

Essays on Behavioural Approach in Finance

DISSERTATION

of the University of St. Gallen,

School of Management,

Economics, Law, Social Sciences

and International Affairs

to obtain the title of

Doctor of Philosophy in Economics and Finance

submitted by

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from

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Dissertation no. 4391

ADAG Copy AG, Zürich, 2015

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St. Gallen, December 9, 2014

The President:

Prof. Dr. Thomas Bieger

Acknowledgements

I am deeply indebted to Prof. Dr. Enrico De Giorgi for giving me the opportunity to work with him in the Institute of Mathematics and Statistics and supervising me tirelessly during my PhD. It had been a great honour and privilege to be surrounded by academics: Prof. Dr. Heinz Müller, Prof. Dr. Francesco Audrino, Prof. Dr. Matthias Fengler, Prof. Dr. Daniel Buncic, Prof. Dr. Claudia Neri, and Prof. Dr. Lorenzo Camponovo, who mentored me and improved my knowledge in different fields. I would like to thank you, Prof. Dr. Thorsten Hens, for providing me with very useful feedback to my PhD thesis. My sincerest gratitude to Prof. Dr. Uwe Sunde, who encouraged me to pursue my PhD after earning my master's degree at HSG. I am also grateful to Prof. Dr. Paolo Porchia who gave me a lot of encouragement especially during my course phase.

During the PhD, especially in the Institute of Mathematics and Statistics, I am very fortunate to build great friendships with kind and intelligent souls: Tatiana, Julius, Kameliya, Pirmin, Simon, Yujia, Katja, Ambika, Markus, Alexander, Constantin, Anselm, and Jacopo. You have enriched me in so many ways, professionally and personally. I hope to always keep in touch, no matter where post-PhD life will lead us. To Fadrina Denoth, many thanks for being there as listening ears.

This PhD journey has been a roller coaster ride, full of ups and downs, and I am truly blessed to have my family and friends supporting me unconditionally. My husband, Lucio, who is always so patient and loving with me. My parents in law, Enzo and Rosetta, who listen throughout my hard days. My family in Bali: my mother, my sister Wathi, my brother Indra and Agus, who always pray for my health and success. My greatest friends in Frankfurt, the Assenheimer and Meffert family. My best friend in Pittsburgh, Derry, who pursues her PhD in Carnegie Mellon University and truly understands what it means to persevere in research.

I would like to dedicate this PhD dissertation to the memory of my loving father, Prof. Dr. I Nyoman Erawan. Without his encouragement and his faith in me, none of this will be possible. I am sure he is watching me from heaven and is very proud of all my accomplishments. My greatest hope is to pass on our legacy to the latest addition in our family, my first child, the soon-to-be-born Matteo.

Zürich, December 2014

Sukma Dewi Erawan

Summary

Chapter 1 provides a first step in formally analyzing the role of behavioural factors as possible main drivers in explaining the annuity market participation puzzle. We present an intertemporal multi-period model which includes behavioural preferences, the Cumulative Prospect Theory (CPT) and narrow framing. Our finding proves that the degree of narrow framing is the only important driver in the investor's decision to hold zero annuities. Specifically, an investor who narrowly frames her assets presents the following behaviour: if her degree of narrow framing is higher, the gradual increment into full annuitization is delayed and much more evident. We show that combining the degree of narrow framing with bequest motives also dampens the desire to invest into annuities. A certain pattern of returns specification provide similar share of post-consumption wealth invested into annuities as compared to the base case. However, it may prolong the delay into annuitization.

Chapter 2 introduces an experimentally supported reference point adaptation into a representative agent, preference-based asset pricing model. This model enables us to economically analyze the impact of the reference point adaptation on the stocks returns' behaviour. Our model is unique as we introduce the dynamics of reference point which is in line with experimental results while keeping loss aversion constant. We capture the impact of prior investment outcomes in terms of the historical reference price of the stocks recalled today. There is an asymmetry in the investor's reference point adaptation because, as she experiences a gain, the investor is very eager in updating her reference point. Conversely, she is more reluctant to lower her reference point after a loss. Our model's main contribution is generating asset prices which are consistent with empirical data. Our results also include conditional volatility which is asymmetrical in the case of gains and losses.

Chapter 3 analyzes an innovative source of data (news-sentiment-driven Thomson Reuters MarketPsych Index (TRMI)) and attempts to discover new factors which might be a consistent source of alpha in time. We use a state-of-the-art technique, Classification and Regression Trees (CART) in finding the predictive power of news sentiment from TRMI. Our results show that the TRMI is a promising new source of alpha. We perform backtesting by implementing two trading strategies (Long/Short and Long Only) using the CART classifiers. We analyze the performance of our base model's trading strategies and compare them with our benchmark models. We observe that our base model's Long Only strategy generally outperforms the benchmark model and the Buy and Hold S&P 500. Our trading strategies' good performance is a new source of alpha, which is not explainable by exposure to common stock risk factors.

