Lu (2012) entirely focusses on settings with homogeneous agents. However, the consideration of research on settings with heterogeneous agents is recommended for future research in the conclusion.[1] Only special cases of the arguably more relevant case where agents can be of different types have been analyzed in the past: In the theoretical literature on contest design, Stein and Rapoport (2004) compare the behavior of asymmetric agents in two-stage contests with different orderings of competition within and between groups. However, since homogeneity is assumed within each group, the major complication that arises in a multi-stage competition between heterogeneous agents is avoided, namely the endogeneity of continuation values in early stages of the game. Other authors focus on specific applications of multi-stage contests: Rosen (1986), for example, uses multi-stage pair-wise elimination contest structure to model a promotion tournament. The analysis mainly concentrates on the case where agents are homogeneous, only numerical examples address heterogeneous settings.[2] A paper by Harbaugh and Klumpp (2005) considers the same contest structure as I do in this paper, but makes two simplifying assumptions: First, agents can only be of two different types, and second, total effort provision by each participant in both stages of the contest is equal to some constant by assumption. In other words, Harbaugh and Klumpp (2005) analyze a version of the model where all agents face the same binding effort endowment (which has no intrinsic value), and then discuss how the endowment is optimally distributed across the two stages. Finally, Klumpp and Polborn (2006) consider heterogeneous contestants in a multi-stage competition, but their contest structure is somewhat different from the one that is analyzed in this paper, because they assume that the same two agents interact repeatedly within stage 1.

Somewhat more is known about the properties of multi-stage contests with heterogeneous agents in a different branch of the contest literature, which uses a perfectly

[1] Another example is the paper by Gradstein and Konrad (1999), where single- and multi-stage contests are compared for the case of homogeneous agents only.

[2] Sherwin Rosen determines the optimal structure of prizes under the assumption that agents are perfectly homogeneous; optimality refers to constant incentives for effort provision across stages. In the last section of the paper, he discusses by use of numerical examples to what extent the results do hold in settings with heterogeneous agents.

discriminating CSF, the so-called "all-pay auction".[3] Moldovanu and Sela (2006) compare one-stage and multi-stage contests and explicitly allow for heterogeneity between the contestants. Groh, Moldovanu, Sela, and Sunde (2012) consider the case of four heterogeneous, optimizing agents in a two-stage pair-wise elimination tournament, as I do in this paper. However, they derive the mixed-strategy equilibrium for the all-pay auction and determine how players should be paired, or seeded, in stage 1 to satisfy four different optimality criteria. Although the baseline situation is the same in their and in my model, the focus is very different: Groh, Moldovanu, Sela, and Sunde (2012) restrict their attention exclusively to the effect which the allocation of player types in stage 1 ("Seeding") has on the properties of a two-stage contest with four agents. The approach in my paper is broader; I analyze comparative statics behavior, and discuss the effect of heterogeneity in multi-stage contests on the structure of optimal prizes. Further, the feedback effect across different branches of the game is considered in some detail, as well as situations with more than two stages. Apart from that, I use a general Tullock CSF which, in contrast to the all-pay auction case, does not restrict the structure of prizes in any dimension, and gives an equilibrium in pure strategies.[4]

I proceed as follows: Section 2 starts by considering the simplest case of a multi-stage pair-wise elimination contest, which is a contest with two-stages and four agents. First, I generally characterize the subgame perfect Nash-equilibrium solution for a discriminatory power r, before I derive the analytical solution for the special case of a lottery CSF ($r = 1$). At the end of the section, extensions of the baseline model to three or more stages are discussed. Section 3 analyzes several properties of the simplest multi-stage contest with two stages. In particular, I present some comparative static results and briefly address the issue of optimal, i.e., effort maximizing prizes. Section 4 concludes.

2.2 Modeling Multi-Stage Elimination Contests

2.2.1 A Two-Stage Tullock Contest with Discriminatory Power r

In a two-stage contest, there are three pair-wise interactions: Two in stage 1, and a third one in stage 2 between the two winners of stage 1. The four agents are assumed to be risk neutral and identical apart from the individual effort productivity parameter $a_i \geq 0$ which determines their type. The higher a_i, the *stronger* (or more productive) is agent i.[5] Agents are perfectly informed about both their own type and the type of

[3] In such a setting, a marginal lead in terms of contest investments implies a winning probability of 1.

[4] A pure strategy equilibrium is an advantage if one intends to test certain predictions of the model in a controlled laboratory experiment, for example.

[5] All productivity parameters are assumed to be non-negative and finite. It is without loss of generality that I model heterogeneity between agents in terms of effort productivity. All the subsequent results do

the remaining three agents participating in the contest. They do not know, however, which agent they will meet in stage 2 of the game, since decisions of all agents in stage 1 are made simultaneously.[6] Therefore, agents make decisions based on expectations with respect to the probability that they meet a certain type in stage 2. Any pair-wise interaction is modeled using linear effort costs and a Tullock contest success function, as in the standard rent-seeking model by Tullock (1980). Assuming that agents i and j optimally choose efforts x_i and x_j, respectively, the winning probability p_{ij} of agent i is defined as

$$p_{ij} = \begin{cases} \frac{a_i x_i^r}{a_i x_i^r + a_j x_j^r} & \text{if} \quad x_i + x_j > 0 \\ \frac{1}{2} & \text{if} \quad x_i + x_j = 0 \end{cases} .$$

r is the discriminatory power of the contest success function, while the parameter a_i (a_j) measures the effort productivity of agent i (j).[7]

Three prizes are awarded in the contest: The prize $P^H \geq 0$ is awarded to the winner of the stage-2 subgame, while the two agents who reach stage 2 each receive $P^L \geq 0$. Consequently, the winner of stage 2 receives $P^L + P^H$, and the overall amount of resources that are used for prizes is equal to the sum $2P^L + P^H$.[8]

It is assumed without loss of generality that agents 1 and 2 meet in one of the two interactions in stage 1, while agents 3 and 4 compete in the remaining one. The equilibrium concept needed to solve this game is subgame perfect Nash: Using backwards induction, I start by solving stage 2 of the game, or, to be precise, all potential constellations of second stage games, and subsequently consider the first stage, taking optimal choices in stage 2 as given.

Stage 2. Since only one agent from each subgame proceeds to the second stage, there are four potential constellations in the stage-2 subgame: (i) agent 1 - agent 3, (ii) agent 1 - agent 4, (iii) agent 2 - agent 3, or (iv) agent 2 - agent 4.[9] All constellations are simple interactions between two heterogeneous agents, a situation which has been studied by Allard (1998) and Nti (1999) in slightly different settings.[10] I will now derive a solution for the general case where agent i meets agent j. It is assumed without loss of generality that agent i is stronger than agent j, i.e., the relation $a_i \geq a_j \geq 0$ does hold. $x_{ij} \geq 0$

also hold if agents have different valuation or cost of effort parameters.

[6]Nothing changes if decisions are made sequentially, as long as no agent is informed about the decision of any other agent before he/she has made his own decision in stage 1.

[7]This contest success function has been axiomatized by Clark and Riis (1998b).

[8]The solution to the game is the same under the assumption that only one or two prizes exist; if two prizes exist, one would be for the winner of the second stage game, and one for the loser of the second stage game.

[9]Note that two or more of these interactions may be strategically equivalent, if at least two agents are of the same type, i.e. if their effort productivity a_i is identical.

[10]Allard (1998) focuses on existence and uniqueness of equilibria in heterogeneous Tullock contests, while Nti (1999) presents extensive comparative static results for heterogeneous two player contests.

($x_{ji} \geq 0$) denotes the effort of agent i (j) in an interaction with agent j (i).[11] The two agents compete for the prize P^H and choose their efforts in such a way as to maximize their expected payoff $\pi_i(i-j)$ and $\pi_j(j-i)$, respectively. Formally, the corresponding optimization problems are

$$\begin{aligned}
\max_{x_i \geq 0} \pi_i(i-j) &= \frac{a_i x_{ij}^r}{a_i x_{ij}^r + a_j x_{ji}^r} P^H - x_{ij}, \\
\max_{x_j \geq 0} \pi_j(j-i) &= \frac{a_j x_{ji}^r}{a_i x_{ij}^r + a_j x_{ji}^r} P^H - x_{ji}.
\end{aligned}$$

First derivatives with respect to the choice variable deliver the following system of first-order conditions:

$$r a_i a_j x_{ij}^{r-1} x_{ji}^r P^H - (a_i x_{ij}^r + a_j x_{ji}^r)^2 = 0 \tag{2.1}$$

$$r a_i a_j x_{ji}^{r-1} x_{ij}^r P^H - (a_i x_{ij}^r + a_j x_{ji}^r)^2 = 0 \tag{2.2}$$

These conditions are necessary and sufficient for the unique pure-strategy equilibrium if the discriminatory power r is not too high.[12] Nti (1999) derived a formal condition which assures that the equilibrium is in pure strategies in two player contests with heterogeneous agents.

Assumption 2.1. *The contest between agents i and j has a unique pure strategy equilibrium, i.e., I assume that the discriminatory power r is not too high relative to the degree of heterogeneity between agents in terms of effort productivity differences. Formally, it must hold that*

$$r \leq \frac{a_j}{a_i} + 1,$$

where $a_i \geq a_j \geq 0$.

For the remainder of this paper, I assume that this condition is satisfied. Under Assumption 2.1, the system of equations (2.1) and (2.2) fully characterizes the equilibrium, i.e., the optimality conditions are necessary and sufficient. Combination of these equations delivers equilibrium efforts

$$x_{ij}^* = x_{ji}^* = r \frac{a_i a_j}{(a_i + a_j)^2} P^H. \tag{2.3}$$

In equilibrium, both agents provide the same effort, even though their productivity parameters a_i and a_j may be different. This does not hold, however, for expected equilibrium

[11]The assumption that $x_{ij} \geq 0$ is standard in the literature, since negative investments into a contest do not make sense.

[12]Mixed strategies which occur in *homogeneous* two player Tullock contests if $r > 2$ are discussed by Baye, Kovenock, and de Vries (1994).

payoffs. Inserting x_{ij}^* and x_{ji}^* in the payoff functions π_i gives the expected equilibrium payoffs

$$\pi_i^*(i-j) = \frac{a_i^2 + (1-r)a_i a_j}{(a_i + a_j)^2} P^H, \quad \text{and} \quad \pi_j^*(j-i) = \frac{a_j^2 + (1-r)a_i a_j}{(a_i + a_j)^2} P^H. \tag{2.4}$$

Recall that agent i is assumed to be stronger than agent j, i.e., the relation $a_i \geq a_j \geq 0$ is assumed to hold. Inspection of the expected equilibrium payoffs reveals that $\pi_i^*(i-j) \geq \pi_j^*(j-j)$, such that the expected equilibrium payoff of the stronger agent is higher, as intuition would suggest. Further, the difference (or ratio) of the expected equilibrium payoffs of the stronger and weaker agent is increasing in the difference (or ratio) of the effort productivity parameters.

Stage 1. Without loss of generality, assume that agent 1 is stronger than agent 2, while agent 3 is stronger than agent 4, i.e. $a_1 \geq a_2 \geq 0$, and $a_3 \geq a_4 \geq 0$ do hold. Recall that there are two interactions on stage 1: One between agents 1 and 2, and another one between agents 3 and 4. I will first consider the former one. Let's assume that y_{ij} is the stage-1 effort by agent i who meets j. Then, agents 1 and 2 face the following maximization problems:

$$\begin{aligned}
\max_{y_{12}\geq 0} \Pi_1 &= \frac{a_1 y_{12}^r}{a_1 y_{12}^r + a_2 y_{21}^r} \left[P^L + \underbrace{\frac{a_3 y_{34}^r}{a_3 y_{34}^r + a_4 y_{43}^r} \pi_1^*(1-3) + \frac{a_4 y_{43}^r}{a_3 y_{34}^r + a_4 y_{43}^r} \pi_1^*(1-4)}_{P_1^c(y_{34}, y_{43})} \right] - y_{12} \\
\max_{y_{21}\geq 0} \Pi_2 &= \frac{a_2 y_{21}^r}{a_1 y_{12}^r + a_2 y_{21}^r} \left[P^L + \underbrace{\frac{a_3 y_{34}^r}{a_3 y_{34}^r + a_4 y_{43}^r} \pi_2^*(2-3) + \frac{a_4 y_{43}^r}{a_3 y_{34}^r + a_4 y_{43}^r} \pi_2^*(2-4)}_{P_2^c(y_{34}, y_{43})} \right] - y_{21}.
\end{aligned}$$

Note that the value of winning stage 1, i.e., the prize for which agents compete in stage 1, consist of two parts: First, each agent who reaches stage 2 receives P^L, and second, each agent i who reaches stage 2 has the chance to win the prize P^H; the winning probability and the effort costs of the stage 2 interaction are included in the expected equilibrium payoffs, $\pi_i^*(i-j)$ (see above). I call this second part of the prize the *continuation value*, i.e., $P_1^c(y_{34}, y_{43})$ and $P_2^c(y_{34}, y_{43})$, respectively, are the continuation values of agents 1 and 2. Due to the assumption that agent 1 is stronger than agent 2, it must hold that $P_1^c(y_{34}, y_{43}) \geq P_2^c(y_{34}, y_{43})$, since the expected equilibrium payoff of a stage-2 participation is higher for agent 1 than for agent 2, no matter whether he/she meets agent 3 or 4 in stage 2.[13] Note, however, that the continuation values also depend on actions of agents 3 and 4

[13]From the discussion in the previous section it is clear that the ratio or difference of expected equilibrium payoffs is increasing in the ratio or difference of productivity parameters, which measure the (relative) strength of an agent.

in the parallel stage-1 interaction, because meeting agent 3 in stage 2 has a different value for agents 1 or 2 than meeting agent 4, at least if agents 3 and 4 are not of the same type. Therefore, in general each agent does not only play a best response to his/her immediate opponent, but in addition, the stage-1 actions of prospective stage-2 opponent(s) matter. This indirect effect, which connects the two stage-1 interactions, makes the continuation values endogenous and constitutes the main difficulty in solving multi-stage contests with heterogeneous agents. If the continuation values were known, the two stage-1 interactions would be independent from one another and the standard solution used for stage 2 could be employed.

Independent of this complication, first-order conditions are still necessary equilibrium conditions. The maximization problems of agents 1 and 2 imply that the following two first-order optimality conditions do hold:

$$
\begin{aligned}
r a_1 a_2 y_{21}^r y_{12}^{r-1}[P^L + P_1^c(y_{34}, y_{43})] &= (a_1 y_{12}^r + a_2 y_{21}^r)^2 \qquad (2.5)\\
r a_1 a_2 y_{12}^r y_{21}^{r-1}[P^L + P_2^c(y_{34}, y_{43})] &= (a_1 y_{12}^r + a_2 y_{21}^r)^2 \qquad (2.6)
\end{aligned}
$$

Combining these equations, one can characterize the ratio of equilibrium efforts:

$$
\frac{y_{21}^*}{y_{12}^*} = \frac{a_4[P^L + \pi_2^*(2-4)]\left(\frac{y_{43}}{y_{34}}\right)^r + a_3[P^L + \pi_2^*(2-3)]}{a_4[P^L + \pi_1^*(1-4)]\left(\frac{y_{43}}{y_{34}}\right)^r + a_3[P^L + \pi_1^*(1-3)]} \equiv G\left(\frac{y_{43}}{y_{34}}\right). \tag{2.7}
$$

Intuitively, the ratio of equilibrium efforts is equal to the ratio of prize valuations, a relation known from Nti (1999). Since agent 1 is at least as strong as agent 2 ($a_1 \geq a_2 \geq 0$), the continuation value for agent 1 is at least as high as the one for agent 2. Consequently, it holds that $0 \leq \frac{y_{21}^*}{y_{12}^*} \leq 1$. Further, note that the ratio of equilibrium efforts $\frac{y_{21}^*}{y_{12}^*}$ is a function of the effort ratio in the parallel stage-1 interaction between agents 3 and 4, which implies that knowledge of equilibrium efforts y_{34}^* and y_{43}^* is, strictly speaking, not necessary to determine the continuation values of agents 1 and 2. All one needs to know is the ratio of equilibrium efforts, which makes intuitive sense, since the ratio of equilibrium efforts determines winning probabilities in pair-wise interaction, i.e., from the perspective of agents 1 and 2, the ratio of effort choices by agents 3 and 4, y_{34} and y_{43}, determines the probability that either one of these agents wins in stage 1 and reaches stage 2.

When considering the stage-1 interaction between agents 3 and 4, one can determine a relation analogous to (2.7), which reads

$$
\frac{y_{43}^*}{y_{34}^*} = \frac{a_2[P^L + \pi_4^*(4-2)]\left(\frac{y_{21}}{y_{12}}\right)^r + a_1[P^L + \pi_4^*(4-1)]}{a_2[P^L + \pi_3^*(3-2)]\left(\frac{y_{21}}{y_{12}}\right)^r + a_1[P^L + \pi_3^*(3-1)]} \equiv R\left(\frac{y_{21}}{y_{12}}\right). \tag{2.8}
$$

Since agent 3 is assumed to be at least as productive as agent 4 ($a_3 \geq a_4 \geq 0$), the ratio of equilibrium efforts $\frac{y_{43}^*}{y_{34}^*}$ is also between zero and one.

Summing up, equations (2.7) and (2.8) show that the ratio of efforts by agents in one stage-1 interaction determines the ratio of equilibrium efforts by agents in the other stage-1 interaction. Said differently, equations (2.7) and (2.8) *independently* ensure that agents 1 and 2, as well as 3 and 4 play mutually best responses for given continuation values, while they *jointly* determine the equilibrium continuation values of the stage-1 subgame. Before further considering the equilibrium of the stage-1 subgame, it must be ensured that the (partial) equilibrium in each of the two pair-wise stage-1 interactions is in pure strategies for given continuation values, since the optimality conditions are otherwise not necessary and sufficient. Intuitively, the degree of heterogeneity between immediate opponents in a stage-1 interaction must not be too high for the given discriminatory power.

Assumption 2.2. *Let $P_1^c(y_{34}^*, y_{43}^*)$, $P_2^c(y_{34}^*, y_{43}^*)$, $P_3^c(y_{12}^*, y_{21}^*)$ and $P_4^c(y_{12}^*, y_{21}^*)$ be the equilibrium continuation values. Then, the following relations are satisfied:*

$$r \leq \frac{a_2}{a_1}\left(\frac{P^L + P_2^c(y_{34}^*, y_{43}^*)}{P^L + P_1^c(y_{34}^*, y_{43}^*)}\right)^r + 1, \qquad \text{and} \qquad r \leq \frac{a_4}{a_3}\left(\frac{P^L + P_4^c(y_{12}^*, y_{21}^*)}{P^L + P_3^c(y_{12}^*, y_{21}^*)}\right)^r + 1.$$

To be precise, assumption 2.2 provides a condition that must be checked *after* equilibrium efforts are determined to ensure that the derived solution is correct. The assumption matters, however, if and only if the impact function of the CSF is convex ($r > 1$); it becomes clear by inspection that the condition is always satisfied for linear and concave impact functions ($r \leq 1$).[14]

To determine equilibrium effort levels of all four players, we need to know either $G^*(\frac{y_{43}^*}{y_{34}^*}) = \frac{y_{21}^*}{y_{12}^*}$, or $R^*(\frac{y_{21}^*}{y_{12}^*}) = \frac{y_{43}^*}{y_{34}^*}$. Once one of the two equilibrium ratios is known, it is straightforward to determine the other one (by use of equation (2.7) or (2.8)). Using the expressions of $R^*(\frac{y_{21}^*}{y_{12}^*})$ and $G^*(\frac{y_{43}^*}{y_{34}^*})$ from (2.7) and (2.8), equilibrium continuation values can be defined as follows:

$$P_1^{c*} = \frac{a_3\pi_1^*(1-3) + a_4\pi_1^*(1-4)R^*(\cdot)}{a_3 + a_4R^*(\cdot)}, \qquad P_2^{c*} = \frac{a_3\pi_2^*(2-3) + a_4\pi_2^*(2-4)R^*(\cdot)}{a_3 + a_4R^*(\cdot)}, \tag{2.9}$$

$$P_3^{c*} = \frac{a_1\pi_3^*(3-1) + a_2\pi_3^*(3-2)G^*(\cdot)}{a_1 + a_2G^*(\cdot)}, \qquad P_4^{c*} = \frac{a_1\pi_4^*(4-1) + a_2\pi_4^*(4-2)G^*(\cdot)}{a_1 + a_2G^*(\cdot)}. \tag{2.10}$$

Once equilibrium continuation values are known, the two stage-1 interactions are independent from one another, and each can be solved along the same lines as the stage-2 interaction in section 2.2.1. The resulting equilibrium effort levels in stage 1 are characterized by

$$y_{12}^* = \frac{a_1a_2[P^L + P_1^{c*}]}{(a_1 + a_2G^*(\cdot))^2}, \qquad y_{21}^* = \frac{a_1a_2[P^L + P_2^{c*}]}{(a_1 + a_2G^*(\cdot))^2}, \tag{2.11}$$

$$y_{34}^* = \frac{a_3a_4[P^L + P_3^{c*}]}{(a_3 + a_4R^*(\cdot))^2}, \qquad y_{43}^* = \frac{a_3a_4[P^L + P_4^{c*}]}{(a_3 + a_4R^*(\cdot))^2}. \tag{2.12}$$

[14]Assumption 2.2 for the existence of a pure strategy equilibrium in stage 1 is analogous to Assumption 2.1, which ensures that a pure-strategy equilibrium exists for convex impact functions in stage 2.

Finally, knowledge of stage-1 equilibrium efforts allows for the determination of expected equilibrium payoffs, total equilibrium effort expenditures, equilibrium winning probabilities, and the like, i.e., for a complete solution of the stage-1 subgame.

For now, I simply assumed that $G^*(\cdot)$ and $R^*(\cdot)$ are known. However, when combining equations (2.7) and (2.8), which implicitly define both $G^*(\cdot)$ and $R^*(\cdot)$, it is impossible to derive a closed form analytical expression for either one of the two ratios in the general case. Two special cases can be solved analytically, however: First, it is possible to solve the game if either $a_1 = a_2$ or $a_3 = a_4$, or both. If this is the case, it is easy to show that at least one of the two ratios is equal to one, such that the missing one can be determined. The intuition for this result is simple: If any two of the agents who interact in stage 1 are equally strong, it is obvious that their continuation values are identical, which implies that their stage-1 efforts are the same. The second special case is less obvious and will be discussed in some detail later in the paper: If $r = 1$, i.e., if a lottery CSF is used, the system of equations (2.7) and (2.8) can be solved analytically.

The Subgame Perfect Nash Equilibrium in Pure-Strategies. Even though a closed form solution for $G^*(\cdot)$ and $R^*(\cdot)$, and therefore for the stage-1 efforts cannot be derived for the general case, the two-stage contest has a subgame perfect Nash-equilibrium under Assumptions 2.1 and 2.2. The equilibrium of a two-stage pair-wise elimination tournament with four agents is given by the strategy profile

$$S^* = \left\{ \left(y_{12}^* \left(y_{21}, \frac{y_{43}}{y_{34}} \right), [x_{13}^*(x_{31}), x_{14}^*(x_{41})] \right); \ \dots \ ; \ \dots \ ; \left(y_{43}^* \left(y_{34}, \frac{y_{21}}{y_{12}} \right), [x_{41}^*(x_{14}), x_{42}^*(x_{24})] \right) \right\},$$

i.e., in stage 1, each agent plays a best response to the chosen effort of his immediate opponent and the ratio of efforts of the remaining two agents in the other stage-1 interaction, as was shown in section 2.2.1. In stage 2, the best response is only with respect to the direct opponent (see section 2.2.1).

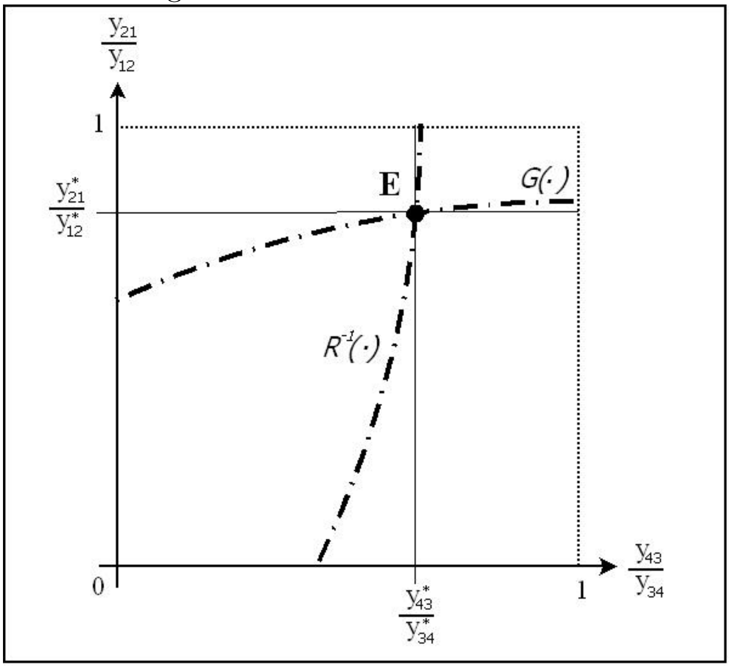
Theorem 2.1 (Existence)**.** *Under Assumptions 2.1 and 2.2, the two stage Tullock contest with four agents and discriminatory power r has a subgame perfect equilibrium in pure strategies, independently of agents' types.*

Proof. See Appendix. □

The intuition for the proof is as follows. Assumption 2.1 ensures that the stage-2 subgame has a unique pure strategy equilibrium. Further, Assumption 2.2 implies that each of the two stage-1 interactions has a unique pure strategy equilibrium as well, conditional on given continuation values.[15] Consequently, what remains to be shown is that the system of equations (2.7) and (2.8) has a solution, which then ensures that equilibrium continuation values are defined. Figure 2.1 plots the two functions $G(\cdot)$ as

[15]This has been proven by Nti (1999) and Cornes and Hartley (2005), for example.

Figure 2.1: Intuition for the Proof



well as $R(\cdot)$ and provides the graphical intuition for the proof, which is that the graphs of $G(\cdot)$ and the inverse function $R^{-1}(\cdot)$ intersect at least once in the $(\frac{y_{43}}{y_{34}}, \frac{y_{21}}{y_{12}})$-space.

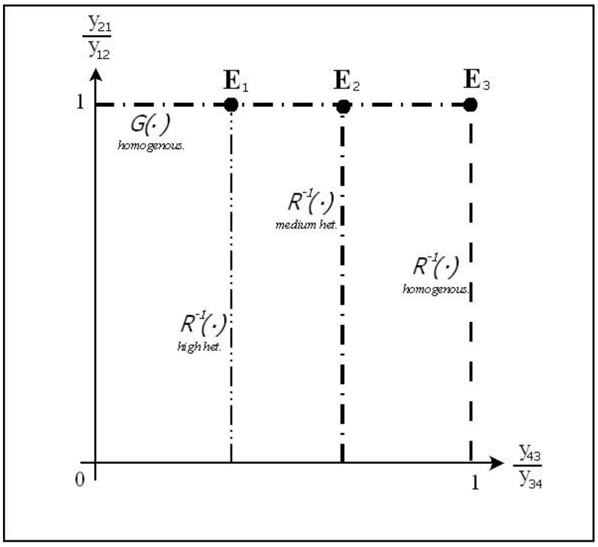
The Interaction Effect Across Stage-1 Pairings. The interaction effect across the two stage-1 interactions is captured by the system of equations (2.7) and (2.8). I use graphical illustrations to explain the forces at work in this model.

It helps to separately consider two effects: The effect of heterogeneity across stages, and the effect of heterogeneity across stage-1 interactions. I start with the effect of heterogeneity across stages and assume that the interaction between agents 1 and 2 is homogeneous, i.e., that these two agents are equally strong, while the degree of heterogeneity between agents 3 and 4 varies. In this case, there is no interaction effect across the stage-1 interactions. Panel (a) of Figure 2.2 illustrates the position of the $G(\cdot)$- and $R^{-1}(\cdot)$-loci in the $(\frac{y_{43}}{y_{34}}, \frac{y_{21}}{y_{12}})$-space under this assumption. $G(\cdot)$ is a horizontal line through the point $\frac{y_{21}}{y_{12}} = 1$, while $R^{-1}(\cdot)$ is a vertical line. The exact position of this line depends on the degree of heterogeneity between agents 3 and 4.[16] As panel (a) of Figure 2.2 shows, the line gradually shifts to the left as the degree of heterogeneity between agents 3 and 4 increases. To understand the position of $R^{-1}(\cdot)$ in the $(\frac{y_{43}}{y_{34}}, \frac{y_{21}}{y_{12}})$-space, recall from the analysis of the stage-2 interaction in section 2.2.1 that the efforts of two players who differ only in terms of their productivity are equal, independently of the degree of heterogeneity. Therefore, deviations of the ratio $\frac{y_{43}}{y_{34}}$ from 1 capture the endogenous heterogeneity between agents 3 and 4 that is caused by different valuations of a participation in stage 2. Said differently, the fact that agents 3 and 4 are (still) of different strength in later

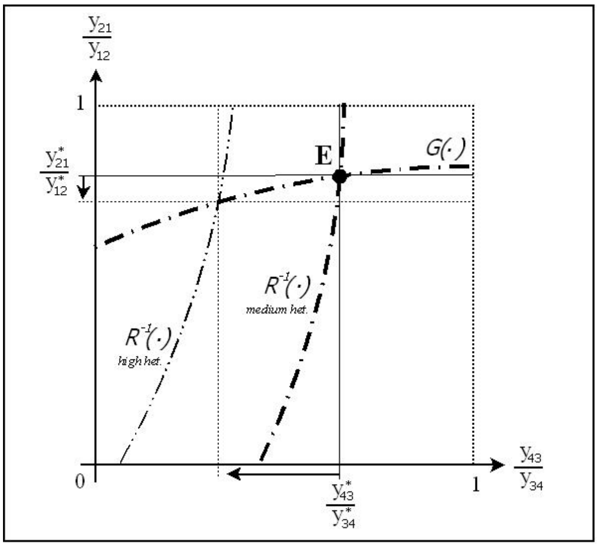
[16]Note that $R^{-1}(\cdot)$ is an inverse correspondence rather than an inverse function in this example.

stages further increases the degree of heterogeneity in early stages of the game through the continuation values.

Figure 2.2: The Interaction Effect across the Two Stage-1 Pairings



(a) One homogeneous stage-1 interaction

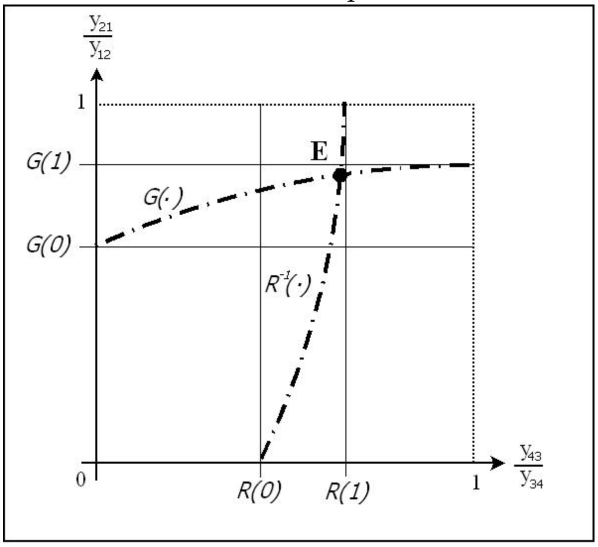


(b) Two heterogeneous stage-1 interactions

Next, consider the interaction effect of heterogeneity across stage-1 pairings. Panel (b) of Figure 2.2 depicts a situation where both stage-1 interactions are heterogeneous. In contrast to panel (a) of the figure where both loci are straight lines, there is a mutual interdependence across stage-1 interactions in panel (b), since both $G(\cdot)$ and $R^{-1}(\cdot)$ are functions with a strictly positive slope. The slopes are strictly positive if and only if the strict relations $a_1 > a_2 \geq 0$ and $a_3 > a_4 \geq 0$ are satisfied, i.e., there is no interdependence if (at least) one of the two stage-1 interactions is homogeneous.[17] Consider the following thought experiment: Assume that the equilibrium is given by the point E in panel (b) of Figure 2.2. Then, an increase in heterogeneity between agents 3 and 4 (such that agent 3 becomes stronger) induces the $R^{-1}(\cdot)$ function to shift to the left, since the equilibrium effort of agent 4 decreases relative to the equilibrium effort of agent 3. This is the heterogeneity effect across stages, which I discussed above. However, due to interaction effect between the two stage-1 pairings, this change affects the behavior of agents 1 and 2, even though their types remain unchanged. Two different channels are important. *First*, there is a direct effect: The intersection of the two functions is moved further to the left, and since $G(\cdot)$ is increasing, the equilibrium ratio of efforts in the second stage-1 interaction decreases, i.e., (the weaker) agent 2 reduces his effort provision relative to the effort of agent 1. The strength of the direct effect depends on the slope of $G(\cdot)$, which is determined by the degree of heterogeneity between agents 3 and 4. For reasonable degrees of heterogeneity between types, $G(\cdot)$ is much flatter than depicted in the plot, such that the change in the ratio $\frac{y^*_{21}}{y^*_{12}}$ due to an increase in the strength of agent 3 is rather small. *Second*,

[17]Then, the ratio $\frac{y_{21}}{y_{12}}$ is equal to one, independent of $\frac{y_{43}}{y_{34}} = 1$.

Figure 2.3: Bounds for the Equilibrium Effort Ratios



there is an indirect effect: Agents 1 and 2 react to the change in behavior of agents 3 and 4; the function $G(\cdot)$ shifts downwards and becomes somewhat steeper, which then further reduces the equilibrium ratio $\frac{y_{21}^*}{y_{12}^*}$. Intuitively, the fact that agent 3 is stronger has a more pronounced negative effect on the continuation value of agent 2 than on the continuation value of agent 1, especially for low values of $\frac{y_{43}^*}{y_{34}^*}$, i.e., when the probability to meet the strong agent 3 on stage 2 is high. Note that this second effect is not depicted in the figure for reasons of clarity.

As the previous discussion indicated, the strength of the interaction effect depends on the slope of $G(\cdot)$ and $R^{-1}(\cdot)$: The steeper $G(\cdot)$ and the flatter $R^{-1}(\cdot)$, the more important is the interdependence effect. One can show formally that the steepness of $G(\cdot)$, which is the equilibrium ratio function for the interaction between agents 1 and 2, is increasing in the degree of heterogeneity between agents 3 and 4.[18] An analogous finding applies to $R^{-1}(\cdot)$. Moreover, it became clear that the heterogeneity effect across stages exists if at least one interaction is between heterogeneous types, while the interaction effect which implies mutual interdependence in stage 1 does only exist if both stage-1 interactions are heterogeneous. Whenever the interaction effect across stage-1 pairings exists, a closed form solution for the general model cannot be derived.

Approximate Solution. Figure 2.3 illustrates that there is a straightforward way to approximate the solution if it cannot be determined analytically. The key insight is that the stage-1 equilibrium effort ratios are bounded. Note that both $\frac{y_{43}^*}{y_{34}^*} \in [R(0), R(1)]$ and $\frac{y_{21}^*}{y_{12}^*} \in [G(0), G(1)]$ must hold. Therefore, one can bound the two equilibrium ratios as follows:

$$\frac{y_{21}^*}{y_{12}^*} \in [G(R(0)), G(R(1))] \qquad \text{and} \qquad \frac{y_{43}^*}{y_{34}^*} \in [R(G(0)), R(G(1))].$$

[18]Details are provided in the Appendix, in particular in the proof of Theorem 2.1.

For low and intermediate degrees of heterogeneity between agents, these bounds are very narrow, such that the equilibrium solution can be approximated with a high precision. The upper bound is a good approximation if heterogeneity between agents 1 and 2 (or agents 3 and 4, respectively), is low, since, intuitively, the continuation values of the two agents do not differ much then. Contrary, the lower bound is preferable if heterogeneity is high due to the same reasoning. However, the approximation becomes most precise for $\frac{y_{21}}{y_{12}} = \frac{a_2}{a_1+a_2}$ as well as $\frac{y_{43}}{y_{34}} = \frac{a_4}{a_3+a_4}$ rather than 1 or 0, respectively. Obviously, both expressions are between zero and one, and in addition, this formulation accounts for the degree of heterogeneity in each stage-1 interaction. Extensive numerical testing suggests that the percentage deviation of this approximated solution from the correct one is below 1 percentage point even for high degrees of heterogeneity. Using this approximation technique, the equilibrium effort ratios satisfy

$$\frac{y_{21}^*}{y_{12}^*} \approx G\left[R\left(\frac{a_2}{a_1+a_2}\right)\right] \qquad \text{and} \qquad \frac{y_{43}^*}{y_{34}^*} \approx R\left[G\left(\frac{a_4}{a_3+a_4}\right)\right].$$

2.2.2 A Two-Stage Tullock Contest for the Lottery CSF

The approximation presented previously is not necessary if a lottery CSF with discriminatory power $r = 1$ is used. For this special case, a closed-form analytical solution can be derived. Imposing the assumption $r = 1$ on equations (2.7) and (2.8) gives the following system of equations:

$$\frac{y_{21}^*}{y_{12}^*} = \frac{a_4[P^L + \pi_2^*(2-4)]\frac{y_{43}}{y_{34}} + a_3[P^L + \pi_2^*(2-3)]}{a_4[P^L + \pi_1^*(1-4)]\frac{y_{43}}{y_{34}} + a_3[P^L + \pi_1^*(1-3)]} \equiv G\left(\frac{y_{43}}{y_{34}}\right) \tag{2.13}$$

$$\frac{y_{43}^*}{y_{34}^*} = \frac{a_2[P^L + \pi_4^*(4-2)]\frac{y_{21}}{y_{12}} + a_1[P^L + \pi_4^*(4-1)]}{a_2[P^L + \pi_3^*(3-2)]\frac{y_{21}}{y_{12}} + a_1[P^L + \pi_3^*(3-1)]} \equiv R\left(\frac{y_{21}}{y_{12}}\right). \tag{2.14}$$

To make the subsequent analysis tractable, I define the functions κ, ϕ, λ, μ, θ, γ, ψ, and ζ, which depend on exogenous heterogeneity and prize parameters only.[19] Then, inserting

[19] The parameters are defined as follows:

$$\begin{aligned}
\kappa &= a_3(P^L + \pi_1^*(1-3))(P^L + \pi_3^*(3-1)) + a_4(P^L + \pi_1^*(1-4))(P^L + \pi_4^*(4-1)) \\
\phi &= a_3(P^L + \pi_1^*(1-3))(P^L + \pi_3^*(3-2)) + a_4(P^L + \pi_1^*(1-4))(P^L + \pi_4^*(4-2)) \\
\lambda &= a_3(P^L + \pi_2^*(2-3))(P^L + \pi_3^*(3-1)) + a_4(P^L + \pi_2^*(2-4))(P^L + \pi_4^*(4-1)) \\
\mu &= a_3(P^L + \pi_2^*(2-3))(P^L + \pi_3^*(3-2)) + a_4(P^L + \pi_2^*(2-4))(P^L + \pi_4^*(4-2)) \\
\theta &= a_1(P^L + \pi_3^*(3-1))(P^L + \pi_1^*(1-3)) + a_2(P^L + \pi_3^*(3-2))(P^L + \pi_2^*(2-3)) \\
\gamma &= a_1(P^L + \pi_3^*(3-1))(P^L + \pi_1^*(1-4)) + a_2(P^L + \pi_3^*(3-2))(P^L + \pi_2^*(2-4)) \\
\psi &= a_1(P^L + \pi_4^*(4-1))(P^L + \pi_1^*(1-3)) + a_2(P^L + \pi_4^*(4-2))(P^L + \pi_2^*(2-3)) \\
\zeta &= a_1(P^L + \pi_4^*(4-1))(P^L + \pi_1^*(1-4)) + a_2(P^L + \pi_4^*(4-2))(P^L + \pi_2^*(2-4)).
\end{aligned}$$

$R(\frac{y_{21}}{y_{12}})$ in (2.13) and $G(\frac{y_{43}}{y_{34}})$ in (2.14), respectively, results in

$$\frac{y_{21}^*}{y_{12}^*} = \frac{a_2\mu\frac{y_{21}}{y_{12}} + a_1\lambda}{a_2\phi\frac{y_{21}}{y_{12}^*} + a_1\kappa} \tag{2.15}$$

$$\frac{y_{43}^*}{y_{34}^*} = \frac{a_4\zeta\frac{y_{43}}{y_{34}} + a_3\psi}{a_4\gamma\frac{y_{43}}{y_{34}^*} + a_3\theta}. \tag{2.16}$$

In equilibrium, it must obviously hold that $\frac{y_{21}}{y_{12}} = \frac{y_{21}^*}{y_{12}^*}$, as well as $\frac{y_{43}}{y_{34}} = \frac{y_{43}^*}{y_{34}^*}$. Imposing this and rearranging gives two quadratic equations

$$\left(\frac{y_{21}^*}{y_{12}^*}\right)^2 + \frac{a_1\kappa - a_2\mu}{a_2\phi}\left(\frac{y_{21}^*}{y_{12}^*}\right) - \frac{a_1\lambda}{a_2\phi} = 0 \qquad \text{and} \qquad \left(\frac{y_{43}^*}{y_{34}^*}\right)^2 + \frac{a_3\theta - a_4\zeta}{a_4\gamma}\left(\frac{y_{43}^*}{y_{34}^*}\right) - \frac{a_3\psi}{a_4\gamma} = 0,$$

respectively, which are independent from one another. Each of these equations can be solved analytically, which gives the equilibrium ratios $\frac{y_{21}^*}{y_{12}^*}$ as well as $\frac{y_{43}^*}{y_{34}^*}$:[20]

$$\frac{y_{21}^*}{y_{12}^*} = \frac{a_2\mu - a_1\kappa + \sqrt{[a_2\mu - a_1\kappa]^2 + 4a_1a_2\phi\lambda}}{2a_2\phi} = G^*\left(\frac{y_{43}^*}{y_{34}^*}\right) \tag{2.17}$$

$$\frac{y_{43}^*}{y_{34}^*} = \frac{a_4\zeta - a_3\theta + \sqrt{[a_4\zeta - a_3\theta]^2 + 4a_3a_4\psi\gamma}}{2a_4\gamma} = R^*\left(\frac{y_{21}^*}{y_{12}^*}\right). \tag{2.18}$$

Even though the bounds for the approximated solution are not needed for this special case, it is instructive to determine them for a comparison of the analytical and the approximate solution. From (2.15) and (2.16), it follows that the equilibrium ratios of effort are bounded as follows if $r = 1$:

$$\frac{y_{21}^*}{y_{12}^*} \in \left[\frac{\lambda}{\kappa}, \frac{a_2\mu + a_1\lambda}{a_2\phi + a_1\kappa}\right] \qquad \text{and} \qquad \frac{y_{43}^*}{y_{34}^*} \in \left[\frac{\psi}{\theta}, \frac{a_4\zeta + a_3\psi}{a_4\gamma + a_3\theta}\right].$$

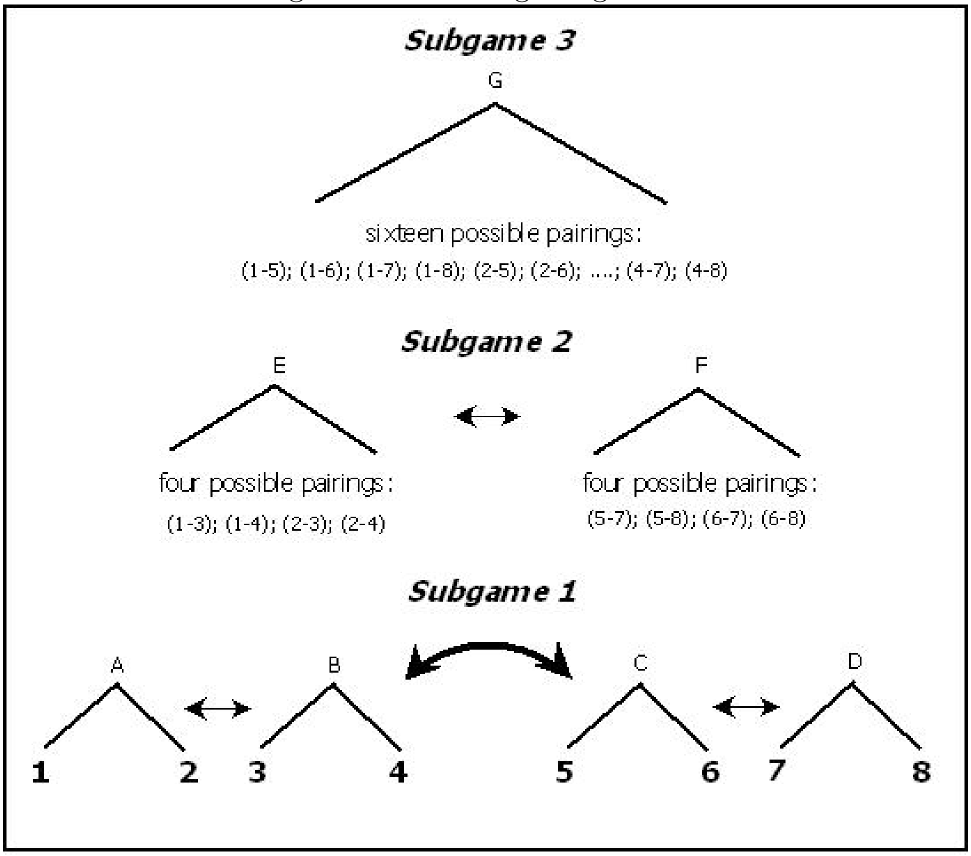
Using the ratios $\frac{y_{21}}{y_{12}} = \frac{a_2}{a_1+a_2}$ as well as $\frac{y_{43}}{y_{34}} = \frac{a_4}{a_3+a_4}$ as suggested in the previous section gives

$$\frac{y_{21}^*}{y_{12}^*} \approx \frac{a_2^2\mu + (a_1^2 + a_1a_2)\lambda}{a_2^2\phi + (a_1^2 + a_1a_2)\kappa} \qquad \text{and} \qquad \frac{y_{43}^*}{y_{34}^*} \approx \frac{a_4^2\zeta + (a_3^2 + a_3a_4)\psi}{a_4^2\gamma + (a_3^2 + a_3a_4)\theta}$$

for the approximated equilibrium effort ratios. Comparing them to the analytical solution in (2.17) and (2.18), one can show that they depend on the same parameters, and have qualitatively identical comparative statics properties. Numerical testing shows that the approximate equilibrium effort levels are always marginally higher than their analytical counterparts.

[20]Note that the quadratic equation has two roots, only one of which is positive. The negative root is irrelevant for the question at hand, since effort choices are restricted on the positive domain.

Figure 2.4: The Eight Agent Case



2.2.3 More Than Two Stages

This section discusses the additional complications which arise in contests with more than two stages. Figure 2.4 illustrates the case of a three-stage contest with eight agents. In this case, there are three instead of two subgames. As for the two-stage contest, the solution of the game is obtained via backwards induction, i.e., start with subgame 3, continue with subgame 2, and finally consider subgame 1.

Subgame 3 has exactly the same structure as the pair-wise interactions in the last stage of the two-stage contest. The only difference is that sixteen rather than four different pairings are possible in the last stage of the three-stage contest. Recall that each of these interactions can be solved analytically for any degree of heterogeneity and any discriminatory power as long as Assumption 2.1 is satisfied. The structure of subgame 2 is identical to the one of stage 1 in the two-stage contest. However, now there are four possible combinations of agents each of the two interaction E and F, which gives *sixteen* different situations in subgame 2, namely $\{(1-3),(5-7)\};\{(1-3),(5-8)\};...$ and so on. Solving all of them, either analytically if $r = 1$, or approximately otherwise, is tedious, but without problems in addition to these that were discussed previously.

A new complication arises in subgame 1, where each interaction depends on all three remaining interactions of the subgame, i.e., agents 1 and 2 in pairing A play a best response to each other, *and* to the ratio of efforts by agents 3 and 4, by 5 and 6, and by 7 and 8. The best way to illustrate this point is to analyze the structure of the continuation values. Denote the effort of agent i in stage 1 by z_i and consider the continuation value P_1 of agent 1 in subgame 3:

$$\begin{aligned} P_1\left(\frac{z_3}{z_4}, \frac{z_5}{z_6}, \frac{z_7}{z_8}\right) &= p_{34}(p_{56}[p_{78} \times S_{1357} + p_{87} \times S_{1358}] + p_{65}[p_{78} \times S^{1367} + p_{87} \times S^{1368}]) \\ &+ p_{43}(p_{56}[p_{78} \times S_{1457} + p_{87} \times S_{1458}] + p_{65}[p_{78} \times S_{1467} + p_{87} \times S_{1468}]) \end{aligned}$$

S_{1ijk} is the expected payoff which a particular constellation of agents in subgame 2 has for agent 1;[21] p_{ij} is the probability that agent i wins against agent j, and it is a function of the ratio $\frac{z_i}{z_j}$. Therefore, as in the case with four agents, probabilities make the continuation values endogenous. However, the continuation value now depends on three endogenously determined probabilities rather than only one. The first step of the solution is to consider the right (1,2,3,4) and left (5,6,7,8) branches separately, i.e., solve the interdependence between interactions A and B as well as between C and D. Each of the two branches is of the same structure as stage 1 in a two-stage contest. Then, however, there is still an interdependence across the two branches. Formally, the problem can be described as follows:

$$\frac{z_2^*}{z_1^*}/\frac{z_4^*}{z_3^*} = \frac{P_2\left(\frac{z_6}{z_5}, \frac{z_8}{z_7}\right) \times P_3\left(\frac{z_6}{z_5}, \frac{z_8}{z_7}\right)}{P_1\left(\frac{z_6}{z_5}, \frac{z_8}{z_7}\right) \times P_4\left(\frac{z_6}{z_5}, \frac{z_8}{z_7}\right)} \equiv H\left(\frac{z_6}{z_5}/\frac{z_8}{z_7}\right) \tag{2.19}$$

$$\frac{z_6^*}{z_5^*}/\frac{z_8^*}{z_7^*} = \frac{P_6\left(\frac{z_2}{z_1}, \frac{z_4}{z_3}\right) \times P_7\left(\frac{z_2}{z_1}, \frac{z_4}{z_3}\right)}{P_5\left(\frac{z_2}{z_1}, \frac{z_4}{z_3}\right) \times P_8\left(\frac{z_2}{z_1}, \frac{z_4}{z_3}\right)} \equiv Q\left(\frac{z_2}{z_1}/\frac{z_4}{z_3}\right). \tag{2.20}$$

This system of equations implicitly defines a solution to subgame 1.[22] However, this solution cannot be determined analytically in general, not even for $r = 1$. Only special cases can be solved, where either interactions A and B, or interactions C and D are homogeneous, or both. In each of these cases, there is no interaction effect across the two branches AB and CD in subgame 3. In all other cases, the solution can only be approximated, either numerically if the productivity for each of the eight agents is known, or analytically, using the technique introduced at the end of section 2.2.2. In the latter case, one obtains

$$\frac{z_2^*}{z_1^*}/\frac{z_4^*}{z_3^*} \approx H\left[Q\left(\frac{a_2}{a_1+a_2}/\frac{a_4}{a_3+a_4}\right)\right] \qquad \text{and} \qquad \frac{z_6^*}{z_5^*}/\frac{z_8^*}{z_7^*} \approx Q\left[H\left(\frac{a_6}{a_5+a_6}/\frac{a_8}{a_7+a_8}\right)\right].$$

With each additional stage that is added to the multi-stage pair-wise elimination contest, the number of potential pairings in later stages of the game increases. It is more problematic, however, that a new interaction effect across the two different branches of

[21]Meeting agent 3 in interaction E, for example, while agents 5 and 7 compete in F, is of value S_{1357} for agent 1.

[22]The proof for this claim includes the same steps as the one for Theorem 1. The expressions do become more complicated, however, since there are eight parameters a_i, $i \in [1, 2, ..., 8]$ to consider rather than only four. Therefore, the formal proof is omitted.

the overall game comes up in the first stage of the game for each additional stage. This complication can only be avoided if all interactions in (at least) one of the two branches are homogeneous, such that the interdependence disappears. A solution for the most general multi-stage pair-wise elimination contest with N players of N different player types, however, cannot be determined, and even if it were known, the resulting expressions would be too complicated to characterize analytically. Therefore, the subsequent discussion section focuses on the properties of a two-stage contest.

2.3 Discussion and Additional Results

If agents are heterogeneous, the multi-stage pair-wise elimination contest has several properties which have not received much attention in the contest literature so far. This section provides an analysis of comparative statics properties, of different player arrangements in stage 1 ("seedings" in short), and of the optimal (total effort maximizing) prize structure.[23] None of these topics is exhausted by the discussion in this section; rather, the main idea is to encourage future research in these dimensions, now that a solution for multi-stage pair-wise elimination tournaments with heterogeneous agents is available.

2.3.1 Comparative Statics

In this section, I will discuss how ceteris paribus variations of an agent's productivity parameter affect the equilibrium outcome of this agent and the outcomes of other agents who are participating in the contest. However, working with either the (approximate or analytical) equilibrium solution in stage 1 is extremely hard, since the resulting expressions are complicated. Therefore, I do not consider the complete equilibrium reaction. Instead, I make the following simplifying assumption: If the productivity of agent 1 or 2 changes (who compete against each other in one of the two stage-1 interactions), the equilibrium efforts of agents 3 and 4 remain unchanged, even though their continuation values are allowed to vary. In effect, I suppress the equilibrium response of agents 3 and 4 and its effect on behavior of agents 1 and 2. For reasons previously discussed, this equilibrium response is usually very small. In this sense what follows is, strictly speaking, not a comparative static analysis, but rather an approximation, which is supported by extensive numerical testing. Finally, note that I only analyze stage 1, which is, however, without loss of generality, since the continuation values account for any change in stage-2 equilibrium behavior.

I start by considering the effect which the change on an agent's productivity has within one of the two stage 1 interactions.

[23] Seedings have been considered previously by Groh, Moldovanu, Sela, and Sunde (2012) for a perfectly discriminating all-pay auction; Rosen (1986) discussed the optimal prize structure of two-stage contests with heterogeneous agents, using numerical evidence.

Proposition 2.1. *The stage-1 winning probability p_{ij} of agent i who meets j in a stage-1 interaction is*

(a) increasing in the own productivity parameter a_i.

(b) decreasing in the productivity parameter a_j of the stage 1 opponent.

The same holds for the expected equilibrium payoff Π_i.

Proof. See Appendix. □

Quite intuitively, an agent benefits both in terms of his/her winning probability and the expected payoff if he/she becomes stronger: First, chances of winning stage 1 are higher, and second, conditional on winning stage 1, the agent fares better in stage 2 as well, independent of the stage-2 opponent. Therefore, the continuation value of this agent increases, which further improves chances to win stage 1. If the stage-1 opponent becomes stronger, however, the opposite holds.

Next, consider the effect of productivity changes in one stage-1 interaction on equilibrium outcomes in the other stage-1 interaction. In this case, it matters whether the productivity of the stronger or of the weaker agent is changed. I consider changes in the productivity of the stronger agent first:

Proposition 2.2. *Increasing the productivity parameter of the* ***stronger*** *agent in either of the two stage-1 interactions reduces the overall winning probabilities and the expected payoffs of the agents who compete in the parallel stage-1 interaction.*

Proof. See Appendix. □

Assuming that agents 1 and 2 compete in one, while agents 3 and 4 compete in the second stage-1 interaction, agents 1 and 2 are potential stage-2 opponents for agents 3 and 4, and vice versa. If the stronger of the two potential stage-2 opponents becomes even stronger, the continuation values of the agents in the other stage-1 interaction fall for two reasons: First, if the stronger agent is met, the expected equilibrium payoff is lower than before due to a lower stage-2 winning probability. Second, the probability to meet the strong rather than the weak agent in stage 2 increases due to Proposition 2.1. As a consequence, the overall winning probability (which is the product of the winning probability in stage 1 and the winning probability in stage 2) decreases, and the expected equilibrium payoff falls. Things may be different if the weaker of the two agents in the other stage-1 interaction becomes stronger, however:

Proposition 2.3. *Increasing the productivity parameter of the* ***weaker*** *agent in any of the two stage-1 interactions may increase or decrease the winning probabilities of agents who compete in the parallel stage-1 interaction; the same holds for expected equilibrium payoffs.*

Proof. See Appendix. □

As in case of Proposition 2.2, the continuation values of the agents competing in the other stage-1 interaction are affected. However, there are two effects which work in opposite directions, such that the direction of the overall effect is ambiguous. First, if the agent whose strength is increased is met in stage 2, both the winning probability and the expected payoff are reduced, which tends to reduce the continuation value. However, at the same time, the probability to meet the weaker agent increases due to Proposition 2.1, and meeting the weaker agent in stage 2 is still better than meeting the stronger one for any of the two agents in the other stage-1 interaction. Which effect dominates depends on the specific values of productivity parameters a_1, a_2, a_3, and a_4. However, it is easy to construct situations where it is beneficial for agents in the other stage-1 interaction if the weak agent becomes stronger.

Without the intuition provided above, this result is surprising, as it implies a non-monotonicity. In a situation, for example with $a_1 \geq a_2 \geq a_3 \geq a_4$, where agents 1 and 2 compete in one, while agents 3 and 4 compete in the second stage-1 interactions, it can be that the weakest agent 4 is better of if the already second strongest agent 2 becomes even stronger. Similar results are impossible in any contest structure where all agents meet one another, either simultaneously in a one-shot contest, or sequentially in a round-robin tournament.

2.3.2 Seedings

For the remainder of this section, I assume without loss of generality that agents are naturally ordered by their strength, i.e., the relation $a_1 \geq a_2 \geq a_3 \geq a_4$ does hold. Note that there are three different ways to seed the four agents in stage 1 of the two-stage contest: (1-4, 2-3), where agents 1 and 2 to compete in one of the stage-1 interactions, whereas agents 3 and 4 participate in the remaining one; (1-3, 2-4), where agents 1 and 3 as well as agents 2 and 4 compete with each other in stage 1, and finally (1-2, 3-4), the setting which I considered when solving the model in section 2.2.1. These three settings, or "Seedings", have different properties, i.e., even if the types of the four agents are left unchanged, seeding them differently changes the properties of the equilibrium.

The properties of different seedings have been analyzed previously by various authors, usually considering the case of four players, since the number of possible seedings explodes with the number of players: With 2^N players, there are $\frac{(2^N)!}{2^{(2^N-1)}}$ different seedings, i.e., there are 3 seedings for 4 players (as seen above), 315 seedings for 8 players, etc. Cases with up to eight players have only been addressed by the statistical literature where agents are not optimizing but probabilities are instead given exogenously; even then the problem is hard to handle analytically.[24] To my knowledge, the paper by Groh, Moldovanu, Sela, and Sunde (2012) is the only one which uses winning probabilities that are endogenously determined by optimizing agents, as in my model. The main difference between the

[24]See for example Schwenk (2000), Hwang (1982) or Horen and Riezman (1985).

approach in this paper and their model is that Groh, Moldovanu, Sela, and Sunde (2012) use a perfectly discriminating all-pay contest success function, rather than an imperfectly discriminating CSF.[25] Therefore, I will investigate in what follows whether, and if so, in how far, properties of different seedings are influenced by the choice of the contest success function.

Groh, Moldovanu, Sela, and Sunde (2012) compare the performance of the three seedings with respect to four optimality, or fairness criteria: (1) maximization of total effort, (2) maximization of the probability of a stage-2 interaction between the two strongest agents, (3) maximization of the winning probability of the strongest agent, and (4) winning probabilities of agents are ordered according to the agent's strength. Criteria (1) to (3) are optimality criteria that are standard in the literature on seedings, but not only there: It is a standard goal of designers in the personnel economics literature to maximize the effort provision of participating employees. Further, in case of promotion tournaments, it might be in the principal's interest to select the most able employees if ability is private information. A similar reasoning applies to designers in sports. Criterium (4), however, is rather a fairness than an optimality criterium: The basic idea is that nobody should have a strategic disadvantage that is so high that the order of ability and winning probability is changed.

Following Groh, Moldovanu, Sela, and Sunde (2012), I use the capital letters A, B, and C to distinguish the three seedings from one another and define *Seeding* A: 1-4, 2-3, *Seeding* B: 1-3, 2-4, and *Seeding* C: 1-2, 3-4. For their CSF specification, and under the assumption $a_1 \geq a_2 \geq a_3 \geq a_4$, Groh, Moldovanu, Sela, and Sunde (2012) find that Seeding A: 1-4, 2-3 satisfies criteria (3) and (4), while Seeding B: 1-3, 2-4 fulfills both (1) and (2); none of the four criteria applies to Seeding C: 1-2, 3-4. If the prize structure in my model with an imperfectly discriminating CSF is specified in the same manner as in Groh, Moldovanu, Sela, and Sunde (2012), I get very similar results, with one notable exception: It still holds that Seeding A: 1-4, 2-3 satisfies criteria (3) and (4), while Seeding B: 1-3, 2-4 fulfills (2). However, criterion (1) may now be satisfied either by Seeding B: 1-3, 2-4, *or* by Seeding C: 1-2, 3-4, depending on the parameters a_1, a_2, a_3, and a_4. This finding is summarized in the subsequent Proposition:

Proposition 2.4. *None of the three Seedings maximizes total expected effort provision for all specifications of heterogeneity* a_1, a_2, a_3 *and* a_4.

Proof. See Appendix. □

Numerical testing suggests that Seeding C: 1-2, 3-4 is optimal with respect to criterion (1) if the difference in strengths between the two weaker and the two stronger agents is not too big; if this difference is extreme, Seeding B: 1-3, 2-4 maximizes total expected

[25] Apart from that, there is one additional restriction on the prize structure in their model that is not needed here. In particular, the prize for winning in stage 1 must be strictly positive in the model by Groh, Moldovanu, Sela, and Sunde (2012) to ensure participation and equilibrium existence in stage 1.

effort provision. This makes sense intuitively: If the difference in strength between the two stronger agents and the two weaker ones is relatively small, the three pair-wise interactions in Seeding C: 1-2, 3-4 are relatively close, such that effort provision is high. If, however, agents 1 and 2 are much stronger than agents 3 and 4, the stage-2 interaction, where most of the effort is provided, is extremely unequal, such that the effort is very low. Then, Seeding B: 1-3, 2-4 ensures a higher effort provision due to the fact that a final between the two strongest agents is extremely likely, a situation in which effort provision is high.

Summing up, it seems that measures which depend on relative effort choices such as probabilities are not affected by the technology of the contest success function, i.e., it seems that the choice of a certain seeding automatically implies that criteria which depend on relative effort provision are satisfied.[26] Yet, this does not hold for absolute measures like total expected effort provision. This implies that any contest designer who is interested in the maximization of total effort provision needs information about the importance of productivity differences for contest outcomes, i.e., on the discriminatory power of the contest success function.

2.3.3 Optimal Prizes

The optimality of the prize structure usually refers to the maximization of total effort provision.[27] It is well known in the contest literature that a unique prize maximizes total effort expenditures in one-shot contests if agents are homogeneous and risk-neutral, while multiple prizes can be optimal in heterogeneous settings. This finding holds for all conventionally used types of contest success functions.[28] However, little is known about the optimal prize structure in multi-stage tournaments, and even less if the agents are heterogeneous.

It is straightforward to show that a unique prize maximizes total effort expenditures in multi-stage pair-wise elimination tournaments with a Tullock CSF if all participating agents are homogeneous; this holds for any discriminatory power r that still allows for an equilibrium in pure strategies.[29] This result, however, may no longer hold if agents are heterogeneous. To be more precise, multiple prizes can be optimal if the interaction in stage 2 is likely to be a pairing between two agents whose strength differs a lot, while at least one of the two stage-1 interactions is fairly homogeneous. An example is a situation with one superstar and three rather weak agents. In such a setting, it can be optimal to have two identical prizes, such that no effort is provided in stage 2 and both

[26]See also the discussion in the conclusion of Groh, Moldovanu, Sela, and Sunde (2012) which relates to the paper by Horen and Riezman (1985).

[27]An exception is Rosen (1986), who considers a prize structure as optimal if effort provision is constant in all stages of a multi-stage pair-wise elimination tournament.

[28]See, for example, Clark and Riis (1996), Clark and Riis (1998a), Clark and Riis (1998c), Krishna and Morgan (1998), or Moldovanu and Sela (2001). An excellent survey is provided by Sisak (2009).

[29]If $r > 2$, the equilibrium is in mixed strategies. See the respective discussion in Baye, Kovenock, and de Vries (1994).

stage-1 interactions become simple static one-shot interactions. The reason is that effort provision in the homogeneous stage-1 interaction is independent of heterogeneity, and therefore constant, while effort provision in the two-stage contest with a unique prize approaches zero as the degree of heterogeneity increases.

Overall, extensive numerical testing suggests that two situations can be distinguished in tournaments with multiple stages: Either a unique prize is optimal, or it is optimal to have two identical prizes such that stage 2 is dropped and only two pair-wise interactions in stage 1 remain. Essentially, this implies that the structure of prizes can be used to change the structure of the contest. This suggests that the joint optimization of contest and prize structure, which has so far only been addressed in homogeneous settings (Fu and Lu 2012), may be an interesting topic for future research.

2.4 Conclusion

This paper characterizes a solution for multi-stage pair-wise elimination contests with heterogeneous agents who differ with respect to their effort effectiveness. Elimination contests have received much attention in the contest literature, since they capture many real life situations, such as promotion and sport tournaments, for example. While attention has focused on cases where all agents are homogeneous so far, this paper discusses the arguably more relevant general case where agents are of different types. I show under which conditions a subgame perfect Nash equilibrium exists when a general Tullock contest success function (CSF) is used. Moreover, the equilibrium solution is derived analytically for the special case of a lottery CSF, and characterized for the remaining cases. So far, a solution to multi-stage pair-wise elimination contests is available only for the perfectly discriminating all-pay auction contest success function (Groh, Moldovanu, Sela, and Sunde 2012). Most contests in reality are, however, imperfectly discriminating. Apart from that, the approach taken here has the advantage that no restrictions on the structure of prizes are needed, and that the equilibrium is in pure strategies, which, for example, facilitates experimental testing of properties predicted by theory.

The main complication that arises in a multi-stage pair-wise elimination contest once agents are allowed to be heterogeneous is that continuation values in early stages become endogenous due to feedback effects across different branches of the game. This paper analyzes these effects in some detail for the most simple multi-stage contest with only two stages. Subsequently, additional complications that arise in more complicated settings with three stages or more were briefly discussed.

A rather short analysis of certain properties of multi-stage pair-wise elimination contest with heterogeneous agents suggests that this contest format has several features that distinctly differ from other contest formats. For example, I show that it can be beneficial for the weakest agent in the contest if some of the other agents becomes even stronger than he/she already is. Or, with respect to the structure of prizes, it seems that a runner-up prize for the loser in stage 2 that is smaller than the main prize is never optimal; either a unique prize or two equal prizes are optimal with respect to effort maximization. These issues certainly deserve more attention in future research. Apart from that, it might be interesting to compare the results of the model presented in this paper to a model in which agents are budget-constrained. Many authors argue that agents face budget constraints in real life (Parco, Rapoport, and Amaldoss 2005, Stein and Rapoport 2005, Amegashie, Cadsby, and Song 2007). The implications of these constraints on behavior of agents in settings with heterogeneous types and imperfectly discriminating CSFs appear not to have been explored yet. This would also help to clarify the robustness of the results by Harbaugh and Klumpp (2005), who consider a special case of the model which is analyzed in this paper and assume that the endowment is of no intrinsic value to simplify their analysis.

Appendix

Proof of Theorem 2.1:

First, note that Assumptions 2.1 ensures that the stage-2 subgame has a unique pure strategy equilibrium. Further, Assumption 2.2 implies that each of the two stage-1 interactions has a unique pure strategy equilibrium, conditional on given continuation values.[30] Consequently, what remains to be proven is that the system of equations (2.7) and (2.8) has at least one solution.