Zusammenfassung

Kapitel 1 stellt einen ersten Schritt in der formalen Analyse der Rolle von “behavioural” Faktoren als mögliche Haupttreiber bei der Erklärung des “Annuity Market Participation Puzzle” dar. Wir präsentieren ein intertemporales Mehrperiodenmodell, das behavioural Preferences, Cumulative Prospect Theory (CPT) und Narrow Framing umfasst. Unsere Resultate zeigen, dass das Ausmaß des Narrow Framing der einzig wichtige Treiber in der Entscheidung des Investors keine Annuities zu halten ist. Genauer gesagt zeigt eine Investorin, die ihre Vermögenswerte “narrowly framed”, das folgende Verhalten: Wenn ihr Narrow Framing Niveau höher ist, wird die graduelle Zunahme auf ausschließlich Annuities verzögert und zeigt sich viel deutlicher. Wir zeigen, dass auch die Kombination des Narrow Framing Niveau mit “bequest Motives” den Wunsch dämpft in Annuities zu investieren.

Kapitel 2 stellt eine experimentell unterstützte Referenzpunktanpassung in einem präferenzbasierten Asset Pricing Model mit repräsentativen Handelnden vor. Dieses Modell ermöglicht es uns, die Auswirkungen der Referenzpunktanpassung auf das Verhalten von Aktienrenditen wirtschaftlich zu analysieren. Unser Modell ist einzigartig, da wir die Dynamik des Referenzpunktes, die im Einklang mit den experimentellen Ergebnissen ist, unter konstanter Risk Aversion einführen. Wir erfassen die Auswirkungen von “prior investment outcomes” in Bezug auf die historischen Referenzpreise der Aktien von heute. Es gibt eine Asymmetrie in Bezug auf die Referenzpunktanpassung des Anlegers, weil der Anleger während er Gewinne macht sehr schnell bei der Anpassung seines Referenzpunktes ist. Umgekehrt ist er eher abgeneigt seinen Referenzpunkt nach einem Verlust zu senken. Unsere Ergebnisse enthalten dabei auch bedingte Volatilität, die asymmetrisch im Fall von Gewinnen und Verlusten ist.

Kapitel 3 analysiert eine innovative Datenquelle (TRMI) und versucht neue Faktoren zu entdecken, die eine konsistente Alpha-Quelle darstellen könnten. Wir verwenden eine state-of-the-art-Technik, CART, bei der Suche nach der Vorhersagekraft des News Sentiment von TRMI. Unsere Ergebnisse zeigen, dass der TRMI eine vielversprechende neue Alpha-Quelle ist. Wir führen das Backtesting durch die Implementierung zweier Handelsstrategien (Long/Short und Long Only) mit den CART Klassifikatoren durch. Wir analysieren die Performance der Handelsstrategien unseres Basismodells und vergleichen sie mit unseren Benchmark-Modellen. Wir beobachten, dass die Long Only Strategie unseres Basismodells das Benchmark-Modell und die Buy and Hold S&P 500 in der Regel übertrifft. Die gute Performance unserer Handelsstrategie ist eine neue Alpha-Quelle, die durch aktienbezogenen Risikofaktoren nicht erklärbar ist.

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Preface

Research in finance or economics can be either positive or normative. Positive research involves a description of what economic agents do based on what the data shows while normative research suggests some theories which describe what economic agents are supposed to be doing. Economists usually assume that both actual and ideal behaviour of economic agents coincide or they impose assumptions on beliefs and preferences which ensure it.

Specifically in the field of asset pricing and household finance, it is evident that there are difficulties in synergizing positive and normative research. Research in asset pricing focuses on how asset prices are determined in the capital markets and describe how average asset returns reflect risks. However, after some years, it became evident that basic facts about the aggregate stock market, the cross-section of average returns and individual trading behaviour are not easily understood as classical theory describes. There are some predictions in the traditional asset pricing framework which are not confirmed by the data. For example, the value premium puzzle shows that assets with a high ratio of price to fundamentals (growth stocks) have lower expected returns relative to assets with a low ratio of price to fundamentals (value stocks). The leverage effect shows that negative news produces a larger increase in volatility as compared to positive news rather than a symmetrical effect as the initial theory predicts.

Research in household finance is primarily concerned about how households use financial instruments to attain their objectives. It is quite different from asset pricing because it puts more emphasis on the behaviour of typical households rather than wealthy and risk-tolerant ones. As suggested by Campbell (2006), a comparison between the positive and normative results in household finance shows that there are larger discrepancies for a minority of households, especially those who are poor and less educated. These discrepancies or investment mistakes are crucial in the field of household finance. For example, the annuity market participation puzzle addresses the fact that households are reluctant to annuitize even if standard economic models suggest that they should do so.