For the proof, I will first show that the functions $G(\cdot)$ and $R(\cdot)$ are either strictly monotonic or constant and equal to one. If at least one of the two functions is constant, there is no interdependence between equations (2.7) and (2.8), and it is straightforward to show that there is a (unique) solution to the system of equations. For the second case where $G(\cdot)$ and $R(\cdot)$ are strictly monotonic, I will show that the graphs of $G(\cdot)$ and the inverse function $R^{-1}(\cdot)$ intersect at least once; the inverse function is defined, since $R(\cdot)$ is strictly monotonic and continuous on the domain $[0,1]$.[31]

Taking the first derivatives of $G(\cdot)$ and $R(\cdot)$ with respect to their only argument, one obtains

$$\frac{\partial G\left(\frac{y_{43}}{y_{34}}\right)}{\partial\left(\frac{y_{21}}{y_{12}}\right)} = a_3 a_4 \frac{[P^L + \pi_1^*(1-3)][P^L + \pi_2^*(2-4)] - [P^L + \pi_1^*(1-4)][P^L + \pi_2^*(2-3)]}{[a_3(P^L + \pi_1^*(1-3)) + a_4(P^L + \pi_1^*(1-4))\left(\frac{y_{21}}{y_{12}}\right)^r]^2} \left(\frac{y_{21}}{y_{12}}\right)^{r-1}$$

$$\frac{\partial R\left(\frac{y_{21}}{y_{12}}\right)}{\partial\left(\frac{y_{43}}{y_{34}}\right)} = a_1 a_2 \frac{[P^L + \pi_3^*(3-1)][P^L + \pi_4^*(4-2)] - [P^L + \pi_3^*(3-2)][P^L + \pi_4^*(4-1)]}{[a_1(P^L + \pi_3^*(3-1)) + a_2(P^L + \pi_3^*(3-2))\left(\frac{y_{43}}{y_{34}}\right)^r]^2} \left(\frac{y_{43}}{y_{34}}\right)^{r-1}$$

Note that the denominator is always positive in both expressions (it is squared). This implies that the sign of the slope is fully determined by the numerator. Equilibrium requires that each of the two ratios of effort must be between zero and one. Therefore, the sign of the numerators of both $G(\cdot)$ and $R(\cdot)$ depends on a difference of two expressions of heterogeneity and prize parameters which are exogenously given. As a consequence, one has to distinguish two cases: For both $G(\cdot)$ and $R(\cdot)$, respectively, it holds that the function is either strictly monotone in the domain of interest (increasing or decreasing), or the slope of the function is always zero. Analysis of $G(\cdot)$ reveals that the slope of $G(\cdot)$ is zero if and only if agents 1 and 2 are of the same player type; (if and only) if this is the case, it holds that $G(\cdot) = \frac{y_{21}^*}{y_{12}^*} = 1$, i.e. the two agents choose the same level of efforts. Similarly, $R(\cdot)$ is equal to one for all values of $\frac{y_{21}}{y_{12}}$ if and only if agents 3 and 4 are of the same type.

This implies that I have to consider three different scenarios for the proof: (1) $G(\cdot)$ and $R(\cdot)$ are equal to one and therefore independent of one another; (2) either $G(\cdot)$ or $R(\cdot)$

[30] This has been proven by Nti (1999) and Cornes and Hartley (2005), for example.

[31] Recall that the equilibrium ratios $\frac{y_{21}^*}{y_{12}^*}$ and $\frac{y_{43}^*}{y_{34}^*}$ must both be between zero and one, since it holds by assumption that (i) agent 1 is stronger than agent 2 ($a_1 \geq a_2$), and (ii) agent 3 is stronger than agent 4 ($a_3 \geq a_4$).

are equal to one, i.e. one of the two relations depends on the other one, but not vice versa; (3) neither $G(\cdot)$ nor $R(\cdot)$ are equal to one, and the two functions are interdependent. It is straightforward to show that a solution to the system consisting of (2.7) and (2.8) does exist in cases (1) and (2); case (3) is somewhat more involved and will be dealt with next.

Due to the previous reasoning, it must be the case that both $G(\cdot)$ and $R(\cdot)$ are strictly monotonic in case (3). Therefore, it is possible to determine the inverse function of $R(\cdot)$. It holds that

$$R^{-1}\left(\frac{y_{21}}{y_{12}}\right) \equiv \sqrt[r]{\frac{a_1}{a_2}\frac{[P^L + \pi_3^*(3-1)]\left(\frac{y_{43}^*}{y_{34}^*}\right) - [P^L + \pi_4^*(4-1)]}{[P^L + \pi_4^*(4-2)] - [P^L + \pi_3^*(3-2)]\left(\frac{y_{43}^*}{y_{34}^*}\right)}} = \left(\frac{y_{21}}{y_{12}}\right). \tag{2.21}$$

By definition of the inverse function, it must hold that $R^{-1}(\frac{y_{21}}{y_{12}})$ is strictly monotonic. Further, $R^{-1}(\frac{y_{21}}{y_{12}})$ has a unique root (in the relevant domain $\frac{y_{43}^*}{y_{34}^*} \in [0,1]$) at $Z = \frac{[P^L+\pi_4^*(4-1)]}{[P^L+\pi_3^*(3-1)]}$, where $0 < Z < 1$. Finally, close inspection of (2.21) reveals that $R^{-1}(\frac{y_{21}}{y_{12}})$ has a pole at $W = \frac{[P^L+\pi_4^*(4-2)]}{[P^L+\pi_3^*(3-2)]}$, $0 < W < 1$. Since $G(0)$ and $G(1)$ are both strictly smaller than 1, it must be that the graphs of the functions $G(\cdot)$ and $R^{-1}(\cdot)$ intersect at least once in the relevant domain $\frac{y_{43}}{y_{34}} \in [0,1]$ by intermediate value theorem, which completes the proof.

Proof of Proposition 2.1:

This proof consists of two parts: In part (1), I will consider stage 1 winning probabilities, whereas I consider expected equilibrium payoffs in part (2). Before I can start with the proof of Proposition 2.1, however, I will derive the respective expressions for the stage 1 equilibrium winning probability p_{ij} and the expected equilibrium payoff in stage 1, Π_i^*. Without loss of generality, I assume that agents 1 and 2 meet in one, while agents 3 and 4 meet in the second stage 1 interaction. Then, the winning probability of agent 1 in stage 1 is defined as

$$p_{12} = \frac{a_1 y_{12}^r}{a_1 y_{12}^r + a_2 y_{21}^r} = \frac{a_1}{a_1 + a_2\left(\frac{y_{21}}{y_{12}}\right)^r}.$$

From equation (2.7), one can show that

$$\frac{y_{21}^*}{y_{12}^*} = \frac{P^L + p_{34} \times \pi_2^*(2-3) + (1-p_{34}) \times \pi_2^*(2-4)}{P^L + p_{34} \times \pi_1^*(1-3) + (1-p_{34}) \times \pi_1^*(1-4)} = \frac{P^L + P_2^c(y_{34}, y_{43})}{P^L + P_1^c(y_{34}, y_{43})}.$$

Note that $P_2^c(y_{34}, y_{43})$ is increasing in a_2, while $P_1^c(y_{34}, y_{43})$ is increasing in a_1. This is because the expected equilibrium payoff of a pair-wise interaction between agents i and j for any agent i is strictly increasing in the effort productivity parameter of agent i, as inspection of equation (2.4) clearly reveals. Note that the effect of changes in a_1 or a_2 on p_{34} is ignored in this analysis; as already mentioned in the paper, the incorporation of this equilibrium reaction effect complicates the expressions to an extent that cannot be characterizes analytically.

(1) Inserting the above expression for the effort ratio in the winning probability gives

$$p_{12} = \frac{1}{1 + \frac{a_2}{a_1}\left(\frac{P^L + P_2^c(y_{34}, y_{43})}{P^L + P_1^c(y_{34}, y_{43})}\right)^r} = \frac{1}{1 + \Phi}, \tag{2.22}$$

where $\Phi = \frac{a_2}{a_1}(\frac{P^L + P_2^c(y_{34}, y_{43})}{P^L + P_1^c(y_{34}, y_{43})})^r$. It is straightforward to show that

$$\frac{\partial \Phi}{\partial a_1} < 0 \quad \text{and} \quad \frac{\partial \Phi}{\partial a_2} > 0 \tag{2.23}$$

do hold. In combination with the fact that $\frac{\partial p_{12}}{\partial \Phi} < 0$, this proves parts (a) and (b) of Proposition 2.1. The winning probabilities of agents 2, 3, and 4, have exactly the same structure, and proving the relations for those expressions goes through the same steps.

(2) Now, I consider the expected payoff, which can be shown to satisfy

$$\Pi_1 = \frac{1 + (1 - r)\Phi}{(1 + \Phi)^2}[P^L + P_1^c(y_{34}, y_{43})]. \tag{2.24}$$

Simple algebra shows that $\frac{\partial \Pi_1}{\partial \Phi} < 0$, which in combination with the results of (2.23) proves the claim. The same holds for the expected equilibrium payoffs of agents 2, 3, and 4, which have the same structure as the one for agent 1.

Proof of Proposition 2.2:

Without loss of generality, I assume that agents 1 and 2 meet in one, while agents 3 and 4 meet in the second stage 1 interaction. Further, agent 1 is stronger than agent 2, while agent 3 is stronger than agent 4, i.e. the relations $a_1 \geq a_2$ and $a_3 \geq a_4$ do hold. Now, I have to proof that both the overall winning probability and the expected equilibrium payoff of agents 3 and 4 are decreasing in a_1. Further, the same must hold for payoffs and probabilities of agents 1 and 2 with respect to a_3. The overall winning probability will be considered in part (1) of the proof; the expected equilibrium payoff follows in part (2).

(1) The overall winning probability for agent 3 is defined as follows:

$$\wp_3 = p_{34}^1 \times [p_{12}^1 \times p_{31}^2 + (1 - p_{12}^1) \times p_{32}^2].$$

p_{34}^1 is the probability that agent 3 wins against his stage 1 opponent 4. Conditional on winning stage 1, agent 3 meets agent 1 with probability p_{12}^1 in stage 2, and with probability p_{31}^2 he wins this stage 2 interaction. With the converse probability, agent 3 meets agent 2 in stage 2, against whom he wins with probability p_{32}^2. As already mentioned in the previous proof, I do not consider the equilibrium response with respect to stage 1 efforts that works across the two stage 1 interactions, i.e. the (extremely weak) effect of a change in a_1 on p_{34}^1 is omitted. Note that $p_{31}^2 \leq p_{32}^2$, i.e. agent 3 has a higher winning probability in stage 2 if he meets agent 2 (who is weaker than agent 1).

From Proposition 2.1 I know that p_{12}^1 is increasing in a_1; p_{32}^2 remains unchanged, but p_{31}^2 decreases. Consequently, the overall winning probability $\wp_3$ of agent 3 is decreasing in the strength of the stronger agent in the other stage 1 interaction, a_1. Going through exactly, the same steps, one can show that the same holds for agent 4. Then, since I did not make any assumptions on the relation between agents 1 and 2 as compared to agents 3 and 4, it can be proven in the same way that the overall winning probability of agents 1 and 2 is decreasing in a_3.

(2) The expected equilibrium payoff for agent 3 is defined as

$$\Pi_3 = \frac{1 + (1-r)\frac{y_{43}}{y_{34}}}{(1 + \frac{y_{43}}{y_{34}})^2}[P^L + P_3^c(y_{12}, y_{21})],$$

where $P_3^c(y_{12}, y_{21}) = p_{12}^1 \times \pi_3^*(3-1) + (1-p_{12}^1) \times \pi_3^*(3-2)$. As in all previous proofs, I ignore the indirect effect across stage 1 interactions on efforts, i.e. I assume that the ratio $\frac{y_{43}}{y_{34}}$ is not affected by a change in a_1. Then, I only have to consider the effect of a change in a_1 on Π_3: From Proposition 2.1, I know that p_{12}^1 is increasing in a_1. Further, note that $\pi_3^*(3-1) \leq \pi_3^*(3-2)$. In addition, $\pi_3^*(3-1)$ is decreasing in a_1. All those effects reduce $P_3^c(y_{12}, y_{21})$. Since Π_3 is increasing in $P_3^c(y_{12}, y_{21})$, it must hold that Π_3 is reduced if a_1 is increasing. Corresponding relations can be shown to hold for the expected payoffs agents 1, 2, and 4.

Proof of Proposition 2.3:

Without loss of generality, I assume that agents 1 and 2 meet in one, while agents 3 and 4 meet in the second stage 1 interaction. Further, agent 1 is stronger than agent 2, while agent 3 is stronger than agent 4, i.e. the relations $a_1 \geq a_2$ and $a_3 \geq a_4$ do hold. Now, I have to proof that both the overall winning probability and the expected equilibrium payoff of agents 3 and 4 may be increasing or decreasing in a_2. Further, the same must hold for payoffs and probabilities of agents 1 and 2 with respect to a_4. The overall winning probability will be considered in part (1) of the proof; the expected equilibrium payoff follows in part (2).

(1) Recall from the proof that the overall winning probability for agent 3 is defined as

$$\wp_3 = p_{34}^1 \times [p_{12}^1 \times p_{31}^2 + (1 - p_{12}^1) \times p_{32}^2].$$

Now, recall from Proposition 2.1 that p_{12}^1 is decreasing in a_2; p_{31}^2 remains unchanged, but p_{32}^2 decreases. Consequently, the total effect on the overall winning probability $\wp_3$ of agent 3 is ambiguous: $\wp_3$ is increasing, since p_{12}^1 decreases; however, at the same time, p_{32}^2 decreases, which tends to decrease $\wp_3$. Going through exactly, the same steps, one can show that the same holds for agent 4. Then, since I did not make any assumptions on the relation between agents 1 and 2 as compared to agents 3 and 4, it can be proven in the same way that the overall winning probability of agents 1 and 2 is decreasing in a_3.

(2) The expected equilibrium payoff for agent 3 is defined as

$$\Pi_3 = \frac{1 + (1-r)\frac{y_{43}}{y_{34}}}{(1 + \frac{y_{43}}{y_{34}})^2}[P^L + P_3^c(y_{12}, y_{21})],$$

where $P_3^c(y_{12}, y_{21}) = p_{12}^1 \times \pi_3^*(3-1) + (1-p_{12}^1) \times \pi_3^*(3-2)$. As in all previous proofs, I ignore the indirect effect across stage 1 interactions on efforts, i.e. I assume that the ratio $\frac{y_{43}}{y_{34}}$ is not affected by a change in a_2. Then, I only have to consider the effect of a change in a_2 on Π_3: From Proposition 2.1, I know that p_{12}^1 is decreasing in a_2, which tends to increase $P_3^c(y_{12}, y_{21})$, since $\pi_3^*(3-1) \leq \pi_3^*(3-2)$. Note, however, that $\pi_3^*(3-2)$ is decreasing in a_2, an effect that tends to reduce $P_3^c(y_{12}, y_{21})$. As a consequence, the total effect of a change in a_2 on $P_3^c(y_{12}, y_{21})$ is ambiguous. Since Π_3 depends linearly on $P_3^c(y_{12}, y_{21})$, the overall effect of a change in a_2 on Π_3 is unclear. Corresponding relations can be shown to hold for the expected payoffs agents 1, 2, and 4.

Proof of Proposition 2.4:

Proposition 2.4 can be proven by the presentation of two examples. Below, I present two different parameterizations for a two stage contest: Total effort provision is maximized in Seeding C: 1-2, 3-4 for the first one, while Seeding B: 1-3, 2-4 maximizes total effort provision for the second one, which proves the claim that there is no Seeding that always maximized total effort provision.

(1) Assume that the vector (a_1, a_2, a_3, a_4) is defined as follows: $(a_1, a_2, a_3, a_4) = (4, 3, 2, 1)$. Under the assumption that $r = 1$ and $P^L = 0$, the solution to the model that was presented in section 2.2.2 shows that total effort expenditures are equal to 0.5986 in Seeding C, to 0.492187 in Seeding B, and to 0.5273 in Seeding A, i.e. total effort provision is maximized in Seeding C: 1-2, 3-4.

(2) Assume that the vector (a_1, a_2, a_3, a_4) is defined as follows: $(a_1, a_2, a_3, a_4) = (200, 70, 20, 1)$. Under the assumption that $r = 1$ and $P^L = 0$, the solution to the model that was presented in section 2.2.2 shows that total effort expenditures are equal to 0.3596 in Seeding C, to 0.3687 in Seeding B, and to 0.3261 in Seeding A, i.e. total effort provision is maximized in Seeding B: 1-3, 2-4.

Chapter 3

Orchestrating Rent-Seeking Contests with Heterogeneous Agents

3.1 Introduction

Contests are situations in which agents compete by expending valuable resources to win a prize. Independently of success or failure, all contestants bear their expenditure costs. Such situations appear in many different areas of economics – including election campaigns, rent seeking competitions, military conflicts, or the competition for bonus payments and promotions on internal labor markets. Given the multiplicity of applications, contests may vary in several dimensions, for example, with respect to their structure. A large proportion of the existing contest design literature analyzes which effect variations of the contest structure have on equilibrium investments by contestants, usually assuming that the central goal of the designer is the maximization of aggregate expenditures, and that contestants are homogeneous. Very little is known, however, about the effect of heterogeneity between contestants on equilibrium behavior in different contest structures, even though interactions between heterogeneous contestants are more likely to be the rule than the exception in reality.

This paper analyzes two prominent contest structures: A static pooling competition where all contestants interact simultaneously, and a dynamic pair-wise elimination format where contestants are split into separate branches and are then sequentially eliminated. In contrast to much of the existing literature, the model in this paper allows for heterogeneous types, i.e., I consider different valuations of contestants: Some contestants attach a high value to the contested prize, while the same prize is worth less to others. Intuitively, the value of being elected in a political campaign, the value of (additional) market shares in advertising wars, or the value of a new patent in R&D competitions depends on the type of the contestant; outside options for politicians, costs structures of firms, or the stock of existing patents in R&D determine the value of the contested good, which I will subsequently refer to as rent. Apart from that, the standard rent-seeking framework with an imperfectly discriminating lottery contest success function (CSF) á la Tullock (1980), linear cost functions and complete information is employed.[1] The question how the contest structure affects overall expenditures by contestants is particularly intriguing for imperfectly discriminating CSFs, since they ensure a positive surplus for participants, i.e., equilibrium contest investments are lower than the value of the rent at stake for this CSF, such that the rent is incompletely dissipated.[2]. Thus, the organizers of the contest may want to modify the design in such a way that the surplus of participants is reduced and overall contest investments increase. An alternative motivation for the subsequent analysis is that a thorough understanding of structural effects on behavior of contestants allows to better understand why particular contest structures are more frequently observed in reality.

Throughout the paper, I assume that the maximization of the rent dissipation rate (which is equal to aggregate expenditures normalized by the contested prize) is the natural goal of the contest designer.[3] Using a lottery contest success function which ensures that the rent dissipation rate in both structures is identical if contestants are homogeneous, I find that the rent dissipation rate is (almost) always higher in the dynamic than in the static format in heterogeneous interactions. Intuitively, the detrimental effect of heterogeneity on contest investments is lower in the dynamic than in the static contest format. While it is well known from previous work that heterogeneity reduces the rent dissipation rate in any immediate interaction, the results indicate that there is a countervailing dynamic effect of heterogeneity across different stages in the dynamic format which works through continuation values: Intuitively, the prospect of facing a low valuation opponent (type `L`) in later stages increases the value of winning and therefore the equilibrium investment levels in early stages for type `H` contestants.[4] Even though equilibrium investment levels for low valuation types are reduced in early stages (since they are likely to meet an opponent with a high valuation in stage 2), the dynamic effect of heterogeneity tends to increase contest investments; the positive dynamic effect of heterogeneity on investments by high valuation contestants dominates the corresponding negative effect on investments by low types, since investments into the contest are linearly increasing in the type specific valuation parameter. While this dynamic heterogeneity effect is always present, the static format may still lead to a higher rent dissipation rate in some cases, namely, if the degree of heterogeneity is so high that low valuation contestants drop-out voluntarily in

[1]The complete information assumption implies that contestants know the type, i.e., the valuation their competitors attach to the rent. This is particularly relevant in small scale contests, and whenever the contestants know each other from previous interactions.

[2]This is not the case in a perfectly discriminating all-pay auction, where the rent is completely dissipated. However, there is a literature on the optimal design in perfectly discriminating contest environments which assumes *incomplete* information, i.e., player types are private information.

[3]This assumption is for simplicity only and without loss of generality. Since only two structures are compared, the results are equally relevant for settings where contest investments are wasteful and one should aim at their minimization.

[4]Using the terminology introduced by Konrad (2010), one might say that heterogeneity reduces the discouragement effect in early stages which is due to future interactions for high valuation contestants.

the static, but not in the dynamic format.

While the technical tools to determine equilibrium behavior by contestants are borrowed from Stein (2002) for the static and from Stracke (2012a) for the dynamic format, this paper is conceptually most closely related to work by Amegashie (1999) and Gradstein and Konrad (1999). While these two papers also compare static and dynamic pair-wise elimination contests, they assume that contestants are homogeneous; both Amegashie (1999) and Gradstein and Konrad (1999) allow for variations of the discriminatory power, however, i.e., they investigate how the noisiness of the contest technology affects behavior in static and dynamic contest formats. I complement and extend their work by allowing for heterogeneous contestants. The consideration of heterogeneity is also a general contribution to the contest design literature which assumes imperfectly discriminating contest success functions. Recent work by Fu and Lu (2012), for example, considers the short-listing procedure by Clark and Riis (1996) rather than pair-wise elimination and determines both the optimal structure of the contest and the effort maximizing prize allocation rule. However, even though they acknowledge the importance of heterogeneity in their conclusion, they assume homogeneity. Different contest structures with heterogeneous agents have so far only been considered for perfectly discriminating all-pay auctions technology by Moldovanu and Sela (2006). However, the authors assume that abilities are private information of contestants in this paper. As a consequence, their result is driven by a mixture of type uncertainty and type heterogeneity. Instead, this paper uses a imperfectly discriminating technology and assumes that types are common knowledge among contestants, which allows for a clean identification of structure specific heterogeneity effects.

The remainder of this paper is structured as follow. The next section derives the equilibrium rent dissipation rates for both contest structures and presents the generic contest design problem, which is subsequently solved in Section 3.3. Section 3.4 discusses the main results and assesses their robustness. Section 3.5 concludes.

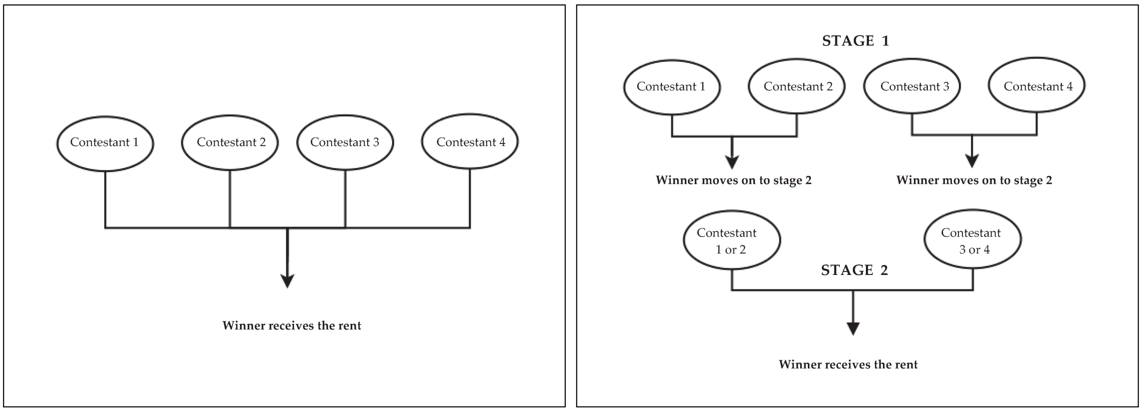
3.2 Theoretical Model

3.2.1 A Generic Contest Design Problem

Consider the problem of a designer who organizes a rent-seeking contest between four risk-neutral contestants in such a way that overall contest investments by participants are maximized; in line with the existing literature, we use the rent dissipation rate as a measure for overall contest investments. The two available design options $\mathcal{S}$ and $\mathcal{D}$ are depicted in Figure 3.1: Either, the recipient of the indivisible rent is determined in a static contest ($\mathcal{S}$), where all contestants interact simultaneously, or in a dynamic elimination contest ($\mathcal{D}$) with three pair-wise interactions on two separate stages.

The (utility) values attached to the rent differ among contestants: Agents of type $\mathtt{H}$

Figure 3.1: Design Options



(a) Static Contest ($\mathcal{S}$) (b) Dynamic Contest ($\mathcal{D}$)

attach the value $v_{\mathtt{H}}$ to the contested rent, while the value of the same rent for agents of type L amounts to $v_{\mathtt{L}}$. Without loss of generality, I assume $v_{\mathtt{H}} \geq v_{\mathtt{L}} > 0$. While valuations parameters are common knowledge among contestants, the designer cannot observe the type of contest participants. She knows, however, that the valuation of agents is either $v_{\mathtt{H}}$ or $v_{\mathtt{L}}$, and second, that the share λ of agents in the overall population $\mathcal{N}$, from which the four contestants are randomly drawn, is of type L.[5] This allows her to use the probability mass function of a binomial distribution to determine the likelihood for an arbitrary configuration with $0 \leq n \leq 4$ contestants of type L and $4 - n$ contestants of type H; the corresponding probability is $f(\lambda, n) = \binom{4}{n}\lambda^n(1-\lambda)^{4-n}$. Defining the rent dissipation rate by n contestants of type L and $4 - n$ contestants of type H in design i as $\mathrm{RD}^i(n)$, the resulting optimization problem of the designer formally reads

$$\max_{i \in \{\mathcal{S}, \mathcal{D}\}} E(\mathrm{RD}^i) \ , \qquad \text{where} \quad E(\mathrm{RD}^i) = \textstyle\sum_{k=0}^{4} f(\lambda, n) * \mathrm{RD}^i(n), \tag{3.1}$$

i.e., the designer chooses any one of the two contest design options $\mathcal{S}$ and $\mathcal{D}$ such that the expected rent dissipation, denoted $E(\mathrm{RD}^i)$, is maximized for a given value of λ.

3.2.2 Equilibrium Behavior by Contestants

Overall, the setup with four contestants of two different types allows for five configurations with differing shares of each type. Throughout, I employ the prominent model of a Tullock (1980) rent-seeking contest with a ratio contest-success-function (CSF) and linear investment costs. The CSF defines an exponential impact function $z(x_i) = x_i^r$, such that winning probabilities of contestants are as follows: If agent i competes against agent j in any one of the three pair-wise interactions of the dynamic contest, or simultaneously against three other agents j, k and l in the static contest, his/her winning probability p_i is given by the ratio of own expenditure impact $z(x_i)$ over expenditure impact by

[5] Since agents are either of type H or of type L, the share of type H agents in $\mathcal{N}$ is $1 - \lambda$.

all contestants $X = \sum_{b \in \mathcal{B}} z(x_b)$, where $\mathcal{B} = \{i, j, k, l\}$ in the static, and $\mathcal{B} = \{i, j\}$ in the dynamic design option. Formally,

$$p_i = \begin{cases} \frac{x_i^r}{x_i^r + \sum_{m \neq i} x_m^r} & \text{if} \quad \sum_{b \in \mathcal{B}} x_b > 0 \\ \frac{1}{\#\mathcal{B}} & \text{if} \quad \sum_{b \in \mathcal{B}} x_b = 0, \end{cases}$$

where $\#\mathcal{B}$ is the number of participants in the contest.[6] The parameter $r \geq 0$ captures the discriminatory power of the CSF and measures the importance of randomness relative to investments in the decision process on an inverse scale.[7] Independent of r, the chosen CSF implies that the winning probability of a contestant is increasing in own investment and decreasing in the investments of the immediate opponent(s). Since investments into the contest are costly for participants, they face a trade-off: Ceteris paribus, increasing the own investment leads to both higher costs and a higher probability of winning. In equilibrium, contestants choose their investment levels such that the marginal costs of investment equal expected marginal gains in terms of a higher probability of winning.

Equilibrium properties of the static and the dynamic contest format are only determined for the special case of a lottery CSF subsequently, i.e., I assume $r = 1$. However, likely effects of variations in the discriminatory power r on the optimal decision of the designer will be discussed in Section 3.4, where the main results are related to previous findings by Gradstein and Konrad (1999).

3.2.2.1 Static Contest ($\mathcal{S}$)

Since the one-stage contest is a simultaneous move game, the solution concept is Nash Equilibrium (NE). Each contestant i maximizes his/her expected payoff by choosing the optimal level of contest investment, taking as given the investment choices of opponents j, k, and l. Formally, the optimization problem of contestant i with valuation v_m ($m = \{\mathtt{H}, \mathtt{L}\}$) is defined as follows:

$$\max_{x_i \geq 0} \Pi_i(x_i, x_j, x_k, x_l) = \frac{x_i}{X} v_m - x_i, \tag{3.2}$$

where $X = x_i + x_j + x_k + x_l$. Since (3.2) is concave in x_i, the first-order condition is both necessary and sufficient for optimality. The partial derivative of (3.2) with respect to x_i reads

$$\frac{\partial \Pi_i(x_i, x_j, x_k, x_l)}{\partial x_i} = \frac{X - x_i}{X^2} v_m - 1,$$

such that the first-order condition for maximization of the objective function (3.2) is either $\frac{\partial \Pi_i(\cdot)}{\partial x_i} = 0$ if $x_i > 0$, or $\frac{\partial \Pi_i(\cdot)}{\partial x_i} \leq 0$ if $x_i = 0$. In equilibrium, contest investments by all contestants with valuation v_m are the same due to symmetry of the objective functions, independent of the particular configuration under consideration. Therefore, the indices i, j, k, and l are dropped and $x_m^*(n)$ defines the equilibrium investment level by an agent

[6] See Skaperdas (1996) for an axiomatization of this CSF.

[7] The decision is independent of investments for $r = 0$, and independent of randomness in the limit for $r \to \infty$.

with valuation v_m in a configuration with n contestants of type $\mathtt{L}$. In configurations where all agents are of the same type ($n = 0$ or $n = 4$), the first-order optimality conditions are binding and all contestants choose the same (strictly positive) equilibrium investment. Using first-order conditions and symmetry delivers

$$x^*_{\mathtt{H}}(0) = \frac{3}{16}v_{\mathtt{H}} \quad \text{and} \quad x^*_{\mathtt{L}}(4) = \frac{3}{16}v_{\mathtt{L}}, \tag{3.3}$$

respectively. When considering the remaining configurations with agents of both types ($0 < n < 4$), however, the n contestants with valuation $v_{\mathtt{L}}$ may optimally choose to invest zero in some settings, and only the equilibrium investment levels of $4 - n$ agents with valuation $v_{\mathtt{H}}$ remain strictly positive. As a consequence, the first-order optimality condition is always binding for high valuation types, but not necessarily for contestants with a low valuation.[8] Considering a configuration with agents of both types and assuming that the first-order conditions are binding for either type, they jointly determine the equilibrium investment ratio:

$$\theta \equiv \frac{x^*_{\mathtt{L}}(n)}{x^*_{\mathtt{H}}(n)} = \frac{v_{\mathtt{H}} - (4-n)(v_{\mathtt{H}} - v_{\mathtt{L}})}{v_{\mathtt{L}} + n(v_{\mathtt{H}} - v_{\mathtt{L}})}. \tag{3.4}$$

Since $x^*_m(n) \geq 0$ and $v_{\mathtt{H}} \geq v_{\mathtt{L}} > 0$ by assumption, $\theta \geq 0$ must hold in an interior NE. This is the case if and only if $\frac{v_{\mathtt{L}}}{v_{\mathtt{H}}} \geq \frac{3-n}{4-n}$. Consequently, contestants with a low valuation invest zero into the contest if their valuation $v_{\mathtt{L}}$ is too low compared to the valuation $v_{\mathtt{H}}$ for a given number of high valuation contestants, i.e., if

$$\frac{v_{\mathtt{L}}}{v_{\mathtt{H}}} \leq \frac{3-n}{4-n}. \tag{3.5}$$

Thus, equilibrium investment levels in configurations with agents of both types are determined by (3.4), (3.5) and the first-order conditions of high and low valuation types, i.e., I obtain

$$x^*_{\mathtt{H}}(n) = \begin{cases} \frac{3-(1-\theta)n}{[4-(1-\theta)n]^2}v_{\mathtt{H}} & (\theta \geq 0) \\ \frac{3-n}{(4-n)^2}v_{\mathtt{H}} & (\theta < 0) \end{cases} \quad \text{and} \quad x^*_{\mathtt{L}}(n) = \begin{cases} \frac{\theta^2(n-1)+\theta(4-n)}{[4-(1-\theta)n]^2}v_{\mathtt{L}} & (\theta \geq 0) \\ 0 & (\theta < 0) \end{cases} \tag{3.6}$$

for contestants of type $\mathtt{H}$ and $\mathtt{L}$, respectively, in configurations with agents of both types ($0 < n < 4$).

3.2.2.2 Dynamic Contest ($\mathcal{D}$)

We employ Subgame Perfect Nash Equilibrium as solution concept due to the dynamic nature of this contest format. The equilibrium is obtained through backward induction. Therefore, I start by analyzing all possible stage-2 interactions. Equilibrium behavior by contestants in all potential stage-1 configurations is considered subsequently; contest investments by contestants i in stage $s = \{1, 2\}$ are denoted x_{is}, and the expected payoff

[8] Formally, this implies $\frac{X - x^*_{\mathtt{H}}}{X^2}v_{\mathtt{H}} - 1 = 0$ and $\frac{X - x^*_{\mathtt{L}}}{X^2}v_{\mathtt{L}} - 1 \leq 0$, respectively.

of i in stage s is defined as Π_{is}.

Stage 2. Consider a pair-wise interaction between contestants i and k and the optimization problem of agent i, who maximizes his/her expected payoff by choosing contest investment x_{i2}, taken as given the investment choice of the opponent k. Formally, the optimization problem of contestant i with valuation v_m ($m = \{\mathtt{H},\mathtt{L}\}$) reads

$$\max_{x_{i2} \geq 0} \Pi_{i2}(x_{i2}, x_{k2}) = \frac{x_{i2}}{X} v_m - x_{i2},$$

where $X = x_{i2} + x_{k2}$. The resulting first-order optimality condition $x_{k2} v_m - X^2 = 0$ is both necessary and sufficient. Moreover, it is always binding in a pair-wise contest for the chosen CSF, independent of the type of contestants i and k.[9] With respect to types, there are three potential constellations: Either both contestants attach the value $v_{\mathtt{H}}$ or $v_{\mathtt{L}}$ to the prize, respectively, or the two contestants have different valuations. In the former two cases, invoking symmetry ($x^*_{i2} = x^*_{k2}$) delivers equilibrium investment levels

$$x^*_{m2}(mm) \equiv x^*_{i2} = x^*_{k2} = \frac{v_m}{4}, \qquad m = \{\mathtt{H},\mathtt{L}\}. \tag{3.7}$$

Inserting equilibrium choices in the objective function gives the expected equilibrium payoff[10]

$$\Pi^*_2(mm) \equiv \Pi^*_{i2}(x^*_{i2}, x^*_{k2}) = \Pi^*_{k2}(x^*_{i2}, x^*_{k2}) = \frac{v_m}{4}, \qquad m = \{\mathtt{H},\mathtt{L}\}. \tag{3.8}$$

In the remaining constellation where stage-2 participants have different valuations, I assume without loss of generality that contestant i (k) has valuation $v_{\mathtt{H}}$ ($v_{\mathtt{L}}$). Combining the respective first-order conditions delivers equilibrium investment levels

$$x^*_{\mathtt{H}2}(\mathtt{LH}) \equiv x^*_{i2} = \frac{v^2_{\mathtt{H}} v_{\mathtt{L}}}{(v_{\mathtt{H}} + v_{\mathtt{L}})^2} \quad \text{and} \quad x^*_{\mathtt{L}2}(\mathtt{LH}) \equiv x^*_{k2} = \frac{v_{\mathtt{H}} v^2_{\mathtt{L}}}{(v_{\mathtt{H}} + v_{\mathtt{L}})^2}, \tag{3.9}$$

respectively. The resulting expected equilibrium payoffs for contestants $i = \mathtt{H}$ and $j = \mathtt{L}$ are

$$\Pi^*_{\mathtt{H}2}(\mathtt{LH}) \equiv \Pi^*_{i2}(x^*_{i2}, x^*_{k2}) = \frac{v^2_{\mathtt{H}}(v_{\mathtt{H}} + v_{\mathtt{L}}) - v^2_{\mathtt{H}}}{(v_{\mathtt{H}} + v_{\mathtt{L}})^2} \tag{3.10}$$

$$\Pi^*_{\mathtt{L}2}(\mathtt{LH}) \equiv \Pi^*_{k2}(x^*_{i2}, x^*_{k2}) = \frac{v^2_{\mathtt{L}}(v_{\mathtt{H}} + v_{\mathtt{L}}) - v^2_{\mathtt{L}}}{(v_{\mathtt{H}} + v_{\mathtt{L}})^2}. \tag{3.11}$$

Stage 1. Assume that contestants i and j, as well as contestants k and l, compete with each other for the right to move on to the next stage in the two pair-wise stage-1 interactions of the dynamic contest. Consider the optimization problem of agent i, who maximizes his/her expected payoff Π_{i1} by choosing contest investment x_{i1}. The

[9] See Cornes and Hartley (2005) for details.

[10] Note that the expected payoff is the same for player i and player k, therefore the indices i and k can be dropped.

(continuation) value of a participation in stage 2 for agent i, denoted C_i, depends on the type of the potential stage-2 opponents k and l, since their type determines the expected stage-2 payoffs $\Pi^*_{i2}(x^*_{i2}, x^*_{k2})$ and $\Pi^*_{i2}(x^*_{i2}, x^*_{l2})$, respectively. Whenever k and l are different types, agent i does not only take the investment x_{j1} by the immediate opponent as given, but also the investment choices x_{k1} and x_{l1} by contestants k and l in the parallel stage-1 interaction; x_{k1} and x_{l1} jointly determine the probability that contestant i interacts with an agent of either type in stage 2. Formally, the general optimization problem of contestant i is defined as follows:

$$\max_{x_{i1}\geq 0} \Pi_{i1}(x_{i1}, x_{j1}, x_{k1}, x_{l1}) = \frac{x_{i1}}{x_{i1}+x_{j1}} \underbrace{\left[\frac{x_{k1}}{x_{k1}+x_{l1}}\Pi^*_{i2}(x^*_{i2}, x^*_{k2}) + \frac{x_{l1}}{x_{k1}+x_{l1}}\Pi^*_{i2}(x^*_{i2}, x^*_{l2})\right]}_{C_i(x_{k1}, x_{l1})} - x_{i1}.$$

The first derivative of $\Pi_{i1}(\cdot)$ with respect to x_{i1} delivers the necessary and sufficient optimality condition

$$\frac{\partial \Pi_{i1}(x_{i1}, x_{j1}, x_{k1}, x_{l1})}{\partial x_{i1}} = \frac{x_{j1}}{(x_{i1}+x_{j1})^2} C_i(x_{k1}, x_{l1}) - 1 = 0, \tag{3.12}$$

which is strictly binding for any type. All possible configurations are separately analyzed below; the stage-1 equilibrium investment level of agent i in a configuration with n contestants of type $\mathtt{L}$ and $4-n$ contestants of type $\mathtt{H}$ is denoted $x^*_{i1}(n)$.

$\mathbf{n=0}$, $\mathbf{n=4}$: All contestants i, j, k, and l are of type $m = \{\mathtt{L},\mathtt{H}\}$. Since agent i knows that the potential stage-2 opponent is always of type $m = \{\mathtt{L},\mathtt{H}\}$, the formal expression for the continuation value simplifies to $C_i = \Pi^*_2(mm)$, where $\Pi^*_2(mm) = \frac{v_m}{4}$ according to (3.8). In equilibrium, the continuation values and investment choices by all contestants are the same due to symmetry of the objective functions; defining $C_m \equiv C_i = C_j = C_k = C_l$ and $x^*_{m1}(n) \equiv x^*_{i1}(n) = x^*_{j1}(n) = x^*_{k1}(n) = x^*_{l1}(n)$, $m = \{\mathtt{L},\mathtt{H}\}$, equation (3.12) gives stage-1 equilibrium investment

$$x^*_{\mathtt{H}1}(0) = \frac{1}{16} v_{\mathtt{H}} \quad \text{and} \quad x^*_{\mathtt{L}1}(4) = \frac{1}{16} v_{\mathtt{L}}, \tag{3.13}$$

respectively.

$\mathbf{n=1}$: Contestants i, k, and l are of type $\mathtt{H}$, only contestant j is of type $\mathtt{L}$. Consider first agents i and j who anticipate that their stage-2 opponent (conditional on winning stage 1) is of type $\mathtt{H}$, independent of stage-1 efforts by k and l. Therefore, it holds that $C_i = \Pi^*_2(\mathtt{HH})$ and $C_j = \Pi^*_{\mathtt{L}2}(\mathtt{LH})$.[11] Combining the first-order conditions of contestants i and j delivers

$$x^*_{i1}(1) = \frac{4v_{\mathtt{H}}^4 v_{\mathtt{L}}^3}{[v_{\mathtt{H}}^3 + 2v_{\mathtt{H}}^2 v_{\mathtt{L}} + v_{\mathtt{H}} v_{\mathtt{L}}^2 + 4v_{\mathtt{L}}^3]^2} \quad \text{and} \quad x^*_{j1}(1) = \frac{16 v_{\mathtt{H}}^3 v_{\mathtt{L}}^6}{[v_{\mathtt{H}} + v_{\mathtt{L}}]^2 [v_{\mathtt{H}}^3 + 2v_{\mathtt{H}}^2 v_{\mathtt{L}} + v_{\mathtt{H}} v_{\mathtt{L}}^2 + 4v_{\mathtt{L}}^3]^2}, \tag{3.14}$$

[11] The respective expressions are defined in (3.8) and (3.11), respectively.

respectively. The optimization problems of agents k and l are symmetric. Both attach the value $C_k = C_l = p_i(\cdot) * \Pi_2^*(\mathtt{HH}) + [1 - p_i(\cdot)] * \Pi_{\mathtt{H}2}^*(\mathtt{LH})$ to a stage-2 participation, where $p_i(\cdot) \equiv p_i(x_{i1}^*(1), x_{j1}^*(1))$ is the probability that type H agent i wins the stage-1 interaction against (type L) agent j. Using (3.12), (3.14), and symmetry gives stage-1 equilibrium investments for agents k and l,

$$x_{k1}^*(1) = x_{l1}^*(1) = \frac{v_{\mathtt{H}}^2[v_{\mathtt{H}} + v_{\mathtt{L}}]^4 + 16v_{\mathtt{H}}^3 v_{\mathtt{L}}^3}{16(v_{\mathtt{H}}[v_{\mathtt{H}} + v_{\mathtt{L}}]^4 + 4v_{\mathtt{L}}^3[v_{\mathtt{H}} + v_{\mathtt{L}}]^2)}. \tag{3.15}$$

n = 2: The configuration $n = 2$ allows for two different settings in the dynamic contest model: Either, stage-1 interactions are mixed (denoted LHLH), or contestants of the same type compete against each other in the two separate stage-1 pairings (denoted LLHH). Setting LHLH occurs with probability 2/3, the probability for the alternative setting LLHH is 1/3.[12] Consequently, the expected stage-1 equilibrium investment for $n = 2$ is defined as

$$x_{b1}^*(2) = \frac{1}{3}x_{b1}^*(\mathtt{LLHH}) + \frac{2}{3}x_{b1}^*(\mathtt{LHLH}), \tag{3.16}$$

where $b = \{i, j, k, l\}$. Equilibrium behavior by contestants in each of the two settings is analyzed below. Consider first setting LLHH, where i and j are of type H, while k and l are of type L. Contestants i and j know that their potential stage-2 opponent is of type L, therefore $C_i = C_j = \Pi_{\mathtt{H}2}^*(\mathtt{LH})$ does hold. Similarly, agents k and l anticipate that they compete with a type H agent in stage 2, conditional on winning stage 1. Thus, $C_k = C_l = \Pi_{\mathtt{L}2}^*(\mathtt{LH})$. Conditions (3.10), (3.11), (3.12), and symmetry ($x_i^* = x_j^*$, as well as $x_k^* = x_l^*$) jointly determine stage-1 equilibrium investment

$$x_{i1}^*(\mathtt{LLHH}) = x_{j1}^*(\mathtt{LLHH}) = \frac{v_{\mathtt{H}}^3}{4[v_{\mathtt{H}} + v_{\mathtt{L}}]^2} \quad \text{and} \quad x_{k1}^*(\mathtt{LLHH}) = x_{l1}^*(\mathtt{LLHH}) = \frac{v_{\mathtt{L}}^3}{4[v_{\mathtt{H}} + v_{\mathtt{L}}]^2}. \tag{3.17}$$

Consider now setting LHLH, where contestants i and k are of type H, while j and l are type L agents. Since each agent of type H interacts with an agent of type L in stage 1, contestants know that their stage-2 opponent may be of either type. Therefore, $C_i = p_k(\cdot) * \Pi_2^*(\mathtt{HH}) + [1 - p_k(\cdot)] * \Pi_{\mathtt{H}2}^*(\mathtt{LH})$ and $C_j = p_k(\cdot) * \Pi_{\mathtt{L}2}^*(\mathtt{LH}) + [1 - p_k(\cdot)] * \Pi_2^*(\mathtt{HH})$, where $p_k(\cdot) \equiv p_k(x_{k1}^*(1), x_{l1}^*(1))$. The optimization problems of contestants i and k, as well as of j and l are symmetric, such that $x_{\mathtt{H}1}^* \equiv x_{i1}^* = x_{j1}^*$ and $x_{\mathtt{L}1}^* \equiv x_{k1}^* = x_{l1}^*$ do hold. In combination

[12] After the first contestant has been chosen randomly from the pool of four agents, the probability that the next agent drawn from the pool of the remaining three contestants is of the same type is 1/3 for $n = 2$ (since only one of the remaining agents is of the same type), while the probability that the next contestant is of the other type is 2/3 (because two of the three remaining agents are of the other type).

with (3.8), (3.10), (3.11), and (3.12), these symmetry conditions deliver[13]

$$x^*_{\mathtt{H}1} \equiv x^*_{i1}(\mathtt{LHLH}) = x^*_{k1}(\mathtt{LHLH}) = \frac{v_{\mathtt{H}}[v_{\mathtt{H}} + v_{\mathtt{L}}]^2 F(v_{\mathtt{H}}, v_{\mathtt{L}})^2 + 4v_{\mathtt{H}}^3 F(v_{\mathtt{H}}, v_{\mathtt{L}})}{4[v_{\mathtt{H}} + v_{\mathtt{L}}]^2[1 + F(v_{\mathtt{H}}, v_{\mathtt{L}})]^3} \tag{3.18}$$

$$x^*_{\mathtt{L}1} \equiv x^*_{j1}(\mathtt{LHLH}) = x^*_{l1}(\mathtt{LHLH}) = \frac{v_{\mathtt{L}}[v_{\mathtt{H}} + v_{\mathtt{L}}]^2 F(v_{\mathtt{H}}, v_{\mathtt{L}}) + 4v_{\mathtt{L}}^3 F(v_{\mathtt{H}}, v_{\mathtt{L}})^2}{4[v_{\mathtt{H}} + v_{\mathtt{L}}]^2[1 + F(v_{\mathtt{H}}, v_{\mathtt{L}})]^3}, \tag{3.19}$$

where $F(v_{\mathtt{H}}, v_{\mathtt{L}})$ is defined as

$$F(v_{\mathtt{H}}, v_{\mathtt{L}}) = \frac{(v_{\mathtt{H}} - v_{\mathtt{L}})(v_{\mathtt{L}} + v_{\mathtt{H}})^2 + \sqrt{64 v_{\mathtt{H}}^3 v_{\mathtt{L}}^3 + (v_{\mathtt{L}} - v_{\mathtt{H}})^2 (v_{\mathtt{L}} + v_{\mathtt{H}})^4}}{8 v_{\mathtt{L}}^3}. \tag{3.20}$$

$\mathbf{n = 3}$: Contestants i, k, and l are of type L, only contestant j is of type H. Consider first agents i and j who anticipate that their stage-2 opponent (conditional on winning stage 1) is of type L, independent of stage-1 investment choices by k and l. Therefore, it holds that $C_i = \Pi_2^*(\mathtt{LL})$ and $C_j = \Pi_{\mathtt{H}2}^*(\mathtt{LH})$. Combining the first-order conditions of contestants i and j delivers

$$x^*_{i1}(3) = \frac{v_{\mathtt{H}}^3 v_{\mathtt{L}}^2 [v_{\mathtt{H}} + v_{\mathtt{L}}]^2}{[4v_{\mathtt{H}}^3 + v_{\mathtt{H}}^2 v_{\mathtt{L}} + 2v_{\mathtt{H}} v_{\mathtt{L}} + v_{\mathtt{H}}^3]^2} \quad \text{and} \quad x^*_{j1}(3) = \frac{4 v_{\mathtt{H}}^6 v_{\mathtt{L}}}{[4v_{\mathtt{H}}^3 + v_{\mathtt{H}}^2 v_{\mathtt{L}} + 2v_{\mathtt{H}} v_{\mathtt{L}} + v_{\mathtt{H}}^3]^2}, \tag{3.21}$$

respectively. The optimization problems of agents k and l are symmetric. Both attach the value $C_k = C_l = p_i(\cdot) * \Pi_2^*(\mathtt{LL}) + [1 - p_i(\cdot)] * \Pi_{\mathtt{L}2}^*(\mathtt{LH})$ to a stage-2 participation, where $p_i(\cdot) \equiv p_i(x^*_{i1}(1), x^*_{j1}(1))$ is the probability that agent i (type L) wins the stage-1 interaction against agent j (type H). Using (3.8), (3.11), (3.12), (3.21), and symmetry delivers

$$x^*_{k1}(3) = x^*_{l1}(3) = \frac{v_{\mathtt{L}}^2 [v_{\mathtt{H}} + v_{\mathtt{L}}]^2 + 16 v_{\mathtt{H}}^3 v_{\mathtt{L}}^3}{16(4v_{\mathtt{H}}^3 [v_{\mathtt{H}} + v_{\mathtt{L}}]^2 + v_{\mathtt{L}}[v_{\mathtt{H}} + v_{\mathtt{L}}]^4)} \tag{3.22}$$

as stage-1 equilibrium investment for agents k and l.

3.2.3 Rent Dissipation Rates

Equilibrium investments by contestants determine the rent dissipation rate, which is defined as the sum of individual investments, normalized by $v_{\mathtt{H}}$, the value of the rent to contestants of type H. Consequently,

$$\mathrm{RD}^{\mathcal{S}}(n) = \frac{1}{v_{\mathtt{H}}} \sum_{b \in \mathcal{B}} x^*_b(n) \qquad (\mathcal{B} = \{i, j, k, l\}) \tag{3.23}$$

gives the rent dissipation rate for a configuration with n contestants of type L and $4 - n$ contestants of type H in the static contest format ($\mathcal{S}$).[14] It is slightly more complicated to

[13] For details on how to solve this slightly more complicated setting, see Höchtl, Kerschbamer, Stracke, and Sunde (2011). Höchtl et al. (2011) assume that types differ with respect to costs (rather than valuations); the necessary steps to derive the equilibrium solution are the same, however. A proof of equilibrium existence and uniqueness is provided in Stracke (2012a).

[14] Individual equilibrium investments $x^*_b(n)$ are defined in (3.3) and (3.6), respectively.

compute the rent dissipation rate in the dynamic contest, even though the concept is the same. However, three different stage-2 interactions may occur from an ex-ante perspective, and equilibrium investment choices vary across settings LL, LH, and HH, such that the rent dissipation rate is an expected value. To circumvent this complication, we define the expected stage-2 equilibrium investment choice by contestant b in a configuration with n contestants of type L as $\bar{x}^*_{b2}(n)$; formally,

$$\bar{x}^*_{i2}(n) = \frac{x^*_{i1}(n)}{x^*_{i1}(n) + x^*_{j1}(n)} \left[\frac{x^*_{k1}(n)}{x^*_{k1}(n) + x^*_{l1}(n)} * x^*_{i2}(\mathtt{IK}) + \frac{x^*_{l1}(n)}{x^*_{k1}(n) + x^*_{l1}(n)} * x^*_{i2}(\mathtt{IL}) \right]$$

is the expected stage-2 investment by contestant i, where I, K, and L are the types of contestants i, k, and l, respectively.[15] Using this measure for stage-2 investment, the rent dissipation rate for a configuration with n contestants of type L and $4-n$ contestants of type H in the dynamic contest format ($\mathcal{D}$) reads[16]

$$\mathrm{RD}^{\mathcal{D}}(n) = \frac{1}{v_{\mathtt{H}}} \left\{ \textstyle\sum_{b\in\mathcal{B}} [x^*_{b1}(n) + \bar{x}^*_{b2}(n)] \right\} \qquad (\mathcal{B} = \{i, j, k, l\}). \tag{3.24}$$

3.3 Optimal Contest Design

3.3.1 Comparison by Configuration

Before the solution to the design problem is presented, I will briefly compare the rent dissipation rates of the static and the dynamic contest by configuration. Essentially, this comparison corresponds to a situation where the designer cannot observe the valuation parameters of the contestants, but she knows the share of high and low valuation types, i.e., she knows that n contestants are of type L, while the remaining $4-n$ contestants are of type H. Through a comparison of (3.23) and (3.24) for $n \in \{0, 1, 2, 3, 4\}$, I determine the optimal contest format for each potential configurations. Figure 3.2 plots the rent dissipation rates both for the static and the dynamic contest format as a function of $\frac{v_{\mathtt{L}}}{v_{\mathtt{H}}}$ and illustrates the following relations:[17]

Proposition 3.1 (Rent Dissipation by Configuration)**.** *The rent dissipation rate in a configuration with n contestants of type L and $4-n$ contestants of type H is*

(a) identical in the static and the dynamic contest for $n = 0$ and $n = 4$, independent of v_H and v_L:

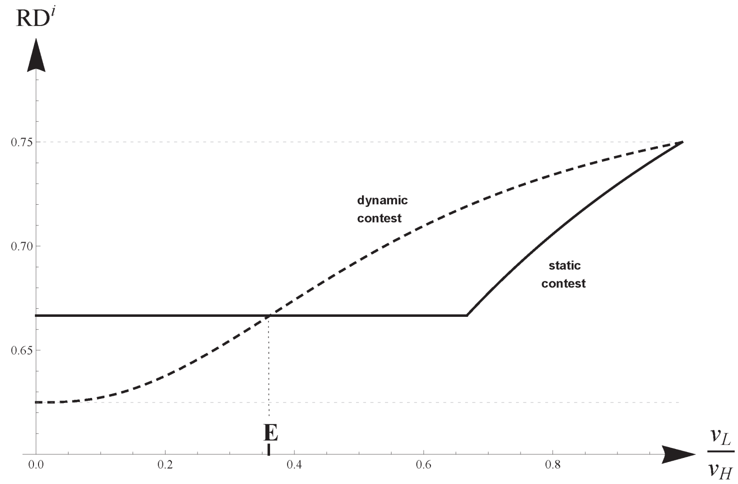
$$\mathrm{RD}^{\mathcal{S}}(0) = \mathrm{RD}^{\mathcal{D}}(0) \qquad \text{and} \qquad \mathrm{RD}^{\mathcal{S}}(4) = \mathrm{RD}^{\mathcal{D}}(4) \quad \forall \;\; 0 \le \frac{v_L}{v_H} \le 1.$$

[15]The formal expressions for $x^*_{i2}(\mathtt{IK})$ and $x^*_{i2}(\mathtt{IL})$ are provided in (3.7) and (3.9), respectively.

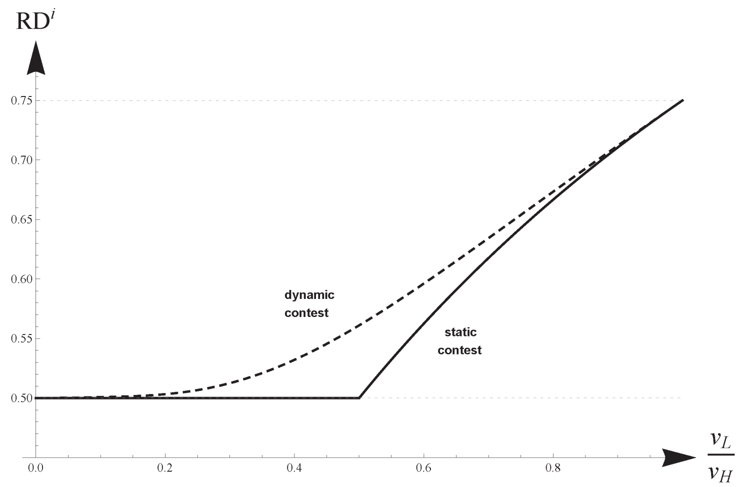
[16]Individual stage-1 equilibrium investments $x^*_{b1}(n)$ are defined in (3.13), (3.15), (3.16), and (3.22).

[17]The ratio of valuations measures the degree of heterogeneity between types on an inverse scale: Heterogeneity is extremely high if the valuations are infinitely far apart ($\frac{v_{\mathtt{L}}}{v_{\mathtt{H}}} = 0$), while heterogeneity approaches zero once valuations do not differ across types ($\frac{v_{\mathtt{L}}}{v_{\mathtt{H}}} = 1$), i.e., the higher $\frac{v_{\mathtt{L}}}{v_{\mathtt{H}}}$, the lower is the degree of heterogeneity between types, and vice versa.

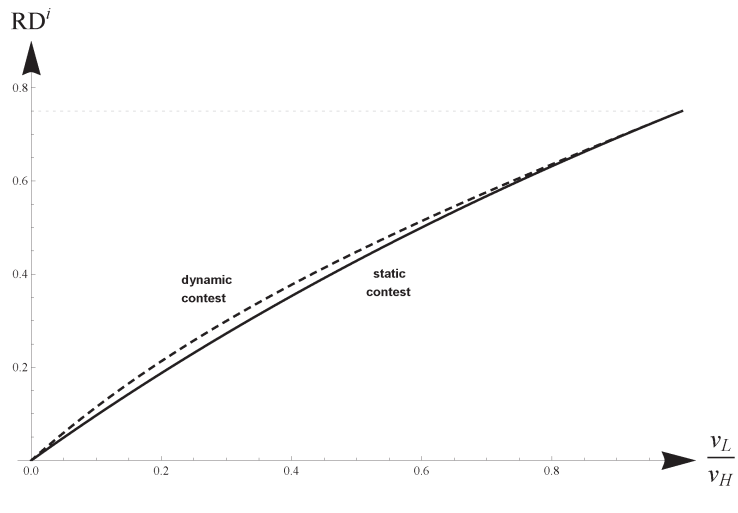
Figure 3.2: Rent Dissipation RD^i by configuration



(a) n=1



(b) n=2



(c) n=3

(b) *higher in the dynamic than in the static contest for* $n = 2$ *and* $n = 3$*, independent of* v_H *and* v_L*:*

$$\mathrm{RD}^{\mathcal{S}}(2) \le \mathrm{RD}^{\mathcal{D}}(2) \quad \text{and} \quad \mathrm{RD}^{\mathcal{S}}(3) \le \mathrm{RD}^{\mathcal{D}}(3) \quad \forall \;\; 0 \le \frac{v_L}{v_H} \le 1.$$

(c) *higher in the dynamic than in the static contest for* $n = 1$ *if* $E \le \frac{v_L}{v_H} \le 1$*, where* $E \approx 0.37$*. Rent dissipation is higher in the static than in the dynamic contest for* $n = 1$ *and* $0 \le \frac{v_L}{v_H} \le E$*:*

$$\mathrm{RD}^{\mathcal{S}}(1) \le \mathrm{RD}^{\mathcal{D}}(1) \;\text{ if }\; E \le \frac{v_L}{v_H} \le 1 \;\; ; \;\; \mathrm{RD}^{\mathcal{S}}(1) \ge \mathrm{RD}^{\mathcal{D}}(1) \;\text{ if }\; 0 \le \frac{v_L}{v_H} \le E.$$

Proof. See Appendix. □

Note that the homogeneous configurations $n = 0$ and $n = 4$ are omitted, as the rent dissipation rates are identical and constant in these cases. The three heterogeneous configurations are separately provided in panels (a), (b), and (c). First, Figure 3.2 shows that the rent dissipation rate is a smooth function of $\frac{v_\mathrm{L}}{v_\mathrm{H}}$ in all configurations of the dynamic contest, while the corresponding function for the static contest has a kink in some cases. The reason is that low valuation types do not participate in the static contest if their valuation v_L is too low relative to v_H for the given number of high valuation types. Below the respective threshold, changes of v_L relative to v_H do not further affect rent dissipation, which explains the kink. Figure 3.2 shows that the participation by type `L` contestants is an issue in the static contest for $n = 1$ and $n = 2$, but not for $n = 3$.[18] In addition, Figure 3.2 provides a graphical representation of Proposition 3.1: The rent dissipation rate in any heterogeneous configurations of the dynamic format is (weakly) higher than in the corresponding configuration of the static contest with one exception: If $n = 1$ and $0 \le \frac{v_\mathrm{L}}{v_\mathrm{H}} \le E$, the static format delivers a higher rent dissipation rate than the dynamic one.

3.3.2 Solution of the Contest Design Problem

Recall from section 3.2.1 that the designer chooses the contest format $i \in \{\mathcal{S}, \mathcal{D}\}$ which maximizes

$$E(\mathrm{RD}^i) = \textstyle\sum_{k=0}^{4} f(\lambda, n) * \mathrm{RD}^i(n).$$

We already know from Proposition 3.1 that the equilibrium rent dissipation rate in any configuration of the dynamic contest is (weakly) higher than in the corresponding configuration of the static contest if $E \le \frac{v_\mathrm{L}}{v_\mathrm{H}} \le 1$ holds. Noting that the central measure of interest to the designer, $E(\mathrm{RD}^i)$, is a composite of rent dissipation rates in each possible configuration, delivers Proposition 3.2:

[18]See also section 3.2.2.1, in particular equation (3.5) for details.

Proposition 3.2 (Expected Rent Dissipation). *Expected rent dissipation in the dynamic contest is higher than in the static contest if $E \leq \frac{v_L}{v_H} \leq 1$, independent of λ:*

$$E(\mathrm{RD}^{\mathcal{D}}) \geq E(\mathrm{RD}^{\mathcal{S}}) \quad \forall \quad \lambda \in [0,1] \quad \text{if} \quad E \leq \frac{v_L}{v_H} \leq 1.$$

Proof. The proof follows directly from Proposition 3.1, which shows that the rent dissipation rate is weakly higher in any configuration of the dynamic than in the static contest if

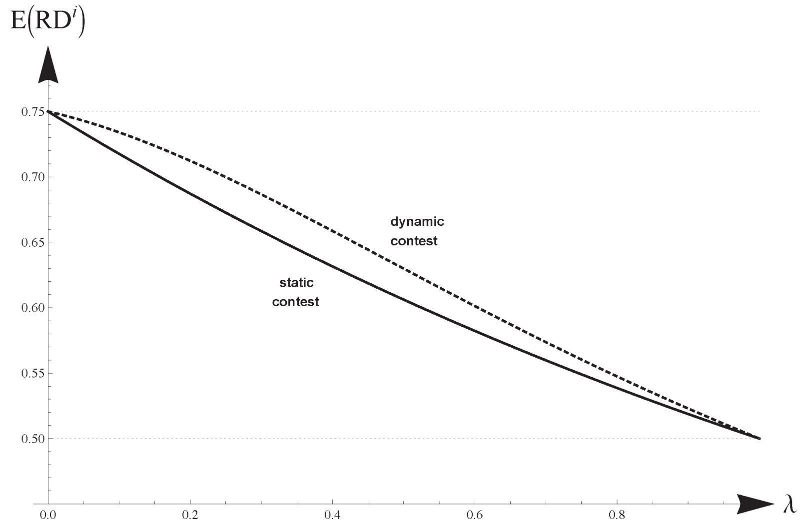
$$\frac{v_{\mathrm{L}}}{v_{\mathrm{H}}} \geq E.$$

$\square$

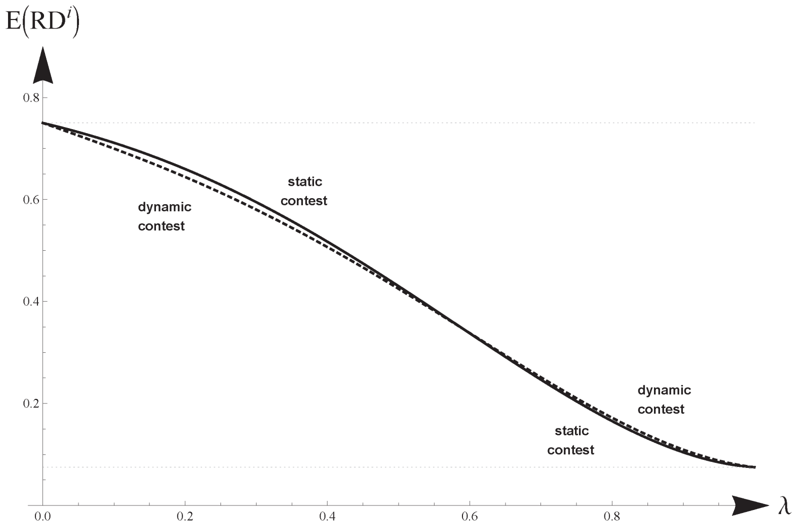
Intuitively, the probability λ to draw a low valuation type, which determines the chances that a particular configuration realizes, is irrelevant for the decision of the designer if $E \leq \frac{v_{\mathrm{L}}}{v_{\mathrm{H}}} \leq 1$, since this implies that rent dissipation in the dynamic contest is (weakly) higher than in the static one in any configuration. Figure 3.3 shows that this changes once the ratio $\frac{v_{\mathrm{L}}}{v_{\mathrm{H}}}$ decreases below its critical value E. While expected rent dissipation is higher in the dynamic than in the static contest for any λ in panel (a), which plots the graphs for $\frac{v_{\mathrm{L}}}{v_{\mathrm{H}}} = \frac{2}{3} \geq E$, the pattern changes in panel (b) where $\frac{v_{\mathrm{L}}}{v_{\mathrm{H}}} = \frac{1}{10} \leq E$. In this case, the expected rent dissipation rate is higher in the static than in the dynamic contest for low values of λ, while the opposite relation holds for high values of λ. The previous comparison by configuration helps to explain this finding: Recall from Proposition 3.1 that the rent dissipation rate in the static contest can only be higher than in the dynamic contest if one low valuation and three high valuation types ($n = 1$) interact. This configuration is particularly likely to occur if the probability to draw a low valuation type (λ) is low, which explains the dominance of the static over the dynamic contest for low values of λ. For higher values of λ, the fact that the rent dissipation rate in the dynamic is higher than in the static contest in configurations $n = 2$ and $n = 3$ becomes more and more important, as the probability that these configurations realize increases with λ. Consequently, expected rent dissipation is higher in the dynamic than in the static contest (even if $\frac{v_{\mathrm{L}}}{v_{\mathrm{H}}} \leq E$) for high values of λ.

Figure 3.4 provides a complete graphical solution of the contest design problem for any $\lambda \in [0,1]$ and $\frac{v_{\mathrm{L}}}{v_{\mathrm{H}}} \in [0,1]$. In line with Proposition 3.2, it is optimal for the designer to choose the dynamic contest if the ratio of valuations is above its critical value E, independent of the probability to draw a low valuation type, λ. When the ratio of valuations v_{L} and v_{H} falls below E, the optimal contest format depends on both λ and $\frac{v_{\mathrm{L}}}{v_{\mathrm{H}}}$. Roughly speaking, the static contest is the dominant option for low values of both λ and $\frac{v_{\mathrm{L}}}{v_{\mathrm{H}}}$, while the dynamic contest delivers a higher expected rent dissipation rate for high values of these parameters.

Figure 3.3: Expected Rent Dissipation $E(\mathrm{RD}^i)$



(a) $\frac{v_{\mathrm{L}}}{v_{\mathrm{H}}} = \frac{2}{3} \geq E$

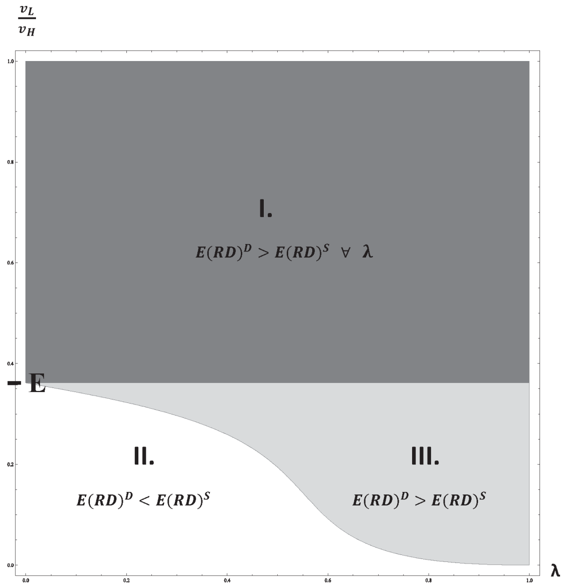


(b) $\frac{v_{\mathrm{L}}}{v_{\mathrm{H}}} = \frac{1}{10} \leq E$

3.4 Discussion of Results

The analysis in the previous section assumes a lottery contest success function, i.e., a discriminatory power of $r = 1$. As previously shown by Amegashie (1999) and Gradstein and Konrad (1999), the rent dissipation rate in both contest structures is identical for this specification of the contest technology if participants are homogeneous. This paper shows that the equality disappears when contestants are heterogeneous. Said differently, I show that the effect of heterogeneity between participants on their equilibrium investments into the contest depends on the structure of the competition. In a static contest, heterogeneity unambiguously reduces contest investments and therefore the rent dissipation rate, as is

Figure 3.4: Graphical Solution of the Design Problem



well known from previous work (Baik 1994, Nti 1999, Stein 2002). In principle, the same holds in any heterogeneous interaction of the dynamic format, but in addition, there is a countervailing dynamic effect of heterogeneity across different stages. Intuitively, the prospect of facing a low valuation opponent (type L) in later stages increases the value of winning and therefore the equilibrium investment levels in early stages for type H contestants. At the same time, equilibrium investment levels for low valuation types are reduced, since they are likely to meet an opponent with a high valuation in stage 2. However, investments into the contest are linearly increasing the valuation parameter, such that the positive dynamic effect of heterogeneity on investments by high valuation contestants dominates the corresponding negative effect on investments by low types.[19] As a consequence, the rent dissipation rate is higher in the dynamic than in the static format, independent of the configuration under consideration. As shown previously, there is one exception to this general result: When the valuation of type L contestants, $v_{\mathtt{L}}$, is very low relative to the valuation of type H contestants, $v_{\mathtt{H}}$, contestants of the former type cease to participate in the static contest. Consequently, the rent dissipation rate in the static format is not affected if the degree of heterogeneity between types is increased above the value where low valuation types drop-out, while higher degrees of heterogeneity do always reduce overall rent dissipation in the dynamic contest. Therefore, the rent dissipation rate may be higher in the static than in the dynamic format if the degree of heterogeneity between types is extremely high.