We believe that in order to address some puzzles in household finance and asset pricing, it is important to explore a behavioural approach. We should move away from the traditional assumptions that individuals behave rationally in terms of their beliefs and preferences. A behavioural alternative is to consider non-standard behavioural models of preferences which incorporate for example, loss aversion and narrow framing. Motivated by this reason, the first two chapters of this thesis uses behavioural models of preferences to address important topics such as the leverage effect and the annuity market

participation puzzle. The last chapter practically uses a behaviourally driven innovative source of data to find a promising source of alpha.

Chapter 1 provides a first step in formally analyzing the role of behavioural factors as possible main drivers in explaining the annuity market participation puzzle. We present an intertemporal multi-period model which includes behavioural preferences, the Cumulative Prospect Theory (CPT) and narrow framing. The model derives the agent's optimal consumption and portfolio choice over three asset classes: liquid bonds, stocks and illiquid constant-life annuity. Narrow framing refers to the idea that the agent evaluates her investment outcomes from different assets in isolation to other existing risks. In this framework, the agent gets both a direct utility from three assets and an indirect utility from the contribution of these assets into her total wealth. The direct utility of gains/losses from these assets follow the CPT which incorporates loss aversion and probability weighting. Our finding proves that the degree of narrow framing is the only important driver in the investor's decision to hold zero annuities. Specifically, an investor who narrowly frames her assets presents the following behaviour: if her degree of narrow framing is higher, the gradual increment into full annuitization is delayed and much more evident. Classical factors such as bequest motives and the asset returns slightly influence the investor's degree of annuitization. We show that combining the degree of narrow framing with bequest motives also dampens the desire to invest into annuities. A certain pattern of returns specification provide similar share of post-consumption wealth invested into annuities as compared to the base case. However, it may prolong the delay into annuitization.

Chapter 2 introduces an experimentally supported reference point adaptation into a representative agent, preference-based asset pricing model. This model enables us to economically analyze the impact of the reference point adaptation on the stocks returns' behaviour. Despite having the same modelling technique as Barberis, Huang, and Santos (2001), our model is unique as we introduce the dynamics of reference point which is in line with experimental results while keeping loss aversion constant. We capture the impact of prior investment outcomes in terms of the historical reference price of the stocks recalled today. There is an asymmetry in the investor's reference point adaptation because, as she experiences a gain, the investor is very eager in updating her reference point. Conversely, she is more reluctant to lower her reference point after a loss. This reference point dynamics is consistent with the experimental results in Arkes, Hirshleifer, Jiang, and Lim (2008), Arkes, Hirshleifer, Jiang, and Lim (2010) and Baucells, Weber, and Welfens (2011). Our model's main contribution is generating asset prices which are consistent with empirical data. Differently from Barberis, Huang, and Santos (2001), our results also include conditional volatility which is asymmetrical in the case of gains and losses. This feature fully describes one of the stylized facts in finance, the leverage effect, which suggests that negative news produces larger increase in volatility as compared to positive news. Moreover, the stock returns produced in our model are high on average, with high volatility and low correlation with consumption growth while the riskless interest rate is low and stable.

Chapter 3 analyzes an innovative source of data (news-sentiment-driven Thomson Reuters MarketPsych Index (TRMI)) and attempts to discover new factors which might be a consistent source of alpha in time. In that regard, we use a state-of-the-art technique, Classification and Regression Trees

(CART) in finding the predictive power of news sentiment from TRMI. Our results show that the TRMI is a promising new source of alpha. The cumulative average hit rates of the 10 sectors CART classifiers are relatively good (on average 58% of the time the model correctly predicts the classification of each sector as outperforming, neutral or under-performing). In terms of variable importance, most of the technical indicators, Fama French factors and classical sentiment indicators are dropped out from the final classifiers. The TRMI variables such as “Price Increase”, “Market Risk”, “Sentiment”, “Gloom”, “Market Forecast”, “Optimism” and “Fear” consistently dominate the weekly average top 10 most important variable for all the 10 sectors. We perform backtesting by implementing two trading strategies (Long/Short and Long Only) using the CART classifiers. The investor’s wealth of these two strategies is constantly higher than the benchmark (Buy and Hold S&P 500). Particularly for the Long/Short strategy, the drawdowns are within acceptable range (the highest drawdown was during November 2008 with around 28%). We analyze the performance of our base model’s trading strategies (Long/Short and Long Only) and compare them with our benchmark model (CART which implements the technical, Fama French and classical sentiment indicators, while excluding all the 23 TRMIs). We observe that our base model’s Long Only strategy generally outperforms the benchmark model and the Buy and Hold S&P 500. Our trading strategies’ good performance is a new source of alpha, which is not explainable by exposure to common stock risk factors. We adjust the performance of these strategies for different risk factors: market in excess of R_f ($MKT-R_f$), value (HML), size (SMB) and momentum (MOM). Our base model is robust even if we differentiate some of its characteristics such as taking excess returns as the dependent variable, or defining only two categories (outperforming or underforming). One caveat of using the TRMI to find a new source of alpha is that the predictive power of these news sentiments are relatively short-lived. Upon testing the strategies with monthly rebalancing, the performance worsen. Hence, the information from this news sentiment data decays relatively quickly. The TRMI serves as an option for the lower-frequency quantitative investors to improve their performance.