A natural question is whether or not the results of this paper survive a generalization of the contest technology. To address this point, I will relate the results of this paper to

[19]This is why the difference in terms of rent dissipation rates between the static and the dynamic format is particularly pronounced if the number of low valuation types is low, i.e., for $n = 1$ and $n = 2$. See Figure 3.2 for details.

previous work by Gradstein and Konrad (1999), who consider exactly the same contest formats, but restrict attention to settings with homogeneous participants. Gradstein and Konrad (1999) find that reductions of the discriminatory power r reduce incentives to invest into the contest in both formats considered in this paper; however, this effect is weaker in the dynamic format. Intuitively, there is a countervailing effect of discriminatory power reductions in the dynamic format, which is very similar to the previously mentioned dynamic heterogeneity effect: If incentives to invest in stage 2 are low due to reductions of the parameter r, this increases the value of participation in stage 2, and therefore incentives to invest in stage 1.[20] As a consequence of this effect, the rent dissipation rate is higher in dynamic than in static contest if the discriminatory power is low ($r < 1$), while the opposite holds if the discriminatory power is high ($r > 1$). Therefore, the result from this paper, which restricts attention to the case where $r = 1$, cannot be generalized in *quantitative* terms: If the discriminatory is sufficiently high, the rent dissipation rate will be higher in static than in dynamic contests, even if contestants are heterogeneous. However, the detrimental effect of heterogeneity on the rent dissipation rate will be lower in the dynamic than in the static contest for any discriminatory power, i.e., the results of this paper do still hold in *qualitative* terms. With respect to the discriminatory power, this implies that the rent dissipation rate is higher in dynamic contests than in static ones even for weakly convex impact functions, i.e., for a discriminatory power r slightly above one, if contestants are heterogeneous.

Another important issue is whether or not the restriction on small scale contests with four participants matters for the result. Even though a formal proof is hard to make due to the then even larger number of potential configurations, I believe that the the main finding of this paper, namely the presence of structure specific heterogeneity effects, carries over to larger contests. Note, however, that the range where low valuation types drop-out from the static contest is increasing with the number of participants.[21] Therefore, the threshold E below which the rent dissipation rate may be higher in the static than in the dynamic contest is likely to increase with the number of contestants. Ultimately, however, the assumption that contestants know the type of their competitors, which allows for the identification of structure specific heterogeneity effects, will restrict the size of the contest. This assumption is very plausible in interactions between a limited number of competitors which know each other well (e.g. due to previous competitions) and react strategically to each other. If the field of contestants is very large, it is more reasonable to assume that all types are drawn from some distribution function, while each contestant has private information about his/her own type.

[20] See Amegashie (2000) for an extensive discussion of this verbal argument.

[21] See Stein (2002) for details.

3.5 Concluding Remarks

This paper analyzes two prominent contest structures: A static pooling competition where all contestants interact simultaneously, and a dynamic pair-wise elimination format where contestants are split into separate branches and are then sequentially eliminated. Using a lottery contest success function which ensures that the rent dissipation rate in both structures is identical if contestants are homogeneous, I find that the rent dissipation rate is almost always higher in the dynamic than in the static format in heterogeneous interactions. Intuitively, the detrimental effect of heterogeneity on contest investments is lower in the dynamic than in the static contest format. While it is well known from previous work that heterogeneity reduces the rent dissipation rate in any immediate interaction, the results indicate that there is a countervailing dynamic effect of heterogeneity across different stages in the dynamic format which works through continuation values: Intuitively, the prospect of facing a low valuation opponent (type $\mathtt{L}$) in later stages increases the value of winning and therefore the equilibrium investment levels in early stages for type $\mathtt{H}$ contestants. Even though equilibrium investment levels for low valuation types are reduced in early stages (since they are likely to meet an opponent with a high valuation in stage 2), the dynamic effect of heterogeneity tends to increase contest investments; the positive dynamic effect of heterogeneity on investments by high valuation contestants dominates the corresponding negative effect on investments by low types, since investments into the contest are linearly increasing in the type specific valuation parameter.

This finding does not imply that dynamic contests do always dominate static ones in terms of their rent dissipation rate when participants are heterogeneous. First, the static format leads to a higher rent dissipation rate if the degree of heterogeneity is so high that low valuation contestants drop-out voluntarily in the static, but not in the dynamic format. Second, the static format does also dominate if the discriminatory power of the contest technology is high. Rather, this paper shows that the effect of heterogeneity on equilibrium investments by participants depends on the structure of the competition. Since interactions between heterogeneous contestants are more likely to be the rule than the exception in reality, the results of this paper raise the question whether or not past findings with respect to the optimal design of contests depend on the homogeneity assumption. I believe that answering this question is a promising, though technically challenging avenue for future research.

Appendix

Lemma 3.1. *Assume without loss of generality that* $v_H = 1$ *and define* $f(v_L) = \frac{v_L^2+2v_L+5}{5v_L^3+2v_L^2+v_L}$. *Then, the relation* $F^*(1, v_L) > f(v_L)$ *does hold for all* $0 < v_L < 1$, *where* $F^*(1, v_L)$ *is defined as in* (3.20). *Furthermore, for* $v_L = 1$ *it holds that* $F^*(1, v_L) = f(v_L) = 1$.

Proof. When assuming symmetry across the two stage-1 interactions of the LHLH setting in the dynamic contest for $n = 2$, one can easily show that the ratio of efforts is proportional to the continuation values of contestants in each interaction, i.e., $\frac{x_{i1}}{x_{j1}} = \frac{v_{\mathrm{H}}}{v_{\mathrm{L}}}\frac{C_i}{C_j}$. The function $F(v_{\mathrm{H}}, v_{\mathrm{L}})$ in equation (3.20) takes the dependence of continuation values on effort decisions by contestants in the parallel stage-1 interaction into account and provides the equilibrium value of this ratio, i.e., $\frac{x^*_{i1}}{x^*_{j1}} = F^*(v_{\mathrm{H}}, v_{\mathrm{L}})$. Consequently, using the normalizing assumption $v_{\mathrm{H}} = 1$, it must hold that

$$F^*(1, v_{\mathrm{L}}) = \frac{1}{v_{\mathrm{L}}}\frac{C_i}{C_j} = \frac{4 + (1+v_{\mathrm{L}})^2 \times \frac{x_{k1}}{x_{l1}}}{(1+v_{\mathrm{L}})^2 v_{\mathrm{L}} + 4v_{\mathrm{L}}^3 \times \frac{x_{k1}}{x_{l1}}}.$$

Note that

$$\frac{\partial F^*(1, v_{\mathrm{L}})}{\partial \frac{x_{j1}}{x_{l1}}} = v_{\mathrm{L}} \frac{(1+v_{\mathrm{L}})^4 - 16v_{\mathrm{L}}^2}{[(1+v_{\mathrm{L}})^2 v_{\mathrm{L}} + 4v_{\mathrm{L}}^3 \times \frac{x_{k1}}{x_{l1}}]^2} > 0$$

if $v_{\mathrm{L}} < 1$. Further, recall that player l has a lower continuation value than player k ($C_k > C_l$), such that $x_{k1} > x_{l1}$ does hold. Therefore, assuming $x_{k1} = x_{l1}$ underestimates $F^*(1, v_{\mathrm{L}})$. Since

$$f(v_{\mathrm{L}}) = \frac{v_{\mathrm{L}}^2 + 2v_{\mathrm{L}} + 5}{5v_{\mathrm{L}}^3 + 2v_{\mathrm{L}}^2 + v_{\mathrm{L}}}$$

is the expression I derive from $F^*(1, v_{\mathrm{L}})$ under this assumption, I have proven $F^*(1, v_{\mathrm{L}}) > f(v_{\mathrm{L}})$. If I assume $v_{\mathrm{L}} = 1$, all contestants are perfectly symmetric, such that $x_{k1} = x_{l1}$ does hold. Consequently, the relation $F^*(1, v_{\mathrm{L}}) = f(v_{\mathrm{L}})$ does hold for $v_{\mathrm{L}} = 1$. □

Lemma 3.2. *Assume without loss of generality that* $v_H = 1$ *and define* $f_{\mathrm{low}}(v_L) = \frac{2}{v_L} - 1$. *Then, the relation* $F^*(1, v_L) < f_{\mathrm{low}}(v_L)$ *does hold for all* $0 < v_L < 1$. *Furthermore, for* $v_L = 1$, *it holds that* $f(v_L) = f_{\mathrm{low}}(v_L)$.

Proof. I start with the relation that I want to prove, namely:

$$\begin{aligned} f(v_{\mathrm{L}}) &> f_{\mathrm{low}}(v_{\mathrm{L}}) \\ \Leftrightarrow v_{\mathrm{L}}^2 + 2v_{\mathrm{L}} + 5 &> (\frac{2}{v_{\mathrm{L}}} - 1)(5v_{\mathrm{L}}^3 + 2v_{\mathrm{L}}^2 + v_{\mathrm{L}}) \\ \Leftrightarrow 3c_{\mathtt{W}}^3 - c_{\mathtt{W}}^2 - 7c_{\mathtt{W}} + 5 &> 0 \end{aligned}$$

I now have to prove that $\phi(c_{\mathtt{W}}) \equiv 3c_{\mathtt{W}}^3 - c_{\mathtt{W}}^2 - 7c_{\mathtt{W}} + 5 > 0$ does always hold for $c_{\mathtt{W}} > 1$. To see this, note that $\phi(\cdot)$ is a cubic function that has a local minimum at $c_{\mathtt{W}} = 1$, and a local maximum at $c_{\mathtt{W}} = -7/9$. Furthermore, $\phi(1) = 0$, which implies that $\phi(c_{\mathtt{W}}) > 0$ for all $c_{\mathtt{W}} > 1$. □

Proof of Proposition 3.1

Parts (a), (b), and (c) will be considered separately in their natural order:

(a) Equations (3.3) and (3.23) determine the rent dissipation rates in the two homogeneous configurations of the static contest, which are formally defined as $\mathrm{RD}^{\mathcal{S}}(0) = \frac{3}{4}$ and $\mathrm{RD}^{\mathcal{S}}(4) = \frac{3v_{\mathtt{L}}}{4v_{\mathtt{H}}}$, respectively. Equilibrium efforts for stages 1 and 2 of the dynamic format are defined in (3.7) and (3.13), respectively. Using the formal definition of the rent dissipation rate provided in (3.24), I obtain $\mathrm{RD}^{\mathcal{D}}(0) = \frac{3}{4}$ and $\mathrm{RD}^{\mathcal{D}}(4) = \frac{3v_{\mathtt{L}}}{4v_{\mathtt{H}}}$. A formal comparison of $\mathrm{RD}^{\mathcal{S}}(0)$ and $\mathrm{RD}^{\mathcal{D}}(0)$, and of $\mathrm{RD}^{\mathcal{S}}(4)$ and $\mathrm{RD}^{\mathcal{D}}(4)$ immediately delivers part (a) of Proposition 3.1.

(b) Below, the two relations $\mathrm{RD}^{\mathcal{D}}(2) > \mathrm{RD}^{\mathcal{S}}(2)$ and $\mathrm{RD}^{\mathcal{D}}(3) > \mathrm{RD}^{\mathcal{S}}(3)$ will be proven separately for all $0 \leq \frac{v_{\mathtt{L}}}{v_{\mathtt{H}}} \leq 1$.

n = 2: I make the normalizing assumption $v_{\mathtt{L}} < v_{\mathtt{H}} = 1$ which simplifies the formal analysis and is without loss of generality. Using the formal expression provided in (3.6) and (3.23), I obtain $\mathrm{RD}^{\mathcal{S}}(2) = \max\{\frac{3v_{\mathtt{L}}}{2+2v_{\mathtt{L}}}, \frac{1}{2}\}$ as the rent dissipation rate in the static contest. The corresponding measure for the rent dissipation rate in the dynamic contest format is more complicated. In fact, I have to account for the rent dissipation rate in the two potential settings `LLHH` and `LHLH`. Formally, $\mathrm{RD}^{\mathcal{D}}(2)$ is defined as

$$\mathrm{RD}^{\mathcal{D}}(2) = \frac{1}{3}\mathrm{RD}^{\mathcal{D}}(\mathtt{LLHH}) + \frac{2}{3}\mathrm{RD}^{\mathcal{D}}(\mathtt{LHLH}).$$

Rent dissipation rates in the two settings are defined by (3.7), (3.9), (3.17), (3.19), and (3.24). Since $\mathrm{RD}^{\mathcal{S}}(2)$ is defined stepwise, I will separately consider the the parameter ranges $0 \leq v_{\mathtt{L}} \leq 0.5$, where the rent dissipation rate equals $\mathrm{RD}^{\mathcal{S}}(2) = 0.5$, and $0.5 \leq v_{\mathtt{L}} \leq 1$, where $\mathrm{RD}^{\mathcal{S}}(2) = \frac{3v_{\mathtt{L}}}{2+2v_{\mathtt{L}}}$.

(i) $0.5 \leq v_{\mathtt{L}} \leq 1$: I have to show that

$$\begin{aligned} \mathrm{RD}^{\mathcal{S}}(2) &< \mathrm{RD}^{\mathcal{D}}(2) \\ \Leftrightarrow \frac{3v_{\mathtt{L}}}{2+2v_{\mathtt{L}}} &< \frac{1}{3}\mathrm{RD}^{\mathcal{D}}(\mathtt{LLHH}) + \frac{2}{3}\mathrm{RD}^{\mathcal{D}}(\mathtt{LHLH}). \end{aligned}$$

The rent dissipation rate in the dynamic format is complicated by the function $F(1, v_{\mathtt{L}})$ in setting `LHLH`, which is defined in equation (5.23). Note that $\mathrm{RD}^{\mathcal{D}}(\mathtt{LHLH})$ is increasing in $F(1, v_{\mathtt{L}})$, since

$$\frac{\partial \mathrm{RD}^{\mathcal{D}}(\mathtt{LHLH})}{\partial F(1, v_{\mathtt{L}})} = \frac{v_{\mathtt{L}}(4 + v_{\mathtt{L}} - 2v_{\mathtt{L}}^2) - 7 - (1 + v_{\mathtt{L}})(3v_{\mathtt{L}} - 1)F(1, v_{\mathtt{L}})}{2(1 + F(1, v_{\mathtt{L}}))^3(1 + v_{\mathtt{L}})^2} > 0.$$

Therefore, it is without loss of generality to replace $F(1, v_{\mathtt{L}})$ by $f_{\text{low}}(v_{\mathtt{L}})$, since $f_{\text{low}}(v_{\mathtt{L}}) \leq$

$F(1, v_L)$ (see Lemmata 3.1 and 3.2). This gives the relation

$$\begin{aligned} \frac{3v_L}{2+2v_L} &< \frac{6+v_L(10+v_L(16+v_L(5+(v_L-2)v_L)))}{12(1+v_L)^2} \\ \Leftrightarrow \quad 0 &< \frac{(v_L-1)^2(6+4v_L+v_L^3)}{12(1+v_L)^2}. \end{aligned}$$

The expression on the right hand side is always positive, which completes this part of the proof.
(ii) $0 \le v_L \le 0.5$: I have to show that

$$\begin{aligned} \mathrm{RD}^{\mathcal{D}}(2) &> \mathrm{RD}^{\mathcal{S}}(2) \\ \Leftrightarrow \frac{1}{3}\mathrm{RD}^{\mathcal{D}}(\mathtt{LLHH}) + \frac{2}{3}\mathrm{RD}^{\mathcal{D}}(\mathtt{LHLH}) &> \frac{1}{2} \end{aligned}$$

This gives

$$\begin{aligned} \frac{F(1,v_L)^2[3v_L^3+8v_L^2+10v_L+5]+F(1,v_L)[6v_L^3+22v_L^2+24v_L+8]+5v_L^3+10v_L^2+8v_L+11}{3[1+F(1,v_L)]^2(1+v_L)} &> 1 \\ \Leftrightarrow F(1,v_L)^2[3v_L^3+5v_L^2+4v_L+2]+F(1,v_L)[6v_L^3+16v_L^2+12v_L+2]+5v_L^3+7v_L^2+2v_L+8 &> 0 \end{aligned}$$

Since $F(1, v_L)$, as it is defined in equation (5.23), is strictly positive, the above relation is always satisfied.
<u>n = 3</u> : I make the normalizing assumption $v_L < v_H = 1$ which simplifies the formal analysis and is without loss of generality. Under this assumption, (3.6) and (3.23) deliver $\mathrm{RD}^{\mathcal{S}}(3) = \frac{3v_L}{3+v_L}$ as the rent dissipation rate in the static contest format. Similarly, (3.9), (3.7), (3.21), (3.22), and (3.24) define the rent dissipation rate in the dynamic format, which is formally defined as follows:

$$\mathrm{RD}^{\mathcal{D}}(3) = \frac{v_L}{8}\left(5+\frac{4}{(1+v_L)^2}+\frac{4(1-v_L)}{4+v_L(1+v_L)^2}\right).$$

I must show that

$$\mathrm{RD}^{\mathcal{S}}(3) \le \mathrm{RD}^{\mathcal{D}}(3)$$

does hold for all $0 \le v_L \le 1$, i.e.,

$$5+\frac{4}{(1+v_L)^2}+\frac{4-4v_L}{4+v_L(1+v_L)^2}-\frac{24}{3+v_L} \quad \ge \quad 0$$

Since $0 \le v_L \le 1$ by assumption, it must hold that

$$\frac{4-4v_L}{4+v_L(1+v_L)^2} \ge 0.$$

Therefore, it is sufficient to show that

$$\begin{aligned}
5+\frac{4}{(1+v_{\mathrm{L}})^2}-\frac{24}{3+v_{\mathrm{L}}} &\geq 0\\
\Leftrightarrow \frac{15-19v_{\mathrm{L}}}{3+v_{\mathrm{L}}}+\frac{4}{(1+v_{\mathrm{L}})^2} &\geq 0\\
\Leftrightarrow \frac{(15-19v_{\mathrm{L}})(1+v_{\mathrm{L}})^2}{(3+v_{\mathrm{L}})(1+v_{\mathrm{L}})^2}+\frac{12+4v_{\mathrm{L}}}{(3+v_{\mathrm{L}})(1+v_{\mathrm{L}})^2} &\geq 0\\
\Leftrightarrow \frac{27+15v_{\mathrm{L}}-23v_{\mathrm{L}}^2-19v_{\mathrm{L}}^3}{(3+v_{\mathrm{L}})(1+v_{\mathrm{L}})^2} &\geq 0\\
\Leftrightarrow 27+15v_{\mathrm{L}}-23v_{\mathrm{L}}^2-19v_{\mathrm{L}}^3 &\geq 0
\end{aligned}$$

Since $0 \leq v_{\mathrm{L}} \leq 1$, this relation is always satisfied; it holds that $27+15v_{\mathrm{L}} = 23v_{\mathrm{L}}^2+19v_{\mathrm{L}}^3$ if $v_{\mathrm{L}} = 1$, while $27+15v_{\mathrm{L}} > 23v_{\mathrm{L}}^2+19v_{\mathrm{L}}^3$ for any $0 < v_{\mathrm{L}} < 1$.

(c) For $\mathbf{n = 1}$, I make the normalizing assumption $v_{\mathrm{L}} < v_{\mathrm{H}} = 1$ which simplifies the formal analysis and is without loss of generality. Under this assumption, (3.6) and (3.23) deliver $\mathrm{RD}^{\mathcal{S}}(1) = \max\{\frac{2}{3}, \frac{3v_{\mathrm{L}}}{1+3v_{\mathrm{L}}}\}$ as the rent dissipation rate in the static contest format. Similarly, (3.9), (3.7), (3.14), (3.15), and (3.24) define the rent dissipation rate in the dynamic format, which is formally defined as follows:

$$\mathrm{RD}^{\mathcal{D}}(1) = \frac{32v_{\mathrm{L}}^5+37v_{\mathrm{L}}^4+68v_{\mathrm{L}}^3+30v_{\mathrm{L}}^2+20v_{\mathrm{L}}+5}{8(1+v_{\mathrm{L}})^2(1+v_{\mathrm{L}}(2+v_{\mathrm{L}}+4v_{\mathrm{L}}^2))}.$$

For this proof, I must show that there is a unique intersection of $\mathrm{RD}^{\mathcal{S}}(1)$ and $\mathrm{RD}^{\mathcal{D}}(1)$ in the range $0 \leq v_{\mathrm{L}} \leq 1$. For $v_{\mathrm{L}} = 1$, it holds that $\mathrm{RD}^{\mathcal{S}}(1) = \mathrm{RD}^{\mathcal{D}}(1) = \frac{3}{4}$, while $\mathrm{RD}^{\mathcal{S}}(1) = \frac{2}{3} > \mathrm{RD}^{\mathcal{D}}(1) = \frac{5}{8}$ for $v_{\mathrm{L}} = 0$. For $0 \leq v_{\mathrm{L}} \leq 1$,

$$\frac{\partial \mathrm{RD}^{\mathcal{D}}(1)}{\partial v_{\mathrm{L}}} \geq 0,$$

and

$$\frac{\partial \mathrm{RD}^{\mathcal{D}}(1)}{\partial v_{\mathrm{L}}} = \frac{21v_{\mathrm{L}}^2+27v_{\mathrm{L}}^3+6v_{\mathrm{L}}^4+18v_{\mathrm{L}}^5-43v_{\mathrm{L}}^6+35v_{\mathrm{L}}^7}{2(1+v_{\mathrm{L}})^3(1+v_{\mathrm{L}}(2+v_{\mathrm{L}}+4v_{\mathrm{L}}^2))^2} > 0$$

do hold. Therefore, there is either one or no intersection between $\mathrm{RD}^{\mathcal{S}}(1)$ and $\mathrm{RD}^{\mathcal{D}}(1)$ in the range $0 < v_{\mathrm{L}} < 1$. When equalizing the two expressions, I find that $\mathrm{RD}^{\mathcal{S}}(1) = \mathrm{RD}^{\mathcal{D}}(1)$ if and only if $v_{\mathrm{L}} = E$, where $E \approx 0.37$. Since the slope of $\mathrm{RD}^{\mathcal{S}}(1)$ at E is zero, while the slope of $\mathrm{RD}^{\mathcal{D}}(1)$ is strictly positive,

$$\mathrm{RD}^{\mathcal{S}}(1) \leq \mathrm{RD}^{\mathcal{D}}(1) \text{ if } v_{\mathrm{L}} \geq E \;\; ; \;\; \mathrm{RD}^{\mathcal{S}}(1) \geq \mathrm{RD}^{\mathcal{D}}(1) \text{ if } v_{\mathrm{L}} \leq E.$$

must hold.

Chapter 4

Ability Matters and Heterogeneity Can Be Good: The Effect of Heterogeneity on the Performance of Tournament Participants

This chapter is based on joint work with Uwe Sunde from the University of St. Gallen (Stracke and Sunde 2012).

4.1 Introduction

Tournaments constitute an important element within the field of Personnel Economics ever since Lazear and Rosen (1981) showed that rank-order tournaments are optimal labor contracts under certain conditions. In particular, if workers' individual effort is not verifiable (e.g., because it is observed with some noise), it might not be possible or optimal to implement piece rates or other pay-for-performance remuneration schemes (Malcomson 1984). As long as an ordinal ranking of workers' performance is still possible, Lazear and Rosen's results show that rank-order tournaments for discrete prizes or bonus payments can be used in such settings as a compensation scheme to provide workers with efficient incentives for effort provision. Tournaments are not only used to model the competition for bonus payments within a company, however. Internal labor markets are often modeled as promotion tournaments along the lines of Rosen (1986). Surveys of the respective literature are provided by McLaughlin (1988) and Prendergast (1999).

One issue that received comparably little attention in the tournament literature is the effect of heterogeneity between participants on their equilibrium performance in dynamic settings, such as promotion tournaments with multiple stages. In reality, workers typically differ in their ability, which implies that virtually all tournaments involve heterogeneous participants. Moreover, empirical evidence by Gibbs and Hendricks (2004) shows that promotion tournaments are an important means for the provision of incentives in organizations. Still, the incentive properties of dynamic tournaments with heterogeneous participants are largely unexplored. It remains an open question whether or not the result by Lazear and Rosen (1981) that effort provision and performance decrease with the degree of heterogeneity in static tournaments carries over to dynamic settings.

This paper takes a closer look at the effects of heterogeneity between participants on overall tournament performance. We consider both a static tournament model and a dynamic model with multiple stages, applying the same tournament setup as in Rosen (1986). The theoretical analysis of these two models delivers three main results: First, we find that the average ability level of tournament participants has a strong impact on their performance, independent of the tournament format. Second, our results show that the incentive effect of heterogeneity on the overall performance of tournament participants depends on the structure of the competition. To isolate the effect of heterogeneity on incentives, we compare homogeneous and heterogeneous situations, where the average ability level of participants is the same. This allows us to separate effects of changes in ability from the effect of variations in the degree of heterogeneity on incentives. The findings show that the incentive effect of heterogeneity on performance is negative in static tournaments, which is in line with the common perception in the literature. However, the opposite holds in the dynamic specification, where heterogeneity has a strictly positive incentive effect on the overall performance of tournament participants. The reason is that heterogeneity increases the value of winning in early stages for strong agents, as they anticipate that it will be easier to win another time in later stages of the tournament due to the presence of weak agents. Therefore, the performance of strong agents is higher in early stages. Finally, the comparison of both, the direct (absolute) ability effect and the incentive effect of heterogeneity through relative ability reveals that the effect of variation in heterogeneity through changes in the average level of absolute abilities dominates the corresponding effect through incentives, if ability and heterogeneity are changed simultaneously.[1]

In the second part of the paper, we provide some experimental evidence regarding the theoretical predictions. The findings provide empirical support for the main qualitative predictions: First, we find that the level of average ability of tournament participants has a strong influence on performance. Second, the incentive effect of heterogeneity on performance is negative in the static tournament treatments, while the effect is positive in dynamic tournaments. The (negative or positive) effect of heterogeneity is much more pronounced than one would expect from the theory. While the theoretical results predict a 4% reduction of overall performance due to heterogeneity for the static tournament, we observe a reduction of more than 15%; similarly, overall performance should be approximately 1.5% higher in the dynamic tournament, but we observe an increase of almost 20%

[1] Average ability and the degree of heterogeneity are changed simultaneously if, for example, a participant i of the tournament is replaced with somebody who has either a higher or a lower ability than i.

in response to a higher degree of heterogeneity. As a consequence, we find little evidence for the third theoretical result that the effect of ability on overall performance dominates the effect of heterogeneity. Instead, the experimental analysis suggests that both effects are equally important under the parametric setup of the experiment.

Our results have several interesting implications for the performance of corporate tournaments. We show that the effect of heterogeneity between participants of a tournament can be affected by the structure of the tournament. Consequently, the specific tournament format plays a role as to whether or not it makes sense for a tournament designing principal to separate or pool different types. In addition, the results also show that the average ability of the workforce participating in a tournament can be as important as negative (or positive) effects of heterogeneity. According to our findings, an increase of the average ability does always have a strictly positive effect, independent of the tournament structure. This suggests that the ability of employees might be more relevant for hiring decisions than potential concerns for the homogeneity among participants of corporate bonus or promotion tournaments for reasons of incentive provision.

This paper complements the existing theoretical and empirical literature on tournaments in several ways. We provide a systematic comparison of the effects of heterogeneity in the two most prominent tournament models in the Personnel Economics literature, the static one-shot tournament along the lines of Lazear and Rosen (1981), and the dynamic multi-stage tournament as suggested by Rosen (1986). Existing theoretical comparisons between static and dynamic tournament models either assume homogeneity of participants (Gradstein and Konrad 1999), or consider the case of a perfectly discriminating all-pay auction (Moldovanu and Sela 2006).[2] The results of this paper also complement earlier studies which suggest that the tournament designing principal has an incentive to induce self-sorting of worker types by ability into different tournaments (O'Keeffe, Viscusi, and Zeckhauser 1984, Bhattacharya and Guasch 1988), or alternatively, if types are observable, to handicap stronger workers (Lazear and Rosen 1981, Gürtler and Kräkel 2010).

Second, this paper is related to existing experimental work on behavior in tournaments. The papers most closely related are the ones by Sheremeta (2010) and Altmann, Falk, and Wibral (2012), who compare static one-stage and dynamic two-stage tournaments with homogeneous participants. We complement their work and additionally consider settings with heterogeneous agents. Further, our experimental analysis is related to research by Bull, Schotter, and Weigelt (1987), Orrison, Schotter, and Weigelt (2004), and Harbring and Lünser (2008), who analyze the behavior in static tournaments. These studies consider homogeneous and heterogeneous treatments, but do not provide a systematic assessment of the strength of the effect of heterogeneity on performance, since average ability of participants is not held constant across treatments. Finally, the paper is also related to the empirical literature that has investigated the performance effects of heterogeneity. In this

[2] In the latter case, it is assumed that signals on the relative performance of tournament participants are always correct and fully informative, an assumption that is likely to be violated in reality.

strand of the literature, field data from sports (Abrevaya 2002, Sunde 2009, Brown 2011) and corporations (Knoeber and Thurman 1994, Eriksson 1999) has been used to test the implications of heterogeneity that follow from static one-stage tournament models. We provide this empirical literature with a new testable hypothesis for dynamic tournaments with multiple stages, which are quite common both in corporate and sport tournaments.

The remainder of the paper is structured as follows. Section 4.2 presents a theoretical analysis of equilibrium behavior in static one-stage and dynamic two-stage tournaments. Section 4.3 presents experimental evidence of tests of the main theoretical predictions, and section 4.4 concludes.

4.2 Theoretical Analysis

We consider two different tournament models. In both models, we allow for ability differences between tournament participants, which we will refer to as "workers" subsequently. The baseline specification is a *static* one-shot tournament, in which two workers compete for some exogeneously given prize P.[3] The prize can be understood as a performance reward for a worker, who receives some bonus payment or a promotion to a better paid position. The second model is a straightforward dynamic extension of the one-shot tournament in the spirit of Rosen (1986). By adding a qualification stage to the static tournament, one can analyze a *dynamic* tournament with two stages. In the first stage of this tournament, four workers compete in two separate pair-wise interactions for a promotion to stage 2. The two losers of the first stage are eliminated from the competition, while the two winning workers are promoted. They encounter each other in stage 2, where they compete for some exogenously given prize P, as in the static tournament model.[4]

The remainder of this section first derives equilibrium solutions for homogeneous and heterogeneous specifications of both tournament models, which allow us to describe optimal behavior of workers in the respective setting. Then, we analyze the effect of heterogeneity on measures of interest for a tournament designing principal in both the static and the dynamic tournament model. The analysis focuses on two central questions: Should a principal separate strong and weak workers from each other, given that both types are employed in his company? And second, should hiring decisions of new workers be influenced by concerns for homogeneity of the workforce? At the end of this theoretical section, we discuss the implications and the robustness of our results for the optimal design of tournaments.

[3]Nothing changes if we assume that an additional prize is available for the worker who places second. Then, P would simply denote the spread between the two prizes.

[4]The analysis does not address the question of optimal prize structures or wage profiles for multi-stage pair-wise elimination tournaments. In fact, however, the model employs a "winner-takes-all" structure that can be shown to maximize overall incentives in all dynamic settings under consideration. For information on optimal wage profiles or prize structures, see Rosen (1986) or Moldovanu and Sela (2001), for example.

4.2.1 Static and Dynamic Tournament Models

Both types of tournament models describe a situation in which a principal awards some valuable prize to the best worker, i.e., to the worker who produces the highest amount of output in a given time frame.[5] We define the individual output produced by a type i worker as $y_i(a_i, x_i) = a_i x_i \geq 0$, where output is the product of ability a_i and effort x_i. Given individual outputs of two workers i and j, the probability p_i that the prize is awarded to worker i equals

$$p_i = \begin{cases} \frac{[y_i(a_i,x_i)]^r}{[y_i(a_i,x_i)]^r + [y_j(a_i,x_i)]^r} & \text{if} \quad y_i(\cdot) + y_j(\cdot) > 0 \\ \frac{1}{2} & \text{if} \quad y_i(\cdot) + y_j(\cdot) = 0 \end{cases} .$$

This formulation is similar to the one used by Rosen (1986) and implies that the principal cannot always perfectly observe which worker produced more, i.e. the monitoring technology is affected by some random component.[6] The parameter r reflects the degree of this randomness: When r approaches infinity, the winning probability of the worker with the higher output converges to 1, implying that the principal can perfectly observe which of the two workers produced more output. For all strictly positive and finite values of r, the monitoring technology implies that the probability to win is greater than 0.5 for the agent whose contribution to aggregate output is higher. Consequently, the winning probability is strictly increasing in the individual output $y_i(a_i, x_i)$, and strictly decreasing in the output $y_j(a_j, x_j)$ produced by the opponent j for all values of $r > 0$.

In both theoretical models considered below, we use this monitoring technology for reasons of analytical tractability.[7]

Model 1: Static Tournament. We start with the static baseline model, where two risk neutral workers compete with each other for some prize P. For simplicity, it is assumed that workers receive no fixed wages. Workers can be of two different types: They are either "strong" (type $\mathtt{S}$), or "weak" (type $\mathtt{W}$). Types may differ with respect to their productive ability a_i ($a_\mathtt{S} \geq a_\mathtt{W}$) or their dis-utility of labor (or effort costs) c_i ($c_\mathtt{S} \leq c_\mathtt{W}$), or both. Compared to weak workers, strong workers either have a higher productivity or a lower dis-utility of labor (or both).[8] Workers are assumed to know their type and the types of their competitors. The two type assumption allows for three different settings: Either, both workers are strong ($\mathtt{SS}$) or weak ($\mathtt{WW}$), or workers are of different types ($\mathtt{SW}$), i.e., we have to consider two homogeneous and one heterogeneous tournament settings. It suffices to solve the general case where workers are allowed to be of different types,

[5] Nothing changes if we assume that each worker also receives some fixed wage payment, since we assume that workers are risk-neutral below.

[6] See Skaperdas (1996) and Clark and Riis (1998b) for an axiomatization of this 'contest success function'.

[7] In the literature, this monitoring technology is usually referred to as the ratio contest success function á la Tullock (1980). Another prominent functional form is the additive noise difference specification. See Hirshleifer (1989) for a comparison of the properties of different contest success functions.

[8] A third possibility would be to model heterogeneity in terms of heterogeneous valuations of prizes. Using this specification would leave the main results unaffected. Details are available upon request.

however, because one can derive the respective expressions for the homogeneous settings by simply imposing the restriction that type specific parameters are equal. Therefore, we start by considering a situation where one worker is of type $\mathtt{S}$, while his opponent is of type $\mathtt{W}$. Formally, the optimization problems can be described as follows:

$$\text{Type } \mathtt{S}: \quad \max_{x_\mathtt{S}} \Pi_\mathtt{S} = \frac{a_\mathtt{S}^r x_\mathtt{S}^r}{a_\mathtt{S}^r x_\mathtt{S}^r + a_\mathtt{W}^r x_\mathtt{W}^r} P - c_\mathtt{S} x_\mathtt{S} \tag{4.1}$$

$$\text{Type } \mathtt{W}: \quad \max_{x_\mathtt{W}} \Pi_\mathtt{W} = \frac{a_\mathtt{W}^r x_\mathtt{W}^r}{a_\mathtt{S}^r x_\mathtt{S}^r + a_\mathtt{W}^r x_\mathtt{W}^r} P - c_\mathtt{W} x_\mathtt{W}, \tag{4.2}$$

where each worker maximizes his expected payoff Π_i by choosing effort x_i. Workers face a trade-off with respect to effort provision: On the one hand, effort increases individual output $y_i(a_i, x_i) = a_i x_i$ and therefore the probability to win the tournament. At the same time, however, each worker bears marginal costs c_i for each unit of effort provided, no matter whether he wins the tournament or not.[9] In equilibrium, workers choose their level of effort provision optimally such that marginal costs equal marginal benefits. Note, however, that first-order conditions are necessary and sufficient for optimal behavior only if the strategic advantage of strong workers is not too high for the given precision of the monitoring technology r. For the necessary conditions to be sufficient, heterogeneity between workers must not exceed a certain threshold if the monitoring technology is relatively precise, otherwise pure strategy equilibria do not exist, as was shown by Nti (1999). Apart from the fact that only mixed-strategy equilibria exist in such a scenario, little is known about the properties of equilibria in tournaments where this restriction is violated, which is why we restrict attention to equilibria in pure strategies throughout the paper.[10] To ensure the existence of equilibria in pure strategies, we impose a parametric restriction on heterogeneity. For notational clarity, denote the relative ability advantage of strong workers in terms of ability and effort costs by

$$\phi = \left(\frac{a_\mathtt{S} c_\mathtt{W}}{a_\mathtt{W} c_\mathtt{S}} \right)^r, \tag{4.3}$$

where $\phi \geq 1$, since by assumptions workers of type $\mathtt{S}$ have a higher productive ability as well as a lower dis-utility of labor (or effort) than workers of type $\mathtt{W}$. Essentially, ϕ measures the degree of heterogeneity in the tournament: If $\phi = 1$, both worker types are identical and the tournament is homogeneous, while high values of ϕ indicate that types differ substantially. We use this measure of heterogeneity to ensure that first-order conditions characterize optimal behavior (and hence the existence of pure strategy equilibria), which

[9]The assumption of constant marginal costs implies a substantial simplification in terms of analytical tractability, but is not central for the main results of this paper.

[10]The design of the experiments presented below ensure that this condition holds. To our knowledge, the only paper which addresses mixed-strategy equilibria in a Tullock contest is the one by Baye, Kovenock, and de Vries (1994).

is the case if and only if the relation

$$r \leq 1 + \frac{1}{\phi} \tag{4.4}$$

is satisfied.[11]

Assumption 4.1. *Relation* (4.4) *is always satisfied, which implies that the degree of heterogeneity between workers (measured by ϕ) is not too high for the given precision r of the monitoring technology.*

Under Assumption 1, the definition of ϕ, and the two first-order optimality conditions which follow from the optimization problem described above, equilibrium efforts are given by

$$x^*_{\mathtt{S}}(\mathtt{SW}) = r\left(\frac{1}{c_{\mathtt{S}}}\right)\frac{\phi}{[1+\phi]^2}P \quad \text{and} \quad x^*_{\mathtt{W}}(\mathtt{SW}) = r\left(\frac{1}{c_{\mathtt{W}}}\right)\frac{\phi}{[1+\phi]^2}P. \tag{4.5}$$

Inserting equilibrium efforts $x^*_{\mathtt{S}}(\mathtt{S},\mathtt{W})$ and $x^*_{\mathtt{W}}(\mathtt{S},\mathtt{W})$ in (4.1) and (4.2) determines the corresponding equilibrium payoffs

$$\Pi^*_{\mathtt{S}}(\mathtt{SW}) = \frac{\phi^2 + (1-r)\phi}{[1+\phi]^2}P \quad \text{and} \quad \Pi^*_{\mathtt{W}}(\mathtt{SW}) = \frac{1 + (1-r)\phi}{[1+\phi]^2}P, \tag{4.6}$$

which solves the heterogeneous interaction (SW). The expressions in (4.5) and (4.6) can then be used directly to characterize equilibrium behavior and outcomes in each of the two homogeneous settings, SS and WW. Recall that $\phi = 1$ by definition in homogeneous specifications. Imposing this assumption on (4.5), we obtain equilibrium efforts

$$x^*_{\mathtt{S}}(\mathtt{SS}) = r\left(\frac{1}{4c_{\mathtt{S}}}\right)P \quad \text{and} \quad x^*_{\mathtt{W}}(\mathtt{WW}) = r\left(\frac{1}{4c_{\mathtt{W}}}\right)P, \tag{4.7}$$

which, when inserted into the formal maximization problems, imply that workers in the homogeneous interactions can expect equilibrium payoffs of

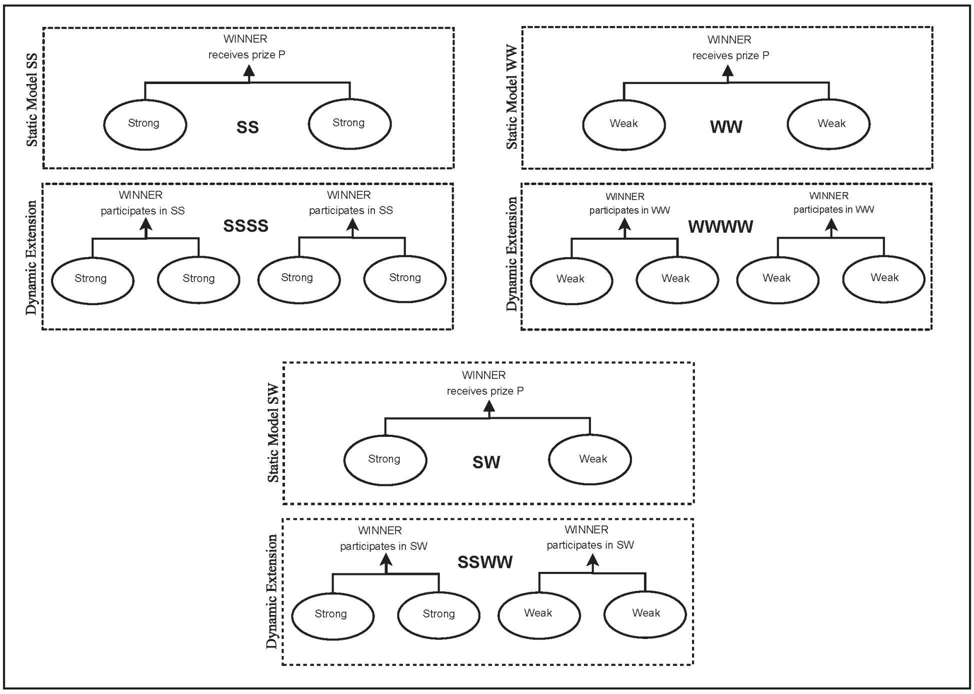
$$\Pi^*_{\mathtt{S}}(\mathtt{SS}) = \frac{2-r}{4}P \quad \text{and} \quad \Pi^*_{\mathtt{W}}(\mathtt{WW}) = \frac{2-r}{4}P. \tag{4.8}$$

Under Assumption 1, these are strictly positive.

Model 2: Dynamic Tournament. The static baseline model can be extended to a dynamic tournament model along the lines of Rosen (1986) by adding a qualification stage to each of the three specifications of the static baseline model, as illustrated in Figure 4.1. In the case of the homogeneous setting with strong workers only (SS), this implies adding two pair-wise stage-1 interactions with two strong workers each; this dynamic setting is

[11] A proof for this claim is provided by Nti (1999).

Figure 4.1: The Dynamic Extension of the Static Baseline Model



denoted SSSS. Similarly, setting WWWW is the dynamic extension of the static model with two weak workers (WW). Analogously, one can add a qualification stage to the static model with heterogenous workers (SW), where two strong and two weak workers compete with each other in stage 1; in what follows, we will refer to this dynamic setting as SSWW. A common feature of these dynamic tournaments is that two workers compete for the right to participate in stage 2 in two separated stage-1 interactions. One worker from each interaction qualifies for stage 2, where the two stage-1 winners compete for prize P as in the static model considered in the previous section; the workers who lost in stage 1 are eliminated from the competition.

All three settings SSSS, SSWW and WWWW are solved via backwards induction due to the dynamic structure of the tournament. Equilibrium efforts in the pair-wise interactions on stage 2 are already known from the analysis of the previous section and are given by the respective expressions in (4.5) and (4.7). In stage 1, the optimization problems differ across specifications, and we start by analyzing setting SSSS. Note that the two stage-1 interactions are fully symmetric, since two workers of the same type compete in each of the two interactions for the right to participate in stage 2. Participation in stage 2 is valuable for workers, because they have a chance to win the prize P only if they reach stage 2. This *continuation value* is given by the payoff that a strong worker can expect in equilibrium if he/she competes with a strong worker for a prize P. Using the results about the expected equilibrium payoff from the previous section in equation (4.8), the continuation value is given by $\Pi^*_{\mathtt{S}}(\mathtt{SS}) = \frac{2-r}{4}P$. Consequently, the two workers in each of the homogeneous stage-1 interactions compete for a prize of value $\Pi^*_{\mathtt{S}}(\mathtt{SS})$. Recall that the equilibrium efforts for two strong workers who compete for a prize P are given in (4.7); replacing P by the expression for $\Pi^*_{\mathtt{S}}(\mathtt{SS})$, we obtain:

$$x^{1*}_{\mathtt{S}}(\mathtt{SSSS}) = r\left(\frac{1}{4c_{\mathtt{S}}}\right)\frac{2-r}{4}P, \tag{4.9}$$

where the superscript 1 indicates that effort is provided in stage 1 of setting SSSS. Note that an analogous line of argument applies to setting WWWW, where weak workers compete in two separate stage-1 interactions for the value of participation in stage 2, which is given by $\Pi^*_{\mathtt{W}}(\mathtt{WW}) = \frac{2-r}{4}P$ according to equation (4.8). Consequently, equilibrium effort in stage 1 by weak workers equals

$$x^{1*}_{\mathtt{W}}(\mathtt{WWWW}) = r\left(\frac{1}{4c_{\mathtt{W}}}\right)\frac{2-r}{4}P. \tag{4.10}$$

Finally, we analyze the slightly more complicated setting SSWW with heterogeneous workers. Note that both stage-1 pairings are between workers of the same type, i.e., strong workers compete in one, while weak workers compete in the other interaction. Therefore, the value of participation in stage 2 depends on the type of a worker. We already saw in the previous section that strong workers can expect a payoff that amounts to $\Pi^*_{\mathtt{S}}(\mathtt{SW}) = \frac{\phi^2+(1-r)\phi}{[1+\phi]^2}P$ in a competition with a weak worker for a prize P. Similarly, a weak worker can expect a payoff that amounts to $\Pi^*_{\mathtt{W}}(\mathtt{SW}) = \frac{1+(1-r)\phi}{[1+\phi]^2}P$ in equilibrium. Consequently, the two strong workers compete for a prize $\Pi^*_{\mathtt{S}}(\mathtt{SW})$ in stage 1, while the prize amounts to $\Pi^*_{\mathtt{W}}(\mathtt{SW})$ in the interaction with weak workers. From the analysis of the static tournament model, we know that this implies equilibrium efforts

$$x^{1*}_{\mathtt{S}}(\mathtt{SSWW}) = r\left(\frac{1}{4c_{\mathtt{S}}}\right)\frac{\phi^2+(1-r)\phi}{[1+\phi]^2}P \quad \text{and} \quad x^{1*}_{\mathtt{W}}(\mathtt{SSWW}) = r\left(\frac{1}{4c_{\mathtt{W}}}\right)\frac{1+(1-r)\phi}{[1+\phi]^2}P. \tag{4.11}$$

This completes the solution of of the static and dynamic tournament models. These solutions constitute the incentive compatibility constraints for a tournament designing principal. Expected equilibrium payoffs are strictly positive for both types and both tournament models under Assumption 1.

4.2.2 Optimal Tournament Design: The Principal's Perspective

This section analyzes how ability and heterogeneity between workers affect output as the central measure of interest of a tournament designing principal. Both the direction and the magnitude of these effects are important to answer the two central questions of our analysis, namely whether or not strong and weak workers should be separated by the principal, and to what extent concerns for the homogeneity of the workforce should affect hiring decisions. We assume that the principal's objective is to maximize profits of the company and abstract from other objective functions that a principal might have. The principal has prior information about the type of each worker. Since total wage costs as well as the price for the output good are assumed to be given exogenously, the principal's problem reduces to a maximization of total output (denoted Y subsequently) produced by all employees.[12] Following the literature on tournament design, we abstract from ability specific tasks or complementarities between output by individual workers and consider the simple case where total output Y equals the sum of individual outputs of all $\mathtt{S}$ and $\mathtt{W}$ type workers. Recall that individual output is given by the product of ability and effort of a worker, i.e. $y_i(a_i, x_i) = a_i x_i$.[13] Then, total output is formally defined by the relation

$$Y = K \cdot y_{\mathtt{S}}(a_{\mathtt{S}}, x_{\mathtt{S}}) + M \cdot y_{\mathtt{W}}(a_{\mathtt{W}}, x_{\mathtt{W}}), \tag{4.12}$$

where K and M are the numbers of $\mathtt{S}$ and $\mathtt{W}$ type workers the principal employs. Total output production for all specifications of the static tournament can then be computed using the expressions for equilibrium efforts given in (4.5) and (4.7) as

$$Y(\mathtt{SS}) = r\left(\frac{a_{\mathtt{S}}}{c_{\mathtt{S}}}\right)\frac{1}{2}P, \quad Y(\mathtt{SW}) = r\left(\frac{a_{\mathtt{S}}}{c_{\mathtt{S}}} + \frac{a_{\mathtt{W}}}{c_{\mathtt{W}}}\right)\frac{\phi}{[1+\phi]^2}P, \quad \text{and} \quad Y(\mathtt{WW}) = r\left(\frac{a_{\mathtt{W}}}{c_{\mathtt{W}}}\right)\frac{1}{2}P. \tag{4.13}$$

It is slightly more complicated to compute total output levels for the dynamic tournament specifications. Note, however, that total output in stage 2 is already known, since the stage-2 interaction is completely identical to the respective static tournament setting. When adding output produced in both stage-1 interactions, we obtain

$$Y(\mathtt{SSSS}) = r\left(\frac{a_{\mathtt{S}}}{c_{\mathtt{S}}}\right)\frac{4-r}{4}P \quad \text{and} \quad Y(\mathtt{WWWW}) = r\left(\frac{a_{\mathtt{W}}}{c_{\mathtt{W}}}\right)\frac{4-r}{4}P \tag{4.14}$$

for the homogeneous specifications, while total output in the heterogeneous case amounts to

$$Y(\mathtt{SSWW}) = r\left[\left(\frac{a_{\mathtt{S}}}{c_{\mathtt{S}}}\right)\frac{\phi^2+(3-r)\phi}{2[1+\phi]^2} + \left(\frac{a_{\mathtt{W}}}{c_{\mathtt{W}}}\right)\frac{1+(3-r)\phi}{2[1+\phi]^2}\right]P. \tag{4.15}$$

We are now in the position to compare total output across different specifications for a particular model. When comparing total output levels of the homogeneous and heterogeneous specifications, the following relations hold for both the static and the dynamic tournament model:[14]

(i) Total output in the homogeneous setting with strong workers is always *higher* than output in the heterogeneous setting with equal shares of strong and weak workers, i.e.

$$Y(\mathtt{SS}) \geq Y(\mathtt{SW}) \quad \text{and} \quad Y(\mathtt{SSSS}) \geq Y(\mathtt{SSWW}).$$

(ii) Total output in the homogeneous setting with weak workers is always *lower* than output in the heterogeneous setting with equal shares of strong and weak workers,

[12] We refrain from modeling the outside options of workers or details of the hiring process. Instead, we assume that workers receive some wage payment by the principal in addition to the tournament compensation. Non-negative expected payoffs are sufficient for participation in the bonus or promotion tournament.

[13] Subsequent results do not depend on this specific functional form assumption. All that is needed is a complementarity between ability and effort.

[14] Relations (i) to (iii) below hold with strict equality whenever strong and weak worker types differ, i.e. if $\phi > 1$.

i.e.

$$Y(\texttt{SW}) \geq Y(\texttt{WW}) \quad \text{and} \quad Y(\texttt{SSWW}) \geq Y(\texttt{WWWW}).$$

(iii) Total output in the homogeneous setting with strong workers is always *higher* than output in the homogeneous setting with weak workers, i.e.

$$Y(\texttt{SS}) \geq Y(\texttt{WW}) \quad \text{and} \quad Y(\texttt{SSSS}) \geq Y(\texttt{WWWW}).$$

The first statement is in line with the standard perception that heterogeneity is associated with lower effort provision by workers and therefore lower total output. The comparison of homogeneous and heterogeneous settings in (ii) tells a very different story, however, indicating that output is always higher in the heterogeneous settings `SW` and `SSWW` compared to the homogeneous settings with weak workers. Finally, relation (iii) states that total output is not the same in two different homogeneous settings, illustrating that the common distinction between homogeneous and heterogeneous tournament settings is sometimes misleading. The intuition for these relations becomes obvious once one separately considers changes of the *average* ability level of all workers who participate in a certain tournament, and changes in terms of *relative* abilities of different worker types: In (iii), for example, settings `SS`/`SSSS` and `WW`/`WWWW` differ only in terms of the average ability level; relative abilities, which are measured by the degree of heterogeneity ϕ, are identical. Strong workers are more productive and face a lower dis-utility of working by definition, such that average ability and therefore total output is higher in the situation where strong workers compete with each other. Consequently, the average level of ability has a strictly positive effect on total output when keeping heterogeneity (or relative abilities) constant. Since this is a general result, we summarize this finding in the following Proposition:

Proposition 4.1 (Ability Effect)**.** *When holding the degree of heterogeneity, as measured by relative abilities ϕ, constant, total output is strictly increasing in the (absolute) ability of each worker in both the static and the dynamic tournament model.*

Proof. See Appendix. □

Note that any ceteris paribus increase in the absolute ability level of any worker type increases the average level of abilities. This fact can explain the seemingly contradictory findings in (i) and (ii). Note that settings `SS`/`SSSS` and `SW`/`SSWW`, as well as settings `WW`/`WWWW` and `SW`/`SSWW` differ in two dimensions, namely in terms of both the average ability level and the degree of heterogeneity ϕ (measured by relative abilities of different worker types). The average ability level is higher in the homogeneous setting with strong workers only as compared to the heterogeneous situation with equal numbers of strong and weak workers (see comparison in (i)). The opposite holds for the comparison in (ii), where the average level of abilities is higher in the heterogeneous specification with both strong and weak workers than in the homogeneous one with weak workers only. Consequently, the two comparisons of homogeneous and heterogeneous tournaments cannot be used to determine the effect of heterogeneity (or relative abilities) on total output, since the average level of abilities changes at the same time.[15] To measure the incentive effect of heterogeneity on total output, however, one has to compare a heterogeneous tournament setting with a homogeneous situation where workers have the *same* average level of ability. In other words, for a meaningful comparison, average ability must be kept constant to isolate the effect of heterogeneity on incentives. Essentially, we use the concept of a mean preserving spread in this comparison, since average ability (the mean) is held constant, while relative ability differences and therefore heterogeneity are increased. Using this approach, the effect of heterogeneity on total output crucially depends on the tournament format:

Proposition 4.2 (Incentive Effect)**.** *When holding the average ability level constant, total output is*

(a) ***decreasing*** *in the degree of heterogeneity between workers (as measured by relative abilities ϕ) in the* ***static*** *tournament model.*

(b) ***increasing*** *in the degree of heterogeneity between workers (as measured by relative abilities ϕ) in the* ***dynamic*** *tournament model.*

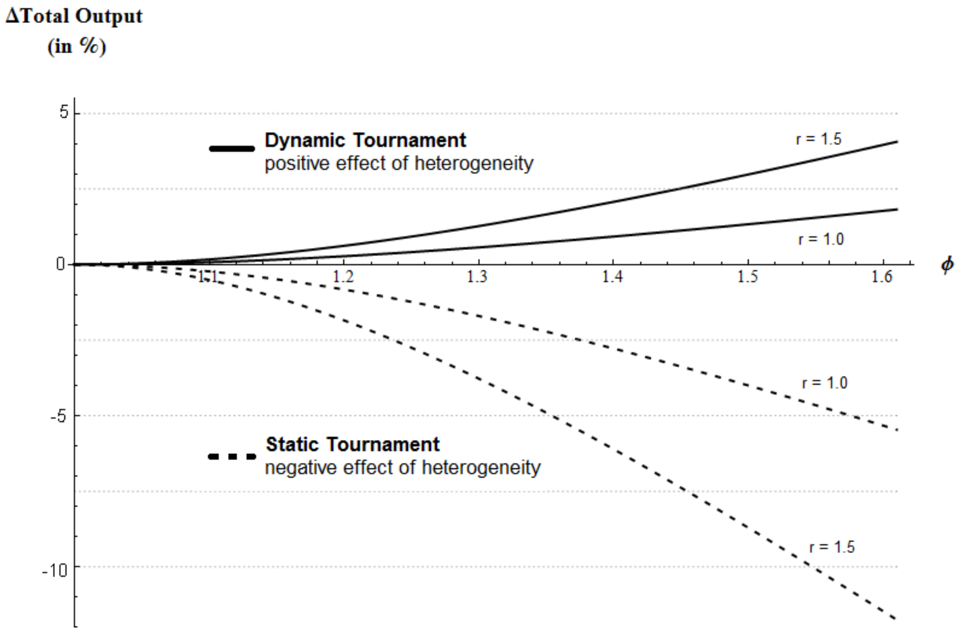
Proof. See Appendix. □

Proposition 4.2 shows that the incentive effect of heterogeneity between workers can have a positive or a negative effect on performance. In case of the static tournament model, we find support for the general perception in the tournament literature that heterogeneity reduces incentives for effort provision, such that total output decreases. Surprisingly, however, the opposite holds for the dynamic tournament, where total output increases in the degree of heterogeneity. Different effects are at work here, which we will analyze separately below. We start with the static tournament model, since it also captures the effect of heterogeneity in stage 2 of the dynamic model.

A higher degree of heterogeneity, or a higher value for ϕ, implies that weak workers reduce effort and therefore the production of output independent of the effort provided by the opponent, i.e., the best response function of weak workers is lower for all effort levels. On the other hand, strong workers also reduce their equilibrium effort to account for the fact that it is easier to win against a relatively weaker opponent. Both effects unambiguously reduce effort provision, and therefore output. There is an opposing effect, however, in the dynamic tournament setting. The fact that workers of both types provide less effort in stage 2 makes it more attractive to reach stage 2, since the promotion to

[15] It is worth noting in this context that on the individual level the effect of heterogeneity is detrimental for individual effort provision in static tournaments since $x^*_{\mathtt{W}}(\mathtt{SW}) < x^*_{\mathtt{W}}(\mathtt{WW})$ and $x^*_{\mathtt{S}}(\mathtt{SW}) < x^*_{\mathtt{S}}(\mathtt{SS})$. Much of the previous literature has focused on this relation without considering the effects on total tournament performance.

Figure 4.2: The Incentive Effect of Heterogeneity on Total Output



the top level position becomes more likely. This holds particularly for strong workers, whose winning probability is increased by a higher relative ability. Consequently, the continuation value of strong workers increases, such that they provide more effort and produce more output in stage 1. Weak workers provide slightly less effort in stage 1, as their continuation value decreases, which means that the effect of heterogeneity on individual output differs across worker types. The overall effect of heterogeneity on total output is unambiguously positive in the dynamic tournament model, however.

It is important to highlight that the effect of heterogeneity is rather small in both the static and the dynamic tournament for modest degrees of heterogeneity. Figure 4.2 shows the percentage change in total output due to changes of the degree of heterogeneity for both tournament formats. One can see, for example, that degrees of heterogeneity in the range of a 10 to 20% difference in abilities between types have almost no detectable effect on total output. For the same average level of abilities, total output in static tournaments is between 1 and 2% lower in a heterogeneous specification where strong workers are 10-20% stronger than weak ones; in dynamic tournaments, output is 0-0.5% higher in the heterogeneous specification for this degree of heterogeneity. Apart from that, the figure also shows that the strength of the heterogeneity effect depends on the precision of the monitoring technology. In line with economic intuition, the positive or negative effect of heterogeneity is stronger when the signal that the principal receives becomes more precise (in the sense of a higher discriminatory power r of the contest success function), since this discourages the weak worker type, whose probability to win by chance is reduced.

While the effect of heterogeneity on total output is small, changes in the average ability of workers have a large effect on total output: A 1% increase (reduction) of the average ability level increases (reduces) total output by 1%. Therefore, the effect of a change in average ability dominates the effect of heterogeneity on total output if both average ability and the degree of heterogeneity are changed simultaneously. Comparison (ii) between settings `WW` and `SW` above illustrates this fact for the static tournament: The positive effect on total output of an increase in the average ability level according to Proposition 4.1 dominates the negative effect of higher heterogeneity (Proposition 4.2), i.e., when moving from a homogeneous situation with two weak workers to a heterogeneous setting with one worker of each type, total output does always increase, irrespective of how small is the difference in ability. The average level of ability is also more important than the degree of heterogeneity in the dynamic tournament specification: When moving from a homogeneous situation with four strong workers to a heterogeneous setting with two workers of each type, total output does always decrease. The negative effect on total output caused by the decrease of the average ability level according to Proposition 4.1 dominates the positive effect of a higher degree of heterogeneity (Proposition 4.2). We summarize these findings in Proposition 3:

Proposition 4.3 (Strength of Ability and Incentive Effects)**.** *If the average ability level and the degree of heterogeneity are changed simultaneously, the effect of the change in absolute average ability on total output is always stronger than the corresponding incentive effect of changes in heterogeneity that works through relative ability. This relation holds in static and dynamic tournaments.*

Proof. See Appendix. □

This finding is particularly important in reality for hiring or firing decisions. The replacement of workers usually implies a simultaneous change of ability and heterogeneity, because it is rather unlikely that a newly hired worker has exactly the same ability level as workers already employed by the company, or as a worker who was recently fired. In such a setting, Proposition 3 tells us that the principal should focus entirely on the absolute ability of this worker, and neglect potential effects on the degree of heterogeneity.

4.2.3 Discussion

The previous analysis of static and dynamic tournaments illustrates that the distinction between the average level of abilities and the degree of heterogeneity in terms of relative abilities is important. We find that individual abilities of a worker determine his general willingness to provide effort in a tournament where ability and effort are complements. Heterogeneity between participants may have a positive or a negative effect on incentives and therefore the production of output, depending on the tournament format: heterogeneity can in fact be good. As a general result, however, changes in the level of average abilities have a stronger impact on effort and total output than corresponding changes in the degree of heterogeneity. Therefore, the theoretical analysis suggests that it is the average ability of a workforce rather than its degree of homogeneity which is crucial for a firm that makes use of tournaments as a compensation scheme. In other words, a principal should be interested mainly in the average ability of his workforce rather than its homogeneity. Whether strong and weak workers are separated into different tournaments does not matter much. It can even be in the interest of the principal not to separate different worker types in dynamic tournaments with multiple stages. While these results suggest that larger heterogeneity can be beneficial for output in the typical static and dynamic tournament settings where ability and effort are complements due to the dominant effect of ability, the results do not imply that greater heterogeneity is always good, however. Under alternative production technologies, e.g., when ability and effort are substitutes, or when firms are primarily interested in balanced competition for different reasons than total output maximization, the results might differ.[16]

Two questions remain: First, it is not clear how general the previous results are, since the theoretical analysis restricted attention to two special cases, namely to static tournaments with two, and to dynamic tournaments with four workers. Second, relatively little is known about the behavior of participants in heterogeneous tournaments, and it cannot be taken for granted that individuals react to heterogeneity in the same way as theory predicts. In fact, some existing evidence suggests that behavior in tournaments with heterogeneous agents strongly deviates from Nash equilibrium predictions.[17] Therefore, we briefly discuss the robustness of our results to changes in the theoretical setup, before we test the relevance of our theoretical results for actual behavior in the next section.

Robustness. The previous theoretical analysis is restricted along two important dimensions: First, we only consider static tournaments with two, and dynamic tournaments with four workers. Second, we assume that there are always equal shares of strong and weak workers in heterogeneous situations. A more general model with both arbitrary numbers of workers and varying shares of worker types can only be analyzed if the generality of the model is restricted in other dimensions, such as the potential for variations in the precision of the monitoring technology (parameter r). However, none of the two restrictions mentioned above affects the main result that the average level of ability has a sizeable effect on total output, while the positive or negative effect of heterogeneity on incentives for effort provision is comparably weak, as we show next.

Two complications arise when one increases the number of workers in the static tournament model and allows for variations in the share of strong and weak workers. First, a closed-form analytical solution is only available for a specific precision of the monitoring technology (lottery contest with $r = 1$) if more than two workers interact.[18] Second, if at least two `S` type workers jointly compete with some workers of type `W`, the latter ones

[16]See, e.g., Gürtler and Kräkel (2011) for a description of alternative tournament settings in which firms might have an incentive to hire low-ability workers.

[17]See for example Bull, Schotter, and Weigelt (1987), or Harbring and Lünser (2008).

[18]See Stein (2002) for a solution of such a model.

might optimally drop out, i.e., their relative costs of participation in the tournament may exceed potential gains, such that they optimally produce nothing. This will, however, only occur in cases where the strategic disadvantage of weak workers is either extremely high, or alternatively, if each weak worker faces a large number of strong opponents, i.e., if weak workers are very rare. Still, the main results are not affected even in these extreme cases, as the analysis of the case with $r = 1$ for situations with four or eight agents and different shares of strong and weak workers shows:[19] Absolute ability does still have a positive effect on total output that is much stronger than the negative effect of heterogeneity. Surprisingly, the relative strength of the effect of heterogeneity on total output is completely independent of the number of competing workers, as long as equal shares of both types compete with each other, as we show in part B of the Appendix. We will come back to this finding in the discussion of the experimental setup.

Finally, consider dynamic promotion tournaments with more than four workers, or with different shares of strong and weak workers. Simply adding stages to the tournament while maintaining the assumption of equal shares of workers of both types implies no qualitative changes as there are simply more pair-wise interactions between workers of the same type before the final stage is reached.[20] One result may change, however, if the shares of strong and weak workers are varied, or if workers are seeded differently in stage 1.[21] Then, heterogeneity can have a negative effect in certain situations, for example if there are three strong and only one weak worker in the tournament. Yet, it is important to stress that the dynamic nature of multi-stage tournaments always reduces the (already relatively weak) negative effect of heterogeneity in comparable static tournaments. The reason is again that the competition for promotions in later stages of the tournament becomes cheaper for strong workers due to the existence of weak workers, which induces the strong workers to increase effort and consequently output production in earlier stages. This effect exists in all dynamic tournaments, but its strength varies across different specifications, such that it overcompensates the negative effect of heterogeneity on total output in later stages in some, but not in all cases.

In summary, the result that heterogeneity can have a positive incentive effect on performance appears to be robust, contrary to the perceived wisdom of a negative effect of heterogeneity on performance. The next section investigates the empirical relevance of this result.

[19] Results are available upon request.

[20] A detailed analysis is available upon request.

[21] Seedings are considered in detail by Höchtl, Kerschbamer, Stracke, and Sunde (2011). Using the solution of the theoretical model where both stage 1 interactions are heterogeneous (setting `SWSW`) from this paper, one can show that the effect of heterogeneity on total output is weakly negative for low degrees of heterogeneity, but strongly positive if heterogeneity is high. Details are available upon request.

4.3 Experimental Evidence

The theoretical analysis in section 4.2 provides several testable hypotheses. We use laboratory experiments to test the theoretical predictions, because the use of experimental methods has the clear advantage that all relevant parameters, in particular in terms of ability, heterogeneity, and the structure of the tournament, are fixed by the experimental design. This allows us to test the theoretical predictions in a direct and controlled way. An investigation using an empirical approach with data from personnel files of companies, like Eriksson (1999), or using data from sports tournaments, would be more difficult because reliable information about absolute levels of ability, an essential component of all our theoretical predictions, is typically not available.[22]

4.3.1 Experimental Design

Following the theoretical model, we assume that agents can be of two different types: Either, they are of the strong type $\mathtt{S}$, or of the weak type $\mathtt{W}$. The cost of effort (or disutility of labor) parameter for weak agents is equal to $c_{\mathtt{W}} = 1.50$ as compared to $c_{\mathtt{S}} = 1.00$ for strong ones. By assumption, productive ability of both agent types is normalized to one ($a_{\mathtt{W}} = a_{\mathtt{S}} = 1$), such that total output equals total effort. For the remainder of this section, we will use the term total output.

Apart from a_i and c_i, there are two additional free parameters in the theoretical model, namely P, which is the value of the bonus payment, or the value of the promotion to the top level position, respectively, and r, which measures the precision of the monitoring technology used by the principal. We set $P = 240$, and $r = 1$. The choice of $r = 1$ has several advantages: First, this case is easy to explain to experimental subjects, which might be the main reason for its popularity in the experimental literature on tournaments.[23] Second, as already mentioned previously, this specification allows us to analytically solve a static tournament model with more than two agents, even if agents are heterogeneous. Therefore, we can consider tournaments with four agents for both one-stage and two-stage tournaments, which facilitates comparison, since the tournaments are completely identical in all but one dimension: The one-stage tournament is static, while the two-stage tournament has a dynamic dimension. A solution to the static tournament with four workers is provided in Appendix B, where we also show that the strength of the effect of the degree of heterogeneity on total output is independent of the number of participating workers.[24]

Overall, we consider six different treatments, three treatments for the static tournament and three treatments for the dynamic tournament. For both tournament formats, we have one treatment with strong agents only, denoted $\mathtt{SSSS}_i$, where $i = 1$ ($i = 2$) in the

[22]For instance, even very rich sports data typically only provide ordinal rankings of ability. See Abrevaya (2002), Brown (2011), or Sunde (2009).

[23]See for example Sheremeta (2011), Sheremeta (2010), and the references provided therein.

[24]See also Stein (2002) for details.