Chapter 1

Dynamic portfolio choice with narrow framing and annuitization

1.1 Introduction

Longevity risk is the risk that an individual underweights her survival probability upon retirement, hence, she is unable to support her consumption expenditure upon outliving herself. Especially in this aging society, longevity risk becomes even more important as the share of population who is exposed to this risk gets considerably larger. For example, Bell and Miller (2002) describe that in the US one tenth of men (women) who are retiring at 65 years old may expect to live for another 27 (30) years respectively. This statistics undeniably portrays the magnitude of the increasing risk of outliving one's retirement wealth.

Life annuity is one of the financial instruments which serves as a hedge for this risk. An individual who purchases a life annuity (the annuitant) pays an annuity premium to the insurance company. In return, the insurance company undertakes the obligation to pay out a stream of income to the annuitant. The annuity premiums paid by those who die earlier are translated as gains to the overall pool of annuitants. These surviving annuitants receive a higher yield (mortality credit) as a compensation for giving up their right to their annuity payments upon death. The older the annuitant gets, the higher the mortality credit that she earns. The mortality credit may even be higher than the risk premia of other financial instruments, such as equity. Therefore, from the annuitant's perspective, life annuity can be considered as an attractive financial instrument for her post-retirement financial planning. From the policymakers' perspective, the annuity provision becomes a very important issue to comprehend in order to create a sound policy for the aging society.

The importance of life annuity has long been supported by classical theorists. In a seminal contribution, Yaari (1965) proves that people should annuitize 100% of their retirement savings. This result is driven by the assumption of no bequest motives, that is, retirees do not leave any remaining wealth after they die to their offspring. Davidoff, Brown, and Diamond (2005) consider weaker assumptions than Yaari (i.e., general utility function which does not necessarily follow the Expected Utility Theory

(EUT) and time additivity) and still conclude that full annuitization is optimal in a complete market case. Upon taking the incomplete market case with bequest motives, they conclude that even though bequest motives decrease the level of annuitization, people should still annuitize a large share of their retirement savings.

Empirical evidence, however, shows that households prefer not to voluntarily buy annuities at retirement. For example, according to LIMRA international the sales of fixed immediate annuities in 2007 were only \$6.5 billion.¹ Based on the U.S. Health and Retirement Survey (HRS), only 1.57% of the respondents declare to obtain life annuity income. Similarly, there is only 8% of the respondents who are in a defined contribution pension plan opting for an annuity payout. According to James and Song (2001), the limited degree of annuitization is not only unique to the U.S. but also throughout other countries such as Canada, UK, Switzerland, Australia, Israel, Chile and Singapore. The discrepancy between the classical theory and the empirical evidence on voluntary annuitization is coined as the annuity market participation puzzle.

So far the traditional explanations on the annuity market participation puzzle are very diverse and not unified. Some researchers suggest that behavioural factors play an important role in addressing this puzzle. For example, Davidoff, Brown, and Diamond (2005, Page 1589) point out that it is definitely important to consider behavioural modeling of annuity demand in order to understand the near absence of voluntary annuitization. Horneff, Maurer, and Stamos (2008, Page 3610) state that although factors such as loadings, poor health, public pensions and bequest motives are plausibly reducing the willingness for annuitization, yet, none of them can really explain the limited voluntary annuitization. Hence, they also suggest that behavioural factors may fully explain the annuity market participation puzzle.

Our research aims at filling this void. We propose a theoretical model with behavioural preferences which include the Cumulative Prospect Theory (CPT) and provide a first step in formally analyzing whether behavioural factors (narrow framing, probability weighting and loss aversion) are indeed possible main drivers in explaining the individual's reluctance into voluntary annuitization. As demonstrated by Tversky and Kahneman (1992), the main characteristics of the CPT is that people have the tendency to be loss averse, that is, they dislike losses more than they value gains. Another of its characteristics is probability weighting, which has been widely used in many financial applications. For example, it describes the investor's behaviour with respect to the low probability payoffs which usually refer to the tails of the assets' returns distribution. It is also very important for portfolio selection as returns often display positive or negative skewness (Barberis and Huang (2008b), De Giorgi and Legg (2012)). Particularly in making an annuity decision, under CPT preferences, a retiree has the tendency to overweight the low probability of dying very soon after purchasing an annuity and underweight the probability of outliving her resources if she does not annuitize. Hence the gain from annuitizing seems to give only a small utility while the loss from dying early appears to have a large disutility.

¹ Fixed immediate annuities are those which pay out a guaranteed constant payment one year after purchase. The number may be an overestimation of the constant-life annuity because it includes non-life contingent and period-certain products.

Many researchers suggest that retirees usually engage in narrow framing when deciding to annuitize their retirement wealth and consider it separately from other investment decisions (e.g., Brown, Kling, Mullainathan, and Wrobel (2008)). The idea of narrow framing is initially introduced in experimental studies suggesting that in most situations, people do not combine a new gamble with existing ones and they simply evaluate a gamble in isolation from other existing risks. In the behavioural finance literature, the framework for dynamic portfolio choice and asset pricing, which includes narrow framing into the utility specification, recently attracts many researchers' attention (e.g., Barberis, Huang, and Thaler (2006), Barberis and Huang (2008a), Barberis and Huang (2009), De Giorgi and Legg (2012)). Narrow framing apparently is able to contribute in explaining some observed features of real portfolios such as the stock market non-participation puzzle (See Mankiw and Zeldes (1991)).

In order to identify the main drivers of the annuity market participation puzzle, our research considers an intertemporal multi-period model where an agent maximizes her utility from consumption and returns of invested post-consumption wealth on three different assets (i.e., bonds, stocks and constant-life annuities). We assume that this agent engages in narrow framing where she evaluates her investment outcomes from different assets in isolation to other existing risks and she gains direct utility from their returns. The direct utility of gains/losses from bonds, stocks and constant-life annuities follow the CPT which incorporates loss aversion and probability weighting.

Our finding proves that the degree of narrow framing is the only important driver in the investor's decision to hold zero annuities. Specifically, an investor who narrowly frames her assets presents the following behaviour: if her degree of narrow framing is higher, the gradual increment into full annuitization is delayed and much more evident. Classical factors such as bequest motives and the asset returns slightly influence the investor's degree of annuitization. We show that combining the degree of narrow framing with bequest motives also dampens the desire to invest into annuities. A certain pattern of returns specification provide similar share of post-consumption wealth invested into annuities as compared to the base case. However, it may prolong the delay into annuitization.

The remaining of this paper is organized as follows. Section 1.2 provides a review on narrow framing concept. Section 1.3 describes our theoretical model and the computational methodology of the maximization problem. Section 1.4 illustrates the results of the model's optimal asset allocation (base case). Section 1.5 performs robustness analysis. Section 1.6 draws the conclusions.

1.2 Narrow framing

Traditional economists typically assume an agent who maximizes her utility functions defined over wealth or consumption, usually based on the principles described in the Expected Utility Theory (EUT). Upon evaluating a new gamble, the agent will combine it with other existing risks to see how it changes the distribution of her future wealth or consumption and whether it increases her welfare. Experimental research, however, observes many occasions where this is not the case. A common observation in an experimental setting shows that, instead of combining a new gamble with existing ones, an agent simply evaluates it in isolation. This idea is coined as narrow framing (Kahneman and

Lovall (1993), Kahneman (2003)).

Tversky and Kahneman (1981) perform an experiment with 150 subjects. The group was asked this question:

Imagine that you face the following situation where you have to firstly examine the two following decisions and then indicate the options you prefer:

Decision (i). Choose between:

(A) A sure gain of \$240.

(B) 25% chance to gain \$1000, and 75% chance to gain nothing.

Decision (ii). Choose between:

(C) A sure loss of \$750.

(D) 75% chance to lose \$1000, and 25% chance to lose nothing.

Tversky and Kahneman (1981) describe that among these 150 subjects, 84% chose (A) and 16% chose (B) in decision (i). In decision (ii), 87% chose (D) while only 13% chose (C). The most surprising result is there are 73% who chose the combination (A) and (D) even if this choice is dominated by the combination of (B) and (C). This means that subjects do not really focus on combining decision (i) and (ii) together in evaluating the outcome and its contribution to final wealth. Instead, they evaluate decision (i) and (ii) separately and these utility appears to depend directly on the outcome of each of decisions (i) and (ii). This experiment shows a typical case of narrow framing.

Specifically for our research at hand, the concept of framing is singled out as one of the most important behavioural explanation for the annuity market participation puzzle. How framing in the context of annuities is formulated determines the agent's decision into annuitization. Brown, Kling, Mullainathan, and Wrobel (2008) perform an internet survey of adults who are at least 50 years old. These adults were asked their preferences on annuitization as annuity is framed in two different contexts. The "consumption frame" describes annuity as giving \$650 of monthly spending for life, while the "investment frame" illustrates annuity as providing a guaranteed monthly return of \$650 for life. Furthermore, it is assumed that there is no residual wealth after death. The results of the survey showed that the majority of the subjects (70%) choose to annuitize in the consumption frame but not in the investment one (21%). This is because under the investment frame, annuity seems unattractive and risky since it has the tendency to lose money due to the uncertainty of time of death. Benartzi, Previtro, and Thaler (2011) support this view by relating this hypothesis into the two kinds of pension plans: the traditional defined benefit (supporting the consumption frame) and cash balance (fostering the investment frame). They finally conclude that framing matters in the annuitization decision.

Our paper goes beyond and formalizes the concept of narrow framing into the preferences specification. We use the common framework which was first introduced by Barberis and Huang (2009) and further improved by De Giorgi and Legg (2012). This new preferences specification is able to shed some lights on some financial applications such as the stock market non-participation puzzle (See Mankiw and Zeldes (1991)). We implement the same tool in order to formally explain the annuity market participation puzzle.