Table 4.1: Theoretical Equilibrium Predictions of Total Output

Static Tournament		*Dynamic Tournament*	
Treatment	Total Output	Treatment	Total Output
$\mathtt{SSSS}_1$	**180**	$\mathtt{SSSS}_2$	**180**
$\mathtt{SSWW}_1$	**144**	$\mathtt{SSWW}_2$	**152**
$\mathtt{WWWW}_1$	**120**	$\mathtt{WWWW}_2$	**120**
mean($\mathtt{SSSS}_1, \mathtt{WWWW}_1$)	**150**	mean($\mathtt{SSSS}_2, \mathtt{WWWW}_2$)	**150**

Note: Equilibrium predictions for total output, i.e. the sum of individuals efforts, in the specific treatment with a single prize of value 240. Note that total effort provision corresponds to total output given the linear production technology with ability normalized to 1 and heterogeneity affecting effort costs. See text for details.

tournament with one (two) stages. All agents are of type W in the second homogeneous treatment $\mathtt{WWWW}_i$, while equal shares are strong and weak in the heterogeneous treatment $\mathtt{SSWW}_i$. A list of these six treatments and the corresponding theoretical equilibrium predictions for total output in each treatment is provided in Table 4.1. This experimental design allows us to test three different hypotheses which directly follow from the theoretical analysis. First, Proposition 4.1 implies that total output is increasing in the absolute ability of each participating worker in both static and dynamic tournaments. Therefore, we should observe that total output is higher in the treatment with four strong agents than in the treatment with four weak ones; theory predicts that output equals 180 units in $\mathtt{SSSS}_i$ for both one-stage and two-stage tournaments, as compared to 120 units in the $\mathtt{WWWW}_i$ treatments.

Hypothesis 4.1 (Ability Effect)**.** *Total output is increasing in the ability of workers in static and in dynamic tournaments. Therefore:*

(a) Total output in $\mathtt{SSSS}_1 >$ *Total output in* $\mathtt{WWWW}_1$

(b) Total output in $\mathtt{SSSS}_2 >$ *Total output in* $\mathtt{WWWW}_2$

Following the order of Propositions in the theoretical analysis, Hypothesis 2 addresses the effect of heterogeneity on total output. According to Proposition 4.2, heterogeneity reduces total output in static tournaments, while heterogeneity has a positive incentive effect through changes in relative ability on total output in dynamic tournaments. Recall that it is essential that the average level of ability remains constant when comparing homogeneous and heterogeneous specifications. This is not the case in any pair-wise comparison of our experimental treatments, which were designed to keep types exactly comparable across the different tournament settings. However, one can easily construct a respective contrast by a simple thought experiment. Suppose that the principal employs eight workers, where four are strong and four are weak. Then, there are (at least) two design options: He can either separate types, which implies that two tournaments with four players each are homogeneous ($\texttt{SSSS}_i$ and $\texttt{WWWW}_i$), or he mixes types and designs two heterogeneous tournaments ($2 \times \texttt{SSWW}_i$). Absolute and average abilities are identical in both options. Therefore, the comparison of these two design options allows us to isolate the effect of heterogeneity on total output, while keeping absolute ability constant. In what follows, we will therefore use the average value of total output in treatments $\texttt{SSSS}_i$ and $\texttt{WWWW}_i$ and compare this value with total output in the heterogeneous treatment $\texttt{SSWW}_i$, which analogously ensures that absolute ability is unchanged. As Table 4.1 shows, the average of total output in the homogeneous treatments equals 150 units in both the one-stage and the two-stage tournament. Heterogeneity reduces total output to 144 units in the static tournament model, while total output increases to a value of 152 units due to heterogeneity in the dynamic tournament model with two stages.

Hypothesis 4.2 (Incentive Effect)**.** *The incentive effect of heterogeneity on total output depends on the tournament format: The effect is negative in static, and positive in dynamic tournaments:*

(a) $mean(\textit{Total output in } \texttt{SSSS}_1 \textit{ and } \texttt{WWWW}_1) > \textit{Total output in } \texttt{SSWW}_1$

(b) $mean(\textit{Total output in } \texttt{SSSS}_2 \textit{ and } \texttt{WWWW}_2) < \textit{Total output in } \texttt{SSWW}_2$

Proposition 4.3 above provides us with another testable hypothesis. According to this Proposition, the effect of changes of the average level of ability on total output is stronger than the corresponding effect of variations in the degree of heterogeneity, if ability and heterogeneity are changed simultaneously. In terms of our experimental treatments, this implies that we have to compare treatments in which ability and heterogeneity effects work in opposite directions. For the static tournament, this is the case if we compare treatments $\texttt{SSWW}_1$ and $\texttt{WWWW}_1$: Average ability is higher in $\texttt{SSWW}_1$, which should increase total output. At the same time, however, heterogeneity is also higher in $\texttt{SSWW}_1$ than in the homogeneous treatment $\texttt{WWWW}_1$, which tends to reduce total output. Theory predicts that the ability effect dominates, since total output amounts to 144 units in $\texttt{SSWW}_1$ and to 120 units in $\texttt{WWWW}_1$, respectively (see Table 4.1). In the dynamic two-stage tournament setting, we have to compare total output of treatments $\texttt{SSSS}_2$ and $\texttt{SSWW}_2$, since a higher level of average ability positively affects total output in $\texttt{SSSS}_2$, while heterogeneity tends to increase total output in $\texttt{SSWW}_2$. Theory predicts that total output amounts to 180 units in $\texttt{SSSS}_2$ as compared to 152 units in $\texttt{SSWW}_2$ (see Table 4.1).

Hypothesis 4.3 (Relative Strength of Ability and Incentive Effect)**.** *When the effects of changes in ability and heterogeneity work in opposite directions, the effect of absolute ability on total output dominates the effect of heterogeneity:*

(a) $\textit{Total output in } \texttt{WWWW}_1 < \textit{Total output in } \texttt{SSWW}_1$

(b) $\textit{Total output in } \texttt{SSSS}_2 > \textit{Total output in } \texttt{SSWW}_2$

The theoretical predictions in Table 4.1 show that there is one additional advantage of considering a static tournament with four rather than two workers in the experimental implementation. According to the theoretical predictions for this specification, total output should be identical in both tournament formats when the worker pool is homogeneous. The design in terms of a static or dynamic tournament should not matter for performance if workers are homogeneous. This theoretical result is well-known and goes back to Gradstein and Konrad (1999).[25] Yet, the table also shows that this result does not hold when workers are of different types: Total output is predicted to be higher in the dynamic setting $\texttt{SSWW}_2$ than in the static specification $\texttt{SSWW}_1$ (152 compared to 144 units, respectively). This constitutes an additional testable hypothesis that serves as a robustness check for our theoretical prediction that the effect of heterogeneity depends on the tournament format.

Hypothesis 4.4 (Heterogeneity and Tournament Format)**.** *Total output is the same in static and dynamic tournament specifications if workers are homogeneous, while total output differs across the two tournament formats if workers are heterogeneous:*

(a) Total output in $\texttt{SSSS}_1$ = *Total output in* $\texttt{SSSS}_2$

(b) Total output in $\texttt{WWWW}_1$ = *Total output in* $\texttt{WWWW}_2$

(c) Total output in $\texttt{SSWW}_1$ < *Total output in* $\texttt{SSWW}_2$

4.3.2 Experimental Implementation

In the experimental sessions, we adopted a between-subject design, such that experimental subjects encountered only one of the six treatments. Each participant played the same tournament 30 times. We use the experimental currency "Taler", where 200 Taler equal 1.00 Euro. As mentioned previously, we define $P = 240$ such that subjects compete for a single prize of 240 Taler in each interaction. Effort provision was implemented in terms of investments into a lottery: Participants were told that they could buy a discrete number of balls in each interaction. The chosen value for P ensures that equilibrium efforts in all stages and both tournaments are positive integers, which implies that the discrete grid has no consequences for the equilibrium strategies; the equilibrium in pure strategies is unique. The balls purchased by the subjects as well as those purchased by their respective opponents were then said to be placed in the same ballot box, out of which one ball was randomly drawn. This setting reflects the experimental implementation of the the monitoring technology with precision $r = 1$ from the theoretical set-up. Players had to buy (and pay for) their desired number of balls *before* they knew whether or not they won the prize in a given tournament. Therefore, each participant received an endowment of 240 Taler in each round to avoid limited liability problems. This endowment could

[25]Experimental tests of this hypothesis are also provided by Sheremeta (2010).

be used to buy balls. In multi-stage treatments, a subject that reached stage 2 could use whatever remained of the endowment to buy balls on the stage-2 interaction. The part of the endowment that a participant did not use to buy balls was added to the payoffs for that round. Since the endowment was as high as the prize that could be won, agents were not budget-constrained at any time. Experimental subjects were told that the endowment could only be used in a given round, transfers across decision rounds were not possible. Therefore, the strategic interaction was the same in each of the 30 decision rounds. Random matching in each round ensured that the same participants did not interact repeatedly. Matching groups corresponded to the entire session. After each decision round, participants were informed about their own decision, the decision(s) of their immediate opponent(s), and about their own payoff. This setting allows for an investigation of whether players learn when completing the task repeatedly. To avoid income effects, however, the participants were told that only four decision rounds (out of 30) would be chosen randomly and paid out at the end of the experiment.

The protocol of an experimental session was as follows for all treatments: First, the participants received some general information about the experimental session. Then, they were given instructions for the respective main treatment (one-stage or two-stage tournament) with four players, which is described above.[26] After each participant confirmed that he/she had understood the instructions on the computer screen, subjects were informed about their type, i.e., about their individual cost parameter ($c_{\mathtt{S}} = 1.00$ or $c_{\mathtt{W}} = 1.50$); the assignment of types was random. Subsequently, participants had to answer a set of control questions to ensure that they had fully understood the instructions. Only once the control questions were answered correctly did the first decision round start. We ran a total of 15 computerized sessions with 20 participants each: Two sessions for each treatment of the static, and three sessions for each treatment of the dynamic tournament. The experiments were implemented using the software z-Tree (Fischbacher, 2007). All 300 participants were students from the University of Innsbruck, which were recruited using ORSEE (Greiner 2004). Each session lasted approximately 1.5 hours, and participants earned between 10-20 Euros (approximately 15 Euros on average).[27]

4.3.3 Experimental Results

Table 4.2 provides session and first round means of total output for each of the six different treatments. Before assessing the empirical validity of Hypotheses 1–4, we compare the session means to the respective theoretical predictions, which are also shown in Table 4.2.

[26] A translated version of the instructions is provided in Appendix C. The orginal instructions (which are in German) are available upon request.

[27] Each experimental sessions consisted of several parts. At the beginning of each session, subjects were told that they would get 3 Euro show-up fee, and that that the experiment consists of three parts. Part 1 is the main treatment which is described above. Risk preferences were elicitated in part 2, and finally distributional preferences in part 3. Subjects received instructions for each part only right before the start of the respective part.

Table 4.2: Total Output

Static Tournament

homogeneous				*heterogeneous*			
Treatment	Data		Theory	Treatment	Data		Theory
	session	1st round			session	1st round	
$\mathtt{SSSS}_1$	**308.69**	**325.60**	180	$\mathtt{SSWW}_1$	**220.33**	**252.40**	144
	(41.35)	(47.62)			(26.49)	(42.43)	
$\mathtt{WWWW}_1$	**215.73**	**190.50**	120				
	(5.40)	(22.03)					
∅	**262.21**	**258.05**	150	∅	**220.33**	**252.40**	144

Dynamic Tournament

homogeneous				*heterogeneous*			
Treatment	Data		Theory	Treatment	Data		Theory
	session	1st round			session	1st round	
$\mathtt{SSSS}_2$	**304.51**	**406.93**	180	$\mathtt{SSWW}_2$	**303.75**	**381.13**	152
	(28.31)	(29.88)			(13.82)	(24.59)	
$\mathtt{WWWW}_2$	**201.88**	**231.40**	120				
	(21.22)	(20.35)					
∅	**253.20**	**319.17**	150	∅	**303.75**	**381.13**	152

Note: The numbers in the column "Data" denote total average output observed in all rounds and the first round of the experimental sessions, respectively. Total output is the sum of individuals outputs (in experimental currency, Taler); standard errors in parantheses, based on 2 (10) independent observations for session (1st round) means of the static tournament, and 3 (15) independent observations for session (1st round) means of the dynamic tournament. The column "Theory" provides the theoretical equilibrium prediction for total output production.

This comparison reveals a high degree of over-provision of effort by experimental subjects: Observed total output over all decision rounds is substantially higher than theory predicts. Such substantial over-provision is, however, not uncommon in tournament experiments. Sheremeta (2010) reports very similar degrees of over-provision for homogeneous one-stage and two-stage tournaments in treatments which are almost identical to the ones implemented here, and presents evidence that the size of the endowment, which is equal to the prize in both his and our treatments, is responsible for this result.[28] Note, however, that both the endowment and the prize for the winner of the tournament are identical in all treatments, such that differences between treatments, on which Hypotheses 1–4 rely, cannot be attributed to the endowment.[29]

One potential reason for the over-provision of effort is that experimental subjects might have difficulties in determining optimal or equilibrium effort levels. Hence, one might conjecture that experimental subjects "learn" over time and reduce their effort provision correspondingly.[30] Figure 4.3 shows that this is indeed the case. The degree of over-provision is lower in later than in earlier decision periods, especially for the dynamic tournaments. However, even in later decision rounds total output is substantially higher than theory predicts. Therefore, it seems that learning reduces over-provision only partly. Importantly, learning has virtually no influence on the qualitative relations between different treatments, as suggested by a closer look at Figure 4.3 and the comparison of session and first round means for total output in Table 4.2.

Hypothesis 1 suggests that total output should be higher if four strong workers compete for a prize than if all workers are weak, independent of the tournament format. Table 4.2 reveals that the experimental results are qualitatively in line with this theoretical prediction, no matter whether session averages or first round means of total output are considered: In the static tournament, session (first round) averages of total output amount to 308.69 (325.60) units if all workers are strong, as compared to 215.73 (190.50) units in setting $\mathtt{WWWW}_1$. Similarly, the session (first round) average of total output in the dynamic tournament equals 304.51 (406.93) units in setting $\mathtt{SSSS}_2$, while weak workers produce 201.88 (231.40) units on average. To determine the statistical significance of this result, we use t-tests and separately consider session and first round means. We find that the difference between total output levels is significantly different (greater) than zero in both tournament formats; the respective p-values of a two-sided test are 0.152 (0.044) for session and 0.002 (0.001) for first round means of the static (dynamic) tournament, respectively.[31]

[28]See also Sheremeta (2011). He shows that a reduction of the endowment causes a proportional reduction of total effort as long as the endowment is not binding for equilibrium effort levels.

[29]This would only be an issue if the endowment is binding in some and not in other treatments. However, the share of experimental subjects who spend their whole endowment does not systematically differ between treatments. If we exclude all observations in which the endowment is binding, for example, total output is somewhat lower in all treatments, but the qualitative findings remain unchanged. Details available upon request.

[30]Bull, Schotter, and Weigelt (1987), for example, find that average effort in simple two-person tournaments converges to equilibrium predictions in homogeneous, but not in heterogeneous settings.

[31]We test and reject the null hypothesis H_0: Total output $\mathtt{WWWW}_i$ = Total output $\mathtt{SSSS}_i$ for $i = 1, 2$. We

We summarize this finding as follows:

Result 4.1. *Ability has a strictly positive effect on total output in both the static and the dynamic tournament treatments with homogeneous participants. The strength of this effect is consistent with the theoretical prediction in both tournament formats.*

Hypothesis 2 makes different predictions for static and dynamic tournaments. According to this hypothesis, heterogeneity is expected to have a negative effect on total output in the static tournament (Hypothesis 2a), compared to a positive effect in dynamic tournaments (Hypothesis 2b). Table 4.2 shows that we observe this pattern in the experimental data: Total output is lower in the heterogeneous than in the homogeneous specifications for the static tournament, no matter whether session (220.33 versus 262.21) or first round means (252.40 and 258.05) are examined. The opposite holds for the dynamic tournament model, where session averages of total output amount to 303.75 units in the heterogeneous, compared to 253.20 units in the homogeneous setting; the corresponding values on the first decision round are 381.13 (heterogeneous) and 319.17 (homogeneous), respectively.

Even though we focus on aggregate outcomes rather than individual effort decisions in this paper, it is interesting to note that strong participants are responsible for the direction of the incentive effect of heterogeneity: In the dynamic tournament, production by strong agents is increasing with heterogeneity, particularly in stage 1 (as theory suggests); the opposite holds in the static tournament, where strong workers produce less in the homogeneous than in the heterogeneous situation. Thus, the reaction of strong agents to heterogeneity crucially depends on the tournament structure. Interestingly, this is not the case for weak agents.[32] Independent of the tournament format, both absolute and relative over-provision by weak agents are higher in heterogeneous than in homogeneous settings. This suggests that weak types try to compensate for their strategic disadvantage in any case.[33]

To examine the statistical significance of both parts of Hypothesis 2, we employ a three sample t-test.[34] Testing indicates that the negative incentive effect of heterogeneity

use the parametric t-test rather than the non-parametric Mann-Whitney U-test throughout the paper for consistency reasons. A non-parametric three sample mean test, which we would need to test Hypothesis 2 (see below), is not available. P-values of the Mann-Whitney U-Test for Hypothesis 1 and 3 are very similar to p-values of the t-test, however, and available upon request.

[32]Further details on individual effort decisions are available upon request.

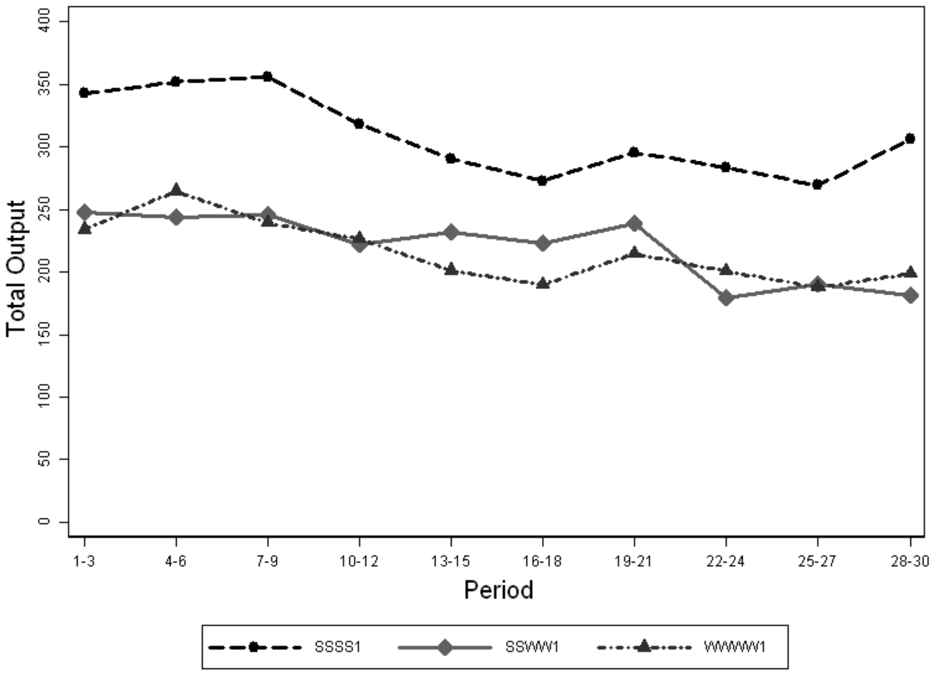
[33]Note that this response of weak agents to a strategic disadvantage has previously been documented for static tournaments with heterogeneous participants (Bull, Schotter, and Weigelt 1987, van Dijk, Sonnemans, and van Winden 2001, Harbring and Lünser 2008).

[34]Formally, we test the hypotheses $\mu(\texttt{SSSS}_1) + \mu(\texttt{WWWW}_1) = 2\mu(\texttt{SSWW}_1)$ and $\mu(\texttt{SSSS}_2) + \mu(\texttt{WWWW}_2) = 2\mu(\texttt{SSWW}_2)$, where $\mu(X)$ is the average total output in setting $X \in \{\texttt{SSSS}_i, \texttt{WWWW}_i, \texttt{SSWW}_i\}$, $i = 1, 2$. The corresponding test statistic is

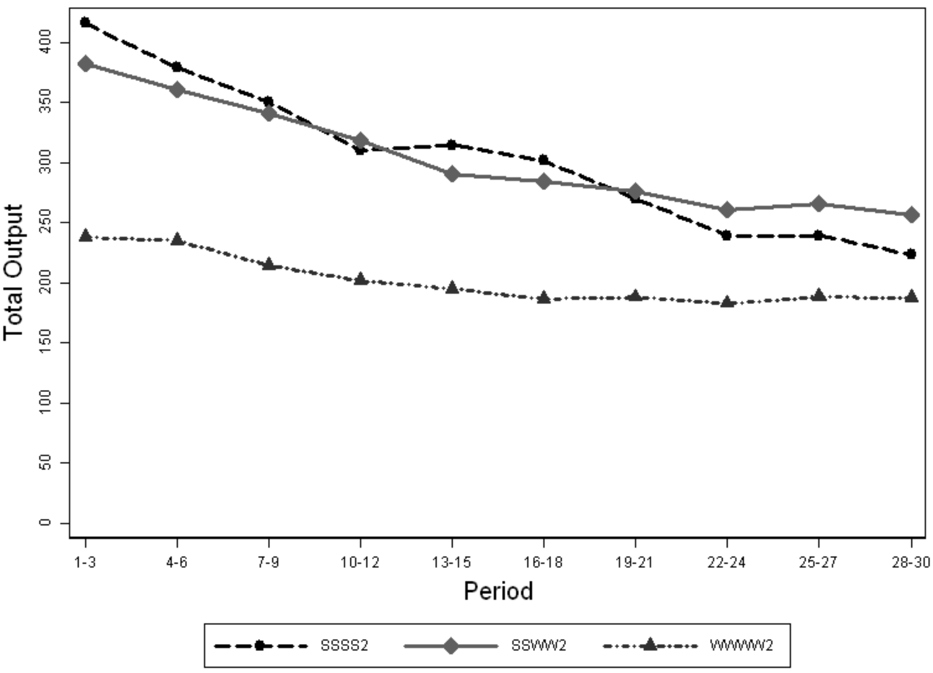
$$T = \frac{\mu(\texttt{SSSS}_i) + \mu(\texttt{WWWW}_i) - 2\mu(\texttt{SSWW})_i}{\sqrt{\frac{\sigma(\texttt{SSSS}_i)^2}{n(\texttt{SSSS}_i)} + \frac{\sigma(\texttt{WWWW}_i)^2}{n(\texttt{WWWW}_i)} + 4\frac{\sigma(\texttt{SSWW}_i)^2}{n(\texttt{SSWW}_i)}}}$$

with $n(\texttt{SSSS}_i) + n(\texttt{WWWW}_i) + n(\texttt{SSWW}_i) - 3$ degrees of freedom.

Figure 4.3: The Effect of Experience on Total Output



(a) Static Tournament



(b) Dynamic Tournament

in the static tournament is insignificant (p-value > 0.10 both for session and first round means), while the positive effect of heterogeneity on total output is significant in the dynamic tournament; the corresponding p-values are 0.065 for session and 0.05 for first round means. Therefore, even though the incentive effect of heterogeneity is insignificant in static tournaments, it is fair to say that the experimental results are qualitatively in line with the theoretical predictions. In summary, we view this evidence to be consistent with Hypothesis 2.

Result 4.2. *The direction of the incentive effect of heterogeneity is in line with the theoretical model. We find that heterogeneity has a*

(a) ***negative*** *effect on total output in the static tournament treatments.*

(b) ***positive*** *effect on total output in the dynamic tournament treatments.*

The data only provide weak evidence for Hypothesis 3, which is on the relative strength of the effect of heterogeneity and ability on total output. According to the theoretical model, the effect of changes in the degree of heterogeneity on total output (be it positive or negative) is always weaker than the corresponding effect of variations in the level of average ability if the degree of heterogeneity and the average ability level are changed jointly. That is, theory predicts for the static tournament that total output is higher in treatment $\texttt{SSWW}_1$ than in $\texttt{WWWW}_1$, since the negative effect of heterogeneity on total output is dominated by the positive effect of higher average ability. For the dynamic tournament, total output in treatment $\texttt{SSSS}_2$ is predicted to be higher than in $\texttt{SSWW}_2$, since the negative effect due to the reduction of average ability is more pronounced than the positive effect of an increase of the degree of heterogeneity when moving from a situation with only strong workers to a setting where equal shares are strong and weak. Figure 4.3 shows that the ability and the incentive effect of heterogeneity are equally strong both for the static and the dynamic tournament, since there is hardly any difference between the total output produced in treatments $\texttt{SSWW}_1$ and $\texttt{WWWW}_1$, or $\texttt{SSSS}_2$ and $\texttt{SSWW}_2$, respectively. When considering the session means, we find that total output equals 215.73 units in treatment $\texttt{WWWW}_1$ and 220.33 units in $\texttt{SSWW}_1$, which is qualitatively in line with the theoretical prediction. However, the difference is statistically insignificant (p-value > 0.10) and much lower than expected (2% rather than 20%).[35] The pattern is almost the same in the dynamic tournament treatments, where the session means of total output amount to 304.51 units in setting $\texttt{SSSS}_2$, compared to 303.75 units in $\texttt{SSWW}_2$ (difference equals 0.2% in the data, compared to 18% predicted by theory). Again, the differences between treatments are insignificant for session and first round means (p-value > 0.10 in both cases).[36]

[35]Even the difference of total output in the first decision round, which is somewhat higher, is statistically insignificant. We test whether H_0 : Total output in $\texttt{SSWW}_1$ - Total output in $\texttt{WWWW}_1 < 0$. H_0 cannot be rejected.

[36]We test whether H_0 : Total output in $\texttt{SSSS}_2$ - Total output in $\texttt{SSWW}_2 < 0$. H_0 cannot be rejected.

These results indicate that either the effect of changes in the level of average ability is much weaker, or that the effect of variations in the degree of heterogeneity is much stronger than theory predicts. Experimental results suggest the latter explanation, since the strength of the pure ability effect is in line with theory, while the incentive effect of heterogeneity is much stronger than expected: Theory predicts that output should be approximately 33% lower in treatment $\mathtt{WWWW}_i$ than in treatment $\mathtt{SSSS}_i$, independent of the tournament format ($i = 1, 2$). In fact, session means of total output are 31% (33%) lower in the static (dynamic) tournament with only weak workers than in the corresponding treatment with only strong workers. The isolated incentive effect of heterogeneity, however, is much more pronounced than expected: While theory predicts that total output decreases by 4% in the static tournament, the session means show a reduction of more than 15%. Similarly, the session means of total output increase by almost 20% in the dynamic tournament, while the increase should be slightly more than 1% according to the theoretical model.[37]

Result 4.3. *When the ability effect and the incentive effect of heterogeneity work in opposite directions, the effects are approximately equally strong and offset each other in both the static and the dynamic tournament treatments; the incentive effect of heterogeneity is stronger than predicted.*

Finally, we briefly consider Hypothesis 4, which makes predictions about the relation between static and dynamic tournaments: Total output should be the same if workers are homogeneous, while this measure is predicted to differ across the two tournament formats if workers are heterogeneous. This pattern emerges from the results in Table 4.2 when considering session means: Total output is similar when only strong or only weak workers compete with each other (308.69 vs. 304.51, and 215.73 vs. 201.88, respectively), while the difference between the two tournament models is remarkable in the heterogeneous settings (220.33 compared to 303.75). Based on two-sided t-tests, we cannot reject the null hypothesis of equal means in either comparison of the homogeneous settings, $\mathtt{SSSS}_1$ and $\mathtt{SSSS}_2$, and $\mathtt{WWWW}_1$ and $\mathtt{WWWW}_2$, respectively. However, the null of equality of total output being equal in settings $\mathtt{SSWW}_1$ and $\mathtt{SSWW}_2$ can be rejected with a p-value of 0.051.[38]

When comparing the first round rather than the session means of the heterogeneous treatments, the results are qualitatively unchanged. Total output equals 252.40 units in the static, compared to 381.13 units in the dynamic tournament. This difference is statistically significant (p-value < 0.01). However, first round means of total output are much higher in the homogeneous treatments of dynamic tournaments (325.60 vs. 406.93, and 190.50 vs. 231.40 for $\mathtt{SSSS}_i$ and $\mathtt{WWWW}_i$ ($i = 1, 2$), respectively), even though the differences are statistically insignificant (p-value > 0.10). While our findings for the homogeneous treatments differ from previous results in the literature when considering session means, it

[37] Both for the static and the dynamic tournament, we compare both the theoretical precision and the session means of $\mathtt{SSWW}_i$ and mean($\mathtt{SSSS}_i, \mathtt{WWWW}_i$) for $i = 1, 2$, respectively.

[38] This p-value refers to a t-test of equality of session means for total output across the two settings.

is interesting to note that our results are comparable with previous findings if we use the first decision period only. For instance, Sheremeta (2010) suggests that effort provision is higher in the two-stage tournament with homogeneous participants due to joy of winning. His experimental design allows for learning, but he uses a mixture of between and within subject comparison, while we employ a between-subject design. Altmann, Falk, and Wibral (2012) also find that effort provision is higher in dynamic tournaments when considering one-shot interactions, and a between subject comparison. The remarkable difference between session and first round means in dynamic tournaments (which we do not observe for static tournaments), suggests that learning patterns may differ between static and dynamic tournaments. This conjecture receives some support when considering Figure 4.3: There is a downward trend of total output in dynamic tournaments, i.e., experimental subjects seem to realize in later rounds that their inital effort provision was too high. This is different in static tournaments, where total output starts at comparably lower initial levels. Note that the differential learning trends in static and dynamic tournaments are of no consequences for the our main results with respect to Hypotheses 1–3, since we compare different specifications of the same tournament format, and learning trends seem to be very similar for different treatments within a certain tournament class.[39]

Result 4.4. *Total output does not differ significantly between static and dynamic tournament treatments if participants are homogeneous. In line with the theoretical predictions, however, there is a difference between the two tournament formats in the heterogeneous treatments; this difference is more pronounced than predicted by theory.*

Overall, Result 4 provides additional support for the theoretical results with respect to the effects of ability and heterogeneity. The data match all qualitative relations not only within, but even across different tournament formats, in particular when using session means.

4.4 Conclusion

This paper analyzes the effect of variations in the degree of heterogeneity between workers on their performance in corporate tournaments. The analysis considers the two most prominent tournament formats in the Personnel Economics literature, namely static one-shot tournaments, in which participants compete for a bonus payment, and dynamic multi-stage tournaments, which are often used to model promotion tournaments in companies with several hierarchy levels. The theoretical results show that the ability of workers who participate in the tournament has a strong impact on the overall level of effort provided, and therefore also on the total amount of output produced. The results also suggest that the common perception that heterogeneity between participants is

[39]The investigation of the dynamic patterns is an interesting topic for future work that directly compares different tournament formats.

detrimental for incentives and performance in tournaments is correct in static one-stage tournaments. In dynamic multi-stage elimination tournaments, however, the effect of heterogeneity on incentives for effort provision is strictly positive, i.e., incentives for effort provision are higher in tournaments with heterogeneous participants than in tournaments with homogeneous agents if the level of average ability is held constant. This is because the negative incentive effect for weak workers is more than compensated for by a higher value of promotions for strong workers, who anticipate that their chances for promotions in the future are higher if there is a chance that they have to compete with a weak opponent in later stages. This possibility strongly increases the value of a promotion today, and consequently incentives to provide effort in early stages of the tournament. From the theoretical analysis, it also follows that the effect of heterogeneity is rather weak in both tournament models, and in particular much weaker than the effect of changes of the average ability level of tournament participants on performance, which is measured by output throughout the paper. The second part of the paper presents evidence from laboratory experiments that is largely consistent with the theoretical predictions. The experimental findings suggest that ability of workers has a strong impact on the performance in both tournament formats. Further, we find that incentives and consequently the overall performance are lower in static tournaments with heterogeneous subjects than in comparable treatments with homogeneous participants. The experimental results suggest, however, that the negative effect of heterogeneity in static tournaments is stronger than predicted by theory. The pattern is similar in our dynamic tournament treatments, where the direction of the heterogeneity effect is in line with the theoretical prediction, but the effect is again stronger than predicted. We find that heterogeneity is associated with a strongly increased effort provision in dynamic tournaments. Concerning the relative strength of ability and heterogeneity effects, we find that the empirical effects of heterogeneity and of changes of the average ability level of tournament participants are of similar size, which is in contrast to the theoretical prediction. However, this suggests that that the influence of the tournament structure on incentives in case of heterogeneity might be much more important in reality than it is in theory.

The results have important implications for both practical applications and future research. The theoretical analysis and the experimental results show that the ability of workers can be very important for performance, potentially more important than a homogeneous workforce, even if tournament compensation schemes are used for the provision of incentives. Second, heterogeneity is not necessarily bad per se in a corporate tournament setting. Whether or not corporations should assign similarly productive employees or different types to the same tournament crucially depends on its structure. An important topic for future research is the surprisingly strong effect of heterogeneity between participants on behavior in the experimental treatments, both in the static tournament, where the effect is negative, and in the dynamic tournament, where the effect is positive.

Appendix

4.A Proofs

Proof of Proposition 1: In the static tournament model, we consider three different specifications, namely SS, WW, and SW. Inspection of equation (4.13) immediately reveals that the respective expressions are strictly increasing in the ability measure $(\frac{a_{\mathtt{S}}}{c_{\mathtt{S}}})$, or $(\frac{a_{\mathtt{W}}}{c_{\mathtt{W}}})$, or both, respectively, when keeping ϕ constant. The same holds for all three settings of the dynamic tournament model, namely SSSS, WWWW, and SSWW, as inspection of equations (4.14) and (4.15) immediately reveals.

Proof of Proposition 2: We separately consider parts (a) and (b) of the Proposition:

(a) From (4.13), total output in the heterogeneous specification SW is given by

$$Y(\mathtt{SW}) = r\left(\frac{a_{\mathtt{S}}}{c_{\mathtt{S}}} + \frac{a_{\mathtt{W}}}{c_{\mathtt{W}}}\right)\frac{\phi}{[1+\phi]^2}P.$$

Recall that $\phi = (\frac{a_{\mathtt{S}}c_{\mathtt{W}}}{a_{\mathtt{W}}c_{\mathtt{S}}})^r$ measures heterogeneity in terms of relative ability. Due to the assumption that workers of type S have a higher productive ability and/or a lower dis-utility of labor than these of type W, $\phi > 1$ holds, which implies that the degree of heterogeneity increases with ϕ. Next, we determine total output in a homogeneous setting where workers have the same average ability level. Workers in the heterogeneous setting have an average ability level of

$$\frac{\bar{a}}{\bar{c}} = \frac{1}{2}\left(\frac{a_{\mathtt{S}}}{c_{\mathtt{S}}} + \frac{a_{\mathtt{W}}}{c_{\mathtt{W}}}\right).$$

Then, it follows from (4.13) that total output in a homogeneous tournament with two workers with ability $\frac{\bar{a}}{\bar{c}}$ amounts to

$$\bar{Y}(\text{hom}) = r\left(\frac{\bar{a}}{\bar{c}}\right)\frac{1}{2}P = r\left(\frac{a_{\mathtt{S}}}{c_{\mathtt{S}}} + \frac{a_{\mathtt{W}}}{c_{\mathtt{W}}}\right)\frac{1}{4}P.$$

To prove that heterogeneity has a negative effect on total output, we have to show that $\bar{Y}(\text{hom}) > Y(\mathtt{SW})$ does always hold, i.e.

$$\begin{aligned} r\left(\frac{a_{\mathtt{S}}}{c_{\mathtt{S}}} + \frac{a_{\mathtt{W}}}{c_{\mathtt{W}}}\right)\frac{1}{4}P &> r\left(\frac{a_{\mathtt{S}}}{c_{\mathtt{S}}} + \frac{a_{\mathtt{W}}}{c_{\mathtt{W}}}\right)\frac{\phi}{[1+\phi]^2}P \\ \Leftrightarrow \frac{1}{4} &> \frac{\phi}{[1+\phi]^2} \\ \Leftrightarrow \phi^2 + 2\phi + 1 &> 4\phi \\ \Leftrightarrow (\phi - 1)^2 &> 0 \end{aligned}$$

Since $\phi > 1$ by construction, this relation is always satisfied. Moreover, the strength of the detrimental effect of heterogeneity is increasing in ϕ, i.e., the higher ϕ, the larger is the difference between the homogeneous and the heterogeneous setting. Therefore, total output is decreasing in the degree of heterogeneity between workers, which completes the proof.

(b) Total output in the heterogeneous specification $Y(\texttt{SSWW})$ is given by

$$Y(\texttt{SSWW}) = r\left[\left(\frac{a_{\texttt{S}}}{c_{\texttt{S}}}\right)\cdot\frac{\phi^2+(3-r)\phi}{2[1+\phi]^2}+\left(\frac{a_{\texttt{W}}}{c_{\texttt{W}}}\right)\cdot\frac{1+(3-r)\phi}{2[1+\phi]^2}\right]P.$$

Recall that $\phi = (\frac{a_{\texttt{S}}c_{\texttt{W}}}{a_{\texttt{W}}c_{\texttt{S}}})^r$ measures heterogeneity in terms of relative ability. Due to the assumption that workers of type S have both a higher productive ability and a lower dis-utility of labor than these of type W, $\phi > 1$ holds. This implies that the degree of heterogeneity increases with ϕ. Next, we have to determine total output in a homogeneous setting where workers have the same average ability level. Workers in the heterogeneous setting have an average ability level of

$$\frac{\bar{a}}{\bar{c}} = \frac{1}{2}\left(\frac{a_{\texttt{S}}}{c_{\texttt{S}}}+\frac{a_{\texttt{W}}}{c_{\texttt{W}}}\right).$$

Then, it follows from (4.14) that total output in a homogeneous tournament with four workers with ability $\frac{\bar{a}}{\bar{c}}$ amounts to

$$\bar{Y}(\text{hom}) = r\left(\frac{\bar{a}}{\bar{c}}\right)\frac{4-r}{4}P = r\left(\frac{a_{\texttt{S}}}{c_{\texttt{S}}}+\frac{a_{\texttt{W}}}{c_{\texttt{W}}}\right)\frac{4-r}{8}P.$$

To prove that heterogeneity has a positive effect on total output, the relation $Y(\texttt{SSWW}) > \bar{Y}(\text{hom})$ has to hold, i.e.

$$\begin{aligned} r\left[\left(\frac{a_{\texttt{S}}}{c_{\texttt{S}}}\right)\cdot\frac{\phi^2+(3-r)\phi}{2[1+\phi]^2}+\left(\frac{a_{\texttt{W}}}{c_{\texttt{W}}}\right)\cdot\frac{1+(3-r)\phi}{2[1+\phi]^2}\right]P \quad &> \quad r\left(\frac{a_{\texttt{S}}}{c_{\texttt{S}}}+\frac{a_{\texttt{W}}}{c_{\texttt{W}}}\right)\frac{4-r}{8}P \\ \Leftrightarrow \left(\frac{a_{\texttt{S}}}{c_{\texttt{S}}}\right)(4\phi^2+(12-4r)\phi)+\left(\frac{a_{\texttt{W}}}{c_{\texttt{W}}}\right)(4+(12-4r)\phi) \quad &> \quad \left(\frac{a_{\texttt{S}}}{c_{\texttt{S}}}+\frac{a_{\texttt{W}}}{c_{\texttt{W}}}\right)(4-r)[1+\phi]^2 \end{aligned}$$

To simplify the subsequent analysis, we make some normalizing assumptions: We assume that $a_{\texttt{W}} = c_{\texttt{W}} = c_{\texttt{S}} = 1$, which is without loss of generality as long as $a_{\texttt{S}} > 1$ does hold. Then, it follows from the definition of ϕ that $\phi = a_{\texttt{S}}^r$. This implies the following relation:

$$\begin{aligned} a_{\texttt{S}}(4a_{\texttt{S}}^{2r}+(12-4r)a_{\texttt{S}}^r)+4+(12-4r)a_{\texttt{S}}^r \quad &> \quad (a_{\texttt{S}}+1)(4-r)[1+a_{\texttt{S}}^r]^2 \\ \Leftrightarrow (a_{\texttt{S}}^r-1)(4a_{\texttt{S}}-r-ra_{\texttt{S}}+a_{\texttt{S}}^r(r+ra_{\texttt{S}}-4)) \quad &> \quad 0 \\ \Leftrightarrow G(a_{\texttt{S}},r) \equiv r(a_{\texttt{S}}^{r+1}-1)-(4-r)(a_{\texttt{S}}^r-a_{\texttt{S}}) \quad &> \quad 0 \end{aligned}$$

Note that $G(a_{\texttt{S}},r)$ is equal to zero if $a_{\texttt{S}} = 1$, while we are interested in the properties of $G(a_{\mathtt{S}}, r)$ when $a_{\mathtt{S}} > 1$ does hold. Therefore, we proceed in two steps: First, we will show that the slope of $G(a_{\mathtt{S}}, r)$ is strictly positive when $a_{\mathtt{S}} = 1$. Second, we will prove that the slope is strictly increasing, which implies that $G(a_{\mathtt{S}}, r)$ is strictly increasing if $a_{\mathtt{S}} > 1$. Since $G(a_{\mathtt{S}}, r)$ is a continuous function, this will prove the claim that $G(a_{\mathtt{S}}, r) > 0 \ \forall \ a_{\mathtt{S}} > 1$.

(i) The first derivative of $G(a_{\mathtt{S}}, r)$ with respect to $a_{\mathtt{S}}$ reads

$$\frac{\partial G(a_{\mathtt{S}}, r)}{\partial a_{\mathtt{S}}} = r(r+1)a_{\mathtt{S}}^{r} - (4-r)(r a_{\mathtt{S}}^{r-1} - 1).$$

For $a_{\mathtt{S}} = 1$, this derivative simplifies to the term $2(r-1)^2$, which is positive. Consequently, the slope of $G(a_{\mathtt{S}}, r)$ at the point $a_{\mathtt{S}} = 1$ is strictly positive.

(ii) The second derivative of $G(a_{\mathtt{S}}, r)$ with respect to $a_{\mathtt{S}}$ reads

$$\begin{aligned}\frac{\partial^2 G(a_{\mathtt{S}}, r)}{\partial a_{\mathtt{S}}^2} &= r(r-4)(r-1)a_{\mathtt{S}}^{r-2} + r^2(1+r)a_{\mathtt{S}}^{r-1} \\ &= r a_{\mathtt{S}}^{r-2}[r^2 - 5r + 4 + (r^2 + r)a_{\mathtt{S}}]\end{aligned}$$

Note that this second derivative is strictly greater than zero since $a_{\mathtt{S}} > 1$.[40] This proves that output is always strictly larger in the heterogeneous than in the homogeneous setting, since the measure of the difference between the two settings is always greater zero, i.e. $G(a_{\mathtt{S}}, r) > 0 \ \forall \ a_{\mathtt{S}} > 1$. Further, since $G(a_{\mathtt{S}}, r)$ is also strictly in increasing in $a_{\mathtt{S}}$, the strength of the positive effect of increases in the degree of heterogeneity.[41] Therefore, we find that total output is strictly increasing in the degree of heterogeneity between workers, which completes the proof.

Proof of Proposition 3: We start by proving the proposition for the static tournament model in part (a) of this proof, and subsequently consider the dynamic tournament model in part (b).

(a) Recall that total output in the heterogeneous setting, $Y(\mathtt{SW})$ equals

$$Y(\mathtt{SW}) = r\left(\frac{a_{\mathtt{S}}}{c_{\mathtt{S}}} + \frac{a_{\mathtt{W}}}{c_{\mathtt{W}}}\right)\frac{\phi}{[1+\phi]^2}P.$$

We know from Propositions 1 and 2 that increases in the ability level have a positive effect on total output, while a higher degree of heterogeneity has a negative effect. To prove the claim that the effect on total output of a change in the average ability level of workers dominates the corresponding effect of a change in the degree of heterogeneity if both ability and heterogeneity are changed simultaneously, we

[40]If $a_{\mathtt{S}}$ were equal to one, one would obtain the relation $r a_{\mathtt{S}}^{r-2}[2(r-1)^2 + 2] > 0$. When $a_{\mathtt{S}} > 1$, the relation is even more positive.

[41]Note that higher values for $a_{\mathtt{S}}$ immediately imply a higher degree of heterogeneity, since average ability is held constant by construction.

compare the heterogeneous setting with a homogeneous setting in which the average level of ability is lower. Proposition 3 is proven if we can show that total output is always higher in the heterogeneous setting than in the homogeneous setting with a lower average ability. Therefore, we show that total output in the heterogeneous setting SW is always higher than in a situation with weak workers only (WW). Recall that total output in the latter case amounts to

$$Y(\mathtt{WW}) = r\left(\frac{a_{\mathtt{W}}}{c_{\mathtt{W}}}\right)\frac{1}{2}P.$$

Consequently, we have to show that $Y(\mathtt{SW}) > Y(\mathtt{WW})$ for all $\frac{a_{\mathtt{S}}}{c_{\mathtt{S}}} > \frac{a_{\mathtt{W}}}{c_{\mathtt{W}}}$. To simplify the subsequent analysis, we assume that $a_{\mathtt{S}} = c_{\mathtt{W}} = c_{\mathtt{S}} = 1$, which is without loss of generality if the relation $0 < a_{\mathtt{W}} < 1$ does hold. This gives

$$\begin{aligned} Y(\mathtt{SW}) &> Y(\mathtt{WW}) \\ \Leftrightarrow r(1+a_{\mathtt{W}})\frac{\phi}{[1+\phi]^2}P &> r\frac{a_{\mathtt{W}}}{2}P \\ \Leftrightarrow 2(1+a_{\mathtt{W}})\frac{\phi}{[1+\phi]^2} - a_{\mathtt{W}} &> 0 \end{aligned}$$

Now, recall that $\phi = \frac{1}{a_{\mathtt{W}}^r}$ by definition. This gives $K \equiv 2a_{\mathtt{W}}^r - a_{\mathtt{W}}^{2r+1} - a_{\mathtt{W}}$. To complete the proof, we have to show that $K > 0$ for all $0 < a_{\mathtt{W}} < 1$. We must take account of one additional constraint, since the derived equilibrium solution is only valid under Assumption 1. Therefore, we have to prove that

$$K = 2a_{\mathtt{W}}^r - a_{\mathtt{W}}^{2r+1} - a_{\mathtt{W}} > 0$$

for all values of $a_{\mathtt{W}}$ and r that satisfy

$$\begin{aligned} &(i) \quad 0 < a_{\mathtt{W}} < 1 \\ &(ii) \quad r \leq 1 + a_{\mathtt{W}}^r \end{aligned}$$

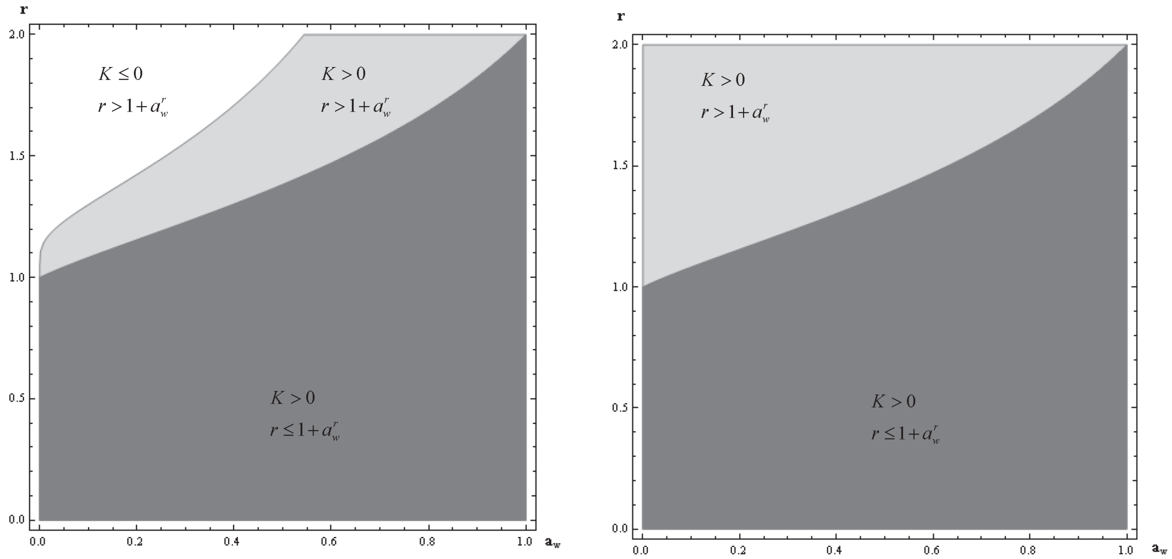
We start by presenting a formal proof for the special case $r = 1$, which we use in our experiments and which is prominent in the literature. For $r = 1$, constraint (ii) is automatically satisfied. K simplifies to

$$K \;=\; 2a_{\mathtt{W}} - a_{\mathtt{W}}^3 - a_{\mathtt{W}} = a_{\mathtt{W}}\underbrace{(1 - a_{\mathtt{W}}^2)}_{>0 \text{ due to (i)}} > 0.$$

Since (ii) is satisfied by definition and $K > 0$ does hold, $Y(\mathtt{SW}) > Y(\mathtt{WW})$ does hold for $r = 1$, independent of the degree of heterogeneity, which completes the proof.

An analytical proof for the general case with variable exponent r and the nonlinear constraint (ii) is more involved. Therefore, we provide a graphical proof by ways of

Figure 4.4: Range Plots



(a) Proof of Proposition 3(a) (b) Proof of Proposition 3(b)

a range plot in panel (a) of Figure 4.4 instead, which indicates for which values of $a_{\mathtt{W}}$ and r all conditions are simultaneously satisfied. Note that we can restrict attention to the parameter space where $0 < a_{\mathtt{W}} < 1$ due to (i) and $0 < r < 2$ due to (ii). Figure 4 provides the plot. The relation $K > 0$ holds in both the light and the dark area, but not in the white one. Therefore, the plot shows that $K > 0$ is not satisfied for small $a_{\mathtt{W}}$ and high values of r. Note, however, that condition (ii) excludes this area, since the pure strategy equilibrium which we consider throughout the paper does not exist for high values of r when heterogeneity is high: The dark area indicates the range in which condition (ii) holds. Consequently, the condition $K > 0$ is less restrictive than $r \leq 1 + a_{\mathtt{W}}^r$, which proves the claim that $Y(\mathtt{SW}) > Y(\mathtt{WW})$ does hold for all values of $a_{\mathtt{W}}$ and r for which the pure strategy equilibrium we consider exists (i.e., under Assumption 1).

(b) Recall that total output in the heterogeneous setting $Y(\mathtt{SSWW})$ equals

$$Y(\mathtt{SSWW}) = r\left[\left(\frac{a_{\mathtt{S}}}{c_{\mathtt{S}}}\right)\cdot\frac{\phi^2 + (3-r)\phi}{2[1+\phi]^2} + \left(\frac{a_{\mathtt{W}}}{c_{\mathtt{W}}}\right)\cdot\frac{1 + (3-r)\phi}{2[1+\phi]^2}\right]P.$$

We know from Propositions 1 and 2 that increases in the ability level as well as increases in the degree of heterogeneity have a positive on total output. To prove the claim that the effect on total output of a change in the average ability level of workers dominates the corresponding effect of a change in the degree of heterogeneity if both ability and heterogeneity are changed simultaneously, we compare the heterogeneous setting with a homogeneous setting in which the average level of ability is higher. Proposition 3 is proven if we can show that total output is always higher in the homogeneous setting with a higher average ability than in the heterogeneous setting. Therefore, we show that total output in the homogeneous setting SSSS is always higher than total output in a situation with both strong and weak workers (SSWW). Recall that total output in the former situation amounts to

$$Y(\mathtt{SSSS}) = r\left(\frac{a_{\mathtt{S}}}{c_{\mathtt{S}}}\right)\frac{4-r}{4}P.$$

Consequently, we have to show that $Y(\mathtt{SSSS}) > Y(\mathtt{SSWW})$ for all $\frac{a_{\mathtt{S}}}{c_{\mathtt{S}}} > \frac{a_{\mathtt{W}}}{c_{\mathtt{W}}}$. To simplify the subsequent analysis, we assume that $a_{\mathtt{S}} = c_{\mathtt{S}} = c_{\mathtt{W}} = 1$, which is without loss of generality if the relation $0 < a_{\mathtt{W}} < 1$ does hold. Recall that $\phi = \frac{1}{a_{\mathtt{W}}^r}$ by definition. Using these relations, we get

$$\begin{aligned}
Y(\mathtt{SSSS}) &> Y(\mathtt{SSWW}) \\
\Leftrightarrow r\frac{4-r}{4}P &> r\left[\frac{1+(3-r)a_{\mathtt{W}}^r}{2[1+a_{\mathtt{W}}^r]^2} + a_{\mathtt{W}}\frac{a_{\mathtt{W}}^{2r}+(3-r)a_{\mathtt{W}}^r}{2[1+a_{\mathtt{W}}^r]^2}\right]P \\
\Leftrightarrow (4-r)[1+a_{\mathtt{W}}^r]^2 &> 2\left[1+(3-r)a_{\mathtt{W}}^r + a_{\mathtt{W}}(a_{\mathtt{W}}^{2r}+(3-r)a_{\mathtt{W}}^r)\right]
\end{aligned}$$

We must take account of one additional constraint, since the equilibrium solution derived is only valid under Assumption 1. Consequently, we have to prove that

$$M \equiv (4-r)[1+a_{\mathtt{W}}^r]^2 - 2\left[1+(3-r)a_{\mathtt{W}}^r + a_{\mathtt{W}}(a_{\mathtt{W}}^{2r}+(3-r)a_{\mathtt{W}}^r)\right] > 0$$

for all values of $a_{\mathtt{W}}$ and r that satisfy

$$\begin{aligned}
&(i) \quad 0 < a_{\mathtt{W}} < 1 \\
&(ii) \quad r \leq 1 + a_{\mathtt{W}}^r
\end{aligned}$$

We start again by presenting a formal proof for the special case $r = 1$, which we use in our experiments and which is prominent in the literature. For $r = 1$, constraint (ii) is automatically satisfied and M simplifies to

$$M = 3(1+a_{\mathtt{W}})^2 - 2[1+2a_{\mathtt{W}}+a_{\mathtt{W}}(a_{\mathtt{W}}^2+2a_{\mathtt{W}})] = (2a_{\mathtt{W}}+1)\underbrace{(1-a_{\mathtt{W}}^2)}_{>0 \text{ due to (i)}} > 0$$

Since (ii) is satisfied by definition and $M > 0$ does hold, this proves that $Y(\mathtt{SSSS}) > Y(\mathtt{SSWW})$ does hold for $r = 1$, independent of the degree of heterogeneity.

An analytical proof for the general case with variable exponent r and the nonlinear constraint (ii) is more involved. Therefore, we provide a graphical proof by ways of a range plot in panel (b) of Figure 4.4 which indicates for which values of $a_{\mathtt{W}}$ and

r all conditions are satisfied. Note that we can restrict attention to the parameter space where $0 < a_{\mathtt{W}} < 1$ due to (i) and $0 < r < 2$ due to (ii). Figure 5 provides the plot. The relation $M > 0$ holds in both the light and the dark area. Therefore, the plot shows that $M > 0$ is satisfied over the whole range of parameter values which are allowed for the pure strategy equilibrium. The dark area indicates the range in which condition (ii) holds. Condition $M > 0$ is thus less restrictive than $r \leq 1 + a_{\mathtt{W}}^r$, which proves the claim that $Y(\mathtt{SSWW}) > Y(\mathtt{WWWW})$ does hold for all values of $a_{\mathtt{W}}$ and r for which the pure strategy equilibrium we consider exists (i.e., under Assumption 1).

4.B Static Tournament with more than two Workers

Assume that there are N participants in the tournament, where equal shares are strong and weak, i.e., there are $\frac{N}{2}$ workers of each type, where $N \geq 2$. If $r = 1$, an arbitrary player s of the strong type faces the following optimization problem:

$$\max_{x_s} \Pi_s = \frac{a_s x_s}{a_s x_s + a_s \sum_{i \neq s} x_i + a_w \sum_{j=1}^{\frac{N}{2}} x_j} P - c_s x_s, \tag{4.16}$$

i.e., player s chooses his effort x_s in such a way as to maximize his expected payoff Π_s, taking as given the effort choices of his opponents of both player types. Similarly, an arbitrary weak player w chooses the optimal level of effort x_w, and his maximization problem reads:

$$\max_{x_w} \Pi_w = \frac{a_w x_w}{a_w x_w + a_s \sum_{i=1}^{\frac{N}{2}} x_i + a_w \sum_{j \neq w} x_j} P - c_w x_w. \tag{4.17}$$

Taking derivatives with respect to x_s and x_w, respectively, gives the first order optimality conditions for an interior Nash equilibrium in which all agents participate:

$$\frac{a_s(a_s \sum_{i \neq s} x_i + a_w \sum_{j=1}^{\frac{N}{2}} x_j)}{(a_s \sum_{i=1}^{\frac{N}{2}} x_i + a_w \sum_{j=1}^{\frac{N}{2}} x_j)^2} P - c_s = 0 \tag{4.18}$$

$$\frac{a_w(a_s \sum_{i=1}^{\frac{N}{2}} x_i + a_w \sum_{j \neq m} x_j)}{(a_s \sum_{i=1}^{\frac{N}{2}} x_i + a_w \sum_{j=1}^{\frac{N}{2}} x_j)^2} P - c_w = 0. \tag{4.19}$$

Next, symmetry among participants of the same type is imposed, i.e. $x_i^* = x_s^* \ \forall i = 1, ..., \frac{N}{2}$, and $x_j^* = x_w^* \ \forall j = 1, ..., \frac{N}{2}$. Equalizing the remaining two first order conditions and using the definition $\phi = \frac{a_w c_s}{a_s c_w}$ gives the equilibrium ratio of efforts, which is characterized by the following relation:

$$\Phi \equiv \frac{a_s x_s^*}{a_w x_w^*} = \frac{1 + \frac{N}{2}(\phi - 1)}{\phi - \frac{N}{2}(\phi - 1)}.$$

Combining this relation with either of the first-order conditions above delivers equilibrium efforts

$$x_s^* = \left(\frac{1}{c_s}\right) \frac{(\frac{N}{2} - 1) + \Phi \frac{N}{2}}{[\frac{N}{2} + \Phi \frac{N}{2}]^2} P, \quad \text{and} \quad x_w^* = \left(\frac{1}{c_w}\right) \frac{\Phi^2(\frac{N}{2} - 1) + \Phi \frac{N}{2}}{[\frac{N}{2} + \Phi \frac{N}{2}]^2} P. \tag{4.20}$$

As a consequence, total output in this heterogeneous tournament amounts to

$$Y(\frac{N}{2} \cdot \mathtt{S}, \frac{N}{2} \cdot \mathtt{W}) = \left(\frac{a_s}{c_s}\right) \frac{(\frac{N}{2} - 1) + \Phi \frac{N}{2}}{\frac{N}{2}[1 + \Phi]^2} P + \left(\frac{a_w}{c_w}\right) \frac{\Phi^2(\frac{N}{2} - 1) + \Phi \frac{N}{2}}{\frac{N}{2}[1 + \Phi]^2} P.$$

From this, one can derive total output for a homogeneous specification with the same average ability. By imposing the condition $\Phi = 1$, we get

$$Y(\texttt{hom}) = \left(\frac{a_s}{c_s}\right)\frac{N-1}{2N}P + \left(\frac{a_w}{c_w}\right)\frac{N-1}{2N}P.$$

To determine the relative strength of the heterogeneity effect on total output in percent, one can compute the difference between output in the heterogeneous and the homogeneous specification, and normalize, i.e.

$$\frac{Y(\frac{N}{2}\cdot\texttt{S}, \frac{N}{2}\cdot\texttt{W}) - Y(\texttt{hom})}{Y(\texttt{hom})}.$$

Inserting the respective expression and simplifying gives

$$Q = -\left[\frac{\left(\frac{a_s}{c_s}\right) - \left(\frac{a_w}{c_w}\right)}{\left(\frac{a_s}{c_s}\right) + \left(\frac{a_w}{c_w}\right)}\right]^2.$$

Assume, for example, that $a_s = a_w = c_s = 1$, while $c_w = 1.5$ as in the experimental treatment in section 3 of the paper. Then, the above expression shows, that total output is 4% lower in a heterogeneous setting where weak agents are 50% weaker than strong ones, compared to a setting with homogeneous participants who have the same average ability level. Obviously, Q is independent of N, i.e., heterogeneity has the same relative strength when equal shares of both strong and weak types participate, independent of the overall number of participants. In particular, this implies that theory predicts the same effects for changes of the degree of heterogeneity in a tournament with two (as in section 2) or four participants (as in the experimental part of the paper).

4.C Experimental Instructions

4.C.1 General Instructions

WELCOME TO THIS EXPERIMENT AND THANK YOU FOR YOUR PARTICIPATION

General Instructions:

You will participate in 3 different experiments today. Please stop talking to any other participant of this experiment from now on until the end of this session. In each of the three experiments, you will have to make certain decisions and may earn an appreciable amount of money. Your earnings will depend upon several factors: on your decisions, on the decisions of other participants, and on random components, i.e. chance. The following instructions explain how your earnings will be determined.

The experimental currency is denoted **Taler**. In addition to your Taler earnings in experiments 1 to 3, you receive 3 EURO show-up fee. You may increase your Taler earnings in experiments 1 to 3, where 2 Taler equal 1 Euro-Cent, i.e.

200 Taler correspond to 1 Euro.

At the end of this experimental session your Taler earnings will be converted into Euro and paid to you in cash.

Before the experimental session starts, you receive a card with your participant number. All your decisions in this experiment will be entered in a mask on the computer, the same holds for all other participants of the experiment. In addition, the computer will determine the random components which are needed in some of the experiments. All data collected in this experiment will be matched to your participant number, **not** to your name or student number. Your participant number will also be used for payment of your earnings at the end of the experimental session. Therefore, your decisions and the information provided in the experiments are completely anonymous; neither the experimenter nor anybody else can match these data to your identity.

We will start with experiment 1, followed by experiments 2 and 3. The instructions for experiments 2 and 3 will only be distributed right before the respective experiment starts, i.e. subsequent to experiments 1 and 2, respectively.

You will receive your earnings in cash at the end of the experimental session.

4.C.2 One-stage Treatment

Experiment 1

Overall, there are 30 decision rounds in Experiment 1. The course of events is the same in each decision round. You will be randomly and anonymously placed into a group of **four participants** in each round, and the identity of participants in your group changes with each decision round.

Course of events in an arbitrary decision round
All four participants of your group receive an **endowment of 240 Taler** at the beginning of a decision round. The endowment can be used to buy a certain amount of balls. The costs for the purchase of a ball are not the same for all participants:
There are equal shares of high (H) and low (L) cost types in each group of four participants, i.e. there are two participants of each type in a group of four. All experimental participants are informed about their type at the beginning of the experiment. Types do not change with decision rounds, such that you face either high or low costs in each of the 30 decision rounds.
Participants of type H have to pay **1.50 Taler** for each ball they buy, i.e.

1 ball costs 1.50 Taler
2 balls cost 3.00 Taler
(and so on)

Participants of type L have to pay **1.00 Taler** for each ball they buy, i.e.

1 ball costs 1.00 Taler
2 balls cost 2.00 Taler
(and so on)

Apart from differences in terms of costs per ball, there is no difference between participants of type H (high cost) or type L (low costs).
When deciding how many balls you want to buy, you do not know the decision of other participants. Also, your decision is not revealed to any other participant. All balls which were bought by four participants of a group are placed into a ballot box. One ball is randomly drawn from the ballot box, and each ball is drawn with the same probability. Assume, for example, that all balls which you bought are green colored. Then, the probability that one of your balls is drawn satisfies

$$\textbf{probability(green ball is drawn)} = \frac{\textbf{\#green balls}}{\textbf{\#green balls + \#balls of other participants in your group}}.$$

where # is short for number. The same probability rule does also hold for other participants in your group. Consequently, the probability that one of your balls in drawn is higher

- the more balls you purchased
- the less balls the other participants in your group purchased.

The computer simulates the random draw of a ball. If all participant of a group of four choose to buy zero balls, each participant wins with the same probability of 25%.

Only the participant whose ball is drawn from the ballot box receives a prize of
240 Taler
in a given decision round. The other participants do not receive any prize.

Your Payoff

Assume that you bought **X** balls in some decision round. There are two possibilities for your payoff:

1) one of your balls was drawn from the ballot box

$$\begin{aligned} \textbf{Your Payoff} &= \textbf{endowment} - \textbf{X} * \textbf{your cost/ball} + \textbf{prize} \\ &= \textbf{240 Taler} - \textbf{X} * \textbf{your cost/ball} + \textbf{240 Taler} \end{aligned}$$

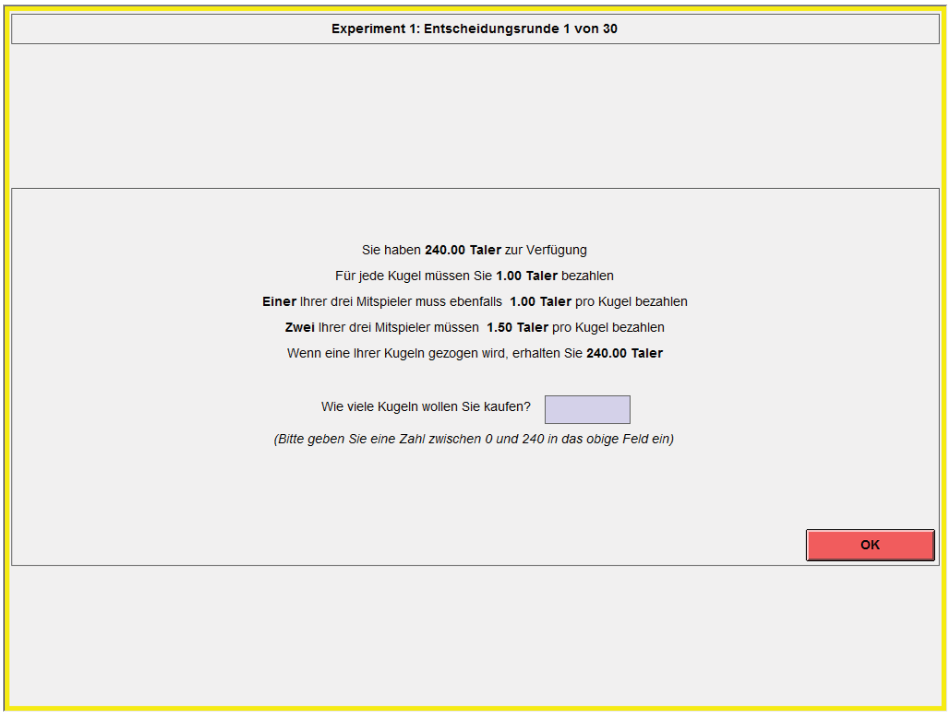
2) none of your balls was drawn from the ballot box

$$\begin{aligned} \textbf{Your Payoff} &= \textbf{endowment} - \textbf{X} * \textbf{your cost/ball} \\ &= \textbf{240 Taler} - \textbf{X} * \textbf{your cost/ball} \end{aligned}$$

Note that your cost/ball are 1.00 Taler (if you are of type L) or 1.50 Taler (if you are of type H), respectively.
Therefore, your payoff is determined by the following components: by the number of balls you buy (**X**); by your cost type (high or low); by a random draw (one of your balls is (not) drawn). The same holds for any other participants of the experiment. Note, however, that costs per ball differ between participants.

Information: You will learn your type before the first decision round starts. The information will be provided on the computer screen. Your type (cost per ball) be same in all 30 decision rounds. At the end of each decision round, you will learn whether or not one of your balls was randomly drawn and how many balls the other participants in your group bought in total. In addition, you will be informed about your payoff.

Decision: In each of the 30 decision rounds you have to decide how many balls you want to buy. You have to enter this number into the respective field on the computer screen. When making this decision, you do know your own type (high or low costs) and the type of the other participants in your group. An example of the decision screen is shown below.



Your Total Payoff: Four out of 30 decision rounds are paid. These rounds are randomly determined, i.e. the probability that some decision round is paid is identical ex-ante for all 30 decision rounds. You will receive the sum of payoffs for the respective decision rounds at the end of the experiment.

Remember:
You receive an endowment of 240 Taler at the beginning of each decision round and have to decide how many balls you want to buy. Overall, there are three additional participants in each group who face the same problem. The identity of these participants is randomly determined in each decision round. However, it always holds that equal shares of participants in a given group are of type **L (1.00 Taler per ball)** and type **H (1.50 Taler per ball)**, respectively.

If you have any questions, please raise your hand now!

4.C.3 Two-stage Treatment

Experiment 1

Overall, there are 30 decision rounds with two stages each in Experiment 1. The course of events is the same in each decision round. You will be randomly and anonymously placed into a group of **four participants** in each round, and the identity of participants in your group changes with each decision round.

Course of events in an arbitrary decision round
All four participants of each group receive an **endowment of 240 Taler** at the beginning of a decision round. The endowment can be used to buy a certain amount of balls in two subsequent stages of a decision round. It is important to note that you receive one endowment only which must suffice to buy balls in both stages. The costs for the purchase of a ball are not the same for all participants:

There are equal shares of high (H) and low (L) cost types in each group of four participants, i.e. there are two participants of each type in a group of four. All experimental participants are informed about their type at the beginning of the experiment. Types do not change with decision rounds, such that you are either a high or a low cost type in each of the 30 decision rounds. The same holds for all other participants of the experiment.