1.3 Model

The model is discrete with $t \in \{0, \dots, T+1\}$ where t is the agent's adult age which is equal to her actual age minus 19. The agent lives up to T years, dies for sure at $T+1$ and has a subjective probability p_t^s to survive from t until $t+1$.

At time $t = 0, \dots, T$, the agent, with wealth W_t , chooses a consumption level C_t and allocate her post-consumption wealth $W_t - C_t$ across n assets with returns $R_{1,t+1}, \dots, R_{n,t+1}$ between time t and $t+1$ and a constant-life annuity with constant payments L_t starting at time $t+1$ until survival and premium A_t at time t .

We denote by $\theta_{i,t}$ the proportion of post-consumption wealth allocated to asset i at time t and define the proportion of post-consumption wealth spent for the life annuity by

$$\theta_{0,t} = \frac{A_t}{W_t - C_t}.$$

In case of survival at time $t+1$, the agent collects payments from life annuities bought at time t or before and the returns from the n assets. Her time $t+1$ wealth is therefore given by

$$(1.1) \quad W_{t+1} = (W_t - C_t) \sum_{i=1}^n \theta_{i,t} R_{i,t+1} + \sum_{s=0}^t L_s = (W_t - C_t) \sum_{i=0}^n \theta_{i,t} R_{i,t+1} + \sum_{s=0}^{t-1} L_s$$

where

$$R_{0,t+1} = \frac{L_t}{A_t}.$$

We denote by ℓ_t the proportion of annuity payments at time t with respect to the overall wealth W_t , that is,

$$\ell_t = \frac{\sum_{s \leq t} L_s}{W_t}.$$

Then Equation (1.1) becomes

$$(1.2) \quad W_{t+1} = (W_t - C_t) \sum_{i=0}^n \theta_{i,t} R_{i,t+1} + W_t \ell_t = (W_t - C_t) \boldsymbol{\theta}'_t \mathbf{R}_{t+1} + W_t \ell_t$$

where $\boldsymbol{\theta}_t = (\theta_{0,t}, \theta_{1,t}, \dots, \theta_{n,t})'$ and $\mathbf{R}_{t+1} = (R_{0,t+1}, R_{1,t+1}, \dots, R_{n,t+1})'$.

Hence, the dynamics of ℓ_{t+1} can be expressed as follows

$$(1.3) \quad \ell_{t+1} = \frac{\sum_{s \leq t+1} L_s}{W_{t+1}} = \frac{(\sum_{s \leq t} L_s) + L_t}{W_{t+1}} = \frac{W_t \ell_t + L_t}{(W_t - C_t) \boldsymbol{\theta}'_t \mathbf{R}_{t+1} + W_t \ell_t} = \frac{(W_t - C_t) \theta_{0,t} R_{0,t+1} + W_t \ell_t}{(W_t - C_t) \boldsymbol{\theta}'_t \mathbf{R}_{t+1} + W_t \ell_t}.$$

If the agent dies between time t and $t+1$, then only returns from the n assets are collected and passed to the children in form of bequest B_{t+1} at time $t+1$. We have

$$(1.4) \quad B_{t+1} = (W_t - C_t) \sum_{i=1}^n \theta_{i,t} R_{i,t+1} = (W_t - C_t) \boldsymbol{\theta}'_{1,t} \mathbf{R}_{1,t+1},$$

where $\boldsymbol{\theta}_{1,t} = (\theta_{1,t}, \dots, \theta_{n,t})'$ and $\mathbf{R}_{1,t+1} = (R_{1,t+1}, \dots, R_{n,t+1})'$.

1.3.1 Mortality

In our model, we differentiate the insurer's view on mortality and the annuitant's beliefs about the health status by applying the Gompertz law. The subjective force of mortality ζ^s and the force of mortality used to compute the annuity premiums ζ^a are given by

$$(1.5) \quad \zeta_t^i = \frac{1}{b^i} \exp\left(\frac{t - m^i}{b^i}\right), \quad i = a, s.$$

m^i and b^i give the shape of the force of mortality function. The survival probabilities are as follows

$$(1.6) \quad \begin{aligned} p_t^i &= \exp\left(-\int_0^1 \zeta_{t+s}^i ds\right), \\ &= \exp\left[-\exp\left(\frac{t - m^i}{b^i}\right) \left(\exp\left(\frac{1}{b^i}\right) - 1\right)\right]. \end{aligned}$$

Furthermore, we model the force of mortality as linear transformation of the force of mortality derived from the average population mortality table ζ_t^{pop} to analyze the impact of good and poor health propositions as follows

$$(1.7) \quad \zeta_t^i = v^i \zeta_t^{\text{pop}} ; \quad p_t^s = (p_t^{\text{pop}})^{v^i}.$$