Participants of type H have to pay **1.50 Taler** for each ball they buy in stage 1 **and** stage 2, i.e.

1 ball costs 1.50 Taler
2 balls cost 3.00 Taler
(and so on)

Participants of type L have to pay **1.00 Taler** for each ball they buy in stage 1 **and** stage 2, i.e.

1 ball costs 1.00 Taler
2 balls cost 2.00 Taler
(and so on)

Apart from differences in terms of costs per ball, there is no difference between participants of type H (high cost) or type L (low costs).
When deciding how many balls you want to buy, you do not know the decision of other participants. Also, your decision is not revealed to any other participant.

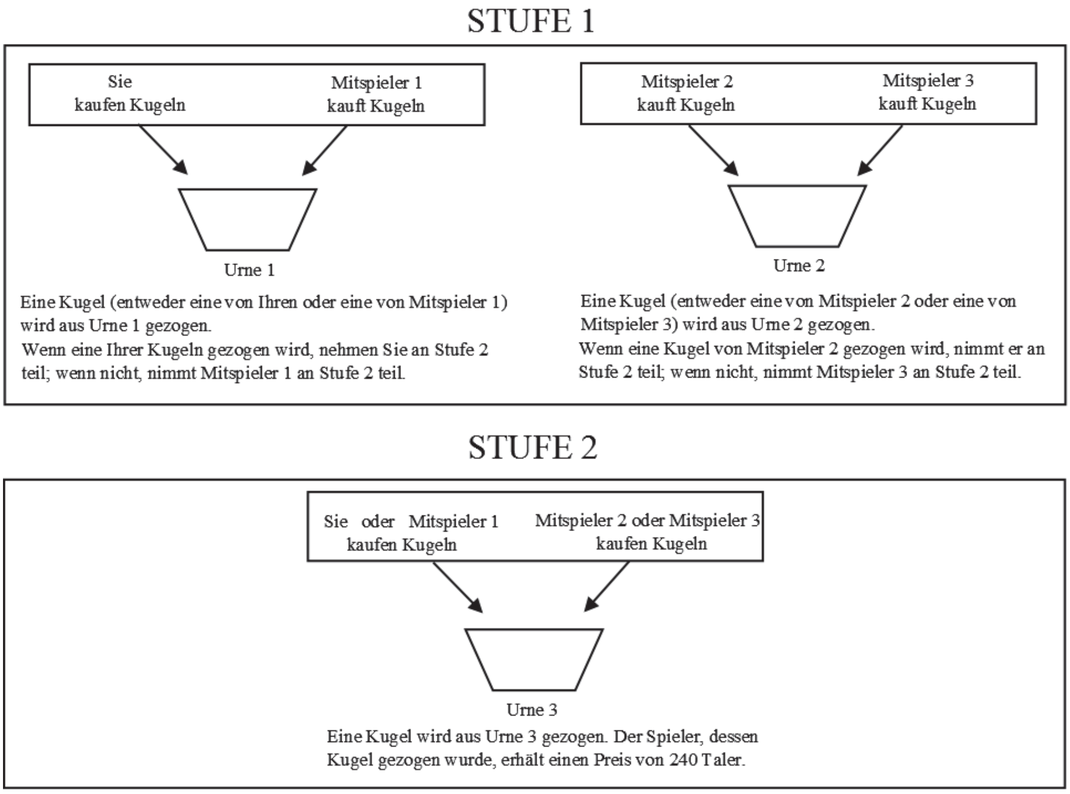
All interactions in the experiment are pairwise. Assume that you are in one group with participant A, participant B, and participant C. Then, you interact with participant A in stage 1, while participants B and C simultaneously meet each other in the second stage 1 interaction. If you reach stage 2, you will interact either with participant B or C, depending on the outcome in the second stage 1 interaction. In stage 1, there are two ballot boxes:

- all balls bought by you or participant A are placed in ballot box 1
- all balls bought by participants B and C are placed in ballot box 2

One ball is randomly drawn from each ballot box, and each ball drawn with the same probability. The two participants whose balls are drawn from ballot box 1 and 2, respectively, reach stage 2; the decision round is over for the other two participants (whose balls were not drawn), i.e. they drop out from this decision round. Any participant has to pay the balls he or she bought in stage 1, whether or not he/she reached stage 2. The respective amount is deducted from the endowment.

The two participants who reached stage 2 do again buy a certain number of balls, using whatever remains from the endowment they received after costs for balls in stage 1 were deducted. The balls are then placed into ballot box 3. One ball is randomly drawn from ballot box 3. The participant whose ball is drawn receives a prize of **240 Taler**. The other participants do not receive any prize in this decision round. Independent of whether or not a participant receives the prize, he/she does always have to pay for the balls bought in stage 2.

Jeder Spieler erhält eine Anfangsausstattung von 240 Talern. Er muss damit alle von ihm in Stufe 1 und Stufe 2 gekauften Kugeln bezahlen.



Let's take a closer look at the random draw of balls from ballot boxes. Assume, for example, that all balls which you bought are green colored, and that you interact with participant A in stage 1. Then, the probability that one of your balls is drawn (such that you make it to stage 2) satisfies

$$\textbf{probability}(\textbf{green ball is drawn}) = \frac{\textbf{\# green balls}}{\textbf{\# green balls} + \textbf{\# balls by participant A}}$$

where # is short for number. The same probability rule does also hold for other participants in your group. Consequently, the probability that one of your balls in drawn is higher

- the more balls you purchased
- the less balls the other participant with whom you interact purchased.

The computer simulates the random draw of a ball. If all participant of a group of four choose to buy zero balls, each participant wins with the same probability of 25%.

Your Payoff

Assume that you bought "**X1**" balls in stage 1, and that you buy "**X2**" balls whenever you reach stage 2. Then, there are three possibilities for your payoff:

1) None of your balls is drawn in stage 1

$$\begin{aligned} \textbf{Your Payoff} &= \textbf{endowment} \quad - \textbf{X1} * \textbf{your cost/ball} \\ &= \textbf{240 Taler} \quad - \textbf{X1} * \textbf{your cost/ball} \end{aligned}$$

2) one of your balls is drawn from the ballot box in stage 1; in stage 2, none of your balls is drawn

$$\begin{aligned} \textbf{Your Payoff} &= \textbf{endowment} \quad - \textbf{X1} * \textbf{your cost/ball} - \textbf{X2} * \textbf{your cost/ball} \\ &= \textbf{240 Taler} \quad - \textbf{X1} * \textbf{your cost/ball} - \textbf{X2} * \textbf{your cost/ball} \end{aligned}$$

3) one of your balls is drawn from the ballot box in stage 1; also, one of your balls is drawn in stage 2

$$\begin{aligned} \textbf{Your Payoff} &= \textbf{endowment} \quad - \textbf{X1} * \textbf{your cost/ball} - \textbf{X2} * \textbf{your cost/ball} + \textbf{prize} \\ &= \textbf{240 Taler} \quad - \textbf{X1} * \textbf{your cost/ball} - \textbf{X2} * \textbf{your cost/ball} + \textbf{240 Taler} \end{aligned}$$

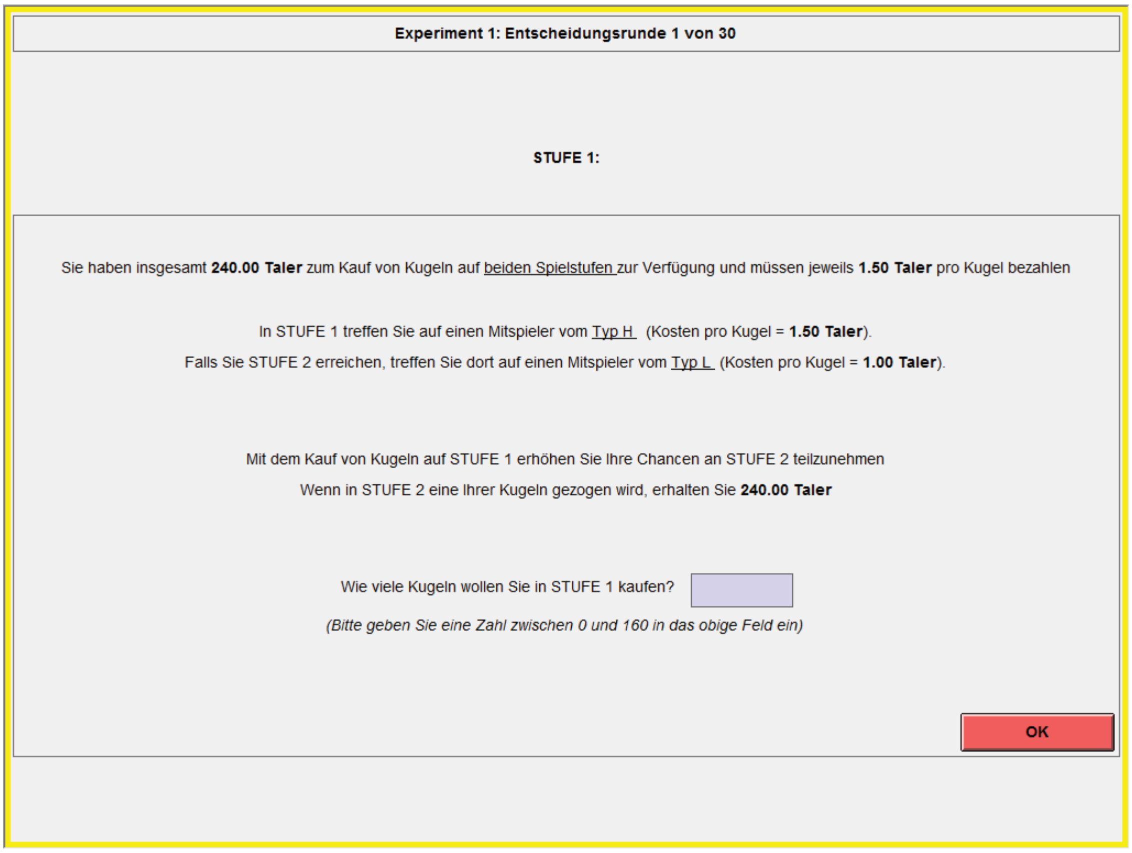
Note that your cost/ball are **1.00 Taler** (if you are of type L) or **1.50 Taler** (if you are of type H), respectively.

Therefore, your payoff is determined by the following components: by the number of balls you buy in stage 1 ("**X1**"); by the number of balls you buy in stage 2 ("**X2**") if you reach it; by your cost type (high or low); by up to two random draws (one of your balls is drawn/not drawn in stage 1 and potentially stage 2). The same holds for any other participants of the experiment. Note, however, that costs per ball differ between participants.

Information: You will learn your type before the first decision round starts. The information will be provided on the computer screen. Your type (cost per ball) will be same in all 30 decision rounds.

- Before making the first decision in stage 1, you will learn the type of participant A whom you meet in stage 1, i.e. you learn whether participant A has to pay 1.00 Taler (type L) or 1.50 Taler (type H) for each ball he/she buys.
- After you made your decision in stage 1, you are informed whether or not you can participate in stage 2, i.e. whether or not one of your balls was drawn from ballot box 1.
- If you did not reach stage 2, you are informed about how many balls participant A bought in stage 1.
- If you reach stage 2, you receive information about the remaining endowment (after costs for the purchase in stage 1 are deducted), and about the type of the other participant whom you meet in stage 2.
- After you made your decision in stage 2, you learn whether or not one of your balls was drawn from ballot box 3 and how many balls the participants who you met in stages 1 and 2, respectively, bought. Further, you learn your payoff for the respective decision round.

Decision: In each of the 30 decision rounds you have to decide how many balls you want to buy in stage 1. If you reach stage 2, you face a similar decision in stage 2. In both cases, you have to enter a number into a field on the computer screen. An example of the decision screen in stage 1 is shown below.



Your Total Payoff: Four out of 30 decision rounds are paid. These rounds are randomly determined, i.e., the probability that some decision round is paid is identical ex-ante for all 30 decision rounds. You will receive the sum of payoffs for the respective decision rounds.

Remember:
You receive an endowment of 240 Taler at the beginning of each decision round and have to decide how many balls you want to buy in stage 1; if you reach stage 2, you have to decide again. Overall, there are three additional participants in each group who face the same problem. The identity of these participants is randomly determined in each decision round. However, it always holds that equal shares of participants in a given group are of type **L (1.00 Taler per ball)** and type **H (1.50 Taler per ball)**, respectively.

If you have any questions, please raise your hand now!

Chapter 5

Incentives versus Selection in Promotion Tournaments: Is It Possible to Kill Two Birds With One Stone?

This chapter is based on joint work with Wolfgang Höchtl from the Austrian National Bank (OeNB), Rudolf Kerschbamer from the University of Innsbruck, and Uwe Sunde from the University of St. Gallen (Höchtl, Kerschbamer, Stracke, and Sunde 2011).

5.1 Introduction

Most employment relationships are characterized by competition among employees for promotion to a better paid, more attractive position. While these promotion tournaments are sometimes just a by-product of a given hierarchical structure, they often are an explicit instrument in the practice of human resource management (HRM) in professional occupations: Think of law firms or consulting firms, for example, where 'up-or-out' promotion policies are the norm and vacant manager or partner position are (almost entirely) filled with insiders. Moreover, tournaments are often used to fill top management positions. The most prominent example is certainly Jack Welch (2001), who designed the competition for his CEO succession about six years before he actually left. Several candidates from inside GE knew that they were competing against each other, and that they would either become the next CEO, or would have to leave the firm.[1] 'Up-or-out' promotion policies are also common in the competition between scientists for (rare) positions at universities: In each year, only the (relatively) best performing PhDs become assistant professors, and only the best among the assistant professors receive a tenured position subsequently, while mediocre staff members have to leave. In all these applications, tournaments are used as a means to achieve two goals: First, the prospect of moving up the ladder to higher levels within the same institution is a strong motivator for employees

[1]See also Konrad (2010) for an extensive discussion of this example.

to exert effort in their current job. Therefore, promotion tournaments help to incentivize employees.[2] Second, the selection of the most able candidate(s) is very important due to the 'up-or-out' nature of the competition. It is quite obvious that institutions intend to promote (and keep) productive employees, whereas the inferior candidates should leave.[3] This raises the question whether promotion tournaments between heterogeneous employees can be designed in such a way that they perform optimally along both dimensions. Can tournaments be used as a device to maximize the incentives for effort provision while at the same time minimizing the probability that the "wrong" contestant wins? Or, in other words, can promotion tournaments be designed in such a way that they kill two birds with one stone?

This paper provides a systematic investigation of how the two criteria 'incentive provision' and 'selection performance' are related to each other. In particular, we investigate how modifications of the tournament structure affect the two aforementioned goals. When comparing a *static* one-shot and a *dynamic* two-stage pair-wise elimination structure, our results indicate that the dynamic format performs better in terms of aggregate equilibrium efforts, while the static tournament dominates with respect to selection. Therefore, it seems that an additional hierarchy level is beneficial for incentive provision, but detrimental for the selection performance of a promotion tournament. In addition, we compare two different seeding variants of the dynamic pair-wise elimination structure, one in which similar workers compete against each other on the first stage, and one in which heterogeneous worker types compete on the first stage of the tournament. Again, one structure (the former one) performs better in terms of aggregate equilibrium effort, while the other one dominates with respect to selection. Overall, these results suggest that the two goals incentive provision and selection are incompatible. While any tournament with heterogeneous participants provides some incentives for effort and some sorting of types, modifications which improve the performance in one will deteriorate the performance in the other dimension.[4] The reason is that the structural variations considered in this paper have similar effects as strategic handicaps á la Lazear and Rosen (1981). Intuitively, tournament structures which amplify the degree of heterogeneity between strong and weak workers perform well in terms of selection, as heterogeneity discourages weak workers

[2]The tournament helps to solve a moral hazard problem. The seminal paper for this application is Lazear and Rosen (1981). Alternatively, the tournament may serve as a commitment device for the principal, see, e.g., Malcomson (1984), and Prendergast (1999) for a survey.

[3]That promotion tournaments provide employees with information about the ability of employees has previously been addressed by Rosen (1986) and Waldman (1990), for example. According to Sherwin Rosen, "the inherent logic [of promotion tournaments] is to determine the best contestants and to promote survival of the fittest" (p.701). Surprisingly, however, Rosen's seminal paper is all about optimal incentive provision across different stages of the tournament.

[4]Even though we concentrate on personnel policies, and in particular on the promotion tournament application throughout this paper, this finding is equally important in the context of rent-seeking contests. Note, however, that the interpretation is different in this case: A conflict between incentive provision and selection in a promotion context (where maximization of aggregate effort is a natural goal) translates into compatibility between the objectives in a rent-seeking contest (where effort inputs are wasteful and the usual objective is their minimization).

relatively more than it induces strong workers to slack off. At the same time, the more a tournament accommodates heterogeneity between types, the better is its performance in the incentive dimension, since heterogeneity decreases the incentives for effort provision for both strong and weak workers in absolute terms. Our results suggest that pooling of types in a simultaneous interaction tends to increase the effective degree of heterogeneity between types, since the one-stage tournament delivers the best selection performance. In contrast, the separation of employees into pair-wise interactions, optimally according to their type (similar rather than heterogeneous types compete in stage 1), reduces the effective degree of heterogeneity between types, which then boosts incentives for effort provision. From a policy perspective, this implies that multiple instruments should be used whenever both goals are equally important, i.e., a promotion tournament alone cannot serve both goals equally well. If the talent of employees is observable, HRM could organize a promotion tournament which maximizes incentives for effort provision between a preselected sample of equally talented employees, for example. Otherwise, some kind of assessment center prior to the promotion competition may serve this function, potentially a tournament that is optimized along its selection dimension.

The question whether promotion tournaments can provide both incentives and sorting was already addressed by Baker, Jensen, and Murphy (1988). However, they interpreted the sorting function in a different way. Baker et al. (1988) investigate in how far promotion tournaments ensure that employees end up in those jobs for which they are best suited, i.e., they assume that skill and human capital requirement differ qualitatively across hierarchy levels.[5] We consider situations where talents requirements are qualitatively identical across hierarchy levels: Skill requirements in law and consultancy firms, for example, do not change by much with positions. Also, top managers and CEOs perform very similar tasks, and both assistant and tenured professors teach and do research. However, the ability to perform the same task is assumed to differ across workers. Therefore, our paper is more related to work by Tsoulouhas, Knoeber, and Agrawal (2007), who study a one-stage promotion tournament where insiders and outsiders compete for a CEO position. Assuming that both the quality of the promoted agent and the provision of incentives matter for the designer, they find that the two goals are conflicting if the ability of outsiders is higher than the ability of insiders. While this result has a similar flavor as the one established in our paper, the focus of their study is different: They analyze optimal handicapping in a setting where selection involves both insiders and outsiders, but only effort provision by insiders is beneficial for the organization. In contrast, we consider within firm competition in different promotion tournament structures. This paper is also related to the contest design literature. Ryvkin and Ortmann (2008) address the selection performance of different tournament structures, but in contrast to our paper

[5] In their words, '...talents for the next level in the hierarchy are not perfectly correlated with talents to be the best performer in the current job' (p. 602). The best salesman, for example, can be a bad manager, which leads to the so-called Peter Principle. See also Prendergast (1993) and Bernhardt (1995), who also consider the matching performance of promotion tournaments.

they discard the effect of this variation on incentives.[6] Groh, Moldovanu, Sela, and Sunde (2012) show that the design of dynamic tournaments can involve a trade-off between incentive provision and selection performance, but they focus on different seedings in a dynamic tournament, while we also investigate how dynamic tournaments relate to static ones when participants are heterogeneous.[7] So far, static and dynamic contests have only been compared in the case of homogeneous participants, see, e.g., Gradstein and Konrad (1999).

The remainder of this paper is structured as follows. The next section introduces the formal model and compares the equilibrium measures for incentive provision and selection performance of different tournament formats. Section 5.3 discusses the intuition for and the implications of our results, and section 5.4 concludes.

5.2 The Model

5.2.1 A Promotion Tournament with Heterogeneous Workers

Consider an institution who uses a promotion tournament to fill some vacant higher-level position that is of value P to employees from lower ranks.[8] For simplicity, assume that four risk neutral workers from the same company compete for the open position on the internal labor market, i.e., while working on their actual position, they are evaluated relative to their colleagues, and the employee with the best performance is promoted at the end of the evaluation period. Workers know that they are being evaluated, and in particular, they are perfectly informed about both their own productivity and the productivity of their colleagues.[9] To keep the theoretical analysis tractable, we assume that workers are of two different types: Equal shares are highly productive ("strong") and less productive ("weak"), respectively. Each worker provides effort to increase his/her chances for a promotion. The organizing entity of the promotion tournament, in short the principal, cannot directly observe individual efforts, but receives a noisy ordinal performance signal instead. As a result, the promotion probability p_i for some arbitrary worker i in a tournament interaction

[6] Brown and Minor (2011) empirically test the selection performance of two-stage pairwise elimination tournaments, which we analyze theoretically.

[7] Another (technical) difference is that they use a perfectly discriminating all-pay-auction framework, while the analysis in this paper uses a standard Tullock contest success function instead.

[8] The value of being promoted may include both monetary components (promotions imply higher wages) and non-monetary aspects (e.g., concerns for status or power). Higher wages may either be chosen by the tournament designing organization, or they may be the result of a signalling value and competition between organizations, i.e., our modelling approach is consistent both with the concept of classic promotion tournaments á la Lazear and Rosen (1981) and market-based tournaments in the spirit of Waldman (1984). For a recent comparison of these two concepts, see Waldman (2011).

[9] This assumption may be problematic in some settings, for example in assessment centers. Note, however, that this paper focusses on within company promotion tournaments. In most professional occupations, the first promotion possibility for new hires is after one or two years. Therefore, workers who compete on the internal labor market for open positions usually know each other due to ongoing interactions in the workplace. The promotion tournament for the succession of Jack Welch, for example, which is presented in the Introduction, lasted six years.

is given by the ratio of own effort x_i over effort provided by the immediate competitor(s), X. Formally, the probability is defined as

$$p_i(x_i, X) = \begin{cases} \frac{x_i}{x_i + X} & \text{if} \quad x_i, X > 0 \\ \frac{1}{\#N} & \text{if} \quad x_i, X = 0 \end{cases}, \tag{5.1}$$

where $\#N$ is the number of workers participating in the tournament interaction. While the promotion probability of a worker is clearly increasing in own effort provision, and decreasing in the effort provided by the immediate opponent(s), the chosen formulation implies that the worker with the highest effort does not always win, i.e., effort does not translate directly into performance. The reason is that the performance signal is distorted by random noise.[10] The principal pursues the following two objectives:

1. Maximize aggregate effort by all workers (Incentive Provision).

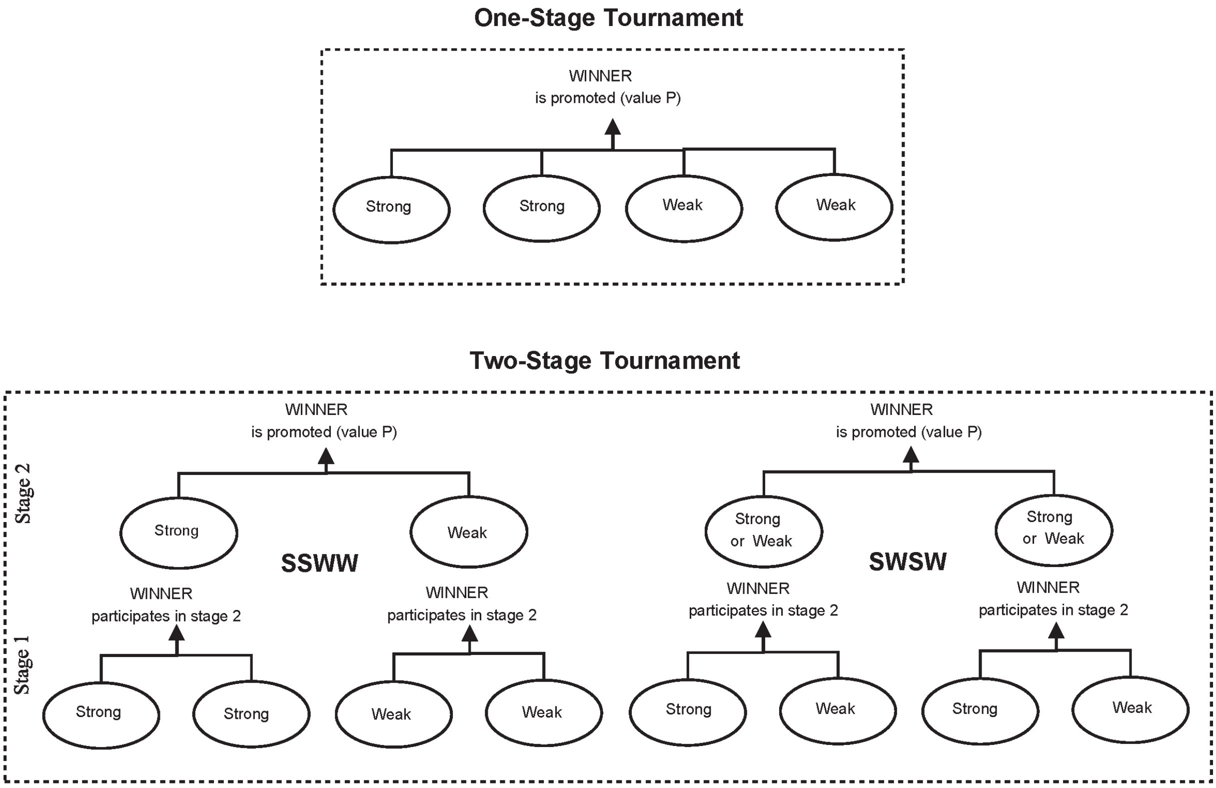
2. Maximize the probability that a strong, productive worker wins (Selection).

Work effort by employees determines output and profits of corporations. Since effort is often costly for the workers and non-contractible at the same time, explicit incentives for effort provision are needed. Therefore, the provision of incentives is an important goal for any corporation, and the prospect of being promoted to a better paid, more attractive position can be used to motivate and incentive workers. Selection performance is usually equally important, however, since the 'up or out' character of the competition implies that only promoted employees stay within the corporation. As able workers are certainly better suited for positions with more responsibilities, institutions intend to promote (and keep) productive employees. This does even hold if the losers of the promotion competition are allowed to stay, since they are certainly discouraged, such that many of them will apply at different companies, i.e., they will leave voluntarily. Thus, promoting the "wrong" worker is costly, and avoiding this cost by implementing a tournament format with optimal selection properties is a natural second objective.

The goal of our analysis is to find out whether the careful design of structural parameters by the principal can ensure that the promotion tournament performs well in both performance dimensions. In particular, we make two comparisons. First, we compare incentive and selection properties of a static (one-stage) tournament and a dynamic (two-stage) pairwise elimination tournament, i.e., we determine the effect pair-wise sequential rather than joint simultaneous performance evaluations by the principal on incentive provision and selection performance in the promotion tournament. The two different tournament formats are depicted in Figure 5.1, which also shows that two different constellations are possible in the dynamic specification: Either a strong worker competes against another strong worker (and a weak worker against another weak worker) in the parallel

[10] We use a so-called Tullock (1980) contest success function (CSF) with discriminatory power one, which can be transformed into an all-pay auction contest success function with multiplicative noise that follows the exponential distribution. See Konrad (2009) for details (p.52f).

Figure 5.1: Design Options Available to the Tournament Designer



stage-1 interactions (setting SSWW); or both stage-1 interactions are mixed in terms of the productivity of the competing workers (setting SWSW). In the comparison of the static and the dynamic tournament format, we assume that workers' types are not observable. Therefore, the seeding in stage 1 is random if the principal decides in favor of the dynamic format; setting SSWW occurs with probability 1/3, setting SWSW with probability 2/3.[11] Second, we compare the performance of settings SSWW and SWSW with respect to incentives and selection. Even though the principal needs to know the worker's types for this structural variation, the selection performance of the promotion tournament is still important, since tournaments have a commitment property. Therefore, the winner of a tournament must be promoted, independent of his/her type. One might certainly argue that a promotion tournament is not the optimal mechanism to assure incentive provision and selection if types are known. However, our approach is positive rather than normative; it is a well known fact that promotion tournaments are widely used, even though theory sometimes suggests other mechanisms.[12] Alternatively, one may also argue that

[11] After the first worker has been chosen randomly from the pool of four workers, the probability that the next worker drawn from the pool of the remaining three workers is of the same type is $1/3$ (since only one of the remaining workers is of the same type), while the probability that the next worker is of the other type is $2/3$ (because two of the three remaining workers are of the other type).

[12] See Baker, Jensen, and Murphy (1988), or Gibbs and Hendricks (2004), for example.

the principal's belief about workers' types is distorted, such that tournament outcomes provide a signal to update this beliefs.

5.2.2 Equilibrium Behavior by Workers

Workers face a trade-off in the one-stage, as well as in each pair-wise interaction of the two-stage tournament: Ceteris paribus, increasing own effort leads to higher costs as well as to a higher probability of winning. In equilibrium, workers choose their efforts such that the marginal cost of effort provision equals the expected marginal gain in terms of a higher probability of being promoted. To introduce heterogeneity, the effort costs of strong workers, $c_{\mathtt{S}}$, are assumed to be lower than the effort costs $c_{\mathtt{W}}$ of weak workers ($c_{\mathtt{S}} \leq c_{\mathtt{W}}$). Intuitively, effort costs are used as an inverse measure for ability.[13]

5.2.2.1 One-Stage Tournament

The one-stage tournament model we consider, denoted $\mathtt{I}$ in the sequel, is a special case of the model developed by (and extensively discussed in) Stein (2002). It is a simultaneous move game, the natural solution concept is therefore Nash Equilibrium (NE). In a NE, each worker i with constant marginal effort costs c_i maximizes his expected payoff $\Pi_i(\mathtt{I})$ by choosing optimal effort $x_i \geq 0$, taking the total effort of all other workers X as given. Formally, the optimization problem of worker i reads as follows:

$$\max_{x_i \geq 0} \Pi_i(x_i, X) = \frac{x_i}{x_i + X} P - c_i x_i.$$

The formal expressions for individual equilibrium efforts of strong and weak workers, $x^*_{\mathtt{S}}(\mathtt{I})$ and $x^*_{\mathtt{W}}(\mathtt{I})$, respectively, are provided in equation (5.11) in the Appendix. Equilibrium efforts determine both the incentive provision and the selection performance of the promotion tournament. Our measure for the incentive provision performance in the one-stage tournament, denoted $\mathcal{E}(\mathtt{I})$, is defined as the sum of individual equilibrium efforts. Since two workers are strong and weak, respectively, we obtain

$$\mathcal{E}(\mathtt{I}) = 2x^*_{\mathtt{S}} + 2x^*_{\mathtt{W}}. \tag{5.2}$$

While the incentive provision measure depends on the absolute value of equilibrium efforts, winning probabilities depend on the ratio of $x^*_{\mathtt{S}}(\mathtt{I})$ and $x^*_{\mathtt{W}}(\mathtt{I})$. To determine the selection performance $\mathcal{S}(\mathtt{I})$, i.e., the probability that a strong worker wins, the equilibrium winning probability of a strong worker must be multiplied by two, since two strong workers participate in the promotion tournament. Thus,

$$\mathcal{S}(\mathtt{I}) = \frac{2x^*_{\mathtt{S}}}{x^*_{\mathtt{S}} + x^*_{\mathtt{W}}}. \tag{5.3}$$

[13] Modeling heterogeneity in terms of effort cost is without loss of generality. Proofs are available from the authors upon request.

5.2.2.2 Two-Stage Tournament

Subgame Perfect Nash Equilibrium is the relevant solution concept for the two-stage tournament, since this structure is a sequential game. Therefore, the equilibrium is obtained through backward induction. First, all possible stage-2 interactions that occur in setting SSWW or SWSW, respectively, must be solved. With four workers of two types, there are three potential stage-2 games, namely SS (both workers are strong), WW (both workers are weak), or SW (one strong and one weak worker). The formal optimization problem of some worker i with effort cost c_i who competes with worker j in stage 2 reads

$$\max_{x_{i2} \geq 0} \Pi_{i2}(x_{i2}, x_{j2}) = \frac{x_{i2}}{x_{i2} + x_{j2}} P - c_i x_{i2},$$

where x_{i2} and x_{j2} are individual efforts by workers i and j, respectively. A detailed solution of all stage-2 games is provided in the Appendix. Note, however, that the equilibrium effort of each worker depends both on his own and on the type of the opponent: $x^*_{\mathtt{S}2}(\mathtt{SS})$ is the equilibrium effort of a strong worker in interaction SS, $x^*_{\mathtt{W}2}(\mathtt{WW})$ the optimal choice of weak workers in interaction WW, while $x^*_{\mathtt{S}2}(\mathtt{SW})$ and $x^*_{\mathtt{W}2}(\mathtt{SW})$ are the equilibrium efforts of strong and weak workers, respectively, in the mixed stage-2 configuration SW.[14] Since stage-2 equilibrium efforts solve the last stage of the game, we can move forward to stage 1.

Setting SSWW. The stage-1 interactions in setting SSWW ensure, as Figure 5.1 shows, that one strong and one weak worker reach stage 2 with certainty.[15] Consequently, SW is the only possible constellation on stage 2. This implies that both strong workers know that, conditional on reaching stage 2, a weak worker will be the opponent, while weak workers anticipate that they will interact with a strong worker if they reach stage 2. The only reward for winning stage 1 is the participation in stage 2, in which workers may then receive the promotion of value P. Thus, the expected equilibrium payoffs of stage-2 interaction SW for strong and weak workers, $\Pi^*_{\mathtt{S}2}(\mathtt{SW})$ and $\Pi^*_{\mathtt{W}2}(\mathtt{SW})$, respectively, determine the continuation values for which workers compete in stage 1. This becomes clear when considering the optimization problem of some strong worker i, who competes with the second strong worker j:

$$\max_{x_{i1} \geq 0} \Pi_i(\mathtt{SSWW}) = \frac{x_{i1}}{x_{i1} + x_{j1}} \Pi^*_{\mathtt{S}2}(\mathtt{SW}) - c_{\mathtt{S}} x_{i1}.$$

Worker i chooses stage-1 effort x_{i1} to increase the probability to participate in stage 2, which is worth $\Pi^*_{\mathtt{S}2}(\mathtt{SW})$ in equilibrium. Similarly, the two weak workers compete for participation in stage 2, which is worth $\Pi^*_{\mathtt{W}2}(\mathtt{SW})$ for them. Let $x^*_{\mathtt{S}1}(\mathtt{SSWW})$ and $x^*_{\mathtt{W}1}(\mathtt{SSWW})$ be the stage-1 equilibrium efforts in setting SSWW by strong and weak workers, respectively,

[14] For formal expressions of equilibrium efforts, see equations (5.12), (5.14), and (5.16), respectively.
[15] Note that Stein and Rapoport (2004) considers a very similar model.

which are determined in the Appendix.[16] Then, the incentive measure in setting SSWW of the two-stage tournament format, denoted $\mathcal{E}(\texttt{SSWW})$, is defined as follows:

$$\mathcal{E}(\texttt{SSWW}) = \underbrace{2[x^*_{\texttt{S}1}(\texttt{SSWW}) + x^*_{\texttt{W}1}(\texttt{SSWW})]}_{\text{stage 1 effort}} + \underbrace{x^*_{\texttt{S}2}(\texttt{SW}) + x^*_{\texttt{W}2}(\texttt{SW})}_{\text{stage 2 effort}} . \tag{5.4}$$

Total effort provision $\mathcal{E}(\texttt{SSWW})$ amounts to individual efforts by two strong and two weak workers in stage 1, and one strong and one weak worker in stage 2. The selection measure, i.e., the probability that a strong worker receives the promotion, is determined by relative effort provision of the stage-2 participants. As mentioned previously, one strong and one weak worker compete in stage 2, independent of stage-1 outcomes. Therefore, the selection measure $\mathcal{S}(\texttt{SSWW})$ depends on the ratio of stage-2 equilibrium efforts $x^*_{\texttt{S}2}(\texttt{SW})$ and $x^*_{\texttt{W}2}(\texttt{SW})$:

$$\mathcal{S}(\texttt{SSWW}) = \frac{x^*_{\texttt{S}2}(\texttt{SW})}{x^*_{\texttt{S}2}(\texttt{SW}) + x^*_{\texttt{W}2}(\texttt{SW})} . \tag{5.5}$$

Setting SWSW. Since both stage-1 interactions are mixed in setting SWSW, the type configuration in stage 2 is uncertain; any one of the three stage-2 games SS, WW, and SW is possible, as Figure 5.1 clearly shows. As a consequence, the solution of this setting is complicated by the fact that stage-1 continuation values are endogenously determined.[17] To illustrate this complication, assume that some strong worker i and an arbitrary weak worker j compete for the right to participate in stage 2. Simultaneously, strong worker k and weak worker l compete for the remaining stage-2 slot in the other stage-1 interaction. Then, the formal optimization problems of workers i and j are as follows:

$$\begin{aligned}
\max_{x_{i1}\geq 0} \Pi_i(\texttt{SWSW}) &= \frac{x_{i1}}{x_{i1}+x_{j1}} \underbrace{\left[\frac{x_{k1}}{x_{k1}+x_{l1}}\pi^*_{\texttt{S}2}(\texttt{SS}) + \frac{x_{l1}}{x_{k1}+x_{l1}}\pi^*_{\texttt{S}2}(\texttt{SW})\right]}_{\equiv P_i(x_{k1},x_{l1})} - c_{\texttt{S}} x_{i1} \\
\max_{x_{j1}\geq 0} \Pi_j(\texttt{SWSW}) &= \frac{x_{j1}}{x_{i1}+x_{j1}} \underbrace{\left[\frac{x_{k1}}{x_{k1}+x_{l1}}\pi^*_{\texttt{W}2}(\texttt{SW}) + \frac{x_{l1}}{x_{k1}+x_{l1}}\pi^*_{\texttt{W}2}(\texttt{WW})\right]}_{\equiv P_j(x_{k1},x_{l1})} - c_{\texttt{W}} x_{j1} .
\end{aligned}$$

[16] Formal expressions for equilibrium efforts are provided in equations (5.19) and (5.20).

[17] In contrast to the one-stage tournament and setting SSWW, the analysis of setting SWSW is novel; a closed-form solution of this model has not been presented in the existing literature. Note, however, that Groh, Moldovanu, Sela, and Sunde (2012) derive a closed-form solution for this setting in an all-pay auction framework, i.e., for the case of a perfectly discriminating contest-success function where the ordinal signal which the principal receives is not distorted by random noise. Rosen (1986) considers exactly the same specification as analyzed here, but provides only numerical simulations and then conjectures (without analytical proof) that certain properties of numerical simulations should hold in general. Finally, Harbaugh and Klumpp (2005) solve a very similar model, but make use of the simplifying assumption that total effort by each participant over both stages of the contest is equal to some constant. In other words, they derive optimal behavior when strong and weak participants face the same binding effort endowment (which has no intrinsic value), and then discuss how the endowment is distributed across the two stages; the effort choice is unrestricted in our model.

Interestingly, the continuation values $P_i(x_{k1}, x_{l1})$ and $P_j(x_{k1}, x_{l1})$ of workers i and j, respectively, depend on the behavior of workers k and l in the other stage-1 interaction. The reason is that expected equilibrium payoffs for workers differ across the three potential stage-2 interactions SS, WW, and SW.[18] Intuitively, the same holds for the continuation values $P_k(x_{i1}, x_{j1})$ and $P_l(x_{i1}, x_{j1})$ of workers k and l in the second stage-1 interaction. Thus, the two heterogeneous stage-1 interactions are linked through endogenously determined continuation values. This interesting technical complication is relegated to the Appendix, which also provides closed-form solutions for stage-1 equilibrium efforts $x^*_{\mathtt{S}1}(\mathtt{SWSW})$ and $x^*_{\mathtt{W}1}(\mathtt{SWSW})$ of strong and weak workers, respectively.[19] Using individual equilibrium efforts, we can compute aggregate effort provision, i.e., the incentive measure $\mathcal{E}(\mathtt{SWSW})$, as follows:

$$\mathcal{E}(\mathtt{SWSW}) = \underbrace{2\{\pi^2 x^*_{\mathtt{S}2}(\mathtt{SS}) + (1-\pi)^2 x^*_{\mathtt{W}2}(\mathtt{WW}) + \pi(1-\pi)[x^*_{\mathtt{S}2}(\mathtt{SW}) + x^*_{\mathtt{W}2}(\mathtt{SW})]\}}_{\text{stage 2 effort}} + \underbrace{2[x^*_{\mathtt{S}1}(\mathtt{SWSW}) + x^*_{\mathtt{W}1}(\mathtt{SWSW})]}_{\text{stage 1 effort}}, \tag{5.6}$$

where $\pi = \frac{x^*_{\mathtt{S}1}(\cdot)}{x^*_{\mathtt{S}1}(\cdot)+x^*_{\mathtt{W}1}(\cdot)}$ is the probability that a strong worker wins against the weak opponent in stage 1; this probability determines the likelihood for a particular stage-2 configuration: The stage-2 participants are both strong with probability π^2, both weak with probability $(1-\pi)^2$, or of different types with probability $2\pi(1-\pi)$. The probability π that a strong worker wins in stage 1 is also relevant for the selection performance of setting SWSW, measured by $\mathcal{S}(\mathtt{SWSW})$. It is defined as

$$\mathcal{S}(\mathtt{SWSW}) = \pi^2 + 2\pi(1-\pi)\frac{x^*_{\mathtt{S}2}(\mathtt{SW})}{x^*_{\mathtt{W}2}(\mathtt{SW}) + x^*_{\mathtt{S}2}(\mathtt{SW})}. \tag{5.7}$$

Intuitively, a strong worker is promoted if either both strong workers win their stage-1 interactions, which happens with probability π^2, or if only one strong worker wins in stage 1, and subsequently also in stage 2.

Random Seeding. If the principal decides in favor of the dynamic format and seeding of types in stage 1 is random, setting SSWW occurs with probability 1/3; the probability of the complementary event that setting SWSW realizes is 2/3. Consequently, the expected incentive provision measure for the two-stage promotion tournament, denoted $\mathcal{E}(\mathtt{II})$, is a weighted average of total effort provision in the two settings. Formally,

$$\mathcal{E}(\mathtt{II}) = \frac{\mathcal{E}(\mathtt{SSWW}) + 2\cdot\mathcal{E}(\mathtt{SWSW})}{3}. \tag{5.8}$$

[18]Conditional on reaching stage 2, workers of both types have a higher expected payoff from meeting a weak rather than a strong opponent, since $\Pi^*_{\mathtt{W}2}(\mathtt{WW}) > \Pi^*_{\mathtt{W}2}(\mathtt{SW})$ and $\Pi^*_{\mathtt{S}2}(\mathtt{SW}) > \Pi^*_{\mathtt{S}2}(\mathtt{SS})$. Details are provided in the Appendix.

[19]See (5.24) and (5.25) for details.

Conceptually, the same holds for the selection measure $\mathcal{S}(\texttt{II})$, which is a weighted average of $\mathcal{S}(\texttt{SSWW})$ and $\mathcal{S}(\texttt{SWSW})$. Formally,

$$\mathcal{S}(\texttt{II}) = \frac{\mathcal{S}(\texttt{SSWW}) + 2 \cdot \mathcal{S}(\texttt{SWSW})}{3} \, . \tag{5.9}$$

5.2.3 Designing the Promotion Tournament

Using results on equilibrium behavior of workers in the different promotion tournament specifications, we will now investigate how structural modifications by the principal affect incentive provision and selection performance. We start with the comparison of one- and two-stage tournaments (I versus II), before the two possible settings of the two-stage tournament, SSWW and SWSW are compared.

One-Stage vs. Two-Stages. Incentive provision and selection performance are identical in the one-stage and the two-stage tournament if all workers are of the same type. The equality in terms of selection performance follows automatically from the homogeneity assumption: Either, all workers are weak, and the probability that a strong worker wins is always zero, or, all workers are strong, and the probability that a strong worker wins must be one. That the tournament structure does not affect aggregate effort provision in the homogeneous case is less obvious. However, as Gradstein and Konrad (1999) established, this holds for the specification we consider.[20] Consequently, the comparison of these two structures also allows us to investigate whether heterogeneity differently affects workers' behavior in one-stage and two-stage tournaments. A formal comparison of the incentive measures $\mathcal{E}(\texttt{I})$ and $\mathcal{E}(\texttt{II})$, and the selection measures $\mathcal{S}(\texttt{I})$ and $\mathcal{S}(\texttt{II})$, respectively, delivers the following Proposition:

Proposition 5.1 (I vs. II). *When the cost of effort is strictly higher for weak than for strong agents ($c_W > c_S$),*

(a) aggregate effort is strictly higher in the two- than in one-stage tournament, i.e.,

$$\mathcal{E}(I) < \mathcal{E}(II) \quad \text{for all} \quad c_W > c_S.$$

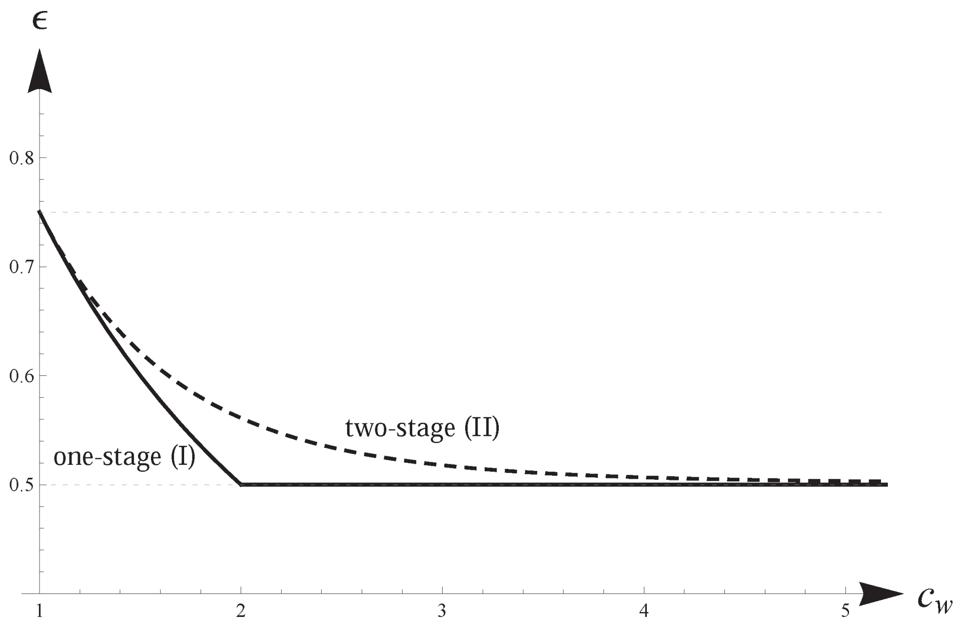
(b) the probability that a strong agent receives the promotion is strictly higher in the one- than in the two-stage tournament, i.e.,

$$\mathcal{S}(I) > \mathcal{S}(II) \quad \text{for all} \quad c_W > c_S.$$

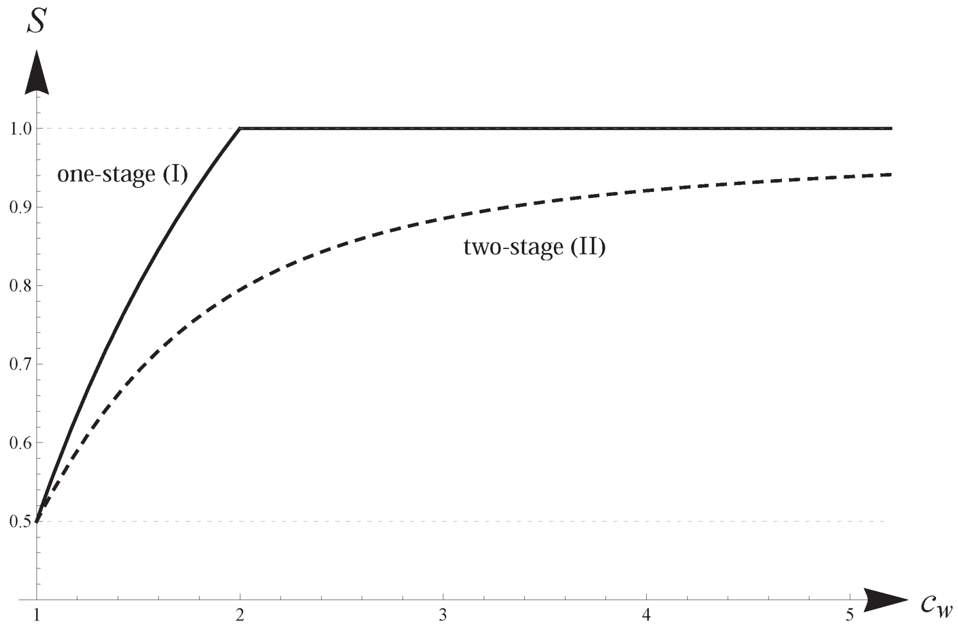
Proof. See Appendix. □

[20] An intuition for this result is provided by Amegashie (2000).

Figure 5.2: Performance in One-Stage and Two-Stage Tournaments



(a) Incentive Provision



(b) Selection

Notes: Panel (a) plots expressions (5.2) and (5.8) with $c_S = 1$ and $P = 1$; panel (b) plots (5.5) and (5.7) under the same assumption.

Panel (a) of Figure 5.2 plots the incentive measures of both tournament formats, $\mathcal{E}(\mathtt{I})$ and $\mathcal{E}(\mathtt{II})$, as a function of the effort costs of weak workers, $c_{\mathtt{W}}$; effort costs for strong types and the value of the promotion are normalized to one, i.e., $c_{\mathtt{S}} = 1$ and $P = 1$. The figure shows that the dotted line for aggregate effort provision in the two-stage tournament is always above the solid line for overall effort provision in the one-stage tournament, as part (a) of Proposition 5.1 suggests. The difference is highest at the kink of the one-stage incentive measure for $c_{\mathtt{W}} = 2$ (where weak workers drop-out voluntarily, see Appendix 5.4 for details), and decreases subsequently. For extremely high values of $c_{\mathtt{W}}$, aggregate effort provision approaches 0.5 in both tournament formats.

The selection performance of both tournament formats is illustrated in panel (b) of Figure 5.2, which plots $\mathcal{S}(\mathtt{I})$ and $\mathcal{S}(\mathtt{II})$, i.e., the probability that a strong worker wins.[21] The figure shows that the one-stage dominates the two-stage tournament in terms of its selection performance; the probability that a strong worker is promoted is strictly higher in I than in II when workers are heterogeneous, i.e., if $c_{\mathtt{W}} > 1$, as suggested by part (b) of Proposition 5.1. This difference is particularly pronounced for relatively low degrees of heterogeneity, since the curve of $\mathcal{S}(\mathtt{I})$ is much steeper initially than the one for $\mathcal{S}(\mathtt{II})$. Only after the kink of the one-stage selection measure at $c_{\mathtt{W}} = 2$ the difference is reduced. However, even if the costs of effort are five times as high for weak than for strong workers, the probability that a strong worker wins is still almost ten percentage points higher in the one-stage than in the two-stage tournament; only when $c_{\mathtt{W}} \to \infty$, both $\mathcal{S}(\mathtt{I})$ and $\mathcal{S}(\mathtt{II})$ approach one.[22]

Setting SSWW vs. Setting SWSW. As in the previous comparison, both the incentive provision and the selection performance are identical in settings SSWW and SWSW if workers are homogeneous. Intuitively, it does not matter how types are seeded if they are all equally talented. We start with a formal comparison of the incentive measures $\mathcal{E}(\mathtt{SSWW})$ and $\mathcal{E}(\mathtt{SWSW})$, and the selection measures $\mathcal{S}(\mathtt{SSWW})$ and $\mathcal{S}(\mathtt{SWSW})$, respectively. The results are summarized in the following Proposition:

Proposition 5.2 (SSWW vs. SWSW). *When the cost of effort is strictly higher for weak than for strong agents ($c_W > c_S$),*

(a) aggregate effort is strictly higher in setting SSWW than in setting SWSW, i.e.,

$$\mathcal{E}(SSWW) > \mathcal{E}(SWSW) \quad \text{for all} \quad c_W > c_S.$$

(b) the probability that a strong agent receives the promotion is strictly higher in setting SWSW than in setting SSWW, i.e.,

$$\mathcal{S}(SSWW) < \mathcal{S}(SWSW) \quad \text{for all} \quad c_W > c_S.$$

[21] The dependent variable is again $c_{\mathtt{W}}$, and effort costs of strong workers are again normalized to one.
[22] Not visible in panel (b) of Figure 5.2, but easy to show formally. Details available upon request.

Proof. See Appendix. □

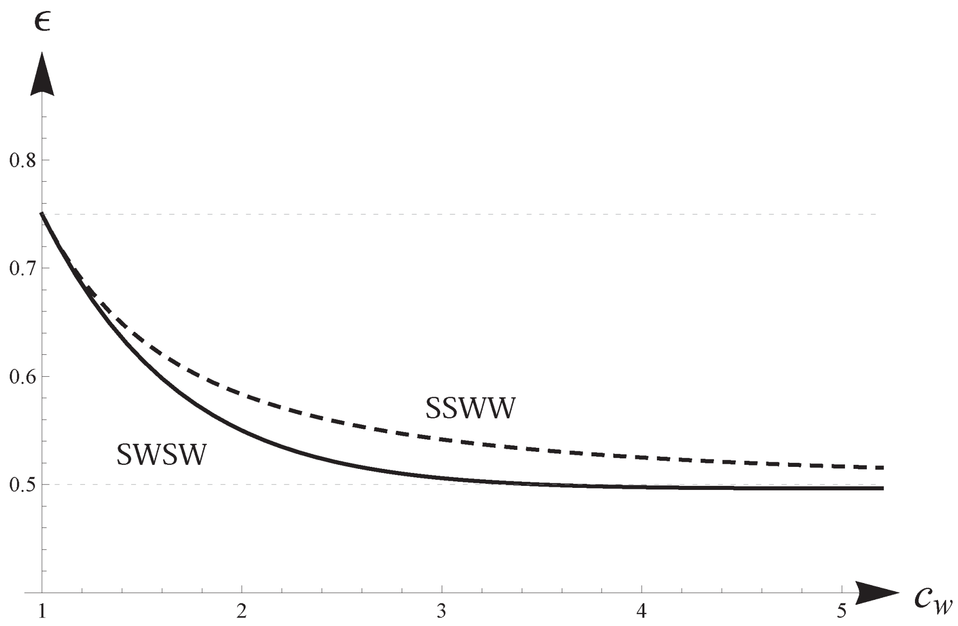
Panel (a) of Figure 5.3 graphically illustrates part (a) of Proposition 5.2 by plotting aggregate (expected) effort, i.e., $\mathcal{E}(\texttt{SSWW})$ and $\mathcal{E}(\texttt{SWSW})$, as a function of the constant marginal cost of effort for the weak workers, $c_{\texttt{W}}$; the effort cost of the strong type and the value of a promotion are normalized to one ($c_{\texttt{S}} = 1$ and $P = 1$). The figure shows that incentives for effort provision are strictly higher in setting SSWW than in setting SWSW. Moreover, one can see that the difference between the two settings is most pronounced for intermediate values of $c_{\texttt{W}}$, since aggregate effort provision in both settings converges towards that of a tournament with strong workers only if $c_{\texttt{W}} \to c_{\texttt{S}}$, while both measures approach 0.5 if $c_{\texttt{W}} \to \infty$. The selection measures $\mathcal{S}(\texttt{SSWW})$ and $\mathcal{S}(\texttt{SWSW})$ are plotted as a function of $c_{\texttt{W}}$ in panel (b) Figure 5.3. As in previous figures, the effort cost of the strong types are normalized to one. The promotion probability for strong workers is clearly higher in setting SWSW than in setting SSWW, as established in part (b) of Proposition 5.2. The difference between the two settings is sizable, both for low and for comparably high degrees of heterogeneity. Only for the limiting case $\frac{c_{\texttt{W}}}{c_{\texttt{S}}} \to \infty$, the selection performance of both settings becomes identical and converges to one.[23] This convergence is much faster in setting SWSW than in SSWW, however.

5.3 Discussion of Results

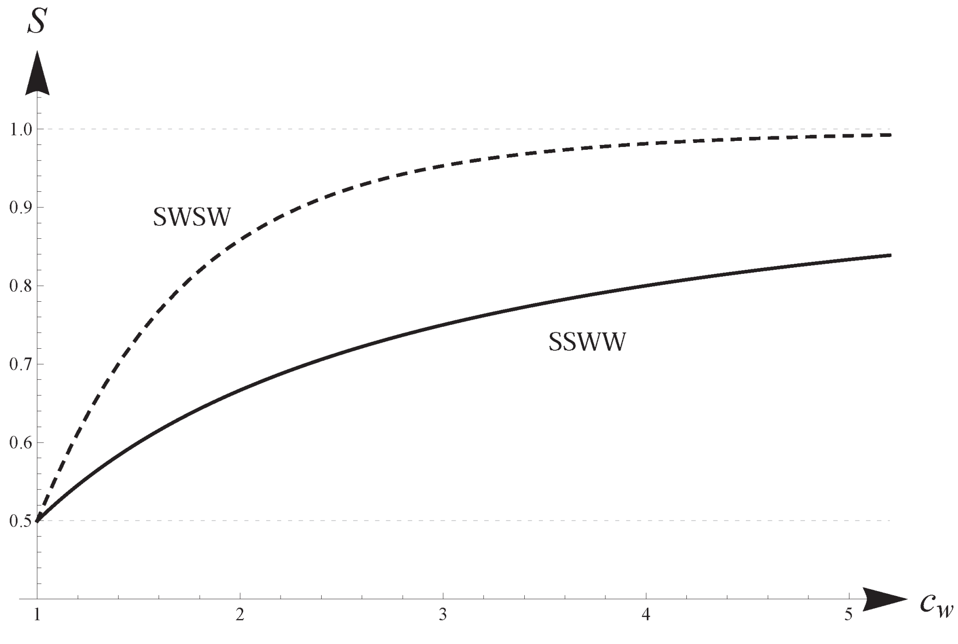
The previous analysis has shown that no structure is optimal both with respect to incentive provision and selection performance. Clearly, any tournament structure provides some incentives for effort and some sorting of types. However, it was shown that modifications which improve the performance in one deteriorate the performance in the other dimension: The two-stage tournament with random seeding dominates the one-stage format in terms of incentive provision, whereas the opposite holds for selection performance (Proposition 5.1). Similarly, the incentive provision properties of setting SSWW are better than they are in setting SWSW; yet, setting SWSW dominates with respect to selection performance (Proposition 5.2). Taken together, these results suggest that the two objectives incentive provision and selection are incompatible. To explain this finding, one has to distinguish *absolute* and *relative* incentives for effort provision. First, note that the ratio of the workers' efforts determines the selection performance: The lower equilibrium efforts of weak workers are *relative* to equilibrium efforts of strong workers, the better is the selection performance of a tournament. In other words, the selection performance is increasing in the degree of heterogeneity between workers, which is also graphically illustrated in panel (b) of Figures 5.2 and 5.3. The higher the effort costs of weak workers are relative to effort costs of strong workers (which are normalized to one), the better does selection work in any tournament format. Second, one should keep in mind that *absolute* incentives

[23]Details of the proof for this claim are available from the authors upon request.

Figure 5.3: Performance in Two-Stage Tournaments by Setting



(a) Incentive Provision



(b) Selection

Notes: Panel (a) plots expressions (5.4) and (5.6), assuming that $c_S = 1$ and $P = 1$; panel (b) plots expressions (5.5) and (5.7) under the same assumption.

for effort provision determine total effort. It is well known that heterogeneity reduces the incentives for effort provision in tournaments; therefore, *absolute* incentives, i.e., the sum of workers' efforts, are decreasing in the degree of heterogeneity. In all tournament formats considered in this paper, total effort provision is lower the higher heterogeneity between types, i.e., the higher the effort costs of weak workers are relative to effort costs of strong workers, as panel (a) of Figures 5.2 and 5.3, respectively, shows. Consequently, tournament structures which amplify the degree of heterogeneity between strong and weak workers perform well in terms of selection, as heterogeneity discourages weak workers relatively more than it induces strong workers to slack off. At the same time, the more a tournament accommodates heterogeneity between types, the better is its performance in the incentive dimension, since heterogeneity decreases the incentives for effort provision for both strong and weak workers in absolute terms.

Essentially, the formal analysis has shown that structural variations of tournaments with heterogeneous workers have similar effects as strategic handicaps. Lazear and Rosen (1981) showed that incentives for effort provision are maximized if strong participants of a tournament are handicapped in such a way that equilibrium winning probabilities are equalized across types. This result already indicated a conflict between the two goals incentive provision and selection performance. This paper shows that structural variation cannot solve this problem. If, for example, a one-stage rather than a two-stage tournament is used to fill a vacancy, this modification works like an inverse handicap: Weak workers are discouraged, since they now compete with two strong workers simultaneously, rather than against one opponent at a time in pair-wise interaction of the two-stage structure; this reduces total effort provision and improves selection. Alternatively, using setting `SSWW` rather than `SWSW` essentially handicaps strong workers: Whereas it is fairly easy for strong workers to reach stage 2 in setting `SWSW` due to the weak stage-1 opponent, it is equally hard for workers of both types to reach stage-2 in setting `SSWW`. Thus, incentives for effort provision are now higher, and selection performance is lower due to structural handicapping of strong types.

Our results do also provide some guidance in situations where the principal is only interested in one of the two objectives we consider: If selection is the only relevant performance measure, all participants should be pooled in a simultaneous interaction.[24] In contrast, separation of tournament participants into pair-wise interactions seems optimal if designers mainly care about incentives for effort provision.[25] Interestingly, anecdotal evidence is in line with these theoretical predictions: First, HRM frequently uses assessment centers, for example, where all applicants interact simultaneously, to fill open entry-level positions. Arguably, the uncertainty about types is particularly high among

[24] The claim that the selection performance is maximized in the one-stage tournament follows from Proposition 5.1(b), which establishes that selection performance is better in the static than in the dynamic tournament with random seeding. In addition, the proof of Proposition 5.1 (b) in the Appendix shows that selection in the static tournament is better than in any one of the two design option `SSWW` and `SWSW`, respectively, of the dynamic format.

[25] This follows directly from Propositions 5.1(a) and 5.2(a).

graduates without professional experience, while effort provision during the application process has no direct value for corporations; this makes the selection of productive types highly important.[26] Second, relative performance evaluations of employees, which may matter for bonus payments or promotions and provide employees with long term incentives, are often separated. For example, it is common that each manager announces who is the relatively best performing employee from his team in meetings of the management division. Subsequently, the available prizes are either shared among several employees, or alternatively, the subsequent relative performance of employees from the preselected sample of top performers is used to award prizes to the overall winner.

5.4 Concluding Remarks

We investigated whether the performance dimensions 'incentive provision' and 'accuracy in selection' are compatible in tournaments with heterogeneous workers. Comparing static one-stage and dynamic two-stage promotion tournaments, as well as two different seeding variants of two-stage promotion tournaments, our results suggest that they are incompatible. Even though any tournament with heterogeneous participants provides some incentives for effort and some sorting of types, modifications which improve the performance in one will deteriorate the performance in the other dimension, i.e., tournament formats that perform better in terms of incentive provision do worse in terms of selecting the best participant, and vice versa. The reason is that structural variations of tournaments with heterogeneous workers have similar effects as strategic handicaps. Intuitively, tournament structures which amplify the degree of heterogeneity between strong and weak workers perform well in terms of selection, as heterogeneity discourages weak workers relatively more than it induces strong workers to slack off. At the same time, the more a tournament accommodates heterogeneity between types, the better is its performance in the incentive dimension, since heterogeneity decreases the incentives for effort provision for both strong and weak workers in absolute terms. Therefore, multiple instruments should be used whenever two two goals are equally important, since a promotion tournament cannot be designed in such a way that it is optimal along both dimensions.

Our results are also important for applications where tournaments between heterogeneous participants are solely used as a means for incentive provision. It is well known, for example, that heterogeneity has detrimental effects on the incentive properties of tournaments. Existing solutions to this problem, such as handicapping á la Lazear and Rosen (1981), or wage discrimination á la Gürtler and Kräkel (2010), require that the principal knows workers' types, which is not always the case. Whenever this information is not available, structural modifications of the tournament structure may be an attractive alternative way of dealing with negative incentive effects of heterogeneity. Therefore, we

[26] In line with this reasoning, separate job interviews are much more common among applicants for higher level positions, which require professional experience and references from previous employers.

believe that the comparison of different tournament structures with heterogeneous participants is a promising topic for future work; most of the existing literature on structural variations of tournaments (or contests) either assumes that participants are homogeneous, or that the ordinal monitoring technology is perfectly precise. Even in the latter case, the effect of tournament structures on heterogeneity is often trivial, since less productive types voluntarily drop-out from the competition.

Appendix

5.A Solution of the One-Stage Tournament

Due to symmetry, it suffices to solve the optimization problem of one strong and one weak worker. Without loss of generality, we consider the strong worker i and the weak worker k and obtain

$$\begin{aligned}\max_{x_i \geq 0} \Pi_i(\mathtt{I}) &= \frac{x_i}{x_i + X} P - c_{\mathtt{S}} x_i,\\ \max_{x_k \geq 0} \Pi_k(\mathtt{I}) &= \frac{x_k}{x_i + X} P - c_{\mathtt{W}} x_k,\end{aligned}$$

where $X = x_j + x_k + x_l$ as in the main text. This leads to the two first-order optimality conditions

$$XP = c_{\mathtt{S}}(x_i + X)^2 \quad \text{and} \quad XP = c_{\mathtt{W}}(x_i + X)^2.$$

Combining these conditions with symmetry (implying $X = 2x_i + 2x_k$) reveals that the relation

$$x_{\mathtt{W}}^* = \frac{2c_{\mathtt{S}} - c_{\mathtt{W}}}{2c_{\mathtt{W}} - c_{\mathtt{S}}} x_{\mathtt{S}}^* \tag{5.10}$$

holds in an interior NE. Since the equilibrium efforts cannot be negative, a corner solution (with $x_{\mathtt{W}}^* = 0$) applies for $c_{\mathtt{W}} \geq 2c_{\mathtt{S}}$. In other words, weak workers drop out from the competition voluntarily for large differences in productivity (for $c_{\mathtt{W}} \geq 2c_{\mathtt{S}}$), leaving the two strong workers as the only contenders for the prize.[27] Taking these considerations into account, the equilibrium efforts of strong and weak workers are given by

$$x_{\mathtt{S}}^*(\mathtt{I}) = \begin{cases} \frac{3(2c_{\mathtt{W}} - c_{\mathtt{S}})}{4(c_{\mathtt{S}} + c_{\mathtt{W}})^2} P & \text{if } \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}} < 2 \\ \frac{1}{4c_{\mathtt{S}}} P & \text{if } \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}} \geq 2 \end{cases} \quad \text{and} \quad x_{\mathtt{W}}^*(\mathtt{I}) = \begin{cases} \frac{3(c_{\mathtt{W}} - 2c_{\mathtt{S}})}{4(c_{\mathtt{S}} + c_{\mathtt{W}})^2} P & \text{if } \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}} < 2 \\ 0 & \text{if } \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}} \geq 2 \end{cases}. \tag{5.11}$$

5.B Solution of the Two-Stage Tournament

5.B.1 Solution for Stage 2

(1) SS: If two strong (type S) workers i and j compete against each other on stage 2, they both face the same maximization problem. Without loss of generality, we consider the optimization by worker i, who maximizes his stage-2 payoff $\pi_{i2}(\mathtt{SS})$ by choosing an optimal level of effort x_{i2}, while taking the effort of his opponent x_{j2} as given.[28] Formally,

[27] See also Stein (2002) for details.

[28] Throughout the paper the first subscript of the variables π and x indicates the player, while the second subscript indicates the stage. The particular tournament environment considered (that is, SS, SW, WW, SSWW, or SWSW) is in parentheses – as in $\pi_{i2}(\mathtt{SS})$ – or is omitted when there is no risk of confusion.

this maximization problem reads

$$\max_{x_{i2}\geq 0} \pi_{i2}(\mathtt{SS}) = \frac{x_{i2}}{x_{i2}+x_{j2}} P - c_{\mathtt{S}} x_{i2} \,.$$

As shown by Cornes and Hartley (2005), any pairwise tournament has a unique interior equilibrium when the lottery CSF is used. Consequently, it suffices to consider first-order conditions which are both necessary and sufficient. Using the first-order condition for worker i ($x_{j2}P - c_{\mathtt{S}}(x_{i2}+x_{j2})^2 = 0$) and invoking symmetry ($x^*_{i2} = x^*_{j2}$) delivers equilibrium efforts

$$x^*_{\mathtt{S}2}(\mathtt{SS}) = x^*_{i2}(\mathtt{SS}) = x^*_{j2}(\mathtt{SS}) = \frac{P}{4c_{\mathtt{S}}}. \tag{5.12}$$

Inserting optimal actions in the objective function gives the payoff that a strong worker can expect in equilibrium if he meets another strong worker on stage 2. Since the expected payoff is the same for workers i and j, the indices can be replaced by $\mathtt{S}$ (indicating strong workers). Equilibrium payoffs then read

$$\pi^*_{\mathtt{S}2}(\mathtt{SS}) = \frac{P}{4} \,. \tag{5.13}$$

(2) $\mathtt{WW}$: Suppose now that two weak (type $\mathtt{W}$) workers k and l compete with each other on stage 2. Without loss of generality, we consider the optimization problem of worker k: $\max_{x_{k2}\geq 0} \pi_{k2}(\mathtt{WW}) = \frac{x_{k2}}{x_{k2}+x_{l2}} P - c_{\mathtt{W}} x_{k2}$. The same steps as in the solution of interaction $\mathtt{SS}$ deliver equilibrium efforts

$$x^*_{\mathtt{W}2}(\mathtt{WW}) = x^*_{k2}(\mathtt{WW}) = x^*_{l2}(\mathtt{WW}) = \frac{P}{4c_{\mathtt{W}}}. \tag{5.14}$$

When inserting these efforts in the objective function, the expected equilibrium payoff for a weak worker in a stage 2 interaction $\mathtt{WW}$ is given by

$$\pi^*_{\mathtt{W}2}(\mathtt{WW}) = \frac{P}{4} \,. \tag{5.15}$$

(3) $\mathtt{SW}$: Finally, consider the situation where a strong worker $\mathtt{S}$ meets a weak worker $\mathtt{W}$ on stage 2. The optimization problems are as follows:

$$\begin{aligned} \max_{x_{\mathtt{S}2}\geq 0} \pi_{\mathtt{S}2}(\mathtt{SW}) &= \frac{x_{\mathtt{S}2}}{x_{\mathtt{S}2}+x_{\mathtt{W}2}} P - c_{\mathtt{S}} x_{\mathtt{S}2} \,, \\ \max_{x_{\mathtt{W}2}\geq 0} \pi_{\mathtt{W}2}(\mathtt{SW}) &= \frac{x_{\mathtt{W}2}}{x_{\mathtt{S}2}+x_{\mathtt{W}2}} P - c_{\mathtt{W}} x_{\mathtt{W}2} \,. \end{aligned}$$

First order conditions are necessary as well as sufficient in *heterogeneous* pairwise interactions (see Nti, 1999, or Cornes and Hartley, 2005). The combination of first-order conditions implies equilibrium efforts

$$x^*_{\mathtt{S}2}(\mathtt{SW}) = \frac{c_{\mathtt{W}}}{(c_{\mathtt{S}}+c_{\mathtt{W}})^2} P \quad \text{and} \quad x^*_{\mathtt{W}2}(\mathtt{SW}) = \frac{c_{\mathtt{S}}}{(c_{\mathtt{S}}+c_{\mathtt{W}})^2} P, \tag{5.16}$$

respectively. Inserting optimal actions in the two objective functions gives the expected payoffs for strong and weak workers in a stage 2 interaction $\mathtt{SW}$:

$$\pi^*_{\mathtt{S}2}(\mathtt{SW}) = \frac{c_{\mathtt{W}}^2}{(c_{\mathtt{S}}+c_{\mathtt{W}})^2}P\,, \tag{5.17}$$

$$\pi^*_{\mathtt{W}2}(\mathtt{SW}) = \frac{c_{\mathtt{S}}^2}{(c_{\mathtt{S}}+c_{\mathtt{W}})^2}P. \tag{5.18}$$

5.B.2 Solution for Stage 1

Setting $\mathtt{SSWW}$. Due to symmetry of the optimization problems, it suffices to solve the optimization problem of one strong worker (i or k), and one weak worker (j or l). Without loss of generality, we consider the maximization problems of workers i and j,

$$\begin{aligned}\max_{x_{i1}\geq 0}\Pi_i(\mathtt{SSWW}) &= \frac{x_{i1}}{x_{i1}+x_{j1}}\pi^*_{\mathtt{S}2}(\mathtt{SW}) - c_{\mathtt{S}}x_{i1}\,,\\ \max_{x_{k1}\geq 0}\Pi_k(\mathtt{SSWW}) &= \frac{x_{k1}}{x_{k1}+x_{l1}}\pi^*_{\mathtt{W}2}(\mathtt{SW}) - c_{\mathtt{W}}x_{k1}\,.\end{aligned}$$

Note that the optimization problem for strong workers is identical to the one considered in stage-2 interaction $\mathtt{SS}$, the only difference is the expected prize, which now amounts to $\pi^*_{\mathtt{S}2}(\mathtt{SW})$ rather than P. Analogously, weak workers face the same situation as in stage-2 interaction $\mathtt{WW}$ with a different prize ($\pi^*_{\mathtt{W}2}(\mathtt{SW})$ instead of P). Consequently, first-order and symmetry conditions deliver stage 1 equilibrium efforts

$$x^*_{\mathtt{S}1}(\mathtt{SSWW}) \equiv x^*_{i1}(\mathtt{SSWW}) = x^*_{j1}(\mathtt{SSWW}) = \frac{c_{\mathtt{W}}^2}{4c_{\mathtt{S}}(c_{\mathtt{S}}+c_{\mathtt{W}})^2}P \tag{5.19}$$

$$x^*_{\mathtt{W}1}(\mathtt{SSWW}) \equiv x^*_{k1}(\mathtt{SSWW}) = x^*_{l1}(\mathtt{SSWW}) = \frac{c_{\mathtt{S}}^2}{4c_{\mathtt{W}}(c_{\mathtt{S}}+c_{\mathtt{W}})^2}P. \tag{5.20}$$

Setting $\mathtt{SWSW}$. We assume (without loss of generality) that workers i and k are strong, whereas workers j and l are weak, and that the two pairwise stage-1 interactions are between workers i and j, and between workers k and l, respectively. We start by considering the decision problem of strong worker i and weak worker j. Both workers choose their optimal stage-1 effort, given equilibrium behavior in any potential stage-2 interaction. The optimization problems are

$$\max_{x_{i1}\geq 0}\Pi_i(\mathtt{SWSW}) = \frac{x_{i1}}{x_{i1}+x_{j1}}\underbrace{\left[\frac{x_{k1}}{x_{k1}+x_{l1}}\pi^*_{\mathtt{S}2}(\mathtt{SS}) + \frac{x_{l1}}{x_{k1}+x_{l1}}\pi^*_{\mathtt{S}2}(\mathtt{SW})\right]}_{\equiv P_i(x_{k1},x_{l1})} - c_{\mathtt{S}}x_{i1}$$

$$\max_{x_{j1}\geq 0}\Pi_j(\mathtt{SWSW}) = \frac{x_{j1}}{x_{i1}+x_{j1}}\underbrace{\left[\frac{x_{k1}}{x_{k1}+x_{l1}}\pi^*_{\mathtt{W}2}(\mathtt{SW}) + \frac{x_{l1}}{x_{k1}+x_{l1}}\pi^*_{\mathtt{W}2}(\mathtt{WW})\right]}_{\equiv P_j(x_{k1},x_{l1})} - c_{\mathtt{W}}x_{j1}\,.$$

The continuation values $P_i(x_{k1},x_{l1})$ and $P_j(x_{k1},x_{l1})$ of workers i and j, respectively, depend on the behavior of workers k and l in the other stage-1 interaction. Similarly, the continuation values $P_k(x_{i1}, x_{j1})$ and $P_l(x_{i1}, x_{j1})$ of workers k and l depend on the behavior of workers i and j, as their formal optimization problems show:

$$\max_{x_{k1}\geq 0} \Pi_k(\mathtt{SWSW}) = \frac{x_{k1}}{x_{k1}+x_{l1}} \underbrace{\left[\frac{x_{i1}}{x_{i1}+x_{j1}}\pi^*_{\mathtt{S}2}(\mathtt{SS}) + \frac{x_{j1}}{x_{i1}+x_{j1}}\pi^*_{\mathtt{S}2}(\mathtt{SW})\right]}_{\equiv P_k(x_{i1},x_{j1})} - c_{\mathtt{S}} x_{k1},$$

$$\max_{x_{l1}\geq 0} \Pi_l(\mathtt{SWSW}) = \frac{x_{l1}}{x_{k1}+x_{l1}} \underbrace{\left[\frac{x_{i1}}{x_{i1}+x_{j1}}\pi^*_{\mathtt{W}2}(\mathtt{SW}) + \frac{x_{j1}}{x_{i1}+x_{j1}}\pi^*_{\mathtt{W}2}(\mathtt{WW})\right]}_{\equiv P_l(x_{i1},x_{j1})} - c_{\mathtt{W}} x_{l1}.$$

Therefore, the two stage-1 interactions are linked through endogenously determined continuation values. The reason is that expected equilibrium payoffs for workers differ across the three potential stage-2 interactions SS, WW, and SW. Conditional on reaching stage 2, workers of both types have a higher expected payoff from meeting a weak rather than a strong opponent, since $\pi^*_{\mathtt{W}2}(\mathtt{WW}) > \pi^*_{\mathtt{W}2}(\mathtt{SW})$ and $\pi^*_{\mathtt{S}2}(\mathtt{SW}) > \pi^*_{\mathtt{S}2}(\mathtt{SS})$. However, each worker takes the probability that the opponent is of a certain type as given, since it is determined in the parallel stage-1 interaction. The first-order conditions for the interaction between workers i and j read

$$x_{j1}P_i(x_{k1}, x_{l1}) - c_{\mathtt{S}}(x_{i1}+x_{j1})^2 = 0 \quad \text{and} \quad x_{i1}P_k(x_{k1}, x_{l1}) - c_{\mathtt{W}}(x_{i1}+x_{j1})^2 = 0.$$

The respective conditions for the other stage-1 interaction between workers k and l are

$$x_{l1}P_j(x_{i1}, x_{j1}) - c_{\mathtt{S}}(x_{k1}+x_{l1})^2 = 0 \quad \text{and} \quad x_{k1}P_l(x_{i1}, x_{j1}) - c_{\mathtt{W}}(x_{k1}+x_{l1})^2 = 0\,.$$

Combining the four conditions, we obtain two expressions that define a relation between equilibrium effort choices of workers *within* each interaction, namely

$$\frac{x_{i1}}{x_{j1}} = \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}}\frac{P_i(x_{k1}, x_{l1})}{P_j(x_{k1}, x_{l1})} \quad \text{and} \quad \frac{x_{k1}}{x_{l1}} = \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}}\frac{P_k(x_{i1}, x_{j1})}{P_l(x_{i1}, x_{j1})}, \tag{5.21}$$

respectively. These expressions show that each stage-1 interaction is a tournament between workers with different costs and endogenously different valuations of winning. While the costs of effort differ by construction, the difference of the value for winning is a result of the tournament structure: Reaching stage 2 is more valuable for strong than for weak workers.

We proceed now to the solution of the problem, which comprises two heterogeneous participants with regard to their effort costs and their valuation. As mentioned previously, any tournament with two heterogeneous participants has a unique, interior equilibrium for the chosen contest success function (Cornes and Hartley 2005, Nti 1999). Consequently, each of the two pairwise stage-1 interactions has a unique equilibrium for each pair of continuation values. What remains to be shown is that the two expressions in (5.21) can be satisfied *jointly* such that both stage-1 interactions are satisfied simultaneously in equilibrium. Inserting the expressions for the continuation values in (5.21) and simplifying gives

$$\frac{x_{i1}}{x_{j1}} = \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}} \frac{(c_{\mathtt{S}} + c_{\mathtt{W}})^2 \frac{x_{k1}}{x_{l1}} + 4c_{\mathtt{W}}^2}{4c_{\mathtt{S}}^2 \frac{x_{k1}}{x_{l1}} + (c_{\mathtt{S}} + c_{\mathtt{W}})^2} \quad \text{and} \quad \frac{x_{k1}}{x_{l1}} = \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}} \frac{(c_{\mathtt{S}} + c_{\mathtt{W}})^2 \frac{x_{i1}}{x_{j1}} + 4c_{\mathtt{W}}^2}{4c_{\mathtt{S}}^2 \frac{x_{i1}}{x_{j1}} + (c_{\mathtt{S}} + c_{\mathtt{W}})^2}. \tag{5.22}$$

System (5.22) consists of two equations in the two unknowns $\frac{x_{i1}^*}{x_{j1}^*}$ and $\frac{x_{k1}^*}{x_{l1}^*}$, respectively. Note that both equations are symmetric, since the two workers in each of the two stage-1 interactions face identical optimization problems. This implies that the conditions $x_{\mathtt{S}1}^* \equiv x_{i1}^* = x_{k1}^*$ and $x_{\mathtt{W}1}^* \equiv x_{j1}^* = x_{l1}^*$ do hold in the symmetric equilibrium.[29] Imposing this condition on (5.22) gives a quadratic equation in $x_{\mathtt{S}1}^*$ and $x_{\mathtt{W}1}^*$:

$$\begin{aligned}
\frac{x_{\mathtt{S}1}^*}{x_{\mathtt{W}1}^*} &= \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}} \frac{(c_{\mathtt{S}} + c_{\mathtt{W}})^2 \frac{x_{\mathtt{S}1}^*}{x_{\mathtt{W}1}^*} + 4c_{\mathtt{W}}^2}{4c_{\mathtt{S}}^2 \frac{x_{\mathtt{S}1}^*}{x_{\mathtt{W}1}^*} + (c_{\mathtt{S}} + c_{\mathtt{W}})^2} \\
\Leftrightarrow \quad 0 &= 4c_{\mathtt{S}}^2 \left[\frac{x_{\mathtt{S}1}^*}{x_{\mathtt{W}1}^*}\right]^2 + \left(1 - \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}}\right)(c_{\mathtt{S}} + c_{\mathtt{W}})^2 \left[\frac{x_{\mathtt{S}1}^*}{x_{\mathtt{W}1}^*}\right] - 4\frac{c_{\mathtt{W}}^3}{c_{\mathtt{S}}} \\
\Leftrightarrow \quad \frac{x_{\mathtt{S}1}^*}{x_{\mathtt{W}1}^*} &= F^*(c_{\mathtt{S}}, c_{\mathtt{W}}),
\end{aligned}$$

where

$$F^*(c_{\mathtt{S}}, c_{\mathtt{W}}) = \frac{(c_{\mathtt{W}} - c_{\mathtt{S}})(c_{\mathtt{S}} + c_{\mathtt{W}})^2 + \sqrt{64 c_{\mathtt{W}}^3 c_{\mathtt{S}}^3 + (c_{\mathtt{S}} - c_{\mathtt{W}})^2 (c_{\mathtt{S}} + c_{\mathtt{W}})^4}}{8c_{\mathtt{S}}^3}. \tag{5.23}$$

$F^*(c_{\mathtt{S}}, c_{\mathtt{W}})$ is the ratio of stage-1 efforts of the two worker types, which is directly proportional to heterogeneity in costs and continuation values, as equation (5.21) shows. Therefore, $F^*(c_{\mathtt{S}}, c_{\mathtt{W}})$ can be interpreted as a measure for both the exogenous heterogeneity in effort costs between strong and weak workers and the endogenous heterogeneity between types that is due to different continuation values in stage 1.

The expression $F^*(c_{\mathtt{S}}, c_{\mathtt{W}})$ allows us to disentangle and solve analytically the two interdependent stage-1 interactions. We start by considering the continuation values which satisfy

$$\begin{aligned}
P_i(x_{\mathtt{S}1}^*, x_{\mathtt{W}1}^*) = P_k(x_{\mathtt{S}1}^*, x_{\mathtt{W}1}^*) &= \frac{(c_{\mathtt{S}} + c_{\mathtt{W}})^2 F^*(c_{\mathtt{W}}, c_{\mathtt{S}}) + 4c_{\mathtt{W}}^2}{4(c_{\mathtt{S}} + c_{\mathtt{W}})^2 [1 + F^*(c_{\mathtt{S}}, c_{\mathtt{W}})]} P, \\
P_j(x_{\mathtt{S}1}^*, x_{\mathtt{W}1}^*) = P_l(x_{\mathtt{S}1}^*, x_{\mathtt{W}1}^*) &= \frac{(c_{\mathtt{S}} + c_{\mathtt{W}})^2 + 4c_{\mathtt{S}}^2 F^*(c_{\mathtt{W}}, c_{\mathtt{S}})}{4(c_{\mathtt{S}} + c_{\mathtt{W}})^2 [1 + F^*(c_{\mathtt{S}}, c_{\mathtt{W}})]} P.
\end{aligned}$$

[29] The symmetric equilibrium exists for any degree of heterogeneity and is unique. Intuitively, one must show that the graphs of the two relations in (5.22) have a unique intersection in the domain defined by $\frac{x_{j1}^*}{x_{i1}^*} \in [0,1]$ and $\frac{x_{l1}^*}{x_{k1}^*} \in [0,1]$. It suffices to consider this domain, since the assumption of lower costs of effort and the resulting higher value of winning of strong workers imply that $x_{i1}^* \geq x_{j1}^*$ and $x_{k1}^* \geq x_{l1}^*$, respectively. This follows from (5.21). See Stracke (2012a) for details and a complete formal proof.

Note that $P_i(x^*_{\mathtt{S}1}, x^*_{\mathtt{W}1}) = P_k(x^*_{\mathtt{S}1}, x^*_{\mathtt{W}1})$ and $P_j(x^*_{\mathtt{S}1}, x^*_{\mathtt{W}1}) = P_l(x^*_{\mathtt{S}1}, x^*_{\mathtt{W}1})$ due to symmetry. Given these continuation values, stage 1 equilibrium efforts can be determined as

$$x^*_{\mathtt{S}1}(\mathtt{SWSW}) \equiv x^*_{i1}(\mathtt{SWSW}) = x^*_{k1}(\mathtt{SWSW}) = \frac{(c_{\mathtt{S}}+c_{\mathtt{W}})^2 F^*(c_{\mathtt{W}},c_{\mathtt{S}})^2 + 4c_{\mathtt{W}}^2 F^*(c_{\mathtt{W}},c_{\mathtt{S}})}{4c_{\mathtt{S}}(c_{\mathtt{S}}+c_{\mathtt{W}})^2[1+F^*(c_{\mathtt{S}},c_{\mathtt{W}})]^3} P \tag{5.24}$$

$$x^*_{\mathtt{W}1}(\mathtt{SWSW}) \equiv x^*_{j1}(\mathtt{SWSW}) = x^*_{l1}(\mathtt{SWSW}) = \frac{(c_{\mathtt{S}}+c_{\mathtt{W}})^2 F^*(c_{\mathtt{W}},c_{\mathtt{S}}) + 4c_{\mathtt{S}}^2 F^*(c_{\mathtt{W}},c_{\mathtt{S}})^2}{4c_{\mathtt{W}}(c_{\mathtt{S}}+c_{\mathtt{W}})^2[1+F^*(c_{\mathtt{S}},c_{\mathtt{W}})]^3} P\,. \tag{5.25}$$

5.C Proofs

Lemma 5.1. *Assume without loss of generality that $c_W \geq c_S = 1$ and define $f(c_W) = \frac{5c_W^3+2c_W^2+c_W}{c_W^2+2c_W+5}$. Then, the relation $F^*(1,c_W) > f(c_W)$ does hold for all $c_W > 1$, where $F^*(1,c_W)$ is defined as in* (5.23)*. Furthermore, for $c_W = 1$ it holds that $F^*(1,c_W) = f(c_W)$.*

Proof. From equation (5.21), we know that $\frac{x_{i1}}{x_{k1}} = \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}}\frac{P_i(x_{j1},x_{l1})}{P_k(x_{j1},x_{l1})}$. Further, equation (5.23) tells us that $\frac{x^*_{i1}}{x^*_{k1}} = F^*(c_{\mathtt{S}}, c_{\mathtt{W}})$. Consequently, using the assumption that $c_{\mathtt{W}} \geq c_{\mathtt{S}} = 1$, it must hold that

$$F^*(1,c_{\mathtt{W}}) = c_{\mathtt{W}}\frac{P_i(x_{j1},x_{l1})}{P_k(x_{j1},x_{l1})} = \frac{4c_{\mathtt{W}}^3 + c_{\mathtt{W}}(1+c_{\mathtt{W}})^2 \times \frac{x_{j1}}{x_{l1}}}{(1+c_{\mathtt{W}})^2 + 4\times\frac{x_{j1}}{x_{l1}}}.$$

Note that

$$\frac{\partial F^*(1,c_{\mathtt{W}})}{\partial \frac{x_{j1}}{x_{l1}}} = \frac{(1+c_{\mathtt{W}})^4 - 16c_{\mathtt{W}}^2}{[(1+c_{\mathtt{W}})^2 + 4\times\frac{x_{j1}}{x_{l1}}]^2} > 0$$

if $c_{\mathtt{W}} > 1$. Further, recall that player l has both higher cost ($c_{\mathtt{W}} > 1$) and a lower continuation value ($P_j > P_l$), such that $x_{j1} > x_{l1}$ does hold. Therefore, assuming $x_{j1} = x_{l1}$ underestimates $F^*(1,c_{\mathtt{W}})$. Since

$$f(c_{\mathtt{W}}) = \frac{5c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}}}{c_{\mathtt{W}}^2 + 2c_{\mathtt{W}} + 5}$$

is the expression we derive from $F^*(1,c_{\mathtt{W}})$ under this assumption, we have proven $F^*(1,c_{\mathtt{W}}) > f(c_{\mathtt{W}})$. If we assume $c_{\mathtt{W}} = 1$, all players are perfectly symmetric, such that $x_{j1} = x_{l1}$ does hold. Consequently, the relation $F^*(1,c_{\mathtt{W}}) = f(c_{\mathtt{W}})$ does hold for $c_{\mathtt{W}} = 1$. □

Lemma 5.2. *Assume without loss of generality that $c_W \geq c_S = 1$ and define $f_{\text{low}}(c_W) = 2c_W - 1$. Then, the relation $F^*(1,c_W) < f_{\text{low}}(c_W)$ does hold for all $c_W > 1$. Furthermore, for $c_W = 1$, it holds that $f(c_W) = f_{\text{low}}(c_W)$.*

Proof. We start with the relation that we want to prove, namely:

$$\begin{aligned} f(c_{\mathtt{W}}) &> f_{\text{low}}(c_{\mathtt{W}}) \\ \Leftrightarrow 5c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}} &> (2c_{\mathtt{W}} - 1)(c_{\mathtt{W}}^2 + 2c_{\mathtt{W}} + 5) \\ \Leftrightarrow 3c_{\mathtt{W}}^3 - c_{\mathtt{W}}^2 - 7c_{\mathtt{W}} + 5 &> 0 \end{aligned}$$

We now have to prove that $\phi(c_{\mathtt{W}}) \equiv 3c_{\mathtt{W}}^3 - c_{\mathtt{W}}^2 - 7c_{\mathtt{W}} + 5 > 0$ does always hold for $c_{\mathtt{W}} > 1$. To see this, note that $\phi(\cdot)$ is a cubic function that has a local minimum at $c_{\mathtt{W}} = 1$, and a local maximum at $c_{\mathtt{W}} = -7/9$. Furthermore, $\phi(1) = 0$, which implies that $\phi(c_{\mathtt{W}}) > 0$ for all $c_{\mathtt{W}} > 1$. $\square$

Lemma 5.3. *Assume without loss of generality that $c_{\mathtt{W}} \geq c_{\mathtt{S}} = 1$ and define $f_{\text{high}}(c_{\mathtt{W}}) = \frac{c_{\mathtt{W}}^3+2c_{\mathtt{W}}^2+c_{\mathtt{W}}}{4}$. Then, the relation $F^*(1, c_{\mathtt{W}}) < f_{\text{high}}(c_{\mathtt{W}})$ does hold for all $c_{\mathtt{W}} > 1$. Furthermore, for $c_{\mathtt{W}} = 1$, it holds that $F^*(1, c_{\mathtt{W}}) = f_{\text{high}}(c_{\mathtt{W}})$.*

Proof. From equation (5.21), we know that $\frac{x_{i1}}{x_{k1}} = \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}} \frac{P_i(x_{j1}, x_{l1})}{P_k(x_{j1}, x_{l1})}$. Further, equation (5.23) tells us that $\frac{x_{i1}^*}{x_{k1}^*} = F^*(c_{\mathtt{S}}, c_{\mathtt{W}})$. Consequently, using the assumption that $c_{\mathtt{W}} \geq c_{\mathtt{S}} = 1$, it must hold that

$$F^*(1, c_{\mathtt{W}}) = c_{\mathtt{W}} \frac{P_i(x_{j1}, x_{l1})}{P_k(x_{j1}, x_{l1})} = \frac{4c_{\mathtt{W}}^3 \times \frac{x_{l1}}{x_{j1}} + c_{\mathtt{W}}(1 + c_{\mathtt{W}})^2}{(1 + c_{\mathtt{W}})^2 \times \frac{x_{l1}}{x_{j1}} + 4}.$$

Note that

$$\frac{\partial F^*(1, c_{\mathtt{W}})}{\partial \frac{x_{l1}}{x_{j1}}} = -\frac{(c_{\mathtt{W}} - 1)^2 c_{\mathtt{W}}(c_{\mathtt{W}}^2 + 6c_{\mathtt{W}} + 1)}{[(1 + c_{\mathtt{W}})^2 \times \frac{x_{l1}}{x_{j1}} + 4]^2} < 0$$

if $c_{\mathtt{W}} > 1$. Further, recall from the main text that player l will never drop out in a pairwise competition for any finite degree of heterogeneity in terms of costs and continuation value, such that $x_{l1} > 0$ does hold. Therefore, assuming $x_{l1} = 0$ (which implies $\frac{x_{l1}}{x_{j1}} = 0$) overestimates $F^*(1, c_{\mathtt{W}})$, since this expression is decreasing in $\frac{x_{l1}}{x_{j1}}$. Since

$$f_{\text{high}}(c_{\mathtt{W}}) = \frac{c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}}}{4}$$

is the expression we derive from $F^*(1, c_{\mathtt{W}})$ under this assumption, we have proven $F^*(1, c_{\mathtt{W}}) < f_{\text{high}}(c_{\mathtt{W}})$. If we assume $c_{\mathtt{W}} = 1$, all players are perfectly symmetric, such that $x_{l1} = x_{j1}$ does hold. When inserting this relation in $F^*(1, c_{\mathtt{W}})$, we see that the relation $F^*(1, c_{\mathtt{W}}) = f(c_{\mathtt{W}})$ does hold for $c_{\mathtt{W}} = 1$. $\square$

Proposition 5.1: *When the cost of effort is strictly higher for weak than for strong agents ($c_{\mathtt{W}} > c_{\mathtt{S}}$),*

(a) aggregate effort is strictly higher in the two- than in one-stage tournament, i.e.,

$$\mathcal{E}(\mathtt{I}) < \mathcal{E}(\mathtt{II}) \quad \text{for all} \quad c_{\mathtt{W}} > c_{\mathtt{S}}.$$

(b) the probability that a strong agent receives the promotion is higher in the one- than in the two-stage tournament, i.e.,

$$\mathcal{S}(\mathtt{I}) > \mathcal{S}(\mathtt{II}) \quad \text{for all} \quad c_{\mathtt{W}} > c_{\mathtt{S}}.$$

Proof. We will separately prove parts (a) and (b) of Proposition 5.1, starting with (a).

(a): To prove the relation $\mathcal{E}(\texttt{II}) > \mathcal{E}(\texttt{I})$ for all $c_{\texttt{W}} > c_{\texttt{S}}$, we assume without loss of generality that $c_{\texttt{W}} > c_{\texttt{S}} = 1$. Recall from (5.2) that $\mathcal{E}(\texttt{I})$ is defined stepwise, i.e., $\mathcal{E}(\texttt{I}) = \max\{\frac{3}{2+2c_{\texttt{W}}}P, \frac{1}{2}P\}$. First, we will consider the range $1 < c_{\texttt{W}} \leq 2$, where $\mathcal{E}(\texttt{I}) = \frac{3}{2+2c_{\texttt{W}}}P$. In the second part of this proof, we will devote attention to $c_{\texttt{W}} > 2$ and $\mathcal{E}(\texttt{I}) = \frac{1}{2}P$.

(i) We consider the range $1 < c_{\texttt{W}} \leq 2$ and want to prove that

$$\begin{aligned} \mathcal{E}(\texttt{I}) &< \mathcal{E}(\texttt{II}) \\ \Leftrightarrow \mathcal{E}(\texttt{I}) &< \frac{2}{3}\mathcal{E}(\texttt{SWSW}) + \frac{1}{3}\mathcal{E}(\texttt{SSWW}). \end{aligned}$$

Recall from the proof of Proposition 1 that the formal expression for $\mathcal{E}(\texttt{SWSW})$ is fairly complicated, in particular due to the $F^*(1, c_{\texttt{W}})$-function. To simplify the subsequent analysis, we will therefore make use again of Lemmata 1/2 and replace $F^*(1, c_{\texttt{W}})$ by $f_{\text{low}}(c_{\texttt{W}}) = 2c_{\texttt{W}} - 1$. This is without loss of generality, since $\mathcal{E}(\texttt{SWSW})$ is strictly increasing in $F^*(1, c_{\texttt{W}})$:

$$\frac{\partial \mathcal{E}(\texttt{SWSW})}{\partial F^*(1, c_{\texttt{W}})} = \frac{(2c_{\texttt{W}}^3 - c_{\texttt{W}}^2 - 4c_{\texttt{W}} + 7)F^*(1, c_{\texttt{W}}) + 3c_{\texttt{W}}^2 + 2c_{\texttt{W}} - 1}{2c_{\texttt{W}}(1 + c_{\texttt{W}})^2(1 + F^*(1, c_{\texttt{W}}))^3} > 0.$$

Note that the denominator is always greater than zero, since we know from Lemma 1 that (a) $\frac{\partial F^*(1, c_{\texttt{W}})}{\partial c_{\texttt{W}}} > 0$ and (b) $F^*(1) = 1$; this implies that the sign of the derivative is determined by the numerator, which is also greater than zero for all $1 < c_{\texttt{W}} \leq 2$. Consequently, effort $\mathcal{E}(\texttt{SWSW})$ is underestimated through the replacement of $F^*(1, c_{\texttt{W}})$ by $f_{\text{low}}(c_{\texttt{W}})$. Inserting $f_{\text{low}}(c_{\texttt{W}})$ and simplifying leaves us with the sufficient condition

$$Q(c_{\texttt{W}}) \equiv \frac{(c_{\texttt{W}} - 1)^2(6c_{\texttt{W}}^3 + 2c_{\texttt{W}}^2 - 9c_{\texttt{W}} + 4)}{12c_{\texttt{W}}^3(1 + c_{\texttt{W}})^2} = \frac{(c_{\texttt{W}} - 1)^2 q(c_{\texttt{W}})}{12c_{\texttt{W}}^3(1 + c_{\texttt{W}})^2} > 0.$$

In the relevant range $1 < c_{\texttt{W}} \leq 2$, the expression $(c_{\texttt{W}} - 1)^2$ in the numerator as well as the denominator $12c_{\texttt{W}}^3(1 + c_{\texttt{W}})^2$ are always greater than zero, such that the sign of $Q(c_{\texttt{W}})$ is determined by the expression $q(c_{\texttt{W}}) \equiv 6c_{\texttt{W}}^3 + 2c_{\texttt{W}}^2 - 9c_{\texttt{W}} + 4$. Note that $q(1) = 3$ and $q(2) = 42$. Since

$$\frac{\partial q(c_{\texttt{W}})}{\partial c_{\texttt{W}}} = 18c_{\texttt{W}}^2 + 4c_{\texttt{W}} - 9 > 0$$

for all $c_{\texttt{W}} > 1$, it holds that $q(c_{\texttt{W}}) > 0$ for all $1 < c_{\texttt{W}} \leq 2$, which immediately implies that $Q(c_{\texttt{W}}) > 0$. This completes the first part of the proof.

(ii) When $c_{\texttt{W}} > 2$, it holds that $\mathcal{E}(\texttt{I}) = \frac{1}{2}P$. We have to prove that the relation $\mathcal{E}(\texttt{II}) > \mathcal{E}(\texttt{I})$ is satisfied. Inserting the respective expressions for $\mathcal{E}(\texttt{II})$ and $\mathcal{E}(\texttt{I})$ gives the condition:

$$\begin{aligned} \frac{(3c_{\texttt{W}}^3 + 6c_{\texttt{W}}^2 + 4c_{\texttt{W}} + 9)F^*(1, c_{\texttt{W}})^2 + (2c_{\texttt{W}}^3 + 14c_{\texttt{W}}^2 + 16c_{\texttt{W}} + 4)F^*(1, c_{\texttt{W}}) + c_{\texttt{W}}^3 + 4c_{\texttt{W}}^2 + 6c_{\texttt{W}} + 3}{3c_{\texttt{W}}(c_{\texttt{W}} + 1)^2(1 + F^*(1, c_{\texttt{W}}))^2} &> 1 \\ \Leftrightarrow B(F^*(1, c_{\texttt{W}}), c_{\texttt{W}}) \equiv (c_{\texttt{W}} + 9)F^*(1, c_{\texttt{W}})^2 - (4c_{\texttt{W}}^3 - 2c_{\texttt{W}}^2 - 10c_{\texttt{W}} - 4)F^*(1, c_{\texttt{W}}) - (2c_{\texttt{W}}^2 - 3)(c_{\texttt{W}} + 1) &> 0 \end{aligned}$$

Note that $B(\cdot)$ is minimized for $F^*(1,c_W)_{\min} = \frac{2c_W^3 - c_W^2 - 5c_W - 2}{9+c_W}$, since

$$\frac{\partial B(\cdot)}{\partial F^*(1,c_W)} = (18+2c_W)F^*(1,c_W) - 4c_W^3 + 2c_W^2 + 10c_W + 4 \quad \text{and} \quad \frac{\partial^2 B(\cdot)}{\partial [F^*(1,c_W)]^2} = 18 + 2c_W > 0.$$

Moreover, note that $F^*(1,c_W)_{\min} > 0$ for all $c_W > 2$, which implies that $B(\cdot)$ is increasing in $F^*(1,c_W)$ in the range which is relevant for this proof. Consequently, when solving the relation $B(\cdot) > 0$ for $F^*(1,c_W)$, we know that $F^*(1,c_W)$ must not be in the range between the two roots, as $B(\cdot)$ is negative here. We obtain

$$\begin{aligned} & B(F^*(1,c_W), c_W) > 0 \\ \Leftrightarrow\ & F^*(1,c_W)^2 - \frac{(4c_W^3 - 2c_W^2 - 10c_W - 4)}{(c_W+9)} F^*(1,c_W) - \frac{(2c_W^2 - 3)(c_W+1)}{(c_W+9)} > 0 \\ \Leftrightarrow\ & F^*(1,c_W) < \frac{2c_W^3 - c_W^2 - 5c_W - 2 - \sqrt{K(c_W)}}{9+c_W} \ \vee\ F^*(1,c_W) > \frac{2c_W^3 - c_W^2 - 5c_W - 2 + \sqrt{K(c_W)}}{9+c_W}, \end{aligned}$$

where

$$K(c_W) = 4c_W^6 - 4c_W^5 - 17c_W^4 + 22c_W^3 + 44c_W^2 - 10c_W - 23.$$

We do only have to consider the second relation, since the first one is always below one for $c_W > 2$, while $F^*(1,c_W) \geq 1$ for all $c_W \geq 1$.[30] To complete the proof, we have to show that

$$F^*(1,c_W) > \frac{2c_W^3 - c_W^2 - 5c_W - 2 + \sqrt{K(c_W)}}{9+c_W}$$

for all $c_W > 2$. Inserting the equilibrium relation $F^*(1,c_W)$ from (5.23) gives:

$$\frac{(c_W-1)(1+c_W)^2 + \sqrt{64c_W^3 + (1-c_W)^2(1+c_W)^4}}{8} > \frac{2c_W^3 - c_W^2 - 5c_W - 2 + \sqrt{K(c_W)}}{9+c_W},$$

which is equivalent to

$$(c_W-1)(1+c_W)^2(9+c_W) + (9+c_W)\sqrt{64c_W^3 + (1-c_W)^2(1+c_W)^4} > 16c_W^3 - 8c_W^2 - 40c_W - 16 + 8\sqrt{K(c_W)}.$$

Rearranging and simplifying gives the condition

$$H(c_W) \equiv \underbrace{c_W^4 - 6c_W^3 + 16c_W^2 + 30c_W - 7}_{\mu(c_W)} + \underbrace{(9+c_W)\sqrt{64c_W^3 + (1-c_W)^2(1+c_W)^4}}_{\gamma(c_W)} - \underbrace{8\sqrt{K(c_W)}}_{\zeta(c_W)} > 0.$$

$H(c_W)$ consists of three parts $\mu(c_W)$, $\gamma(c_W)$, and $\zeta(c_W)$. Close inspection of $\mu(c_W)$ reveals that $\mu(c_W)$ is strictly increasing and greater than zero for all $c_W > 2$.[31] Consequently, it is

[30] $F^*(1,1) = 1$; also, we know from Lemma 1 that $\frac{\partial F^*(1,c_W)}{\partial c_W} > 0$. Therefore, $F^*(1,c_W) \geq 1$ for all $c_W \geq 1$.
[31] $\mu(2) = 85$, and $\mu'(c_W) = 4c_W^3 - 18c_W^2 + 32c_W + 30$.

a sufficient condition for $H(c_{\mathtt{W}}) > 0$ to show that $\gamma(c_{\mathtt{W}}) > \zeta(c_{\mathtt{W}})$ in the range $c_{\mathtt{W}} > 2$:

$$\begin{aligned} (9+c_{\mathtt{W}})\sqrt{64c_{\mathtt{W}}^3+(1-c_{\mathtt{W}})^2(1+c_{\mathtt{W}})^4} &> 8\sqrt{K(c_{\mathtt{W}})} \\ \Leftrightarrow (9+c_{\mathtt{W}})^2[64c_{\mathtt{W}}^3+(1-c_{\mathtt{W}})^2(1+c_{\mathtt{W}})^4] &> 64K(c_{\mathtt{W}}) \\ \Leftrightarrow c_{\mathtt{W}}^8+20c_{\mathtt{W}}^7-140c_{\mathtt{W}}^6+460c_{\mathtt{W}}^5+2086c_{\mathtt{W}}^4+3436c_{\mathtt{W}}^3-2860c_{\mathtt{W}}^2+820c_{\mathtt{W}}+1553 &> 0 \end{aligned}$$

A sufficient condition for the above relation to hold is

$$\begin{aligned} 20c_{\mathtt{W}}^7-140c_{\mathtt{W}}^6+460c_{\mathtt{W}}^5+2086c_{\mathtt{W}}^4+3436c_{\mathtt{W}}^3-2860c_{\mathtt{W}}^2 &> 0 \\ \Leftrightarrow c_{\mathtt{W}}^2[20c_{\mathtt{W}}^5-140c_{\mathtt{W}}^4+460c_{\mathtt{W}}^3+2086c_{\mathtt{W}}^2-2860] &> 0. \end{aligned}$$

Since $c_{\mathtt{W}} > 2$ by assumption, we are left with

$$20c_{\mathtt{W}}^5-140c_{\mathtt{W}}^4+460c_{\mathtt{W}}^3+2086c_{\mathtt{W}}^2-2860 > 0.$$

For $c_{\mathtt{W}} > 2$, it must hold that $2086c_{\mathtt{W}}^2-2860 > 0$, such that we can drop those two expressions without loss of generality. We get

$$\begin{aligned} 20c_{\mathtt{W}}^5-140c_{\mathtt{W}}^4+460c_{\mathtt{W}}^3 &> 0 \\ \Leftrightarrow c_{\mathtt{W}}^3[20c_{\mathtt{W}}^2-140c_{\mathtt{W}}+460] &> 0 \\ \Leftrightarrow c_{\mathtt{W}}^2-7c_{\mathtt{W}}+23 &> 0, \end{aligned}$$

which is greater than zero for all $c_{\mathtt{W}} > 2$. This completes part (a) of the proof.

(b): Recall from part (b) of Proposition 5.2 that selection in setting SSWW is always dominated by selection in SSWW. Consequently, it is sufficient to show that $\mathcal{S}(\mathtt{I}) > \mathcal{S}(\mathtt{SWSW})$ to prove part (b) of Proposition 5.1, since $\mathcal{S}(\mathtt{II})$ is a composite measure of $\mathcal{S}(\mathtt{SWSW})$ and $\mathcal{S}(\mathtt{SSWW})$. We start with the relation which we want to prove:

$$\begin{aligned} \mathcal{S}(\mathtt{I}) &> \mathcal{S}(\mathtt{SWSW}) \\ \Leftrightarrow \min\left\{\frac{2c_{\mathtt{W}}-c_{\mathtt{S}}}{c_{\mathtt{S}}+c_{\mathtt{W}}},1\right\} &> \frac{(c_{\mathtt{S}}+c_{\mathtt{W}})F^*(1,c_{\mathtt{W}})^2+2c_{\mathtt{W}}F^*(1,c_{\mathtt{W}})}{(c_{\mathtt{S}}+c_{\mathtt{W}})[1+F^*(1,c_{\mathtt{W}})]^2} \end{aligned}$$

Since $\mathcal{S}(\mathtt{I})$ is defined stepwise, we have to proceed in two steps. First, we start with the case where $1 < c_{\mathtt{W}} \leq 2$ such that $\mathcal{S}(\mathtt{I}) = \frac{2c_{\mathtt{W}}-c_{\mathtt{S}}}{c_{\mathtt{S}}+c_{\mathtt{W}}}$, before we consider $c_{\mathtt{W}} > 2$ and $\mathcal{S}(\mathtt{I}) = 1$.

(i) We assume without loss of generality that $c_{\mathtt{S}} = 1$ and consider the range $1 < c_{\mathtt{W}} \leq 2$. Then, we get

$$\begin{aligned} \mathcal{S}(\mathtt{I}) &> \mathcal{S}(\mathtt{SWSW}) \\ \Leftrightarrow \frac{2c_{\mathtt{W}}-1}{1+c_{\mathtt{W}}} &> \frac{(1+c_{\mathtt{W}})F^*(1,c_{\mathtt{W}})^2+2c_{\mathtt{W}}F^*(1,c_{\mathtt{W}})}{(1+c_{\mathtt{W}})[1+F^*(1,c_{\mathtt{W}})]^2} \\ \Leftrightarrow (2c_{\mathtt{W}}-1)[1+F^*(1,c_{\mathtt{W}})]^2 &> (1+c_{\mathtt{W}})F^*(1,c_{\mathtt{W}})^2+2c_{\mathtt{W}}F^*(1,c_{\mathtt{W}}) \end{aligned}$$

Rearranging gives the condition

$$N(c_{\mathtt{W}}) = (c_{\mathtt{W}} - 2)[F^*(1, c_{\mathtt{W}})]^2 + 2(c_{\mathtt{W}} - 1)F^*(1, c_{\mathtt{W}}) + 2c_{\mathtt{W}} - 1 > 0$$

Recall from equation (5.23) that the expression for $F^*(1, c_{\mathtt{W}})$ is fairly complicated. To simplify the subsequent analysis, we make use of Lemma 3, where we established that $F^*(1, c_{\mathtt{W}}) < f_{\text{high}}(c_{\mathtt{W}})$ for all $c_{\mathtt{W}} > 1$. Since $\mathcal{S}(\mathtt{SWSW})$ is strictly increasing in $F^*(1, c_{\mathtt{W}})$, it is sufficient for the proof if we use the much simpler expression $f_{\text{high}}(c_{\mathtt{W}})$, as this tends to reduce the difference between the one-stage and the two-stage tournament in terms of selection:

$$\frac{\partial \mathcal{S}(\mathtt{SWSW})}{\partial F^*(1, c_{\mathtt{W}})} = \frac{2(c_{\mathtt{W}} + F^*(1, c_{\mathtt{W}})}{(1 + c_{\mathtt{W}})[1 + F^*(1, c_{\mathtt{W}})]^3} > 0.$$

This leaves us with

$$\begin{aligned}
\bar{N}(c_{\mathtt{W}}) &= (c_{\mathtt{W}} - 2)\left[\frac{c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}}}{4}\right]^2 + 2(c_{\mathtt{W}} - 1)\frac{c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}}}{4} + 2c_{\mathtt{W}} - 1 \\
&= \frac{(c_{\mathtt{W}} - 2)\left[c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}}\right]^2 + 8(c_{\mathtt{W}} - 1)(c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}}) + 32c_{\mathtt{W}} - 16}{16} \\
&= \frac{(c_{\mathtt{W}} - 1)[(c_{\mathtt{W}} - 1)(c_{\mathtt{W}} + 2)(c_{\mathtt{W}}(c_{\mathtt{W}} - 1)^2 + 4)c_{\mathtt{W}} + 16]}{16}.
\end{aligned}$$

Recall that we must show that $\bar{N}(c_{\mathtt{W}}) > 0$ holds for all $1 < c_{\mathtt{W}} \leq 2$. Note that $\bar{N}(1) = 0$ and $\bar{N}(2) = 12$. Therefore, the proof is complete if we can show that $\bar{N}(c_{\mathtt{W}})$ is strictly increasing in the relevant range. Since $c_{\mathtt{W}} > 1$, the factor $(c_{\mathtt{W}} - 1)$ in the expression of $\bar{N}(c_{\mathtt{W}})$ is always positive and can be disregarded in the subsequent analysis of the slope. Subsequently, we use the simpler expression

$$\hat{N}(c_{\mathtt{W}}) = \frac{(c_{\mathtt{W}} - 1)(c_{\mathtt{W}} + 2)(c_{\mathtt{W}}(c_{\mathtt{W}} - 1)^2 + 4)c_{\mathtt{W}} + 16}{16}.$$

When computing the first derivative of $\bar{N}(c_{\mathtt{W}})$ with respect to $c_{\mathtt{W}}$, we obtain

$$\frac{\partial \hat{N}(c_{\mathtt{W}})}{\partial c_{\mathtt{W}}} = \frac{6c_{\mathtt{W}}^5 + 15c_{\mathtt{W}}^4 + 4c_{\mathtt{W}}^3 + 3c_{\mathtt{W}}^2 + 4c_{\mathtt{W}} - 8}{16},$$

which is clearly positive for all values in the range $1 < c_{\mathtt{W}} \leq 2$. This proves the first part of the Proposition.

(ii) We assume without loss of generality that $c_{\mathtt{S}} = 1$. Then, a comparison of $\mathcal{S}(\mathtt{I})$ and $\mathcal{S}(\mathtt{SWSW})$ in the range $c_{\mathtt{W}} > 2$ gives

$$\begin{aligned}
\mathcal{S}(\mathtt{I}) &> \mathcal{S}(\mathtt{SWSW}) \\
\Leftrightarrow \quad 1 &> \frac{(1 + c_{\mathtt{W}})F^*(1, c_{\mathtt{W}})^2 + 2c_{\mathtt{W}}F^*(1, c_{\mathtt{W}})}{(1 + c_{\mathtt{W}})[1 + F^*(1, c_{\mathtt{W}})]^2} \\
\Leftrightarrow (1 + c_{\mathtt{W}})[1 + F^*(1, c_{\mathtt{W}})]^2 &> (1 + c_{\mathtt{W}})F^*(1, c_{\mathtt{W}})^2 + 2c_{\mathtt{W}}F^*(1, c_{\mathtt{W}})
\end{aligned}$$

Rearranging gives the condition

$$M(c_{\mathtt{W}}) = 2F^*(1, c_{\mathtt{W}}) + c_{\mathtt{W}} + 1 > 0.$$

As in the first part of this proof, we substitute $f_{\text{high}}(c_{\mathtt{W}})$ for $F^*(1, c_{\mathtt{W}})$, which gives

$$\begin{aligned}\bar{M}(c_{\mathtt{W}}) &= 2\frac{c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}}}{4} + c_{\mathtt{W}} + 1 \\ &= \frac{c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + 3c_{\mathtt{W}} + 2}{2}.\end{aligned}$$

$\bar{M}(c_{\mathtt{W}})$ is clearly positive for all $c_{\mathtt{W}} > 2$, which proves the second part of the Proposition. □

Proposition 5.2: *When the cost of effort is strictly higher for weak than for strong agents ($c_{\mathtt{W}} > c_{\mathtt{S}}$),*

(a) aggregate effort is strictly higher in setting SSWW than in setting SWSW, i.e.,

$$\mathcal{E}(\mathtt{SSWW}) > \mathcal{E}(\mathtt{SWSW}) \quad \text{for all} \quad c_{\mathtt{W}} > c_{\mathtt{S}}.$$

(b) the probability that a strong agent receives the promotion is strictly higher in setting SWSW than in setting SSWW, i.e.,

$$\mathcal{S}(\mathtt{SSWW}) < \mathcal{S}(\mathtt{SWSW}) \quad \text{for all} \quad c_{\mathtt{W}} > c_{\mathtt{S}}.$$

Proof. We will separately prove parts (a) and (b) of Proposition 5.2. We start with part (a) below.

(a): To prove the relation $\mathcal{E}(\mathtt{SSWW}) > \mathcal{E}(\mathtt{SWSW})$ for all $c_{\mathtt{W}} > c_{\mathtt{S}}$, we assume without loss of generality that $c_{\mathtt{W}} > c_{\mathtt{S}} = 1$. In the proof, we will proceed in two steps. First, we derive a necessary and sufficient condition in terms of the function $F^*(1, c_{\mathtt{W}})$ for the relation $\mathcal{E}(\mathtt{SSWW}) > \mathcal{E}(\mathtt{SWSW})$ to hold. Second, we prove that the equilibrium function $F^*(1, c_{\mathtt{W}})$, which was derived in (5.23), indeed satisfies this condition. We start with the relation which we want to prove:

$$\begin{gathered}\mathcal{E}(\mathtt{SSWW}) > \mathcal{E}(\mathtt{SWSW}) \\ \Leftrightarrow \frac{c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}(1 + c_{\mathtt{W}}) + 1}{2c_{\mathtt{W}}(1 + c_{\mathtt{W}})^2} > \frac{(1 + c_{\mathtt{W}})^2[1 + [1 + F^*(1, c_{\mathtt{W}})]c_{\mathtt{W}}F^*(1, c_{\mathtt{W}})] + 4c_{\mathtt{W}}[c_{\mathtt{W}}^2 + (1 + c_{\mathtt{W}})F^*(1, c_{\mathtt{W}})]}{2c_{\mathtt{W}}(1 + c_{\mathtt{W}})^2[1 + F^*(1, c_{\mathtt{W}})]^2}\end{gathered}$$

Multiplying both sides by $2c_{\mathtt{W}}(1 + c_{\mathtt{W}})^2[1 + F^*(1, c_{\mathtt{W}})]^2$ and rearranging gives

$$F^*(1, c_{\mathtt{W}})^2 + \frac{c_{\mathtt{W}}^3 - 2c_{\mathtt{W}}^2 - c_{\mathtt{W}} + 2}{c_{\mathtt{W}} + 1}F^*(1, c_{\mathtt{W}}) - \frac{3c_{\mathtt{W}}^3 - c_{\mathtt{W}}^2}{c_{\mathtt{W}} + 1} \quad > \quad 0$$

Solving for $F^*(1, c_{\mathtt{W}})$ gives us two conditions:

$$F^*(1, c_{\mathtt{W}}) < \frac{-c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}} - 2 - R(c_{\mathtt{W}})}{2c_{\mathtt{W}} + 2} \quad \vee \quad F^*(1, c_{\mathtt{W}}) > Z(c_{\mathtt{W}}) \equiv \frac{-c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}} - 2 + R(c_{\mathtt{W}})}{2c_{\mathtt{W}} + 2},$$

where

$$R(c_{\mathtt{W}}) = \sqrt{c_{\mathtt{W}}^6 - 4c_{\mathtt{W}}^5 + 14c_{\mathtt{W}}^4 + 16c_{\mathtt{W}}^3 - 11c_{\mathtt{W}}^2 - 4c_{\mathtt{W}} + 4}.$$

We do only have to consider the second relation, since the first one is below one for some values of $c_{\mathtt{W}}$, while $F^*(1, c_{\mathtt{W}}) \geq 1$ for all $c_{\mathtt{W}} \geq 1$.[32] This completes the first part of the proof. We now have to prove that

$$F^*(1, c_{\mathtt{W}}) > Z(c_{\mathtt{W}}) \equiv \frac{-c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}} - 2 + R(c_{\mathtt{W}})}{2c_{\mathtt{W}} + 2} \tag{5.26}$$

for all $c_{\mathtt{W}} > 1$. From Lemmata 1 and 2 we know that $F^*(1, c_{\mathtt{W}}) > f_{\text{low}}(c_{\mathtt{W}})$. Consequently, a sufficient condition for (5.26) is given by $f_{\text{low}}(c_{\mathtt{W}}) > Z(c_{\mathtt{W}})$. Rearranging this condition gives

$$c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}} > R(c_{\mathtt{W}}).$$

Squaring both sides leaves us with[33]

$$\begin{aligned} 2c_{\mathtt{W}}^5 - 2c_{\mathtt{W}}^4 - 3c_{\mathtt{W}}^3 + 3c_{\mathtt{W}}^2 + c_{\mathtt{W}} - 1 &> 0 \\ \Leftrightarrow 2(c_{\mathtt{W}} - 1)^2(c_{\mathtt{W}} + 1)(c_{\mathtt{W}} - \frac{1}{\sqrt{2}})(c_{\mathtt{W}} + \frac{1}{\sqrt{2}}) &> 0. \end{aligned}$$

This relation is always satisfied if $c_{\mathtt{W}} > 1$, which completes part (a) of this proof.

(b): In part (b) of this proof, we first derive a necessary and sufficient condition which assures that the relation $\mathcal{S}(\mathtt{SSWW}) < \mathcal{S}(\mathtt{SWSW})$ does hold in terms of the function $F^*(1, c_{\mathtt{W}})$. Then, we prove that $F^*(1, c_{\mathtt{W}})$ satisfies this condition.

(i) As previously, we assume that $c_{\mathtt{W}} > c_{\mathtt{S}} = 1$ does hold without loss of generality. Consequently, we can use the expressions in equations (5.5) and (5.7) in what follows. We start with the relation which we want to prove:

$$\begin{aligned} \mathcal{S}(\mathtt{SWSW}) &> \mathcal{S}(\mathtt{SSWW}) \\ \Leftrightarrow (1 + c_{\mathtt{W}})F^*(1, c_{\mathtt{W}})^2 + 2c_{\mathtt{W}}F^*(1, c_{\mathtt{W}}) &> c_{\mathtt{W}}F^*(1, c_{\mathtt{W}})^2 + 2c_{\mathtt{W}}F^*(1, c_{\mathtt{W}}) + c_{\mathtt{W}} \\ \Leftrightarrow F^*(1, c_{\mathtt{W}})^2 &> c_{\mathtt{W}} \\ \Leftrightarrow F^*(1, c_{\mathtt{W}}) < -\sqrt{c_{\mathtt{W}}} &\vee F^*(1, c_{\mathtt{W}}) > \sqrt{c_{\mathtt{W}}} \end{aligned}$$

Note that it is sufficient to show that $F^*(1, c_{\mathtt{W}}) > c_{\mathtt{W}}$, since $c_{\mathtt{W}} > \sqrt{c_{\mathtt{W}}}$ for $c_{\mathtt{W}} > 1$.

(ii) From Lemma 1, we know that $F^*(1, c_{\mathtt{W}}) > f(c_{\mathtt{W}})$. We will now prove that $f(c_{\mathtt{W}}) > c_{\mathtt{W}}$ for $c_{\mathtt{W}} > 1$ to complete the proof. $f(c_{\mathtt{W}}) > c_{\mathtt{W}}$ implies that

$$\frac{5c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}}}{c_{\mathtt{W}}^2 + 2c_{\mathtt{W}} + 5} > c_{\mathtt{W}}$$

[32] $F^*(1, 1) = 1$; also, we know from Lemma 1 that $\frac{\partial F^*(1, c_{\mathtt{W}})}{\partial c_{\mathtt{W}}} > 0$. Therefore, $F^*(1, c_{\mathtt{W}}) \geq 1$ for all $c_{\mathtt{W}} \geq 1$.

[33] Squaring is without loss of generality here, since we are only interested in solutions for $c_{\mathtt{W}} > 1$.

does hold. Rearranging gives

$$\begin{aligned} 5c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}} &> c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + 5c_{\mathtt{W}} \\ \Leftrightarrow c_{\mathtt{W}}(c_{\mathtt{W}}^2 - 1) &> 0 \\ \Leftrightarrow c_{\mathtt{W}} > 1 &\vee -1 < c_{\mathtt{W}} < 0 \end{aligned}$$

This proves the claim $\mathcal{S}(\mathtt{SWSW}) > \mathcal{S}(\mathtt{SSWW})$ for all $c_{\mathtt{W}} > 1$. $\square$

Chapter 6

Optimal Prizes in Dynamic Elimination Contests: An Experimental Analysis

This chapter is based on joint work with Wolfgang Höchtl from the Austrian National Bank (OeNB), Rudolf Kerschbamer from the University of Innsbruck, and Uwe Sunde from the University of St. Gallen (Stracke, Höchtl, Kerschbamer, and Sunde 2012).

6.1 Introduction

Contests are situations in which agents compete by expending valuable resources to win a prize. Such situations appear in many different areas of economics – including election campaigns, R&D competitions, military conflicts, or the competition for bonus payments and promotions on internal labor markets. Given the multiplicity of applications, contests may vary in several dimensions, for example, with respect to the number of participants, the number of prizes, or with respect to their structure. The effect of different modeling choices in these dimensions on behavior of contest participants has been studied extensively in theoretical work, which typically determines the optimal contest design with respect to a given optimality criterion.[1] Two criteria are particularly prominent in the literature on optimal prizes in dynamic contests, namely the *maximization of aggregate incentives* (operationalized as the sum of efforts provided by all agents across all stages of the contest), and the *maintenance of incentives across stages* of the contest (operationalized as constant individual efforts over stages). A common motivation for both objectives is that effort provision by contestants is valuable for the entity organizing the contest, henceforth called the contest designer. The maximization of aggregate incentives is a natural objective of the contest designer, in particular when efforts across stages are additively separable, see Sisak (2009) for an excellent survey of the literature addressing this criterion. Alternatively, complementarities between the efforts at different stages can imply that incentive maintenance across stages is the relevant criterion for the contest

[1]See Konrad (2009) for a literature review.

designer. The classical reference for this case is Rosen (1986), who argued that incentive maintenance is particularly important in corporate tournaments in which workers are incentivized by wage increases that are associated with promotions to higher hierarchy levels within the same organization.

In this paper, we study the optimal design of a two-stage elimination contest with four homogeneous participants. Assuming that the overall prize money is fixed, our analysis first replicates the result of Fu and Lu (2012) that a "winner-takes-all" structure with a single prize for the winner of the final round maximizes total effort under the standard assumption of rational and risk-neutral contestants.[2] Then, we derive the prize structure that ensures incentive maintenance across stages in the sense of Rosen (1986). This structure turns out to be a format with multiple prizes, where the winner of the final receives most of the prize money, while a smaller part is assigned to the runner-up prize. Thus, the theoretical analysis shows that there is a trade-off between the two optimality criteria 'maximization of aggregate efforts' and 'incentive maintenance across stages' in the standard benchmark of a pair-wise elimination contest:[3] The **single-prize** format (abbreviated as **SP** in the sequel) maximizes aggregate efforts, while the **multiple-prizes** format (abbreviated as **MP**) delivers constant effort across stages.

We test these predictions in lab experiments. In line with the theoretical model, we find that total effort is higher in **SP** than in **MP**. However, the observed difference between treatments is smaller than predicted and statistically insignificant at conventional levels. On the other hand, incentive maintenance across stages in **MP** holds almost exactly as predicted by theory. A closer look at the disaggregate data reveals that risk-aversion of experimental subjects can account for the departure from the theoretical prediction in the total effort dimension. Specifically, we find that total effort provision by risk-averse subjects is higher (and not lower) in the **MP** than in **SP** format, while the behavior of risk-neutral and risk-loving subjects is in line with the theoretical prediction. Intuitively, the **MP** format is more attractive for risk-averse subjects, since the runner-up prize provides insurance against situations where costly effort is provided but no prize is won, while such insurance is not important for risk-neutral participants. Overall, the results of this paper suggest that there is a trade-off between the two goals 'total effort provision' and 'constant effort across stages' under the standard assumption of risk-neutral contestants, but this trade-off might be mitigated if contestants are sufficiently risk-averse. In such a case, a format that awards multiple prizes might well be the dominant option in both performance dimensions.

Our results contribute to the recent literature on the behavior in contests. So far, the

[2]To be precise, we consider a pair-wise elimination rather than pyramid contest. However, the result by Fu and Lu (2012) carries over to this format, since the underlying economic intuition is exactly the same.

[3]This trade-off exists if effort provision by contest participants is costly. As shown by Matros (2005), a "winner-takes-all" structure maximizes aggregate incentives and ensures incentive maintenance if contestants receive an endowment (which cannot be cashed out) and are then asked to allocate it across different stages of a contest.

experimental literature has mainly focused on static contests.[4] Exceptions are the studies of Altmann, Falk, and Wibral (2012) and Sheremeta (2010), which both compare static (one-shot) and dynamic (two-stage) contests. The paper by Altmann et al. (2012) considers a prize structure which predicts incentive maintenance across stages in the theoretical benchmark, and one of their main findings in the experiments is that effort provision by subjects in the first stage is much higher than in the second stage. Sheremeta (2010), on the other hand, investigates a single-prize two-stage contest format and compares it to an analogous one-stage contest interaction. Our paper combines the two approaches and analyzes a systematic variation of the prize structure in dynamic contests. Moreover, our paper is the first experimental test of the result by Fu and Lu (2012) that a "winner-takes-all" prize structure maximizes total effort in dynamic Tullock contests with homogeneous participants. Finally, this paper is also related to recent work by Delfgaauw, Dur, Non, and Verbeke (2012), who investigate whether a more convex prize spread affects relative effort exertion across different stages of a dynamic contest. Using data from a field experiment, they find that the effect of the prize structure on relative effort provision across stages is rather weak. The same effect appears to be much stronger in our experimental data. A likely explanation for this difference in magnitude could be that our prize spread variation is more extreme, since we compare a “winner-takes-all” structure with a multiple prizes setting, while Delfgaauw et al. (2011) investigate the effects of a more modest variation of prizes in a setting with multiple prizes.

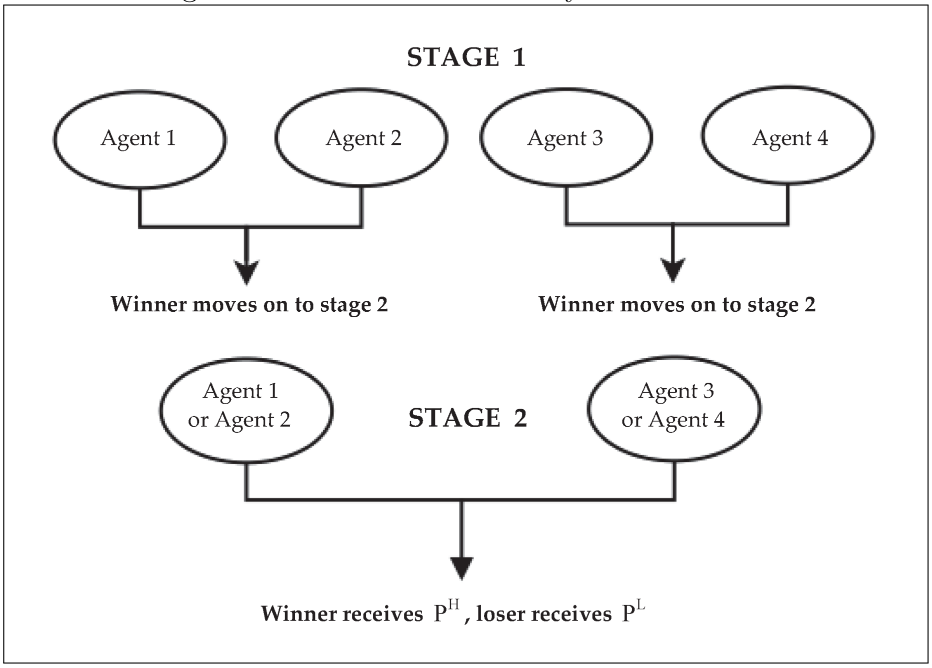
The remainder of this paper is organized as follows: Section 6.2 derives the theoretical benchmark for a simple dynamic contest model. Section 6.3 outlines the experimental design and derives our main hypotheses. The experimental results are presented and discussed in Section 6.4. Section 6.5 concludes.

6.2 A Simple Dynamic Contest Model

Set-up. We consider a simple two-stage pair-wise elimination contest where four identical agents compete for two prizes. In the first stage, there are two pair-wise interactions, and in the second stage, the winners of the two stage-1 interactions compete against each other. Figure 6.1 illustrates the sequence of events: In stage 1, two pairs of agents compete simultaneously for the right to move on to stage 2. Participation in stage 2 is valuable, since two prizes are awarded to the participants of this stage: The loser of the stage-2 interaction receives the prize P^L, while P^H is awarded to the winner, where $P^H > P^L \geq 0$. In each of the three interactions of this contest model, two risk-neutral agents independently choose their effort level to maximize their expected payoffs. The effort of agent i in stage $s \in \{1, 2\}$ is denoted x_{si}. For each invested unit, agents incur constant marginal costs of one. The benefit of effort provision is that the probability to win an interaction is increasing in the amount invested into the contest. Thus, agents face a trade-off. For

[4] See, for example, Harbring and Irlenbusch (2003), Harbring and Lünser (2008), or Sheremeta (2011).

Figure 6.1: Structure of the Dynamic Contest



simplicity, we assume that the probability to win is given by a lottery contest success function á la Tullock (1980).[5] That is, given investments x_{si} and x_{sj} by agents i and j in stage s, the probability that agent i wins in stage s equals

$$p_{si}(x_{si}, x_{sj}) = \begin{cases} \frac{x_{si}}{x_{si}+x_{sj}} & \text{if} \quad x_{si}, x_{sj} > 0 \\ \frac{1}{2} & \text{if} \quad x_{si}, x_{sj} = 0 \end{cases} .$$

Equilibrium. Due to the dynamic structure of the contest, the equilibrium concept is Subgame Perfect Nash. The equilibrium is determined by applying backward induction. Since all agents are identical, the identity of the agents who compete in stage 2 does not affect the solution. Therefore, without loss of generality, it is assumed that agents i and j interact in stage 2. The formal optimization problem for agent i reads

$$\begin{aligned} \max_{x_{2i}} \Pi_{2i}(x_{2i}, x_{2j}) &= \frac{x_{2i}}{x_{2i}+x_{2j}} P^H + \left(1 - \frac{x_{2i}}{x_{2i}+x_{2j}}\right) P^L - x_{2i} \\ &= \frac{x_{2i}}{x_{2i}+x_{2j}} (P^H - P^L) + P^L - x_{2i}, \end{aligned}$$

and delivers the first-order condition[6]

$$\frac{\partial \Pi_{2i}(x_{2i}, x_{2j})}{\partial x_{2i}} = \frac{x_{2j}}{(x_{2i}+x_{2j})^2}(P^H - P^L) - 1 = 0.$$

[5]For an axiomatization of this technology, see Skaperdas (1996).
[6]The first-order condition is necessary and sufficient – see Perez-Castrillo and Verdier (1992) for details.

Using symmetry leads to equilibrium efforts

$$x_2^* \equiv x_{2i}^* = x_{2j}^* = (P^H - P^L)/4. \tag{6.1}$$

Inserting equilibrium efforts in the objective functions gives the expected stage-2 equilibrium payoff

$$\Pi_2^* \equiv \Pi_{2i}(x_{2i}^*, x_{2j}^*) = \Pi_{2j}(x_{2i}^*, x_{2j}^*) = (P^H + 3P^L)/4. \tag{6.2}$$

Consequently, reaching stage 2 has value Π_2^* for an agent participating in stage 1. Agent k will take this value into account when choosing his stage-1 effort x_{1k}. As in stage 2, the identity of agents does not matter in stage 1, since all agents are identical by assumption. Without loss of generality, consider the interaction between agents k and l. Agent k faces the optimization problem

$$\begin{aligned} \max_{x_{1k}} \Pi_{1k}(x_{1k}, x_{1l}) &= \frac{x_{1k}}{x_{1k} + x_{1l}} \Pi_2^* - x_{1k} \\ &= \frac{x_{1k}}{x_{1k} + x_{1l}} \left(\frac{P^H + 3P^L}{4} \right) - x_{1k}. \end{aligned}$$

As in the solution of stage 2 above, the first-order condition together with symmetry yields the equilibrium efforts on stage 1 as

$$x_1^* \equiv x_{1k}^* = x_{1l}^* = \left(P^H + 3P^L\right)/16. \tag{6.3}$$

Optimal Prize Structures. Assuming that the overall prize money is fixed, we consider two goals of the contest designer: maximization of aggregate incentives, and maintenance of incentives across stages. Assuming that P units are available as total prize money, it holds that $P^H = P - P^L$. Inserting this expression into (6.1) and (6.3), we obtain as stage-1 and stage-2 equilibrium efforts

$$x_1^* = \frac{P + 2P^L}{16} \qquad \text{and} \qquad x_2^* = \frac{P - 2P^L}{4}, \tag{6.4}$$

respectively. Since four agents provide effort in stage 1, while only two of them reach stage 2, total effort $\mathcal{E}$ amounts to

$$\mathcal{E} = \frac{3P - 2P^L}{4}. \tag{6.5}$$

This expression confirms that that total effort is maximized in a "winner-takes-all" contest, i.e., if $P^L = 0$ and $P^H = P$ (since $\mathcal{E}$ is strictly decreasing in P^L).[7] With respect to the criterion of incentive maintenance across stages, equalizing the expressions for stage-1

[7] One can easily show that this result does not hinge on the number of stages and/or the specific lottery contest success function considered here. In fact, a "winner-takes-all" contest maximizes total effort in any Tullock contest with pair-wise elimination, provided the equilibrium is in pure strategies, which exists if the contest success function involves sufficient noise in terms of low discriminatory power.