1.3.2 Investor's preferences

At time t , the agent frames assets $m + 1, \dots, n$ and life annuities bought at time t narrowly. Her utility at time t is then given as follows

$$(1.8) \quad V_t = H \left(C_t, \left(p_t^s \mathbb{E}_t \left(V_{t+1}^{1-\gamma} \right) + (1 - p_t^s) \kappa \mathbb{E}_t \left(B_{t+1}^{1-\gamma} \right) \right)^{\frac{1}{1-\gamma}} + b_0 \left(p_t^s \sum_{i=m+1}^n U_t(G_{i,t+1}) + U_t(G_{0,t+1}) \right) \right),$$

where

$$(1.9) \quad H(C, x) = ((1 - \beta p_t^s) C^\rho + \beta x^\rho)^{\frac{1}{\rho}}, \quad 0 < \beta < 1, 0 \neq \rho < 1,$$

$$(1.10) \quad G_{i,t+1} = \theta_{i,t} (W_t - C_t) (R_{i,t+1} - R_{i,z}), \quad i = 0, m + 1, \dots, n,$$

and p_t^s is the subjective probability that the agent will survive from the period t until $t + 1$, with $p_T^s = 0$, that is, T is small enough so that we can assume that the agent will not survive after period T . κ is a parameter to account for bequests and γ is a parameter of risk aversion where $\gamma > 0$ and $\gamma \neq 1$. The variable $G_{i,t+1}$ gives gains and losses of asset i with respect to a given reference point $R_{i,z}$. In our numerical analysis, we assume $R_{i,z} = R_f$ for all $i = 0, m + 1, \dots, n$, where $R_f > 0$ is the risk-free gross returns.

In Equation (1.8), $\left(p_t^s \mathbb{E}_t \left(V_{t+1}^{1-\gamma} \right) + (1 - p_t^s) \kappa \mathbb{E}_t \left(B_{t+1}^{1-\gamma} \right) \right)^{\frac{1}{1-\gamma}}$ is the certainty equivalent at time t of the investor's (random) utility at time $t + 1$ conditioned on information available at time t . Equation (1.8) is thus a recursive utility specification that allows for narrow framing, that is, the investor can also get utility directly from assets $i = m + 1, \dots, n$ and the constant-life annuity bought

at time t . The utility function U_t is defined as follows. For a random variable G_{t+1} with cumulative distribution F_t at time t we have

$$(1.11) \quad U_t(G_{t+1}) = \int_{-\infty}^0 \bar{v}(x) \frac{d}{dx} [w^-(F_t(x))] dx + \int_0^{\infty} \bar{v}(x) \frac{d}{dx} [-w^+(1 - F_t(x))] dx.$$

where

$$(1.12) \quad \bar{v}(x) = \begin{cases} x, & x \geq 0 \\ \lambda x, & x < 0 \end{cases},$$

and the probability weighting functions are

$$(1.13) \quad w^+(p) = \frac{p^{\delta^+}}{(p^{\delta^+} + (1-p)^{\delta^+})^{1/\delta^+}}, \quad \delta^+ \in (0.3, 1]$$

$$(1.14) \quad w^-(p) = \frac{p^{\delta^-}}{(p^{\delta^-} + (1-p)^{\delta^-})^{1/\delta^-}}, \quad \delta^- \in (0.3, 1].$$

The function U_t corresponds to the CPT value function, consistent with the motivation given by Barberis and Huang (2009) suggesting that CPT is the natural choice to be coupled with narrow framing. Note that when $\delta^+ = \delta^- = 1$ (no probability weighting, that is, $w^+(p) = w^-(p) = p$ for all $p \in [0, 1]$), then

$$U_t(G_{i,t+1}) = \mathbb{E}_t [\bar{v}(G_{i,t+1})]$$

which is the case considered by Barberis and Huang (2009).

As shown in Barberis and Huang (2008b), if $1 < 2 \min(\delta^+, \delta^-)$, which is the case in many calibrations of CPT (see Abdellaoui 2000), then the function U_t can be written as

$$(1.15) \quad U_t(G_{t+1}) = - \int_{-\infty}^0 w^-(F_t(x)) d\bar{v}(x) + \int_0^{\infty} w^+(1 - F_t(x)) d\bar{v}(x)$$

When $G_{i,t+1}$ is given by Equation (1.10) with a fixed reference point $R_{i,z}$, $\theta_{i,t} > 0$ and $W_t > C_t$, then the following holds:

$$F_{i,t}(x) = \mathbb{P}[G_{i,t+1} \leq x] = \mathbb{P}\left[R_{i,t+1} \leq \frac{x}{\theta_{i,t}(W_t - C_t)} + R_{i,z}\right] = F_{i,t+1}^R\left(\frac{x}{\theta_{i,t}(W_t - C_t)} + R_{i,z}\right)$$