Table 6.1: Parametrization and Theoretical Predictions

	Single Prize (**SP**)	Multiple Prizes (**MP**)
Total Effort ($\mathcal{E}$)	180	144
Stage-1 Effort (x_1^*)	15	24
Stage-2 Effort (x_2^*)	60	24
Prizes (P, P^L, P^H)	(240,0,240)	(240,72,168)

and stage-2 effort given in (6.4) implies a runner-up prize of $P^L = 3P/10$, and a winner prize $P^H = 7P/10$. Thus, there is a trade-off between the two goals: While total effort is maximal with a single prize equal to the total prize money for the winner of the final, incentive maintenance across stages requires two prizes: One equal to 30% of the prize money for the loser of the final, and one equal to the rest for the winner of the final.

6.3 Design of the Experiments

Experimental Parameters and Treatments. We consider two treatments with different prize structures. Independent of the treatment, the total prize money available, P, amounts to 240 units, which implies that $P^H + P^L = 240$ must hold. As shown above, total effort is predicted to be maximized in a "winner-takes-all" contest, i.e., by setting $P^L = 0$ and $P^H = 240$. This prize structure is implemented in the single prize treatment **SP**. With respect to the "incentive maintenance across stages" criterion, our results above imply a runner-up prize of $P^L = 72$, and a winner prize $P^H = 168$. We implement this prize structure in the multiple-prizes treatment **MP**.

Testable Hypotheses. Table 6.1 shows the theoretical predictions for both treatments with respect to total effort and individual effort provision in each stage. As derived above, total effort is higher in **SP** than in **MP**. Therefore, the comparison of total effort in treatments **SP** and **MP** allows us to test the hypothesis:

Hypothesis 6.1 (Total Effort Maximization). *Total effort provided by all four participants in both stages is higher in* ***SP*** *than in* ***MP****:*

$$\mathcal{E}^{\mathrm{SP}} > \mathcal{E}^{\mathrm{MP}}$$

Apart from information on total effort provision, Table 6.1 provides the individual equilibrium effort levels in each stage of both treatments. First, individual effort provision by participants in the **MP** treatment is predicted to be the same in both stages, which leads to Hypothesis 2.

Hypothesis 6.2 (**Incentive Maintenance**). *Individual efforts are identical across stages in* ***MP****:*

$$x_1^{\mathrm{MP}} = x_2^{\mathrm{MP}}$$

Second, individual effort in stage 1 is higher in treatment **MP** than in **SP**, while the opposite holds for stage-2 effort. The formal expressions in (6.4) show why this is the case: Stage-1 effort is strictly increasing in the runner-up prize P^L, since a high runner-up prize makes participation in stage 2 more valuable. Stage-2 effort is, however, decreasing in P^L. The reason is that each participant of stage 2 has the runner-up prize for sure, such that the two participants compete only for the residual prize $P^H - P^L$. We call this mechanism the "Runner-up Prize Effect" and test it in Hypothesis 3:

Hypothesis 6.3 (**Runner-up Prize Effect**). *In stage 1, individual effort provision is higher in* ***MP*** *than in* ***SP****, while the opposite holds for stage-2 effort:*

$$\begin{aligned} &\text{(a)} \quad x_1^{\mathrm{MP}} > x_1^{\mathrm{SP}} \\ &\text{(b)} \quad x_2^{\mathrm{MP}} < x_2^{\mathrm{SP}}. \end{aligned}$$

Note that the strength of the "Runner-up Prize Effect" is at the heart of the result that a "winner-takes-all" prize structure maximizes total effort. Intuitively, we consider a setting where the higher effort exertion in early stages cannot compensate for the lower effort exertion in later stages, even though the number of participants is higher in early stages. As shown by Fu and Lu (2012) and Krishna and Morgan (1998), this relation holds whenever the contest technology is sufficiently noisy.[8]

Implementation. We adopt a between-subject design; that is, our experimental subjects encountered either the **MP** or the **SP** treatment. The protocol of an experimental session was the same for both treatments: First, participants received some general information about the experimental session. Then, instructions for the respective treatment (either **SP** or **MP**) were distributed.[9] After each participant confirmed that he/she had read and understood the instructions, participants had to correctly answer a set of control questions. Only then did the first decision round start. Overall, each subject participated in 30 decision rounds with different opponents. After the main treatment, we first elicited risk preferences using a standard incentivized procedure, and then asked participants to

[8]Fu and Lu (2012) show that a "winner-takes-all" prize structure maximizes total effort in any dynamic (pyramid) contest with risk-neutral and homogeneous participants as long as the impact function is not too convex (see their Proposition 4 for details). Krishna and Morgan (1998) consider a difference (rather than ratio) contest success function with additive noise and find that multiple prizes maximize total effort in sequential elimination contests only if the noise parameter has very narrow bounds.

[9]A translated version of the instructions is provided in the Appendix. The original instructions, which are in German, are available from the authors upon request.

fill out a questionnaire (voluntary and non-incentivized). Only thereafter participants were informed about their payoff in the experimental session. We ran a total of 8 computerized sessions with 20 participants each. The experiment was programmed in z-Tree (Fischbacher 2007). All 160 participants were students from the University of Innsbruck, which were recruited using ORSEE (Greiner 2004). Each session lasted approximately 70 minutes in total (including the distribution of instructions at the beginning and the payment at the end), and participants earned between 9-13 Euro (approximately 11 Euro on average).[10]

Treatments. Each participant played the same contest game 30 times, knowing that the identities of his/her opponents are randomly determined in each decision round. We used the experimental currency "Taler", where 200 Taler corresponded to 1.00 Euro. The only variation across the two treatments **SP** and **MP** concerned the prize structure; everything else was kept constant. The role of investments into the contest (effort) was explained to subjects using an analogy between the chosen contest success function and a lottery. Participants were told that they could buy a discrete number of balls in each interaction.[11] The balls purchased by the subjects as well as those purchased by their respective opponents were then said to be placed in the same ballot box, out of which one ball would be randomly drawn subsequently. This replicates the ratio contest success function Ãă la Tullock (1980) from the theoretical set-up. Players had to buy (and pay for) their desired number of balls before they knew whether or not they won a pair-wise interaction in the contest. For this purpose, each participant received an endowment of 240 Taler in each round. This endowment could be used to buy balls on both stages, i.e., a subject that reached stage 2 could use whatever remained of his/her endowment to buy balls in the stage-2 interaction. The part of the endowment that a participant did not use to buy balls was added to the payoffs for that round. Since the endowment was as high as the total prize money P, agents were not budget-constrained at any time.[12] Experimental subjects were told that the endowment could only be used in a given round, that is, that transfers across decision rounds were not possible. Therefore, the strategic interaction is the same in each of the 30 decision rounds. Random matching in each round ensured that the same participants did not interact repeatedly; matching groups corresponded to the entire session. After each decision round, participants were informed about their own decision, the decision(s) of their immediate opponent(s), and about their own payoff. This allows for an investigation of whether players learn when completing the task repeatedly.

[10]In two out of three sessions of the **SP** treatments, an additional experiment was conducted *after* the risk-elicitation part. This experiment was entirely unrelated to the tournament experiment and subjects were not informed about what to expect in this second experiment. All they knew is that the session also included a third part, rather than only two parts. These sessions where approximately 15 minutes longer, and payoffs in this additional experiment amounted to approximately 2.50 Euros on average.

[11]The chosen prizes ensured that equilibrium investments in both stages of both contest specifications were positive integers, which implies that the discrete grid had no consequences for the equilibrium strategies; the equilibrium in pure strategies is unique in both treatments.

[12]This is also confirmed by the experimental data on effort.

In order to minimize the potential impact of income effects, participants were told that only four decision rounds (out of 30) would be randomly chosen and paid out at the end of the experiment.

Elicitation of Risk Attitudes. We used a choice list similar to the one employed by Dohmen, Falk, Huffman, and Sunde (2010) to elicit risk attitudes.[13] Specifically, each subject was exposed to a series of 21 binary choices between a cash gamble and a safe payoff. While the cash gamble remained the same in all 21 binary choices – it always gave either 400 Taler or 0 Taler, each with 50 percent probability – the safe payoff increased in steps of 20 Taler from 0 Taler in the first choice to 400 Taler in the last choice. Given this design, a decision maker whose preferences satisfy ordering (completeness and transitivity) and strict monotonicity switches exactly once from the cash gamble to the safe payoff. For subjects who switch exactly once we use the first choice scenario in which the subject decides in favor of the save payoff as our measure of risk attitude (we do not classify subjects with multiple switching points).

6.4 Experimental Results

Our main experimental results are summarized in Table 6.2. The table displays the theoretical predictions from Section 6.2 as well as observed means for stage-1, stage-2, and total effort provision in both treatments. The data match all qualitative relations that were predicted, even though the empirically observed efforts exceed their theoretical counterparts quite substantially in quantitative terms. This finding of quantitative over-provision is in line with much of the existing experimental literature and will be discussed at the end of this section.

6.4.1 Baseline Results Regarding the Hypotheses

We proceed in the same order as in Section 6.3, starting with the comparison of total effort between treatments. In line with the theoretical prediction, total effort is higher in **SP** than in **MP** (304.513 compared to 277.861, see Table 6.2 for details). However, the difference is smaller than predicted in relative terms (total output in **MP** is only 10% lower than in **SP**, while theory predicts that it is 25% lower) and the difference is not statistically significant at conventional levels. Indeed, the p-value for a test of the null of equality of session means is above 0.10 both for the parametric t-test and the non-

[13] In the Dohmen, Falk, Huffman, and Sunde (2010) procedure, each subject is exposed to a series of choices between a safe payment (which is systematically varied) and a binary lottery (which remains constant across choices). This is cognitively simpler than the procedure employed by Holt and Laury (2002), where a subject is confronted with a series of choices between two binary lotteries that are both varied systematically. The instructions which experimental subjects received right before the risk-elicitation part are provided in the Appendix.

Table 6.2: Experimental Results

	SP			MP		
	N	Data	Theory	N	Data	Theory
Total Effort ($\mathcal{E}$)	3	**304.513**	180	5	**277.861**	144
		(28.314)			(6.796)	
Stage-1 Effort (x_1^*)	60	**33.660**	15	100	**45.238**	24
		(2.911)			(2.957)	
Stage-2 Effort (x_2^*)	60	**85.134**	60	100	**45.976**	24
		(4.658)			(2.614)	

Note: The numbers in the columns "Data" denote averages over all rounds of the experimental sessions. Total effort is the sum of individual efforts over subjects and stages, and stage-1 (stage-2) effort is individual effort in that stage (in experimental currency, Taler). Standard errors in parentheses. The column "Theory" provides the theoretical equilibrium prediction for the respective effort measure.

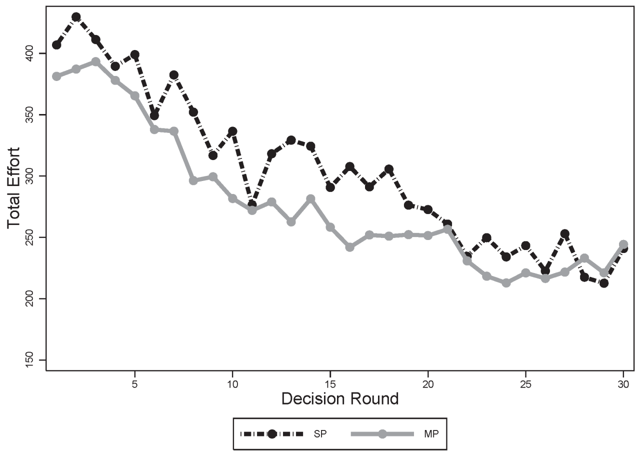
parametric Mann-Whitney-U-test (MWU-test).[14] Figure 6.2 plots the evolution of total effort over time from round 1 to 30 and shows two things: First, we observe that total effort is decreasing over time in both treatments. It seems that participants realize after a few rounds that they initially provided too much effort, even though total effort in later decision rounds is still well above the risk-neutral benchmark in both treatments. Second, total effort in both treatments becomes very similar in later rounds of the experiment, i.e., even the small initial difference in total effort across treatments disappears in later rounds of the experiment. We summarize our findings with respect to total effort provision as follows:

Result 6.1 (Total Effort Maximization). *Total effort over all contestants and both stages of the contest is higher in **SP** than in **MP**, in line with the theoretical prediction. However, the difference is smaller than predicted and not significantly different from zero.*

Hypothesis 2 is concerned with the maintenance of incentives across stages and states that individual efforts in **MP** are not expected to differ across stages. According to Table 6.2, this is exactly what we observe in the experiment: Subjects invest approximately 45 units of effort in both stages, and we cannot reject the null of equality of individual session means ($p < 0.001$). Figure 6.3 plots the stage-1 and stage-2 effort choices in treatment **MP** over the different rounds of the experiment and might help to explain why we observe incentive maintenance, while Altmann, Falk, and Wibral (2012) do not. Altmann et al. employ an experimental design where participants interact only once. In

[14]The p-values are 0.2825 (t-test) and 0.4561 (MWU-test). In the following, we only report p-values for the non-parametric MWU-test unless noted otherwise.

Figure 6.2: Total Effort by Decision Round and Treatment



contrast, in our experiment the same contest is repeated 30 times with random matching.[15] If we only consider the first decision round, the data replicate the pattern observed by Altmann, Falk, and Wibral (2012): In this round, subjects choose, on average, an effort of 65.75 in stage 1, compared to 59.16 in stage 2 in treatment **MP**.[16] However, this pattern disappears and is even reversed in later rounds, as Figure 6.3 shows. In fact, the equality of stage-1 and stage-2 efforts can be rejected in some of the first seven decision rounds, while equality cannot be rejected in any subsequent round. Finally, Figure 6.3 illustrates that both stage-1 and stage-2 efforts are decreasing with experience in the experiment, but remain well above the theoretical benchmark even in the last decision round. This gives our second result:

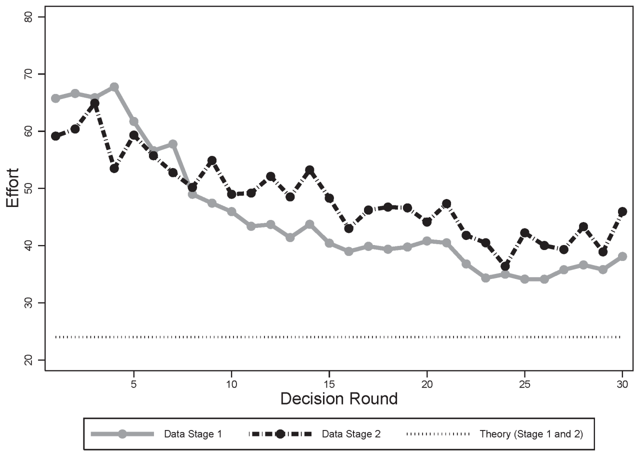
Result 6.2 (Incentive Maintenance). *Efforts are approximately identical across stages in **MP** when considering session means. In the initial decision rounds, however, effort provision is somewhat higher in stage 1 than in stage 2.*

Our third and last hypothesis addresses the effect of the runner-up prize. Theory predicts that a runner-up prize increases individual effort in stage 1, while at the same time decreasing stage-2 effort. Table 6.2 shows that this pattern is present in the experimental data: Effort provision by experimental subjects in stage 1 is higher in **MP** than in **SP** (45.238 vs. 33.660), and equality of mean effort can be rejected at the 1% level. In contrast, stage-2 effort is higher in **SP** than in **MP** (85.134 vs. 45.976), and again the difference is highly significant ($p < 0.01$). Figure 6.4 illustrates that this pattern is present in each single decision round when comparing individual session means; only in the very first rounds, stage-1 efforts are rather similar across treatments. In addition, Figure 6.4 shows that individual efforts in the last decision rounds are much closer to the theoretical

[15] Another difference of their experimental design is that they use a 'difference' contest success function rather than the 'ratio' technology we employ. For a theoretical comparison of these technologies, see Hirshleifer (1989).

[16] This difference is significantly different from zero at the 5%-level.

Figure 6.3: Individual Effort in Treatment MP by Decision Round



prediction in **SP** than in **MP**; this holds both in stage 1 and in stage 2.[17] Summing up, our findings are well in line with Hypothesis 3:

Result 6.3 (Runner-up Prize Effect). *The comparison of efforts in a given stage across treatments shows that the introduction of a runner-up prize has the predicted effect: Stage-1 effort is higher in **MP** than in **SP**, while stage-2 effort is higher in **SP** than in **MP**.*

6.4.2 Discussion and Additional Results

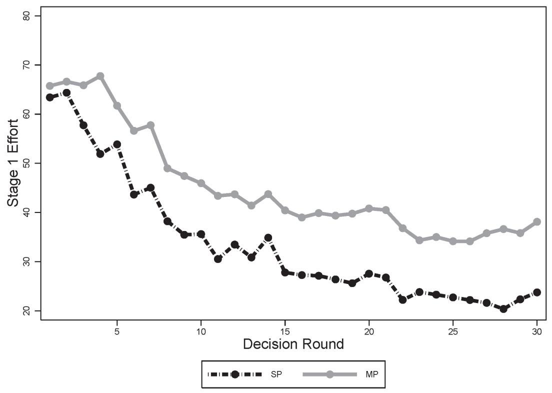
Risk Preferences. Overall, the choices of 138 participants exhibit a unique switching point in the risk-preference elicitation procedure, while 9 (13) subjects in treatment **SP** (**MP**) have multiple switching points. Considering only subjects with a unique switching point, Table 6.3 disaggregates the data into two classes of risk preferences, namely risk-averse subjects and risk-neutral or risk-loving subjects.[18] We find that incentive maintenance across stages holds for both risk classes. Moreover, when comparing efforts in a given stage across treatments, Table 6.3 shows that the runner-up prize increases stage-1 effort but decreases stage-2 effort, independent of risk-attitudes. Interestingly, however, the relation of total effort provision across treatments differs between risk-averse subjects on the one hand and risk-neutral or risk-loving subjects on the other hand: In line with the theoretical benchmark, risk-neutral (and risk-loving) subjects provide more effort in **SP** than in **MP** on average.[19] However, total effort provision by risk-averse subjects is

[17]In stage 1 of the **SP** (**MP**) treatment, effort approaches 20 (40) in the experiment, compared to a theoretical prediction of 15 (24). Similarly, in stage 2, effort approaches 65 (45) in treatment **SP** (**MP**), compared to a prediction of 60 (24).

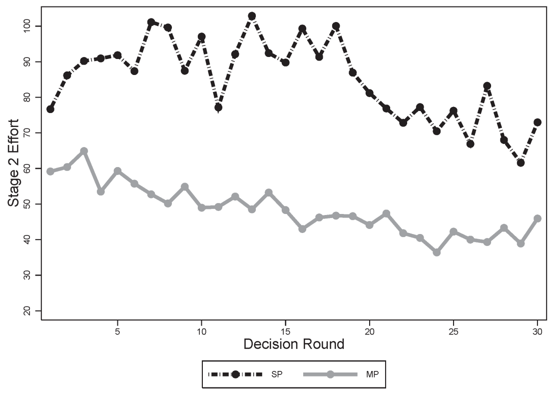
[18]Risk-loving and risk-neutral subjects are pooled, since less than 20% of all subjects are risk-loving. Moreover, risk-neutral and risk-loving subjects show fairly similar behavior, such that this pooling does not affect the results. Details are available from the authors upon request.

[19]In fact, the difference across treatments for the class of risk-neutral and risk-loving subjects amounts to 20%, which is relatively close to the 25% difference predicted by theory.

Figure 6.4: Individual Effort by Stage, Decision Round, and Treatment



(a) Stage-1 Effort



(b) Stage-2 Effort

higher in **MP** than in **SP** (304.344 versus 291.764). This suggests that risk-attitudes are a potential explanation for the result that the difference in total effort provision across treatments is insignificant in the aggregate. It seems that risk-averse subjects value the insurance provided by the runner-up prize in **MP** higher than risk-neutral and risk-loving subjects, while the higher prize for the overall winner in **SP** is especially attractive for risk-neutral and risk-loving subjects. Figure 6.5(a) shows how this effect evolves over the rounds in the two treatments: Initially, total effort provision by risk-averse subjects is higher in **SP** than in **MP**. Subsequently, total effort provision declines much faster in **SP** than in **MP**, however, and in the second half of the experimental sessions, total effort is always higher in the **MP** treatment. Figure 6.5(b) shows that the pattern is more stable for the class of risk-neutral and risk-loving subjects, who consistently provide more effort in the single-prize than in the multiple-prizes treatment.

Over-provision of Effort. As mentioned at the beginning of this section, we observe a substantial amount of effort over-provision relative to the theoretical prediction, with total effort in the experimental session being between 70% and 90% higher than predicted. This finding complements earlier evidence on over-provision in contest experiments – see Davis and Reilly (1998), Gneezy and Smorodinsky (2006), or Sheremeta (2010), for instance.[20] Several explanations have been put forward in the literature to explain this phenomenon. First, the endowment that experimental subjects receive at the beginning of each decision round may lead to over-provision if subjects perceive the endowment as 'play money' (Thaler and Johnson 1990). In this case, subjects provide more effort due to this perception than they would without an endowment. In line with this argument, observed effort choices in experiments without endowments are often much closer to the

[20] Sheremeta (2010), for example, reports similar degrees of over-provision. In his single-prize treatment with two stages, which is almost identical to our **SP** treatment, total effort is on average almost 90% higher than theory predicts.

Table 6.3: Results by Risk Attitude

	risk-averse		*risk-neutral/loving*	
	SP	**MP**	**SP**	**MP**
Stage-1 Effort (x_1^*)	33.649	51.710	27.330	39.830
	(4.391)	(7.355)	(3.223)	(2.747)
Stage-2 Effort (x_2^*)	78.584	48.752	89.226	42.078
	(7.449)	(4.641)	(4.391)	(3.505)
Total Effort ($\mathcal{E}$)	**291.764**	**304.344**	**287.772**	**243.476**

Note: The numbers for stage-1 and stage-2 effort denote session averages by risk averse or risk neutral/loving participants. In **SP**, 21 (30) subjects are risk-averse (-neutral/loving), compared to 31 risk-averse and 56 risk-neutral/loving subjects in **MP**. Total effort is the sum of individual efforts (in experimental currency, Taler). Standard errors in parentheses.

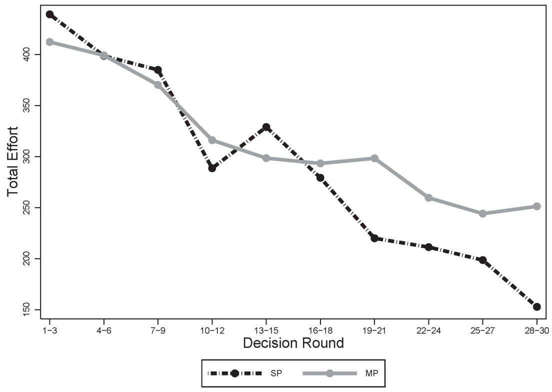
theoretical prediction.[21] In our experiments, we explicitly decided to use endowments to avoid negative payoffs for the losers of a contest and the associated problem of limited liability. Arguably, we could also have solved this issue through additional prizes for the losers, as in Altmann, Falk, and Wibral (2012). Then, however, the contrast between a single- and a multiple-prizes treatment, which is central for our research question, would be less clear. A second explanation for over-provision is that subjects experience a 'joy of winning' in strategic interactions, which amplifies the valuation of prizes awarded in contests. Since individual efforts are strictly increasing in the prizes at stake, non-monetary values of winning can rationalize over-provision of effort. Sheremeta (2011) experimentally elicits a measure for the 'joy of winning' and finds that it is highly correlated with the amount of effort provided by individual subjects. This supports the hypothesis that the 'joy of winning' is at least partly responsible for over-provision relative to the benchmark. According to Potters, de Vries, and van Winden (1998), a third explanation for over-provision might be that experimental subjects are prone to make mistakes in experimental settings. If this is the case, a higher endowment increases the chance to make mistake. Sheremeta (2010) varies the endowment and finds evidence that is in line with this argument.[22]

It is important to note that none of these potential explanations for over-provision predicts a systematic difference between the two treatments contrasted here, since the 'joy of winning' is unlikely to differ systematically across treatments, and both the en-

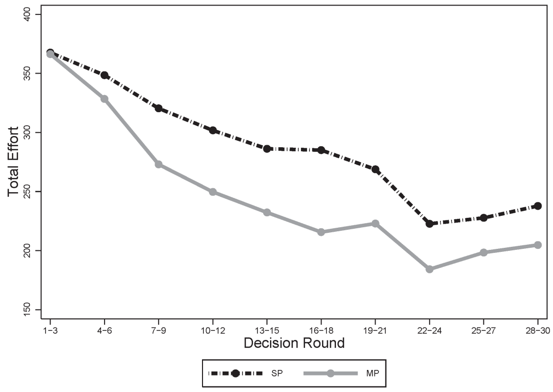
[21]See Altmann, Falk, and Wibral (2012), for example.

[22]Sheremeta (2010) uses the concept of a Quantal Response Equilibrium (QRE) by McKelvey and Palfrey (1995), which allows for mistakes of decision makers. He finds that a reduction of the endowment causes a proportional reduction of total effort, even if the endowment is not binding for equilibrium effort levels. Low (though non-binding) endowments even lead to under-provision of effort.

Figure 6.5: Total Effort by Risk Attitude, Decision Round, and Treatment



(a) Risk-Averse



(b) Risk-Neutral (or -Loving)

dowment and the overall amount available for prizes are identical in the two treatments we consider.[23]

6.5 Conclusion

This paper has tested the impact of variations in the prize structure on effort decisions in dynamic contests. Specifically, we have compared two prize-structures: A "winner-takes-all" setting that is predicted to maximize total effort, and a structure with multiple prizes which is predicted to ensure incentive maintenance across stages. We have tested (i) whether total effort is indeed higher in the single-prize treatment; (ii) whether incentive maintenance is observed in the multiple-prizes treatment; and (iii) whether a runner-up prize increases stage-1 and decreases stage-2 efforts as theory predicts. We found strong evidence in support of (ii) and (iii). The evidence for (i) – that total effort is higher in the single-prize treatment – is mixed at best: Even though total effort is somewhat higher in the single-prize than in the multiple-prizes treatment, the difference across treatments is less pronounced than predicted by theory and statistically insignificant. When controlling for risk-attitudes of experimental subjects, our evidence suggests that risk-averse subjects value the insurance effect of the runner-up prize in the multiple-prizes treatment and consequently provide more effort in that environment than in a contest with a single prize. At the same time, the behavior of risk-neutral and risk-loving subjects is qualitatively in line with the theoretical prediction, which explains the mixed findings in this dimension in the aggregate. Overall, our results indicate that the format with multiple prizes does not perform substantially worse in the total effort dimension, and significantly better

[23]The explanation based on errors would only be an issue if, e.g., the endowment were to bind more often in one than in the other treatment. However, the share of experimental subjects who spend their entire endowment is very low and does not systematically differ between the two treatments. If we exclude, for instance, all observations in which the endowment is binding, total output is somewhat lower in both treatments, but the qualitative findings remain unchanged. Details are available upon request.

in terms of eliciting constant effort across different stages of the contest. Our findings also suggest a more systematic investigation of the role of risk attitudes for behavior in dynamic contests as a fruitful direction for future research.

Appendix

6.A Experimental Instructions

The experimental instructions consist of three parts: First, experimental subjects receive some general information about the experimental session. Then, they are informed about the main treatment (Experiment 1), which is either the **SP** or the **MP** specification (both versions are provided). Finally, subjects receive instructions for the elicitation of risk attitudes (Experiment 2).

WELCOME TO THIS EXPERIMENT AND THANK YOU FOR YOUR PARTICIPATION

General Instructions:

You will participate in 2 different experiments today. Please stop talking to any other participant of this experiment from now on until the end of this session. In each of the two experiments, you will have to make certain decisions and may earn an appreciable amount of money. Your earnings will depend upon several factors: on your decisions, on the decisions of other participants, and on random components, i.e. chance. The following instructions explain how your earnings will be determined.

The experimental currency is denoted **Taler**. In addition to your Taler earnings in experiments 1 and 2, you receive 3 EURO show-up fee. You may increase your Taler earnings in experiments 1 and 2, where 2 Taler equal 1 Euro-Cent, i.e.

200 Taler correspond to 1 Euro.

At the end of this experimental session your Taler earnings will be converted into Euro and paid to you in cash.

Before the experimental session starts, you receive a card with your participant number. All your decisions in this experiment will be entered in a mask on the computer, the same holds for all other participants of the experiment. In addition, the computer will determine the random components which are needed in some of the experiments. All data collected in this experiment will be matched to your participant number, **not** to your name or student number. Your participant number will also be used for payment of your earnings at the end of the experimental session. Therefore, your decisions and the information provided in the experiments are completely anonymous; neither the experimenter nor anybody else can match these data to your identity.

We will start with experiment 1, followed by experiment 2. The instructions for experiment 2 will only be distributed right before this experiment starts, i.e. subsequent to experiment 1.

You will receive your earnings in cash at the end of the experimental session.

<u>Experiment 1</u> [*SP Treatment*]

Overall, there are 30 decision rounds with two stages each in Experiment 1. The course of events is the same in each decision round. You will be randomly and anonymously placed into a group of **four participants** in each round, and the identity of participants in your group changes with each decision round.

Course of events in an arbitrary decision round

All four participants of each group receive an **endowment of 240 Taler** at the beginning of a decision round. The endowment can be used to buy a certain amount of balls in two subsequent stages of a decision round. It is important to note that you receive one endowment only which must suffice to buy balls in both stages. The costs for the purchase of a ball are the same for all participants: Participants have to pay **1.00 Taler** for each ball they buy in stage 1 **or** stage 2, i.e.

1 ball costs 1.00 Taler
2 balls cost 2.00 Taler
(and so on)

When deciding how many balls you want to buy, you do not know the decision of other participants. Also, your decision is not revealed to any other participant.

All interactions in the experiment are pair-wise. Assume that you are in one group with participant A, participant B, and participant C. Then, you interact with participant A in stage 1, while participants B and C simultaneously meet each other in the second stage 1 interaction. If you reach stage 2, you will interact either with participant B or C, depending on the outcome in the second stage 1 interaction. In stage 1, there are two ballot boxes:

- all balls bought by you or participant A are placed in ballot box 1
- all balls bought by participants B and C are placed in ballot box 2

One ball is randomly drawn from each ballot box, and each ball drawn with the same probability. The two participants whose balls are drawn from ballot box 1 and 2, respectively, reach stage 2; the decision round is over for the other two participants (whose balls were not drawn), i.e. they drop out from this decision round. Any participant has to pay the balls he or she bought in stage 1, whether or not he/she reached stage 2. The respective amount is deducted from the endowment.

The two participants who reached stage 2 do again buy a certain number of balls, using whatever remains from the endowment they received after costs for balls in stage 1 were deducted. The balls are then placed into ballot box 3. One ball is randomly drawn from ballot box 3. The participant whose ball is drawn receives a prize of **240 Taler**. The other participants do not receive any prize in this decision round. Independent of whether or not a participant receives the prize, he/she does always have to pay for the balls bought in stage 2.

Let's take a closer look at the random draw of balls from ballot boxes. Assume, for example, that all balls which you bought are green colored, and that you interact with participant A in stage 1. Then, the probability that one of your balls is drawn (such that you make it to stage 2) satisfies

$$\mathbf{probability(green\ ball\ is\ drawn)} = \frac{\mathbf{\#\ green\ balls}}{\mathbf{\#\ green\ balls + \#\ balls\ by\ participant\ A}}$$

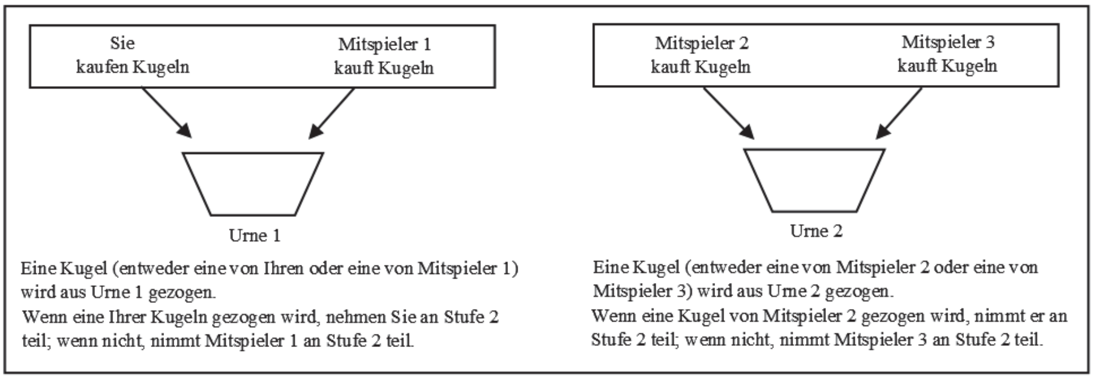
where # is short for number. The same probability rule does also hold for other participants in your group. Consequently, the probability that one of your balls in drawn is higher

- the more balls you purchased
- the less balls the other participant with whom you interact purchased.

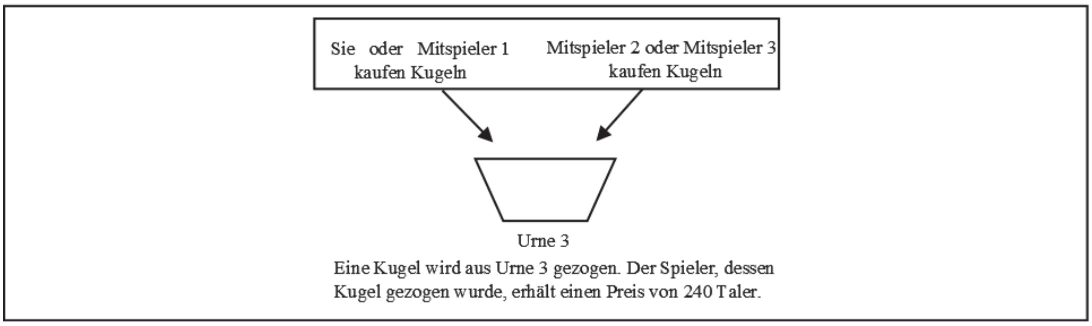
The random draw is simulated by the computer according to the procedures outlined above. If both participants of a pairing choose to buy zero balls, each participant wins with a probability of 50%.

Jeder Spieler erhält eine Anfangsausstattung von 240 Talern. Er muss damit alle von ihm in Stufe 1 und Stufe 2 gekauften Kugeln bezahlen.

STUFE 1



STUFE 2



Your Payoff

Assume that you bought "**X1**" balls in stage 1, and that you buy "**X2**" balls whenever you reach stage 2. Then, there are three possibilities for your payoff:

1) None of your balls is drawn in stage 1

$$\textbf{Your Payoff} = \textbf{endowment} - \textbf{X1} * \textbf{1 Taler}$$
$$= \textbf{240 Taler} - \textbf{X1} * \textbf{1 Taler}$$

2) one of your balls is drawn from the ballot box in stage 1; in stage 2, none of your balls is drawn

$$\textbf{Your Payoff} = \textbf{endowment} - \textbf{X1} * \textbf{1 Taler} - \textbf{X2} * \textbf{1 Taler}$$
$$= \textbf{240 Taler} - \textbf{X1} * \textbf{1 Taler} - \textbf{X2} * \textbf{1 Taler}$$

3) one of your balls is drawn from the ballot box in stage 1; also, one of your balls is drawn in stage 2

$$\textbf{Your Payoff} = \textbf{endowment} - \textbf{X1} * \textbf{1 Taler} - \textbf{X2} * \textbf{1 Taler} + \textbf{prize}$$
$$= \textbf{240 Taler} - \textbf{X1} * \textbf{1 Taler} - \textbf{X2} * \textbf{1 Taler} + \textbf{240 Taler}$$

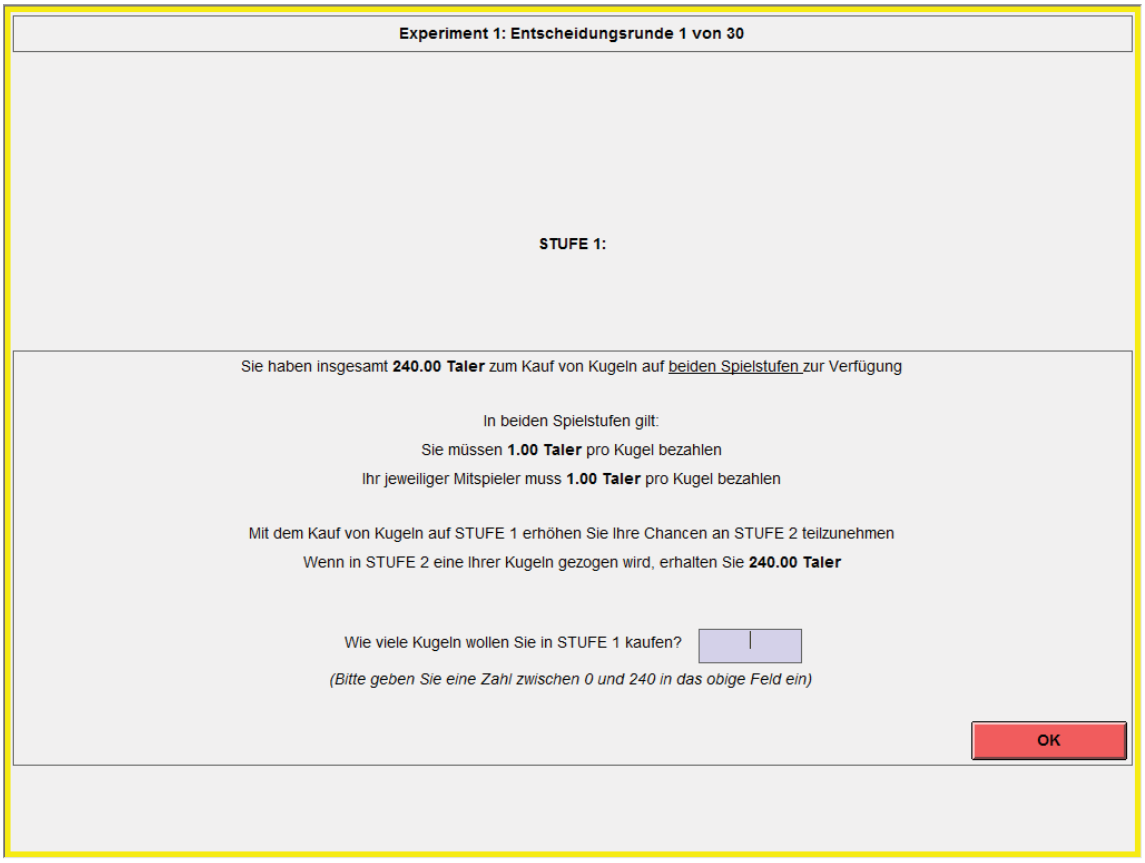
Therefore, your payoff is determined by the following components: by the number of balls you buy in stage 1 ("**X1**"); by the number of balls you buy in stage 2 ("**X2**") if you reach it; by up to two random draws (one of your balls is drawn/not drawn in stage 1 and potentially stage 2). The same holds for any other participants of the experiment.

Information:

- After you made your decision in stage 1, you are informed whether or not you can participate in stage 2, i.e. whether or not one of your balls was drawn from ballot box 1.
- If you did not reach stage 2, you are informed about how many balls participant A bought in stage 1.

- If you reach stage 2, you receive information about the remaining endowment (after costs for the purchase in stage 1 are deducted.
- After you made your decision in stage 2, you learn whether or not one of your balls was drawn from ballot box 3 and how many balls the participants who you met in stages 1 and 2, respectively, bought. Further, you learn your payoff for the respective decision round.

Decision: In each of the 30 decision rounds you have to decide how many balls you want to buy in stage 1. If you reach stage 2, you face a similar decision in stage 2. In both cases, you have to enter a number into a field on the computer screen. An example of the decision screen in stage 1 is shown below.



Your Total Payoff: Four out of 30 decision rounds are paid. These rounds are randomly determined, i.e., the probability that some decision round is paid is identical ex-ante for all 30 decision rounds. You will receive the sum of payoffs for the respective decision rounds.

Remember:
You receive an endowment of 240 Taler at the beginning of each decision round and have to decide how many balls you want to buy in stage 1; if you reach stage 2, you have to decide again. Overall, there are three additional participants in each group who face the same problem. The identity of these participants is randomly determined in each decision round. Every participant has to pay **1.00 Taler** for each ball he/she buys in stage 1 **or** stage 2.

If you have any questions, please raise your hand now!

<u>Experiment 1</u> [*MP Treatment*]

Overall, there are 30 decision rounds with two stages each in Experiment 1. The course of events is the same in each decision round. You will be randomly and anonymously placed into a group of **four participants** in each round, and the identity of participants in your group changes with each decision round.

Course of events in an arbitrary decision round

All four participants of each group receive an **endowment of 240 Taler** at the beginning of a decision round. The endowment can be used to buy a certain amount of balls in two subsequent stages of a decision round. It is important to note that you receive one endowment only which must suffice to buy balls in both stages. The costs for the purchase of a ball are the same for all participants: Participants have to pay **1.00 Taler** for each ball they buy in stage 1 **or** stage 2, i.e.

1 ball costs 1.00 Taler
2 balls cost 2.00 Taler
(and so on)

When deciding how many balls you want to buy, you do not know the decision of other participants. Also, your decision is not revealed to any other participant.

All interactions in the experiment are pair-wise. Assume that you are in one group with participant A, participant B, and participant C. Then, you interact with participant A in stage 1, while participants B and C simultaneously meet each other in the second stage 1 interaction. If you reach stage 2, you will interact either with participant B or C, depending on the outcome in the second stage 1 interaction. In stage 1, there are two ballot boxes:

- all balls bought by you or participant A are placed in ballot box 1
- all balls bought by participants B and C are placed in ballot box 2

One ball is randomly drawn from each ballot box, and each ball drawn with the same probability. The two participants whose balls are drawn from ballot box 1 and 2, respectively, reach stage 2; the decision round is over for the other two participants (whose balls were not drawn), i.e. they drop out from this decision round. Any participant has to pay the balls he or she bought in stage 1, whether or not he/she reached stage 2. The respective amount is deducted from the endowment.

The two participants who reached stage 2 do again buy a certain number of balls, using whatever remains from the endowment they received after costs for balls in stage 1 were deducted. The balls are then placed into ballot box 3. One ball is randomly drawn from ballot box 3. The participant whose ball is drawn receives the main prize of **168 Taler**. The other participant of stage 2, whose ball is not drawn from ballot box 3, receives a runner-up prize of **72 Taler.** Independent of the prize which a stage 2 participant receives, he/she does always have to pay for the balls bought in stage 2. Participants who did not reach stage 2 do not receive any prize.

Let's take a closer look at the random draw of balls from ballot boxes. Assume, for example, that all balls which you bought are green colored, and that you interact with participant A in stage 1. Then, the probability that one of your balls is drawn (such that you make it to stage 2) satisfies

$$\textbf{probability(green ball is drawn)} = \frac{\textbf{\# green balls}}{\textbf{\# green balls + \# balls by participant A}}$$

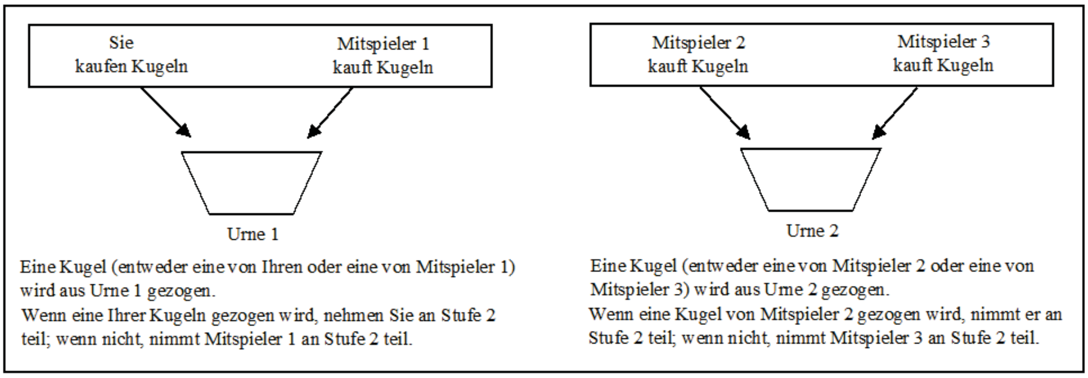
where # is short for number. The same probability rule does also hold for other participants in your group. Consequently, the probability that one of your balls in drawn is higher

- the more balls you purchased
- the less balls the other participant with whom you interact purchased.

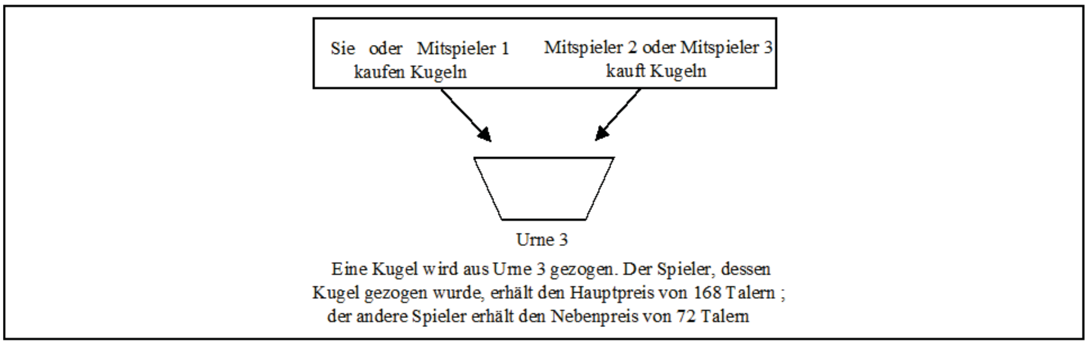
The random draw is simulated by the computer according to the procedures outlined above. If both participants of a pairing choose to buy zero balls, each participant wins with a probability of 50%.

Jeder Spieler erhält eine Anfangsausstattung von 240 Talern. Er muss damit alle von ihm in Stufe 1 und Stufe 2 gekauften Kugeln bezahlen.

STUFE 1



STUFE 2



Your Payoff

Assume that you bought "**X1**" balls in stage 1, and that you buy "**X2**" balls whenever you reach stage 2. Then, there are three possibilities for your payoff:

1) None of your balls is drawn in stage 1

$$\begin{aligned}\textbf{Your Payoff} &= \textbf{endowment} - \textbf{X1} * \textbf{1 Taler}\\ &= \textbf{240 Taler} - \textbf{X1} * \textbf{1 Taler}\end{aligned}$$

2) one of your balls is drawn from the ballot box in stage 1; in stage 2, none of your balls is drawn

$$\begin{aligned}\textbf{Your Payoff} &= \textbf{endowment} - \textbf{X1} * \textbf{1 Taler} - \textbf{X2} * \textbf{1 Taler} + \textbf{runner up prize}\\ &= \textbf{240 Taler} - \textbf{X1} * \textbf{1 Taler} - \textbf{X2} * \textbf{1 Taler} + \textbf{72 Taler}\end{aligned}$$

3) one of your balls is drawn from the ballot box in stage 1; also, one of your balls is drawn in stage 2

$$\begin{aligned}\textbf{Your Payoff} &= \textbf{endowment} - \textbf{X1} * \textbf{1 Taler} - \textbf{X2} * \textbf{1 Taler} + \textbf{main prize}\\ &= \textbf{240 Taler} - \textbf{X1} * \textbf{1 Taler} - \textbf{X2} * \textbf{1 Taler} + \textbf{168 Taler}\end{aligned}$$

Therefore, your payoff is determined by the following components: by the number of balls you buy in stage 1 ("**X1**"); by the number of balls you buy in stage 2 ("**X2**") if you reach it; by up to two random draws (one of your balls is drawn/not drawn in stage 1 and potentially stage 2). The same holds for any other participants of the experiment.

Information:

- After you made your decision in stage 1, you are informed whether or not you can participate in stage 2, i.e. whether or not one of your balls was drawn from ballot box 1.
- If you did not reach stage 2, you are informed about how many balls participant A bought in stage 1.

- If you reach stage 2, you receive information about the remaining endowment (after costs for the purchase in stage 1 are deducted.
- After you made your decision in stage 2, you learn whether or not one of your balls was drawn from ballot box 3 and how many balls the participants who you met in stages 1 and 2, respectively, bought. Further, you learn your payoff for the respective decision round.

Decision: In each of the 30 decision rounds you have to decide how many balls you want to buy in stage 1. If you reach stage 2, you face a similar decision in stage 2. In both cases, you have to enter a number into a field on the computer screen. An example of the decision screen in stage 1 is shown below.

Your Total Payoff: Four out of 30 decision rounds are paid. These rounds are randomly determined, i.e., the probability that some decision round is paid is identical ex-ante for all 30 decision rounds. You will receive the sum of payoffs for the respective decision rounds.

Remember:
You receive an endowment of 240 Taler at the beginning of each decision round and have to decide how many balls you want to buy in stage 1; if you reach stage 2, you have to decide again. Overall, there are three additional participants in each group who face the same problem. The identity of these participants is randomly determined in each decision round. Every participant has to pay **1.00 Taler** for each ball he/she buys in stage 1 **or** stage 2. Two prizes are awarded: the main prize of 168 Taler for the participant whose ball is drawn in stage 2, and the runner-up prize of 72 Taler for the other participant of the stage 2 interaction.

If you have any questions, please raise your hand now!

Experiment 2

In Experiment 2, you will face **21 decisions**. Each decision is a **choice between option 1 and option 2**. Each choice affects you own payoff, but not the payoff of any other participant of the experiment. When choosing option 1, your payoff is affected by chance, while option 2 implies a certain payment. You may be asked, for example, whether you prefer option 1, in which you receive either 400 Taler or 0 Taler with a 50% chance, or if you rather like option 2, which implies a sure payoff of c Taler. In the experiment, you will have to choose the option you prefer. This decision problem would be presented to you as follows:

Option 1	**Option 2**	Your Choice
with 50% probability **400 Taler** with 50% probability **0 Taler**	with certainty ***c* Taler**	Option 1 ◯ ◯ Option 2

As previously mentioned, you will encounter 21 decision problems of this kind. Your payoff from Experiment 2 is determined as follows:
At the end of all experiments, one of the 21 decision problems will be randomly chosen for each experimental participant. The option you chose in this decision problem determines your payoff. Assume, for example, that the previous example is chosen for you, and that you preferred option 1 over option 2. Then, you would receive 400 Taler or 0 Taler, each with a probability of 50%. Whether you receive 400 Taler or 0 Taler is determined by a simulated random draw of the computer.

Chapter 7

Timing Effects in Dynamic Elimination Contests: Immediate versus Delayed Rewards

This chapter is based on joint work with Rudolf Kerschbamer from the University of Innsbruck and Uwe Sunde from the University of St. Gallen (Stracke, Kerschbamer, and Sunde 2012).

7.1 Introduction

Strategic decisions of economic agents are often made in a contest environment – think of spending choices for R&D or advertising by firms, of election campaigns in politics, or even of employees' effort choices in consulting firms. In all these situations, agents compete by expending valuable resources to win a reward, such as a patent, market shares, a promotion, and so forth. Abstracting from the particularities of applications and focusing on the strategic decision, contest theory has extensively studied the effect of different modelling choices with respect to the number of participants, the number of rewards, or the structure of the competition, for example, on equilibrium behavior of contestants. These theoretical investigations help to better understand observed behavior by decision makers, and to structure contests in such a way that they serve the designer's objective, which is often the maximization (or the minimization) of overall contest investments by participants. Many theoretical predictions were recently tested empirically, using both data from the lab and from the field.

In this paper, we use experimental methods to study which effect the timing of rewards has on the behavior of agents in two-stage pair-wise elimination contests. Comparing a treatment where agents receive an *immediate* reward for winning stage 1 (**IR**) with a specification where the reward for winning stage 1 is only awarded after the stage-2 interaction is completed (**DR**), we find that stage-1 effort choices by experimental subjects are higher in the delayed than in the immediate reward treatment, while effort provision in stage 2 does not differ between treatments. Contest theory predicts that the two treatments are strategically identical in both stages if agents are risk neutral, or if agents *jointly* evaluate the payoff of both interactions. Therefore, it comes as a surprise that average differences of stage-1 effort choices between treatments are fully explained by risk attitudes: While the stage-1 effort choices of risk averse subjects in the delayed are much higher than in the immediate reward treatment, there is no difference across treatments for risk neutral subjects. This pattern can only be rationalized if experimental subjects *separately* evaluate the payoff of each stage. In this case, delayed rewards provide an insurance for risk averse decision makers, such that the effort choice in stage 1 should indeed differ across treatments for risk averse, but not for risk neutral decision makers. Generally speaking, our findings suggest that the presence of risk in future periods generates an effect that works against standard discounting due to impatience. Consequently, time and risk preferences jointly determine optimal behavior in inter-temporal, risky environments. This finding implies that it will often be hard to disentangle the two components outside the lab.

The paper contributes to the empirical literature on behavior in dynamic contests. While much of the traditional experimental literature concentrates on the simplest case with two contestants (Bull, Schotter, and Weigelt 1987, Harbring and Lünser 2008), recent work by Sheremeta (2010) and Altmann, Falk, and Wibral (2012) considers the two-stage contest with four participants. Yet, both papers keep the structure of rewards fixed across treatments and compare strategically equivalent static and dynamic contests instead. The only experimental paper which systematically varies rewards in a dynamic contest is our companion paper Stracke, Höchtl, Kerschbamer, and Sunde (2012), in which we compare a winner-takes-all contest and a treatment with runner-up prize. Using data from the field, a similar question was previously addressed by Delfgaauw, Dur, Non, and Verbeke (2012).[1] This treatment variation, however, changes the amounts that subjects receive for certain outcomes, while the present paper leaves total amounts unaffected. Instead, we investigate whether or not it matters for subjects that their reward is delayed.

The remainder of this paper is organized as follows: Section 7.2 derives the theoretical benchmark for a simple dynamic contest model. Section 7.3 outlines the experimental design and derives our main hypotheses. The experimental results are presented and discussed in Section 7.4. Section 7.5 concludes.

[1]To be precise, Delfgaauw, Dur, Non, and Verbeke (2012) consider two multiple prizes treatments with different runner-up prizes, rather than one treatment with multiple prizes and one winner-takes-all contest.

7.2 A Simple Dynamic Elimination Contest

We consider a simple two-stage pair-wise elimination contest with four identical agents. There are two pair-wise interactions in the first stage of this contest, and one additional interaction between the two winners of stage 1 in the second stage. In each of these three interactions, two risk-neutral agents independently choose the optimal level of effort provision such that their expected payoff is maximized. The effort provided by agent $i \in \{1,2,3,4\}$ in stage $s \in \{1,2\}$ is denoted x_{si}. For each unit of effort provided, agents incur constant marginal costs of one. The benefit is that the probability to win an interaction is increasing in own effort provision. Thus, agents face a trade-off. For simplicity, we assume that the probability to win is given by a lottery contest success function á la Tullock (1980). That is, given individual efforts x_{si} and x_{sj} by agents i and j in stage s, the probability that agent i wins in stage s equals

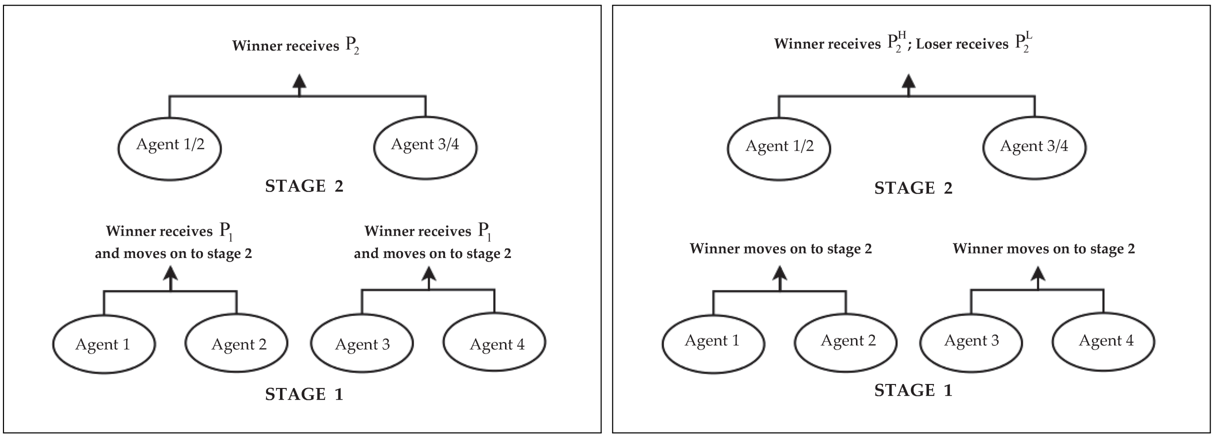
$$p_i(x_{si}, x_{sj}) = \begin{cases} \frac{x_{si}}{x_{si}+x_{sj}} & \text{if} \quad x_{si} + x_{sj} > 0 \\ \frac{1}{2} & \text{if} \quad x_{si} = x_{sj} = 0 \end{cases} .$$

The formal expression shows that the probability to win is increasing in own effort and decreasing in the effort provided by the opponent.

We compare two variants of the two-stage elimination contest, which are both displayed in Figure 7.1: In stage 1 of the specification depicted in Panel (a), agents 1 and 2 as well as agents 3 and 4 compete for a prize P_1 and for the right to move on to stage 2. Participation in stage 2 is valuable, since the winner of the overall contest receives the prize P_2 in addition to P_1. Since winning stage 1 is immediately rewarded with the prize P_1, this is the "Immediate Reward" (**IR**) specification. Panel (b) illustrates the details of the second setting we consider, the so-called "Delayed Reward" (**DR**) specification. In this case, there is no immediate reward for winning stage 1; instead, the loser of the stage-2 interaction receives the prize P_2^L, while P_2^H is awarded to the overall winner of the contest. Since both stage-2 participants won their stage-1 interaction, and any participant of stage 2 knows for sure that he/she will at least receive P_2^L, this prize can be understood as a delayed compensation for winning stage 1. In the remainder of this section, we will derive equilibrium efforts for agents in the **IR** and the **DR** specification, respectively. The equilibrium concept is subgame perfect Nash in both cases, since the contest is inherently dynamic. Therefore, we will start by solving the stage-2 interaction, before considering the parallel stage-1 pairings.

Solving Stage 2. Since all agents are identical by assumption, the identity of the agents who participate in stage 2 does not affect the solution. Therefore, it is assumed that agents i and j interact in stage 2 without loss of generality. The formal optimization problem

Figure 7.1: Structure of the Dynamic Contest



(a) Immediate Reward (**IR**)

(b) Delayed Reward (**DR**)

for agent i in the **IR** specification reads

$$\max_{x_{2i}\geq 0} \Pi_{2i}(x_{2i}, x_{2j}) = \frac{x_{2i}}{x_{2i} + x_{2j}} P_2 - x_{2i}$$

and leads to the first-order condition $x_{2j}P_2 - (x_{2i} + x_{2j})^2 = 0$. This condition is necessary and sufficient for the unique interior equilibrium.[2] Using symmetry leads to stage-2 equilibrium effort

$$x_2^*(\mathbf{IR}) \equiv x_{2i} = x_{2j} = \frac{P_2}{4}. \tag{7.1}$$

In stage 2 of the **DR** specification, agent i faces the optimization problem

$$\begin{aligned} \max_{x_{2i}\geq 0} \Pi_{2i}(x_{2i}, x_{2j}) &= \frac{x_{2i}}{x_{2i} + x_{2j}} P_2^H + \left(1 - \frac{x_{2i}}{x_{2i} + x_{2j}}\right) P_2^L - x_{2i} \\ &= \frac{x_{2i}}{x_{2i} + x_{2j}} [P_2^H - P_2^L] + P_2^L - x_{2i}. \end{aligned}$$

When combining the first-order optimality and the symmetry condition, we obtain the stage-2 equilibrium effort

$$x_2^*(\mathbf{DR}) \equiv x_{2i} = x_{2j} = \frac{P_2^H - P_2^L}{4}. \tag{7.2}$$

Finally, inserting equilibrium efforts in the respective objective functions gives expected stage-2 equilibrium payoffs for both specifications. We obtain

$$\Pi_2^*(\mathbf{IR}) = \frac{P_2}{4} \qquad \text{and} \qquad \Pi_2^*(\mathbf{DR}) = \frac{P_2^H + 3P_2^L}{4},$$

respectively, where $\Pi_2^* \equiv \Pi_{2i}^*(x_{2i}^*, x_{2j}^*) = \Pi_{2j}^*(x_{2i}^*, x_{2j}^*)$ and $\Psi_2^* \equiv \Psi_{2i}^*(x_{2i}^*, x_{2j}^*) = \Psi_{2j}^*(x_{2i}^*, x_{2j}^*)$. Under the assumption that future payoffs are not discounted, participation in stage 2

[2]See Perez-Castrillo and Verdier (1992) for details.

is worth Π_2^* in **IR** and Ψ_2^* in **DR** for agents who interact in stage 1 of the respective specification. Agents will take the value of a participation in stage 2 into account when choosing stage-1 effort x_{1i}.

Solving Stage 1. The identity of agents does not matter for the stage-1 solution, since all agents are identical by assumption. Therefore, it is without loss of generality to consider the interaction between agents k and l. Agent k in specification **IR** faces the optimization problem

$$\max_{x_{1k}\geq 0} \Pi_{1k}(x_{1k}, x_{1l}) = \frac{x_{1k}}{x_{1k}+x_{1l}}[P_1 + \Pi_2^*(\mathbf{IR})] - x_{1k}.$$

As previously mentioned, stage-1 winners in **IR** are immediately rewarded with the prize P_1. Moreover, they take the value of a participation in stage 2, Π_2^*, into account. Therefore, the overall value of winning stage 1 amounts to the sum $P_1 + \Pi_2^*(\mathbf{IR})$. The first-order and the symmetry condition $x_{1k}^* = x_{1l}^*$ jointly determine stage-1 equilibrium effort; inserting the formal expression for $\Pi_2^*(\mathbf{IR})$ gives

$$x_1^*(\mathbf{IR}) \equiv x_{1k}^* = x_{1l}^* = \frac{4P_1 + P_2}{16}. \tag{7.3}$$

The optimization problem in specification **DR** is slightly different, since stage-1 winners are not immediately rewarded. Instead, the only prize for winning stage 1 is the right to participate in stage 2. Consequently, the formal optimization problem of agent k reads

$$\max_{x_{1k}\geq 0} \Pi_{1k}(x_{1k}, x_{1l}) = \frac{x_{1k}}{x_{1k}+x_{1l}}\Pi_2^*(\mathbf{DR}) - x_{1k}.$$

In equilibrium, participants of this specification will choose stage-1 effort

$$x_1^*(\mathbf{DR}) \equiv x_{1k}^* = x_{1l}^* = \frac{(P_2^H + 3P_2^L)}{16}. \tag{7.4}$$

7.3 Design of the Experiments

Treatment Design. The two specifications **IR** and **DR** of the dynamic pair-wise elimination contest constitute our experimental treatments. It is assumed that 240 units of prize money are available in both treatments, i.e., $P_2 + 2P_1 = 240$ holds in the **IR** specification, while $P_2^H + P_2^L = 240$ must be satisfied in the **DR** treatment. We use these conditions to substitute for P_2 and P_2^H, respectively, in the formal expressions for stage-2 effort, which are provided in (7.1) and (7.2), respectively. As a result, we obtain stage-2 equilibrium efforts

$$x_2^*(\mathbf{IR}) = \frac{240 - 2P_1}{4} \quad \text{and} \quad x_2^*(\mathbf{DR}) = \frac{240 - 2P_2^L}{4}.$$

Interestingly, stage-2 efforts are identical across treatments under the assumption $P_1 = P_2^L$. This finding makes sense intuitively, as the two panels in Figure 7.1 and the subsequent discussion illustrate: In the **IR** specification, each of the two stage-1 winners already received the prize P_1 prior to the interaction in stage 2, which leaves the amount $240-2P_1$ for the prize P_2. Equilibrium effort provision amounts to a quarter of the gain for winning stage 2, P_2. Note that the gain for winning stage 2 is not equal to the prize P_2^H in the **DR** specification. The prize P_2^H amounts to $P_2^H = 240 - P_2^L$. Yet, both participants of stage 2 know that they have the amount P_2^L for sure, such that they only compete for the difference $P_2^H - P_2^L$. The prize for winning stage 2 is $240 - 2P_2^L$ units higher than the prize of losing, and in equilibrium, contest participants choose a quarter of this gain as their stage-2 effort.

When inserting the assumption that 240 units of prize money are available in each treatment into the formal expressions for stage-1 equilibrium effort, provided in (7.3) and (7.4), respectively, we obtain

$$x_1^*(\mathbf{IR}) = \frac{240 + 2P_1}{16} \quad \text{and} \quad x_1^*(\mathbf{DR}) = \frac{240 + 2P_2^L}{16}.$$

As for stage-1 efforts, we find that stage-2 efforts are also identical across treatments under the assumption $P_1 = P_2^L$. Intuitively, participants in the **IR** specification know that they have P_1 for sure when winning stage 1; they may gain P_2 in addition if they win stage 2. Similarly, the **DR** specification ensures that every stage-2 participant will at least receive P_2^L, while winning stage-2 delivers the additional gain $P_2^H - P_2^L$.

In addition to individual stage-1 and stage-2 efforts, we consider one additional incentive measure, namely total effort provision by all participants in both stages of the contest (denoted $\mathcal{E}$). When adding individual stage-1 effort by four agents and stage-2 effort provision by two agents, we obtain

$$\mathcal{E}^*(\mathbf{IR}) = \frac{720 - 2P_1}{4} \quad \text{and} \quad \mathcal{E}^*(\mathbf{DR}) = \frac{720 - 2P_2^L}{4}.$$

Since stage-1 and stage-2 efforts are identical across treatments for $P_1 = P_2^L$, it comes as no surprise that the same holds for the aggregate measures $\mathcal{E}^*(\mathbf{IR})$ and $\mathcal{E}^*(\mathbf{DR})$ under this assumption.

In the experimental treatments, we assume that $P_1 = P_2^L = 72$. Table 7.1 shows the resulting equilibrium predictions for this parametrization, which form the basis for the set of hypotheses which we will test in the remainder of this paper. First of all, the theoretical model predicts that total effort $\mathcal{E}$ should be the same in both treatments, which gives Hypothesis 7.1:

Hypothesis 7.1 (Total Effort Equality)**.** *Total effort provision by all four participants*

Table 7.1: Theoretical Prediction and Parametrization

	Immediate Reward (**IR**)	Delayed Reward (**DR**)
Total Effort ($\mathcal{E}^*$)	144	144
Stage-1 Effort (x_1^*)	24	24
Stage-2 Effort (x_2^*)	24	24
Rewards	P_1=72, P_2=96	P_2^L=72, P_2^H=168

in both stages is identical across treatments:

$$\mathcal{E}(\mathbf{IR}) = \mathcal{E}(\mathbf{DR}).$$

Even though total effort is a common measure of interest in contests, the comparison of this measure across treatments will not allow us to determine the reason for potential differences. Therefore, we do also consider individual efforts in both stages. Recall from the previous analysis that stage-1 and stage-2 effort are identical across treatments under the assumption $P_1 = P_2^L$, which is satisfied in the parametrization we consider (see Table 7.1):

Hypothesis 7.2 (Individual Effort Equality)**.** *Individual effort provision is identical across treatments in both stages:*

$$\begin{aligned} &(a) \qquad x_1(\mathbf{IR}) = x_1(\mathbf{DR}) \\ &(b) \qquad x_2(\mathbf{IR}) = x_2(\mathbf{DR}). \end{aligned}$$

Finally, the chosen parametrization allows us to test one additional hypothesis. As Table 7.1 shows, incentives for effort provision are maintained across stages, such that individual effort provision is identical across stages in each treatment. While Hypotheses 7.1 and 7.2 hold whenever $P_1 = P_2^L$, incentive maintenance can be observed only if 30% of the overall prize money are allocated to prizes P_1 and P_2^L, respectively. In fact, this is the reason why we choose $P_1 = P_2^L = 72$. Consequently, our last hypothesis is as follows:

Hypothesis 7.3 (Incentive Maintenance)**.** *Individual effort provision is identical across stages within each treatment:*

$$\begin{aligned} &(a) \qquad x_1(\mathbf{IR}) = x_2(\mathbf{IR}) \\ &(b) \qquad x_1(\mathbf{DR}) = x_2(\mathbf{DR}). \end{aligned}$$

Experimental Implementation. We ran a total of 10 computerized sessions with 20 participants each using the software z-Tree (Fischbacher 2007). All 200 participants were students from the university of Innsbruck, which were recruited with ORSEE (Greiner 2004). Each session lasted approximately 70 minutes (including distribution of instructions and payment at the end), and participants earned between 9-13 Euro (approximately 11 Euro on average).

We adopted a between-subject design, such that experimental subjects encountered either the "Immediate Reward" (**IR**) or the "Delayed Reward" treatment (**DR**). Each subject encountered the same contest game 30 times. We use the experimental currency "Taler", where 200 Taler equal 1.00 Euro. Effort provision was modeled by using an analogy between the chosen contest success function and a lottery: Participants were told that they could buy a discrete number of balls in each interaction.[3] The balls purchased by the subject as well as those purchased by their respective opponents were then said to be placed in the same ballot box, out of which one ball was randomly drawn subsequently. This setting reflects the experimental implementation of the ratio contest success function á la Tullock (1980) from the theoretical set-up. Players had to buy (and pay for) their desired number of balls before they knew whether or not they won a pair-wise interaction in the contest. Therefore, each participant received an ex-ante endowment of 240 Taler in each round to avoid limited liability problems. This endowment could be used to buy balls in both stages, i.e., a subject that reached stage 2 could use whatever remained of the endowment (after the costs for the desired number of balls in stage 1 were deducted) to buy balls in the stage-2 interaction. Subjects knew that the share of the endowment which they did not use to buy balls was added to the payoffs of that round. Therefore, purchasing balls implies real monetary costs. Since the endowment was as high as the sum of all prizes that were awarded in the contest, agents were not budget-constrained at any time. Experimental subjects were told that the endowment could only be used in a given round, such that the strategic interaction was the same in each (of all 30) decision round. Random matching ensured that the same participants did not interact repeatedly; matching groups corresponded to the entire session. After each decision round, participants were informed about their own decision, the decision of their immediate opponent in stage 1 and stage 2 (if applicable), and about their own payoff. This setting allows for an investigation of whether experimental subjects learn when completing the same task repeatedly. To avoid income effects, however, the participants were told that only four decision rounds (out of 30) would be chosen randomly and paid out at the end of the experiment.

The procedures in an experimental session were as follows for all treatments: First, the participants received some general information about the experimental session. Then, instructions for our main treatment, the two-stage contest with four agents described above, were distributed. After each participant confirmed that he/she had read the instructions on the computer screen, subjects had to answer a set of control questions correctly to ensure that they had fully understood the instructions. Only then did the first decision

[3] The chosen prizes ensured that equilibrium investments in all stages and both contests are positive integers, which implies that the discrete grid has no consequences for the equilibrium strategies; the equilibrium in pure strategies is unique in both treatments.

round start. Overall, each subject participated in 30 decision rounds which were identical. After the main treatment, we used a choice list similar to the one employed by Dohmen, Falk, Huffman, and Sunde (2010) to elicit risk attitudes.[4] Specifically, each subject was exposed to a series of 21 binary choices between a cash gamble and a safe payoff. While the cash gamble remained the same in all 21 binary choices – it always gave either 400 Taler or 0 Taler, each with 50 percent probability – the safe payoff increased in steps of 20 Taler from 0 Taler in the first choice to 400 Taler in the last choice. Given this design, a decision maker whose preferences satisfy ordering (completeness and transitivity) and strict monotonicity switches exactly once from the cash gamble to the safe payoff. For subjects who switch exactly once we use the first choice scenario in which the subject decides in favor of the save payoff as our measure of risk attitude (we do not classify subjects with multiple switching points). Finally, individual information with respect to certain socio-economic characteristics was collected in a questionnaire (non incentivized, voluntary participation). Only thereafter were participants informed about their payoff in the experimental session.

7.4 Results

Main Results. The main results of the experimental sessions are summarized in Table 7.2, which provides the average of (five independent) session means for total effort in each treatment, as well as means for stage-1 and stage-2 effort on the individual level. In addition, theoretical predictions from the model in section 7.2 are given. When comparing the experimental observations with the theoretical benchmark, we find that subjects provide much more effort than predicted by theory in both treatments, i.e., we observe a high degree of over-provision. Such substantial over-provision in contests is, however, not uncommon in experimental settings – see Stracke, Höchtl, Kerschbamer, and Sunde (2012) for an overview of potential explanations for this phenomenon.

The first important result is that total effort differs across treatments: In treatment **IR**, agents invest slightly more than 231 Taler (experimental currency units) on average in both stages of the contest, compared to almost 278 Taler in treatment **DR**. The difference between treatments is significantly different from zero, such that Hypothesis 7.1 can be rejected; the respective p-values are 0.029 for a t-test and 0.047 for a non-parametric 'Mann-Whitney-U'-test (MWU-test). Figure 7.2 plots a local polynomial smooth of total effort provision over the course of an experimental session in both treatments.[5] The figure shows two things: First, total effort provision is much lower in later than in early decision

[4]In the Dohmen, Falk, Huffman, and Sunde (2010) procedure, each subject is exposed to a series of choices between a safe payment (which is systematically varied) and a binary lottery (which remains constant across choices). This is cognitively simpler than the procedure employed by Holt and Laury (2002), where a subject is confronted with a series of choices between two binary lotteries that are both varied systematically. The instructions which experimental subjects received right before the risk-elicitation part are provided in the Appendix.

[5]We use an Epanechnikov kernel function, and a "rule-of-thumb" (ROT) bandwidth estimation.

Table 7.2: Experimental Results

	IR			DR		
	N	Data	Theory	N	Data	Theory
Total Effort ($\mathcal{E}^*$)	5	**231.879** (15.989)	144	5	**277.861** (6.796)	144
Stage 1 Effort (x_1^*)	100	**35.494** (1.874)	24	100	**45.238** (2.957)	24
Stage 2 Effort (x_2^*)	100	**43.589** (1.852)	24	100	**45.976** (2.614)	24

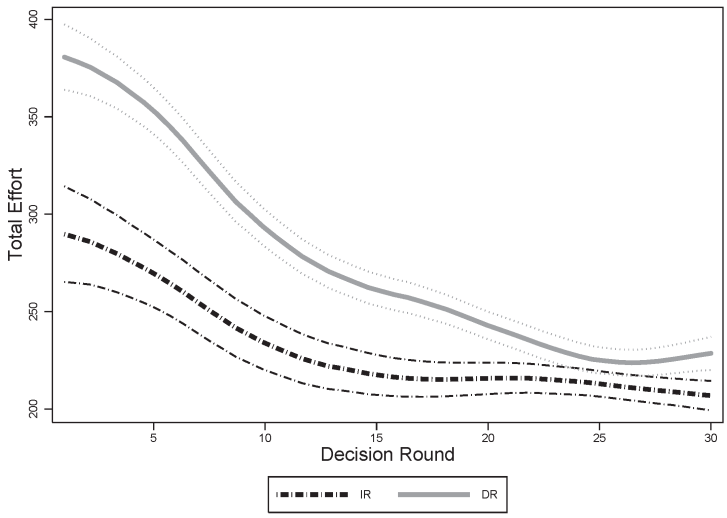
Note: The numbers in the columns "Data" denote (total) average effort observed in all rounds of the experimental sessions. Total effort is the sum of individuals efforts (in experimental currency, Taler). Standard errors in parentheses. The column "Theory" provides the theoretical equilibrium prediction for the respective effort measure.

rounds. With respect to the issue of over-provision previously discussed, this suggests that experimental participants learn that their initial effort choices where too high. However, even though over-provision is reduced, total effort is still way higher than theory predicts even in the very last decision round; the theoretical prediction is 144, and we observe more than 200. Second, Figure 7.2 shows that the difference across treatments is particularly strong in early decision rounds, decreasing until decision round 25, and then increasing again in the last few rounds. It is important to emphasize, however, that total effort provision is significant higher in the **DR** than in the **IR** in all decision rounds except for one, as the 95% confidence bounds show which the figure provides. This implies that the difference between the two treatments with respect to total effort is decreasing over time, but present in all decision rounds. Summing up, we find the following with respect to Hypothesis 7.1:

Result 7.1 (Total Effort Equality)**.** *The equality of total effort provision across treatments can be rejected; total effort provision is significantly higher in treatment **DR** than in treatment **IR**.*

This result immediately raises the question whether the difference across treatments in terms of observed total effort provision is due to unequal behavior in stage 1, in stage 2, or in both stages. The results in Table 7.2 clearly suggest that differences in stage 1 are the driving force of Result 7.1: While effort provision in stage 2 is very similar across treatments (45.976 in treatment **DR** as compared to 43.589 in treatment **IR**), stage-1 effort is much lower in the **IR** than in **DR** treatment. Formal testing confirms the relevance of the observed pattern: We can reject the equality of stage-1 efforts across treatments (p-value 0.006 both for the t-test and the MWU-test), while the small difference between

Figure 7.2: Total Effort by Decision Round



investments in stage 2 is insignificant (p-value 0.460 for the t-test, and 0.9698 for the MWU-test). Figure 7.3 confirms that the observed pattern is not only present in means over all decision rounds of the experimental treatment, but that the pattern is stable over the course of an experimental session: As panel (a) shows, the difference between stage-1 efforts is somewhat higher in early than in later decision rounds, but it can be observed in all periods. Panel (b) indicates that there is no systematic difference between stage-2 efforts across treatments; if anything, stage-2 efforts in treatment **DR** are marginally higher at the beginning, but even this small difference disappears over time. In addition, Figure 7.3 shows that all effort measures are decreasing over the course of an experimental session. This effect is stronger in the **DR** treatment, particularly in stage 1.

Summing-up, we reject Hypothesis 7.2 (a), while experimental evidence is well in line with Hypothesis 7.2 (b):

Result 7.2 (Individual Effort Equality)**.** *Individual effort equality across treatments can be rejected in stage 1, but not for stage 2. Average individual efforts are*

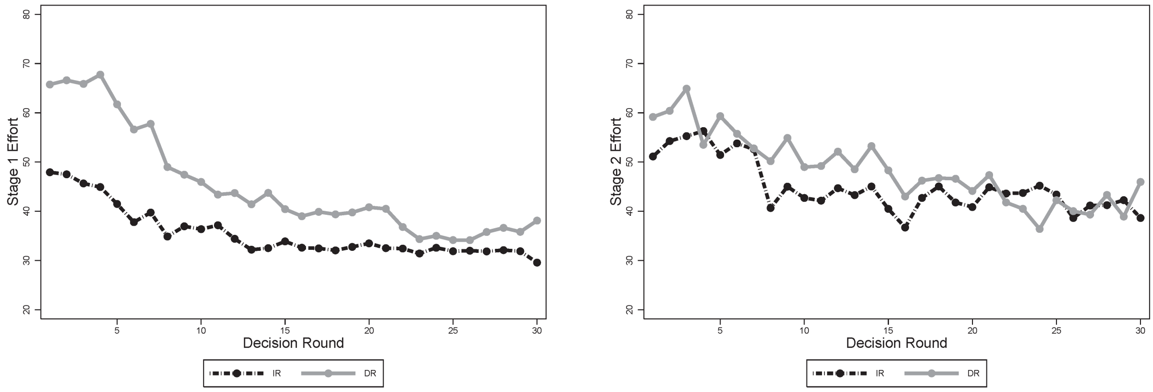
(a) significantly higher in stage 1 of the ***DR*** *than in stage 1 of the* ***IR*** *treatment.*

(b) practically identical in stage 2 of treatments ***IR*** *and* ***DR****.*

Interestingly, stage-1 effort is lower in the treatment with an immediate reward for winning stage 1 than in the setting where the reward is delayed. This suggests that we certainly do not observe discounting of future rewards. Rather, it seems that subjects prefer late over immediate rewards, which contradicts standard economic intuition.

Next, consider Hypothesis 7.3, according to which efforts are predicted to be the same across stages within each treatment. Table 7.2 shows that stage-1 and stage-2 effort choices differ substantially in treatment **IR** (35.494 vs. 43.589), while there is no difference in treatment **DR** (45.238 vs. 45.976). Testing shows that mean equality across stages can

Figure 7.3: Individual Effort by Stage and Decision Round



(a) Stage 1 Effort

(b) Stage 2 Effort

be rejected for treatment **IR** (p-value 0.002 for the t-test and 0.000 for the MWU-test), but not for treatment **DR**. Figure 7.4 shows that this pattern is comparably stable over the course of an experimental session. Panel (a) plots observed stage-1 and stage-2 effort for the **IR** treatment, and shows that average effort provision in stage 2 is higher than (average) effort provision in stage 1 in every decision round; the difference is particularly pronounced at the beginning of an experimental session. Similarly, panel (b) shows that stage-1 effort is slightly higher than stage-2 effort in treatment **DR** in early decision rounds, but in this case the order is reversed over the course of the experiment, such that stage-2 effort is marginally higher than stage-1 effort after decision round 7. Figure 7.4 also provides the prediction of the theoretical model for stage-1 and stage-2 equilibrium effort. When comparing theoretical predictions and observed effort choices, it becomes clear that observed effort choices are well above their equilibrium prediction in all decision rounds, event though stage-1 and stage-2 effort choices are decreasing over the course of an experimental session in both treatments. This suggests that experimental subjects realize that their initial effort choices are too high, but choices do still not converge towards equilibrium predictions as, for example, in Bull, Schotter, and Weigelt (1987). Overall, our findings with respect to the incentive maintenance Hypothesis are as follows:

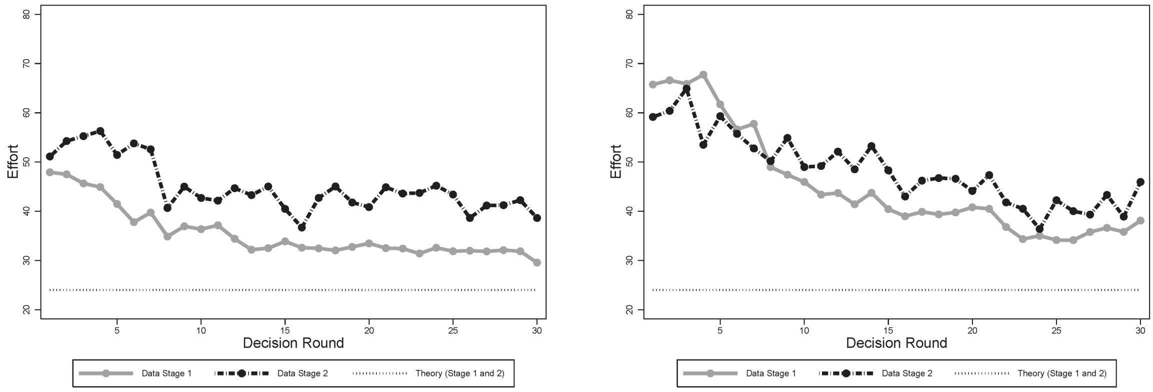
Result 7.3 (Incentive Maintenance)**.** *We can reject incentive maintenance across stages in treatment **IR**, but not in treatment **DR**. Average individual efforts are*

*(a) significantly lower in stage 1 than in stage 2 of treatment **DR**.*

*(b) almost identical in both stages of treatment **IR**.*

It should be mentioned that the incentive maintenance hypothesis in a two-stage contest has already been examined by Altmann, Falk, and Wibral (2012) in a delayed reward framework. However, they consider an experimental design where participants interact only once, while we repeat the same contest 30 times with random matching. If we

Figure 7.4: Individual Effort by Treatment and Decision Round



(a) Immediate Reward (**IR**) (b) Delayed Reward (**DR**)

consider the first decision round only, we can replicate the pattern they observe: In this decision round, subjects choose the effort 65.75 in stage 1 of the **DR** treatment, compared to 59.16 in stage 2 of this treatment.[6] However, this pattern disappears over time and is even reversed in later decision rounds, as previously discussed and depicted in Panel (b) of Figure 7.4.[7] Overall, it seems that the number of repetitions, as well as the timing of rewards for winning stage 1 determines whether or not incentive maintenance across stages can be observed in experimental treatments; our findings with respect to individual and total effort equality across treatments are, however, relatively stable over the course of the experimental session(s).

Behavioral Differences in Stage 1. As established in Result 7.2 (a), the effort choice of experimental subjects in stage 1 differs across treatments. To be precise, effort choices in the **IR** treatment are lower than in the **DR** treatment on average. This finding is contrary to standard economic intuition, which suggests that, if there is any difference at all, future rewards should be discounted, such that stage-1 effort choices are higher in the delayed reward than in the immediate reward case. Subsequently, we will investigate whether individual characteristics help to explain differential stage-1 choices across treatments.

As mentioned in the discussion of the experimental design, we elicit risk attitudes and ask subjects to fill out a questionnaire at the end of an experimental session.[8] Moreover, we count the number of mistakes when subjects answer a set of control questions prior to the first decision round.[9] Table 7.3 provides information on gender, field of study, the last math grade in school, risk attitudes and the number of mistakes when answering the

[6]This difference is significantly different from zero at the 5%-level.

[7]For details, see also our companion paper Stracke, Höchtl, Kerschbamer, and Sunde (2012), where we compare the **DR** treatment with a ”winner-takes-all“ treatment.

[8]Filling out the questionnaire was voluntary and non incentivized.

[9]The translated version of all control questions is provided in section B of the Appendix; the original German version is available from the authors upon request.

Table 7.3: Individual Characteristics by Treatment

	IR			DR		
	N	mean	S.D.	N	mean	S.D.
Econ Department	100	**0.47**	0.502	100	**0.46**	0.501
Gender (male=1)	100	**0.59**	0.494	100	**0.49**	0.502
Math Grade	96	**2.427**	1.129	98	**2.360**	1.115
Risk Attitude	90	**10.289**	4.453	87	**10.126**	4.220
Control Questions	100	**1.030**	1.087	100	**1.200**	1.287

control questions for each treatment.[10] The table indicates that there are slightly more male students in the **IR** than in the **DR** treatment; yet, the difference across treatments is insignificant.[11] Risk attitudes, the share of students from the econ department, the average math grade, and the number of incorrectly answered control questions are almost identical in both treatments. The absolute values suggest that subjects are weakly risk-averse, fairly good in mathematics, often from the econ department, and able to correctly answer the control questions.[12]

In a first step, we investigate whether the observed difference of stage-1 efforts is due to unclear instructions. Recall that control questions were used to ensure that each subject correctly understands the strategic situation. Therefore, we employ the number of incorrectly answered control questions to address this potential issue. Table 7.4 provides the means of stage-1 effort in both treatments for different sub-samples. However, excluding subjects who have problems with the control questions leaves the main result unaffected; stage-1 effort choices by experimental subjects are significantly higher in the delayed reward treatment even if only subjects who answer all questions correctly are considered.

Next, we investigate whether gender, risk attitudes, math grades or the field of study systematically affect the behavior of experimental subjects. To address this question, a regression analysis is performed; average stage-1 effort choices by experimental subjects over the course of an experimental session are regressed on the respective individual characteristic, on a treatment dummy, and on an interaction between the two. The

[10] In the questionnaire, we also collect information on nationality and the "Abitur"-grade. The nationality of students does not affect their decision; since the Abitur-grade is highly correlated with the math grade, we omit this information in the subsequent analysis. Details are available from the authors upon request.

[11] The p-value of a Mann-Whitney-U test is 0.1502.

[12] A switching point at 11 is optimal for risk-neutral individuals; therefore, values below (above) 11 indicate that a subject is risk averse (loving). Grades in mathematics are on a scale from 1 to 6, where 1 is the best grade.

Table 7.4: Stage-1 Effort by Treatment

	IR		DR		p-value
	N	mean	N	mean	MWU-test
all observations	100	**35.494**	100	**45.238**	0.006
# mistakes ≤ 3	99	**35.391**	93	**44.096**	0.013
# mistakes ≤ 2	96	**35.478**	90	**41.517**	0.018
# mistakes ≤ 1	73	**32.894**	70	**41.450**	0.024
# mistakes = 0	33	**28.510**	32	**44.000**	0.004

results are provided in Table 7.5: Specification (1) considers the effect of being a student from the econ department on stage-1 behavior. The variable *econ* equals one if a student is enrolled in the Econ Department, and zero otherwise. Interestingly, the level effect of *econ* is insignificant, whereas both the treatment dummy *DR* and the interaction term *econ*DR* are highly significant; the fact that both coefficients have opposing signs and are approximately of the same size suggests that the stage-1 difference across treatments can almost completely be explained by the field of study of experimental subjects. Figure 7.5 illustrate this finding graphically and show that the pattern is present in any decision round: There is hardly any observable difference across treatments **IR** and **DR** among the group of students from the econ department after the initial periods of a session, as panel (a) clearly shows. In contrast, panel (b) indicates that subjects from other departments choose much higher stage-1 effort levels in any decision round.

When considering the math score in specification (2) or the gender effect in specification (3) in Table 7.5, the estimated coefficients have the same pattern as for the *econ* dummy:[13] The treatment dummy *DR* is positive (i.e., subjects in the delayed reward treatment choose higher stage-1 efforts on average), while the interaction effects *mathsc*DR* and *male*DR*, respectively, have a negative sign, which implies that the difference across treatments is lower for male subjects and for subjects with a high math score, i.e., for students who are good in math.[14] Even though the effects are fairly stable across all decision rounds of an experimental session, as panels (c) to (f) in Figure 7.5 show, they are much less significant than when considering the field of study, however.[15]

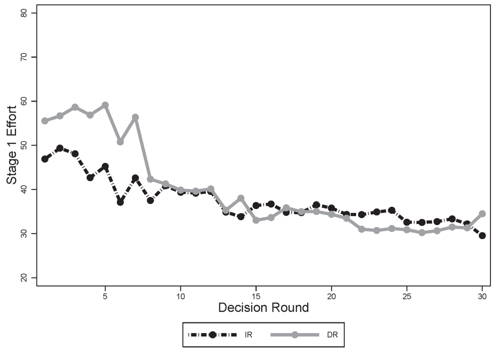
Specification (4) in Table 7.5 investigates whether risk attitudes of experimental subjects differentially affect stage-1 efforts in the two treatments. The employed risk measure

[13]Note that *mathsc* measures the math score rather than the math grade. The math score is defined as 6 – last math grade. We employ this normalization to make coefficients comparable across specifications.

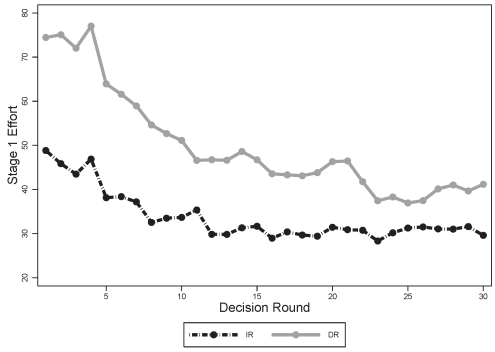
[14]The coefficient estimate is higher for the dummy variable *male* than for *mathsc* which allows for values between five and zero.

[15]For the graphical illustration of the math score, we split the sample at the mean. We refer to subjects above the mean as being good or better in math, while those below the mean are satisfactory or worse.

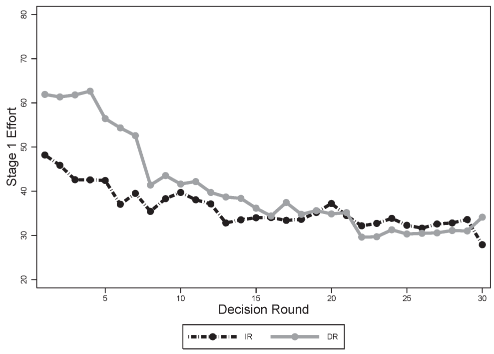
Figure 7.5: Stage-1 Effort by Decision Round



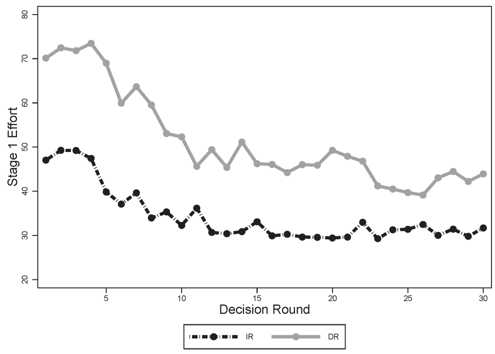
(a) Econ Department



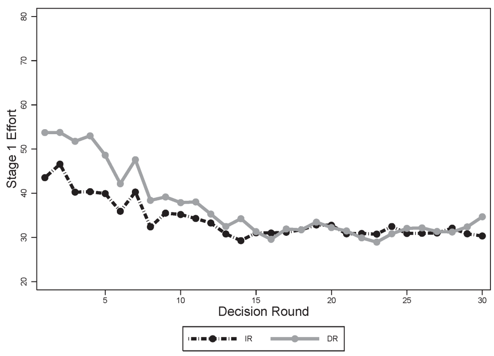
(b) Other Department



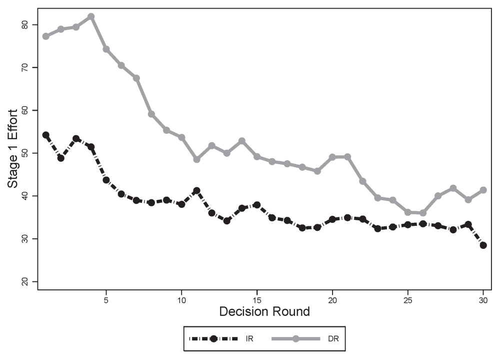
(c) Math good or better



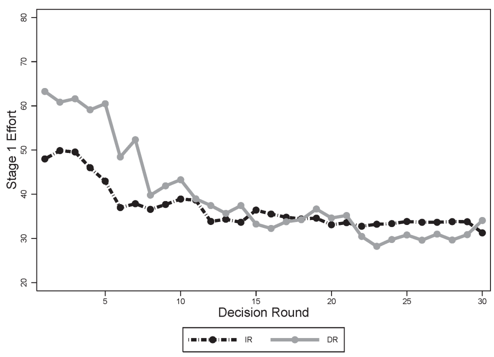
(d) Math satisfactory or worse



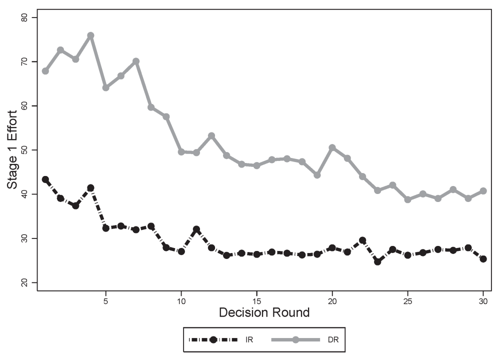
(e) Male



(f) Female

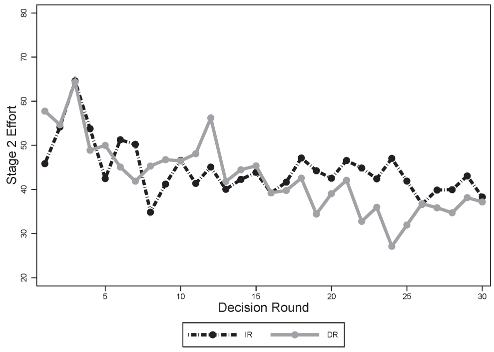


(g) Risk neutral or loving

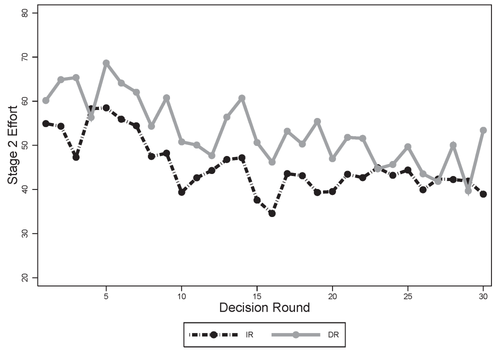


(h) Risk averse

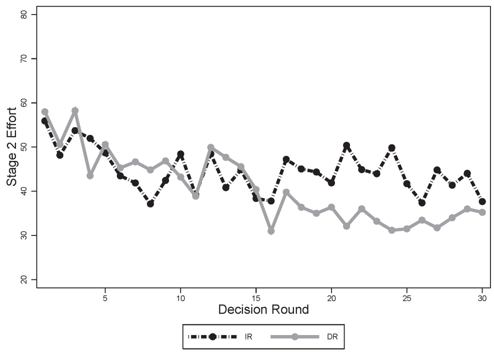
Figure 7.6: Stage-2 Effort by Decision Round



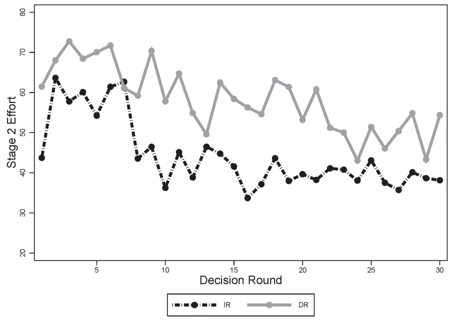
(a) Econ Department



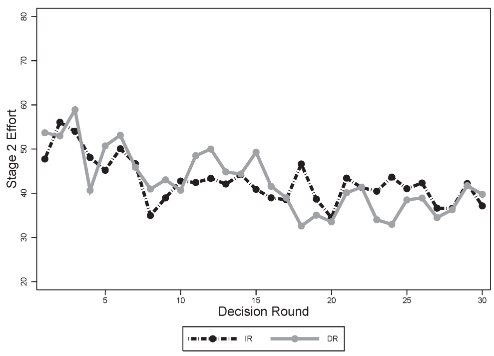
(b) Other Department



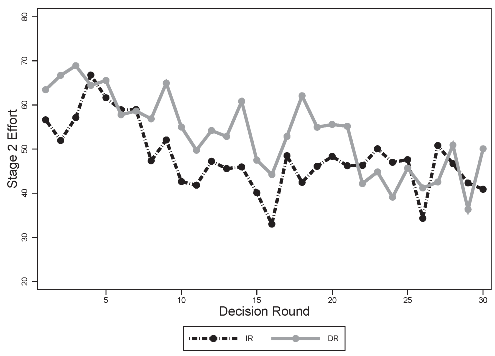
(c) Math good or better



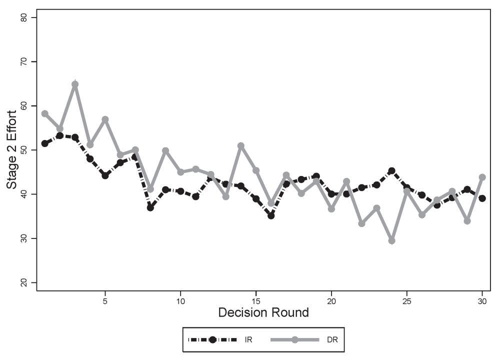
(d) Math satisfactory or worse



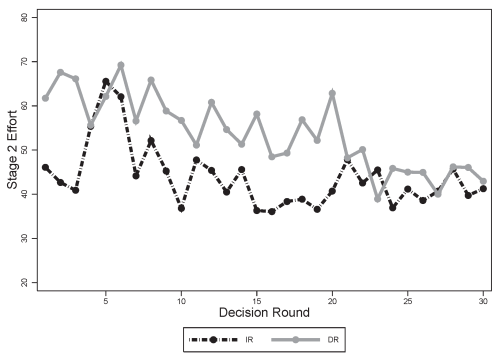
(e) Male



(f) Female



(g) Risk neutral or loving



(h) Risk averse

Table 7.5: Individual Characteristics and Treatment Differences

Dependent Variable: Individual Stage-1 Effort						
	(1)	(2)	(3)	(4)	(5)	(6)
econ	3.711	-	-	-	4.664	4.664
	(0.452)				(0.381)	(0.378)
mathsc	-	-0.166	-	-	-0.249	-0.249
		(0.942)			(0.308)	(0.304)
male	-	-	-3.720	-	-5.639	-5.639
			(0.450)		(0.308)	(0.304)
risk	-	-	-	0.383	0.506	0.506
				(0.510)	(0.401)	(0.398)
DR	16.155***	21.784*	15.424***	31.529***	47.822***	77.421***
	(0.001)	(0.074)	(0.003)	(0.001)	(0.003)	(0.006)
*econ*DR*	−13.858**	-	-	-	-7.008	-55.571
	(0.048)				(0.362)	(0.705)
*mathsc*DR*	-	-3.201	-	-	-2.260	-7.106
		(0.320)			(0.440)	(0.355)
*male*DR*	-	-	−12.351*	-	-8.498	-10.948
			(0.075)		(0.275)	(0.737)
*risk*DR*	-	-	-	−2.192**	−2.102**	−5.577**
				(0.017)	(0.017)	(0.019)
Fully int.	Yes	Yes	Yes	Yes	No	Yes
# Obs.	200	194	200	171	171	171

Note: p-values are provided in parentheses.

risk is the switching point where students with consistent risk-preferences decide in favor of the save payment for the first time; low (high) values of *risk* indicate that a subject is risk-averse (-loving). As for all individual characteristics previously considered, risk attitudes have no (significant) level effect per se. The treatment dummy *DR* and the interaction term *risk*DR* have very strong and highly significant effects, however. The coefficient signs suggest that risk-averse (rather than risk-neutral) subjects are responsible for the treatment effect we observe in stage 1. Panels (g) and (h) of Figure 7.5 show that this is indeed the case. When separately considering risk-averse and risk-neutral (or -loving) subjects, we find that there is no difference across treatments after the initial decision rounds for risk-neutral subjects. In contrast, there is a pronounced difference with respect to stage-1 effort choices across treatments for risk-averse students.

Overall, the regression analysis suggests that all observable characteristics, and in particular the field of study and risk attitudes, affect the behavior of experimental subjects in stage 1. To ensure that we do not simply pickup effects which generally hold across the two treatments to explain the difference in stage 1, Figure 7.6 plots the same characteristics for stage 2 which were presented for stage 1 in Figure 7.5. Interestingly, we do not find differences across types and treatments in stage 2 which are comparable to those present in stage 1.[16] This suggests that at least some of the previously considered observable characteristics are responsible for differential behavior in stage 1. Specifications (5) in Table 7.5, which jointly estimates all interaction coefficients, shows that the effect is mainly driven by risk attitudes. While the effect of the treatment dummy is still highly significant and all other interaction coefficients do have the expected negative sign, the interaction term *risk*IR* is the only one with a significant coefficient. Specification (6) shows that this does not change when estimating the fully interacted model.[17]

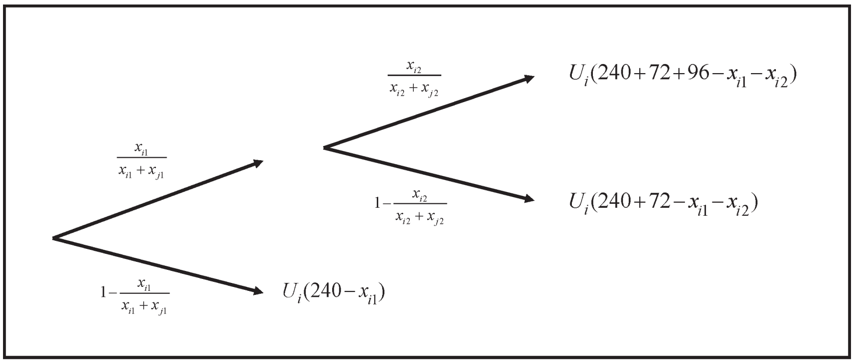
This raises the question why risk attitudes differentially affect behavior across treatments in stage 1. At first sight, this seems to be unreasonable, since the overall outcomes do not differ across treatments: The overall winner in the delayed reward treatment receives 168 Taler, compared to $72 + 96 = 168$ Taler in the immediate reward specification. Similarly, participants have 72 Taler for sure when reaching stage 2 – either because they received this reward after winning stage 1 (**IR**), or because they receive this reward in the worst possible stage-2 outcome (**DR**). Panels (a) and (b) of Figure 7.7 illustrate the decision for decision makers with an arbitrary (potentially non-linear) utility function, and the outcomes are clearly the same across treatments. Note, however, that this representation assume that decision makers *jointly* evaluate stage-1 and stage-2 outcomes. Panels (c) and (d) depict the situation when each stage is *separately* evaluated. While the utility when losing stage 1 is the same in both treatments for some arbitrary agent i, winning stage 1 is worth $U_i(72 + \mathbf{CV_{IR}} - x_{i1})$ in the immediate reward, compared to $U_i(\mathbf{CV_{DR}} - x_{i1})$ in the delayed reward specification. Since contest theory predicts that efforts are increasing in the reward, our experimental results suggest that the value of winning stage 1 is lower in the **IR** than in the **DR** treatment for concave utility functions, while the respective values are the same for linear utility functions. In fact, this is exactly what we find: The value of winning in treatment **IR** equals $72 + \mathbf{CV_{IR}}$, compared to $\mathbf{CV_{DR}}$ in treatment **DR**. The continuation value terms correspond to the certainty equivalent of the respective stage-2 lottery. For simplicity, assume that all agents in a particular treatment choose the same stage-2 effort x_{i2}, such that each of the two potential states in stage 2 realizes with probability $\frac{1}{2}$. Then, we have to show that the relation

$$\underbrace{U^{-1}\left(\frac{1}{2}[U(X) + U(Y)]\right)}_{=\mathbf{CV_{IR}}} + 72 \quad \leq \quad \underbrace{U^{-1}\left(\frac{1}{2}[U(X+72) + U(Y+72)]\right)}_{=\mathbf{CV_{DR}}},$$

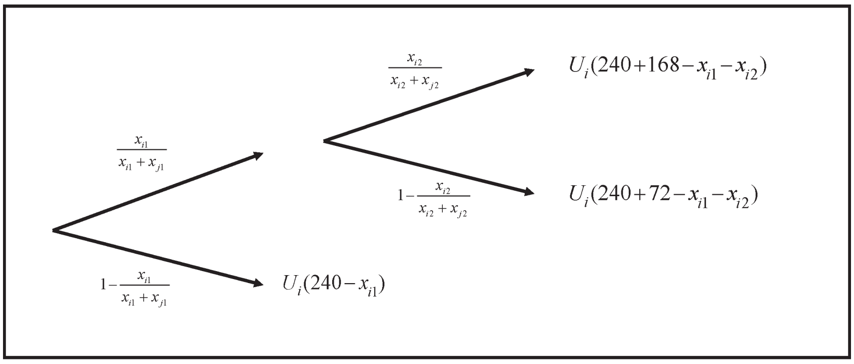
[16]When using regression analysis, all treatment dummy and interaction term coefficients are insignificant. Details available from the authors upon request.

[17]The coefficients of the additional interaction terms are not reported. Note, however, that they are all insignificant. Details are available from the authors upon request

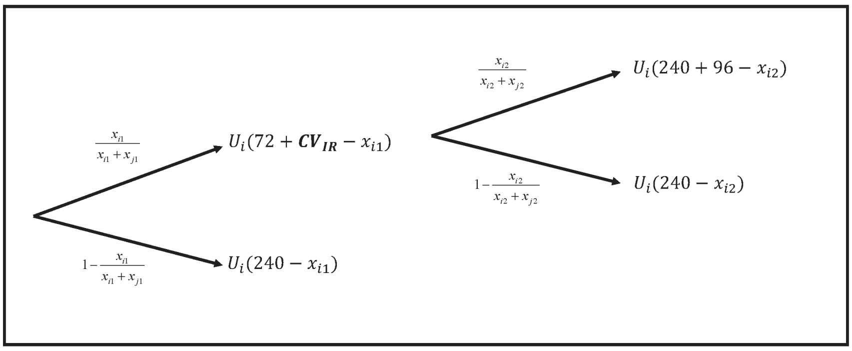
Figure 7.7: Decision Problem for Arbitrary Utility Functions



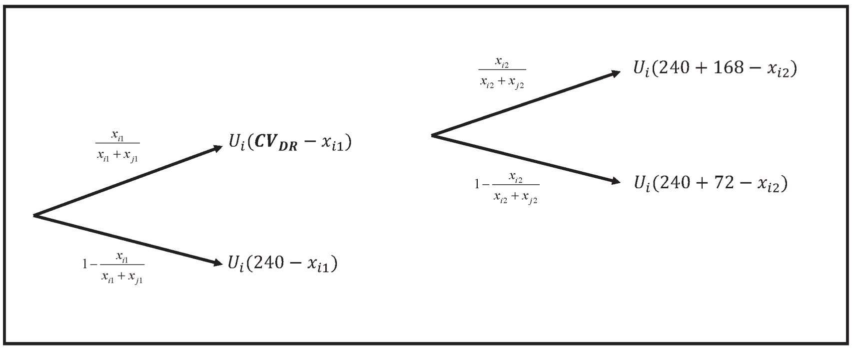
(a) Joint Evaluation of Stages: Immediate Reward (**IR**)



(b) Joint Evaluation of Stages: Delayed Reward (**DR**)



(c) Separate Evaluation of Stages: Immediate Reward (**IR**)



(d) Separate Evaluation of Stages: Delayed Reward (**DR**) where $X = 240 + 96 - x_{i2}$ and $X = 240 - x_{i2}$, respectively, holds with equality for linear, and with strict inequality for concave utility functions. The first part is obvious, since the utility function vanishes in the linear case, such that rearranging of terms delivers equality. It is somewhat harder to show that strict inequality holds for concave utility functions. For illustrative purposes, consider the case where $u_i(x) = \ln(x)$. This gives

$$\begin{aligned} e^{\frac{1}{2}(\ln[X+72]+\ln[Y+72])} &> e^{\frac{1}{2}(\ln[X]+\ln[Y])} + e^{\ln[72]} \\ \Leftrightarrow \sqrt{X+72}\sqrt{Y+72} &> \sqrt{X}\sqrt{Y} + 72 \\ \Leftrightarrow \sqrt{XY + (X+Y)72 + 5184} &> \sqrt{X}\sqrt{Y} + \sqrt{5184} \end{aligned}$$

Squaring both sides, which is without loss of generality, since X and Y are strictly positive, delivers:

$$\begin{aligned} XY + (X+Y)72 + 5184 &> XY + 2\sqrt{X}\sqrt{Y}72 + 5184 \\ \Leftrightarrow X - 2\sqrt{X}\sqrt{Y} + Y &> 0 \\ \Leftrightarrow \left(\sqrt{X} - \sqrt{Y}\right)^2 &> 0. \end{aligned}$$

Clearly, this relation is always satisfied. Intuitively, the result is driven by the curvature of the utility function: If the 72 Taler reward is added in both potential stage-2 states (winning or losing), as in treatment **DR**, this effectively reduces the risk in stage 2. The reason is that both states, winning and losing, are shifted upwards into a range of the utility function that is flatter than in the **IR** treatment, where the utility values of losing and winning are lower. Therefore, shifting the 72 Taler reward to stage 2, as in the delayed reward treatment, provides valuable insurance to risk averse decision makers. Consequently, the value of winning stage 1 is higher in the delayed than in the immediate reward treatment, which implies that theory predicts higher stage-1 effort in the former than in the latter specification, i.e., theory predicts the pattern which we observe in the experimental data.

Summing up, our results indicate that experimental subjects separately evaluate each of the two stages. While they clearly take continuation values into account, it is the payoff in each of the two stages which delivers utility, and not the aggregate payoff in both stages. In other words, winning or losing a given stage matters, not only the overall payoff subjects finally obtain.

7.5 Conclusion

This paper has analyzed which effect the timing of rewards has on the behavior of agents in two-stage pair-wise elimination contests. Comparing a treatment where agents receive an *immediate* reward for winning stage 1 (**IR**) with a specification where the reward for winning stage 1 is only awarded after the stage-2 interaction is completed (**DR**), we find that stage-1 effort choices by experimental subjects are higher in the delayed than in the immediate reward treatment, while effort provision in stage 2 does not differ between treatments. Theory predicts that the two treatments are strategically identical in both stages if agents are risk neutral, or if agents *jointly* evaluate the payoff of both interactions. Therefore, the suggestion of the empirical analysis that the treatment effect is due to risk preferences of experimental subjects seems counterintuitive at first. However, average differences of stage-1 effort choices between treatments are fully explained by choices of risk averse subjects; while their stage-1 effort choices in the delayed are much higher than in the immediate reward treatment, there is no difference across treatments for risk neutral subjects. This pattern is consistent with theoretical predictions only if experimental subjects *separately* evaluate the payoff of each stage. We show that delayed rewards provide an insurance for risk averse decision makers in this case, such that stage-1 effort choices should indeed differ across treatments for risk averse, but not for risk neutral decision makers. Generally speaking, this finding suggests that the presence of risk in the future generates an effect that works against standard discounting due to impatience. Consequently, time and risk preferences jointly determine optimal behavior in inter-temporal, risky environments. This finding implies that it will often be hard to disentangle the two components outside the lab.

It would be an interesting topic for future research to investigate whether the insurance effect of delayed rewards is influenced by the presence of strategic risk with respect to the effort choice of the opponent. To address this question, one could simply analyze whether the shifting of rewards across stages affects the value which subjects attach simple payoff equivalent lotteries with several stages and without strategic risk.

Appendix

7.A Experimental Instructions

The experimental instructions consist of three parts: First, experimental subjects receive some general information about the experimental session. Then, they are informed about the main treatment (Experiment 1), which is either the **IR** or the **DR** specification (both versions are provided). Finally, subjects receive instructions for the elicitation of risk attitudes (Experiment 2).

WELCOME TO THIS EXPERIMENT AND THANK YOU FOR YOUR PARTICIPATION

<u>**General Instructions:**</u>

You will participate in 2 different experiments today. Please stop talking to any other participant of this experiment from now on until the end of this session. In each of the two experiments, you will have to make certain decisions and may earn an appreciable amount of money. Your earnings will depend upon several factors: on your decisions, on the decisions of other participants, and on random components, i.e. chance. The following instructions explain how your earnings will be determined.

The experimental currency is denoted **Taler**. In addition to your Taler earnings in experiments 1 and 2, you receive 3 EURO show-up fee. You may increase your Taler earnings in experiments 1 and 2, where 2 Taler equal 1 Euro-Cent, i.e.

200 Taler correspond to 1 Euro.

At the end of this experimental session your Taler earnings will be converted into Euro and paid to you in cash.

Before the experimental session starts, you receive a card with your participant number. All your decisions in this experiment will be entered in a mask on the computer, the same holds for all other participants of the experiment. In addition, the computer will determine the random components which are needed in some of the experiments. All data collected in this experiment will be matched to your participant number, **not** to your name or student number. Your participant number will also be used for payment of your earnings at the end of the experimental session. Therefore, your decisions and the information provided in the experiments are completely anonymous; neither the experimenter nor anybody else can match these data to your identity.

We will start with experiment 1, followed by experiment 2. The instructions for experiment 2 will only be distributed right before this experiment starts, i.e. subsequent to experiment 1.

You will receive your earnings in cash at the end of the experimental session.

Experiment 1

Overall, there are 30 decision rounds with two stages each in Experiment 1. The course of events is the same in each decision round. You will be randomly and anonymously placed into a group of **four participants** in each round, and the identity of participants in your group changes with each decision round.

Course of events in an arbitrary decision round

All four participants of each group receive an **endowment of 240 Taler** at the beginning of a decision round. The endowment can be used to buy a certain amount of balls in two subsequent stages of a decision round. It is important to note that you receive one endowment only which must suffice to buy balls in both stages. The costs for the purchase of a ball are the same for all participants: Participants have to pay **1.00 Taler** for each ball they buy in stage 1 **or** stage 2, i.e.

1 ball costs 1.00 Taler
2 balls cost 2.00 Taler
(and so on)

When deciding how many balls you want to buy, you do not know the decision of other participants. Also, your decision is not revealed to any other participant.

All interactions in the experiment are pairwise. Assume that you are in one group with participant A, participant B, and participant C. Then, you interact with participant A in stage 1, while participants B and C simultaneously meet each other in the second stage 1 interaction. If you reach stage 2, you will interact either with participant B or C, depending on the outcome in the second stage 1 interaction. In stage 1, there are two ballot boxes:

- all balls bought by you or participant A are placed in ballot box 1
- all balls bought by participants B and C are placed in ballot box 2

One ball is randomly drawn from each ballot box, and each ball drawn with the same probability. The two participants whose balls are drawn from ballot box 1 and 2, respectively, receive an intermediate prize of **72 Taler** and reach stage 2; the decision round is over for the other two participants (whose balls were not drawn), i.e. they drop out from this decision round and receive no prize. Any participant has to pay the balls he or she bought in stage 1, whether or not he/she reached stage 2. The respective amount is deducted from the endowment.

The two participants who reached stage 2 do again buy a certain number of balls, using whatever remains from the endowment they received after costs for balls in stage 1 were deducted. This implies that the intermediate prize which stage 2 participants received at the end of stage 1 cannot be used to purchase balls. All purchased balls are placed into ballot box 3. One ball is randomly drawn from ballot box 3. The participant whose ball is drawn receives the main prize of **96 Taler**. Independent of whether or not a participant receives the main prize, he/she does always have to pay for the balls bought in stage 2.

Let's take a closer look at the random draw of balls from ballot boxes. Assume, for example, that all balls which you bought are green colored, and that you interact with participant A in stage 1. Then, the probability that one of your balls is drawn (such that you make it to stage 2) satisfies

$$\textbf{probability(green ball is drawn)} = \frac{\textbf{\# green balls}}{\textbf{\# green balls + \# balls by participant A}}$$

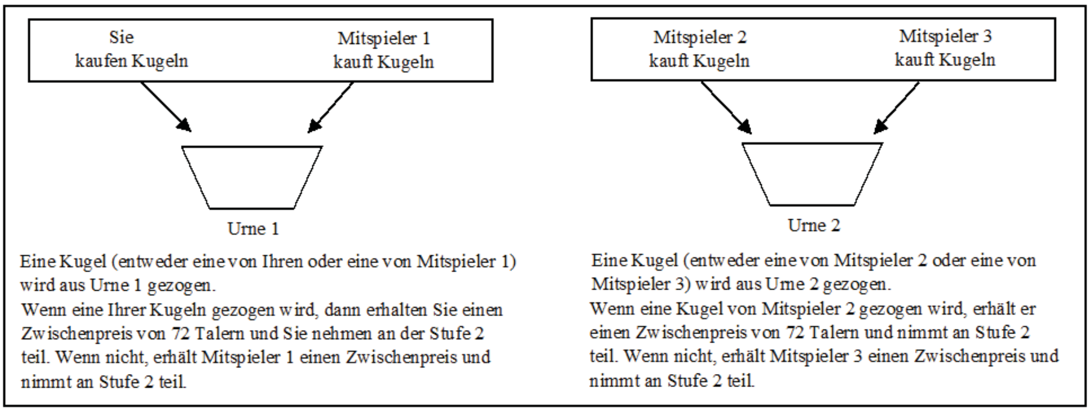
where # is short for number. The same probability rule does also hold for other participants in your group. Consequently, the probability that one of your balls in drawn is higher

- the more balls you purchased
- the less balls the other participant with whom you interact purchased.

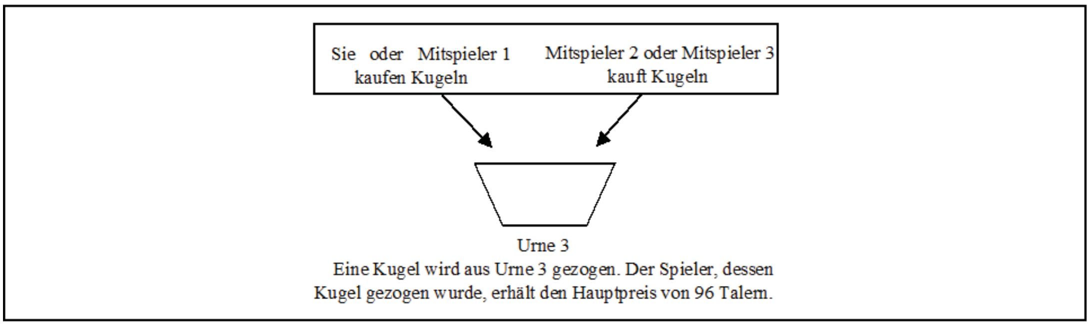
The computer simulates the random draw of a ball. If all participant of a group of four choose to buy zero balls, each participant wins with the same probability of 25%.

Jeder Spieler erhält eine Anfangsausstattung von 240 Talern. Er muss damit alle von ihm in Stufe 1 und Stufe 2 gekauften Kugeln bezahlen.

STUFE 1



STUFE 2



Your Payoff

Assume that you bought "**X1**" balls in stage 1, and that you buy "**X2**" balls whenever you reach stage 2. Then, there are three possibilities for your payoff:

1) None of your balls is drawn in stage 1

$$\textbf{Your Payoff} = \textbf{endowment} - \textbf{X1} * \textbf{1 Taler}$$
$$= \textbf{240 Taler} - \textbf{X1} * \textbf{1 Taler}$$

2) one of your balls is drawn from the ballot box in stage 1; in stage 2, none of your balls is drawn

$$\textbf{Your Payoff} = \textbf{endowment} - \textbf{X1} * \textbf{1 Taler} - \textbf{X2} * \textbf{1 Taler} + \textbf{intermediate prize}$$
$$= \textbf{240 Taler} - \textbf{X1} * \textbf{1 Taler} - \textbf{X2} * \textbf{1 Taler} + \textbf{72 Taler}$$

3) one of your balls is drawn from the ballot box in stage 1; also, one of your balls is drawn in stage 2

$$\textbf{Your Payoff} = \textbf{endowment} - \textbf{X1} * \textbf{1 Taler} - \textbf{X2} * \textbf{1 Taler} + \textbf{main prize}$$
$$= \textbf{240 Taler} - \textbf{X1} * \textbf{1 Taler} - \textbf{X2} * \textbf{1 Taler} + \textbf{168 Taler}$$

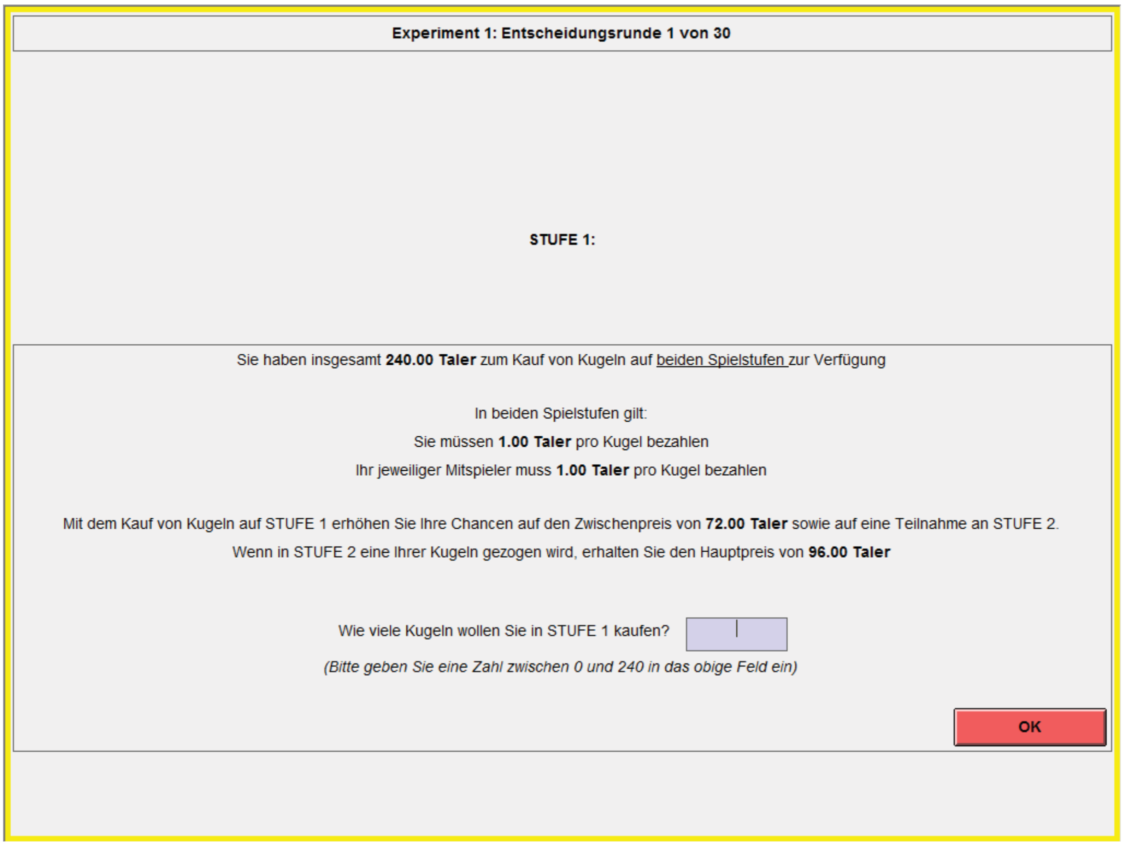
Therefore, your payoff is determined by the following components: by the number of balls you buy in stage 1 ("**X1**"); by the number of balls you buy in stage 2 ("**X2**") if you reach it; by up to two random draws (one of your balls is drawn/not drawn in stage 1 and potentially stage 2). The same holds for any other participants of the experiment.

Information:

- After you made your decision in stage 1, you are informed whether or not you can participate in stage 2, i.e. whether or not one of your balls was drawn from ballot box 1.
- If you did not reach stage 2, you are informed about how many balls participant A bought in stage 1.

- If you reach stage 2, you receive information about the remaining endowment (after costs for the purchase in stage 1 are deducted.
- After you made your decision in stage 2, you learn whether or not one of your balls was drawn from ballot box 3 and how many balls the participants who you met in stages 1 and 2, respectively, bought. Further, you learn your payoff for the respective decision round.

Decision: In each of the 30 decision rounds you have to decide how many balls you want to buy in stage 1. If you reach stage 2, you face a similar decision in stage 2. In both cases, you have to enter a number into a field on the computer screen. An example of the decision screen in stage 1 is shown below.



Your Total Payoff: Four out of 30 decision rounds are paid. These rounds are randomly determined, i.e., the probability that some decision round is paid is identical ex-ante for all 30 decision rounds. You will receive the sum of payoffs for the respective decision rounds.

Remember:
You receive an endowment of 240 Taler at the beginning of each decision round and have to decide how many balls you want to buy in stage 1; if you reach stage 2, you have to decide again. Overall, there are three additional participants in each group who face the same problem. The identity of these participants is randomly determined in each decision round. Every participants has to pay **1.00 Taler** for each ball he/she buys in stage 1 **or** stage 2. If one of your balls is drawn in stage 1, you receive an intermediate prize of 72 Taler. If, in addition, one of your balls is drawn in stage 2, you additionally receive the main prize of 96 Taler.

If you have any questions, please raise your hand now!

Experiment 1

Overall, there are 30 decision rounds with two stages each in Experiment 1. The course of events is the same in each decision round. You will be randomly and anonymously placed into a group of **four participants** in each round, and the identity of participants in your group changes with each decision round.

Course of events in an arbitrary decision round

All four participants of each group receive an **endowment of 240 Taler** at the beginning of a decision round. The endowment can be used to buy a certain amount of balls in two subsequent stages of a decision round. It is important to note that you receive one endowment only which must suffice to buy balls in both stages. The costs for the purchase of a ball are the same for all participants: Participants have to pay **1.00 Taler** for each ball they buy in stage 1 **or** stage 2, i.e.

1 ball costs 1.00 Taler
2 balls cost 2.00 Taler
(and so on)

When deciding how many balls you want to buy, you do not know the decision of other participants. Also, your decision is not revealed to any other participant.

All interactions in the experiment are pairwise. Assume that you are in one group with participant A, participant B, and participant C. Then, you interact with participant A in stage 1, while participants B and C simultaneously meet each other in the second stage 1 interaction. If you reach stage 2, you will interact either with participant B or C, depending on the outcome in the second stage 1 interaction. In stage 1, there are two ballot boxes:

- all balls bought by you or participant A are placed in ballot box 1
- all balls bought by participants B and C are placed in ballot box 2

One ball is randomly drawn from each ballot box, and each ball drawn with the same probability. The two participants whose balls are drawn from ballot box 1 and 2, respectively, reach stage 2; the decision round is over for the other two participants (whose balls were not drawn), i.e. they drop out from this decision round. Any participant has to pay the balls he or she bought in stage 1, whether or not he/she reached stage 2. The respective amount is deducted from the endowment.

The two participants who reached stage 2 do again buy a certain number of balls, using whatever remains from the endowment they received after costs for balls in stage 1 were deducted. The balls are then placed into ballot box 3. One ball is randomly drawn from ballot box 3. The participant whose ball is drawn receives the main prize of **168 Taler**. The other participant of stage 2, whose ball is not drawn from ballot box 3, receives a runner-up prize of **72 Taler.** Independent of the prize which a stage 2 participant receives, he/she does always have to pay for the balls bought in stage 2. Participants who did not reach stage 2 do not receive any prize.

Let's take a closer look at the random draw of balls from ballot boxes. Assume, for example, that all balls which you bought are green colored, and that you interact with participant A in stage 1. Then, the probability that one of your balls is drawn (such that you make it to stage 2) satisfies

$$\textbf{probability}(\textbf{green ball is drawn}) = \frac{\textbf{\# green balls}}{\textbf{\# green balls} + \textbf{\# balls by participant A}}$$

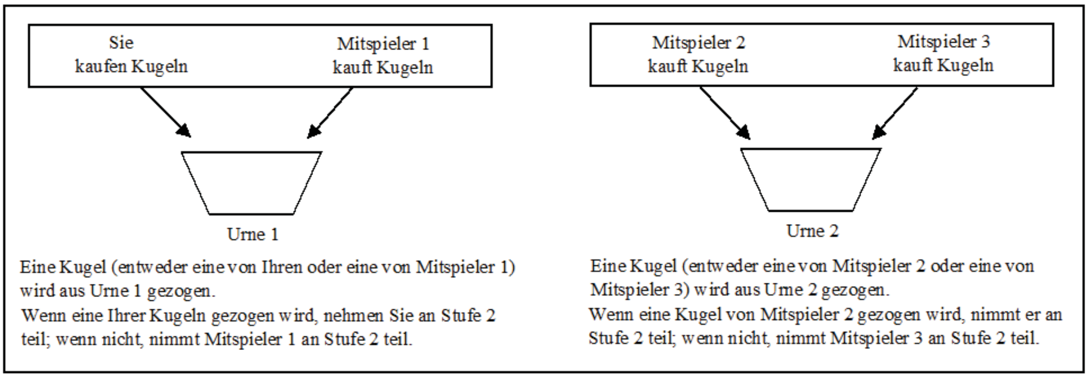
where # is short for number. The same probability rule does also hold for other participants in your group. Consequently, the probability that one of your balls in drawn is higher

- the more balls you purchased
- the less balls the other participant with whom you interact purchased.

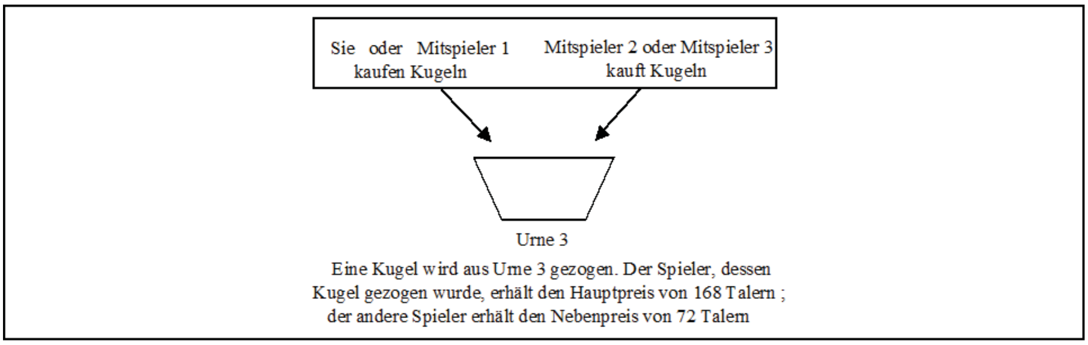
The computer simulates the random draw of a ball. If all participant of a group of four choose to buy zero balls, each participant wins with the same probability of 25%.

Jeder Spieler erhält eine Anfangsausstattung von 240 Talern. Er muss damit alle von ihm in Stufe 1 und Stufe 2 gekauften Kugeln bezahlen.

STUFE 1



STUFE 2



Your Payoff

Assume that you bought "**X1**" balls in stage 1, and that you buy "**X2**" balls whenever you reach stage 2. Then, there are three possibilities for your payoff:

1) None of your balls is drawn in stage 1

$$\textbf{Your Payoff} = \textbf{endowment} - \textbf{X1} * \textbf{1 Taler}$$
$$= \textbf{240 Taler} - \textbf{X1} * \textbf{1 Taler}$$

2) one of your balls is drawn from the ballot box in stage 1; in stage 2, none of your balls is drawn

$$\textbf{Your Payoff} = \textbf{endowment} - \textbf{X1} * \textbf{1 Taler} - \textbf{X2} * \textbf{1 Taler} + \textbf{runner up prize}$$
$$= \textbf{240 Taler} - \textbf{X1} * \textbf{1 Taler} - \textbf{X2} * \textbf{1 Taler} + \textbf{72 Taler}$$

3) one of your balls is drawn from the ballot box in stage 1; also, one of your balls is drawn in stage 2

$$\textbf{Your Payoff} = \textbf{endowment} - \textbf{X1} * \textbf{1 Taler} - \textbf{X2} * \textbf{1 Taler} + \textbf{main prize}$$
$$= \textbf{240 Taler} - \textbf{X1} * \textbf{1 Taler} - \textbf{X2} * \textbf{1 Taler} + \textbf{168 Taler}$$

Therefore, your payoff is determined by the following components: by the number of balls you buy in stage 1 ("**X1**"); by the number of balls you buy in stage 2 ("**X2**") if you reach it; by up to two random draws (one of your balls is drawn/not drawn in stage 1 and potentially stage 2). The same holds for any other participants of the experiment.

Information:

- After you made your decision in stage 1, you are informed whether or not you can participate in stage 2, i.e. whether or not one of your balls was drawn from ballot box 1.
- If you did not reach stage 2, you are informed about how many balls participant A bought in stage 1.

- If you reach stage 2, you receive information about the remaining endowment (after costs for the purchase in stage 1 are deducted.
- After you made your decision in stage 2, you learn whether or not one of your balls was drawn from ballot box 3 and how many balls the participants who you met in stages 1 and 2, respectively, bought. Further, you learn your payoff for the respective decision round.

Decision: In each of the 30 decision rounds you have to decide how many balls you want to buy in stage 1. If you reach stage 2, you face a similar decision in stage 2. In both cases, you have to enter a number into a field on the computer screen. An example of the decision screen in stage 1 is shown below.

Your Total Payoff: Four out of 30 decision rounds are paid. These rounds are randomly determined, i.e., the probability that some decision round is paid is identical ex-ante for all 30 decision rounds. You will receive the sum of payoffs for the respective decision rounds.

Remember:
You receive an endowment of 240 Taler at the beginning of each decision round and have to decide how many balls you want to buy in stage 1; if you reach stage 2, you have to decide again. Overall, there are three additional participants in each group who face the same problem. The identity of these participants is randomly determined in each decision round. Every participants has to pay **1.00 Taler** for each ball he/she buys in stage 1 **or** stage 2. Two prizes are awarded: the main prize of 168 Taler for the participant whose ball is drawn in stage 2, and the runner-up prize of 72 Taler for the other participant of the stage 2 interaction.

If you have any questions, please raise your hand now!

Experiment 2

In Experiment 2, you will face **21 decisions**. Each decision is a **choice between option 1 and option 2**. Each choice affects you own payoff, but not the payoff of any other participant of the experiment. When choosing option 1, your payoff is affected by chance, while option 2 implies a certain payment. You may be asked, for example, whether you prefer option 1, in which you receive either 400 Taler or 0 Taler with a 50% chance, or if you rather like option 2, which implies a sure payoff of c Taler. In the experiment, you will have to choose the option you prefer. This decision problem would be presented to you as follows:

Option 1	**Option 2**	Your Choice
with 50% probability **400 Taler** with 50% probability **0 Taler**	with certainty ***c*** **Taler**	Option 1 ◯ ◯ Option 2

As previously mentioned, you will encounter 21 decision problems of this kind. Your payoff from Experiment 2 is determined as follows:
At the end of all experiments, one of the 21 decision problems will be randomly chosen for each experimental participant. The option you chose in this decision problem determines your payoff. Assume, for example, that the previous example is chosen for you, and that you preferred option 1 over option 2. Then, you would receive 400 Taler or 0 Taler, each with a probability of 50%. Whether you receive 400 Taler or 0 Taler is determined by a simulated random draw of the computer.

7.B Control Questions

Overall, participants of the experimental sessions where asked to answer seven control questions. Each control questions provided participants with three alternative answers, only one of which is correct. Five questions where identical across treatment, two were different. We start with the questions which were identical in both treatments:

1. How many other experimental participants do you meet in each stage-1 interaction?
    - 1 other participant.
    - 2 other participant.
    - 3 other participant.
2. How many Taler does it cost you to buy 10 balls?
    - 7.50 Taler.
    - 10.00 Taler.
    - 15.00 Taler .
3. Assume that you bought 100 balls in stage 1. Then, what is the maximal number of balls that you can still buy in stage 2?
    - 140 balls.
    - 160 balls.
    - 200 balls.
4. What is the probability that one of your balls is drawn in stage 1 if all participants of the experiment bought 20 balls in stage 1?
    - 25%.
    - 50%.
    - 55%.
5. What is your payoff in some decision round if you bought 30 balls in stage 1 and 50 balls in stage 2, and one of your balls is drawn in stage 1 and in stage 2?
    - 328 Taler.
    - 362 Taler.
    - 408 Taler.

The two remaining questions in treatment IR are:

6. In which case do you receive the 72 Taler prize? If
   - you buy at least 80 balls in stage 1.
   - you buy more balls in stage 1 than anybody else.
   - one of your balls is drawn from your stage-1 ballot box.
7. In which case do you additionally receive the main prize of 96 Taler? If
   - you reach stage 2.
   - you buy more balls in both stages than anybody else.
   - one of your balls is drawn from the stage-2 ballot box.

The two remaining questions in treatment DR are:

6. In which case do you receive the runner-up prize of 72 Taler? If
   - you buy at least 80 balls in stage 1.
   - you reach stage 2 and a ball from the other stage-2 participant is drawn.
   - one of your balls is drawn from the stage-2 ballot box.
7. In which case do you receive the main prize of 168 Taler? If
   - you reach stage 2.
   - you buy more balls in both stages than anybody else.
   - one of your balls is drawn from the stage-2 ballot box.

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