and thus

$$\begin{aligned} U_t(G_{i,t+1}) &= - \int_{-\infty}^0 w^-\left(F_{i,t+1}^R\left(\frac{x}{\theta_{i,t}(W_t - C_t)} + R_{i,z}\right)\right) d\bar{v}(x) \\ &\quad + \int_0^{\infty} w^+\left(1 - F_{i,t+1}^R\left(\frac{x}{\theta_{i,t}(W_t - C_t)} + R_{i,z}\right)\right) d\bar{v}(x) \\ &= \theta_{i,t}(W_t - C_t) \left[- \int_{-\infty}^0 w^-(F_{i,t+1}^R(y + R_{i,z})) d\bar{v}(y) \right. \\ &\quad \left. + \int_0^{\infty} w^+(1 - F_{i,t+1}^R(y + R_{i,z})) d\bar{v}(y) \right] \\ &= \theta_{i,t}(W_t - C_t) U_t(R_{i,t+1} - R_{i,z}). \end{aligned}$$

Note that $U_t(G_{i,t+1}) = \theta_{i,t}(W_t - C_t)U_t(R_{i,t+1} - R_{i,z})$ obviously holds also when $\theta_{i,t} = 0$ or $W_t = C_t$. Similarly, for $\theta_{i,t} < 0$ and $W_t > C_t$ we have

$$U_t(G_{i,t+1}) = |\theta_{i,t}|(W_t - C_t)U_t(R_{i,t+1} - R_{i,z}).$$

Finally, Equation (1.8) can be written as

$$(1.16) \quad \begin{aligned} V_t = & H \left(C_t, \left(p_t^s \mathbb{E}_t \left(V_{t+1}^{1-\gamma} \right) + (1 - p_t^s) \kappa \mathbb{E}_t \left(B_{t+1}^{1-\gamma} \right) \right)^{\frac{1}{1-\gamma}} + \right. \\ & \left. + b_0 (W_t - C_t) \left(p_t^s \sum_{i=m+1}^n |\theta_{i,t}| U_t(R_{i,t+1} - R_{i,z}) + |\theta_{0,t}| U_t(R_{0,t+1} - R_{0,z}) \right) \right). \end{aligned}$$

We conclude the specification of the model with the pricing of the constant-life annuity. An actuarially fair priced constant-life annuity with payments L_t starting at time $t + 1$ and premium A_t at time t satisfies

$$(1.17) \quad A_t = \sum_{s=t+1}^T \frac{1}{R_f^{s-t}} \prod_{u=t}^s p_u^a L_t,$$

where $\prod_{u=t}^s (p_u^a)$ is the probability of surviving from t to s and R_f is the risk-free gross returns. It follows that in case of survival at time $t + 1$ we have

$$(1.18) \quad R_{0,t+1} = \frac{L_t}{A_t} = \left(\sum_{s=t+1}^T \frac{1}{R_f^{s-t}} \prod_{u=t}^s p_u^a \right)^{-1} = \frac{1}{h_t},$$

where

$$h_t = \sum_{s=t+1}^T \frac{1}{R_f^{s-t}} \prod_{u=t}^s p_u^a$$

is the so called annuity factor.

When the constant-life annuity is not actuarially fair priced, the annuity factor h_t is increased by a proportion $1 + \delta$, where $\delta > 0$ represents administrative costs incurred by the insurance company. In that case, the annuity factor becomes $(1 + \delta) h_t$.

Finally, the CPT utility on constant-life annuities bought at time t is

$$U_t(R_{0,t+1} - R_{0,z}) = w^+(p_t^s) \left(\frac{1}{h_t} - r_{0,z} \right) - \lambda w^-(1 - p_t^s) r_{0,z}.$$

1.3.3 The maximization problem

From Equation (1.16) we derive the Bellman equation

$$\begin{aligned}
J_t(W_t, \ell_t) &= \max_{C_t, \theta_t} H \left(C_t, \left(p_t^s \mathbb{E}_t (J_{t+1}(W_{t+1}, \ell_{t+1})^{1-\gamma}) + (1 - p_t^s) \kappa \mathbb{E}_t (B_{t+1}^{1-\gamma}) \right)^{\frac{1}{1-\gamma}} + \right. \\
&\quad \left. + b_0 (W_t - C_t) \left(p_t^s \sum_{i=m+1}^n |\theta_{i,t}| U_t(R_{i,t+1} - R_{i,z}) + |\theta_{0,t}| U_t(R_{0,t+1} - R_{0,z}) \right) \right), \\
&= \max_{C_t, \theta_t} H \left(C_t, \left(p_t^s \mathbb{E}_t (J_{t+1}(W_{t+1}, \ell_{t+1})^{1-\gamma}) + (1 - p_t^s) \kappa \mathbb{E}_t \left(((W_t - C_t) \theta'_{1,t} \mathbf{R}_{1,t+1})^{1-\gamma} \right) \right)^{\frac{1}{1-\gamma}} + \right. \\
&\quad \left. + b_0 (W_t - C_t) \left(p_t^s \sum_{i=m+1}^n |\theta_{i,t}| U_t(R_{i,t+1} - R_{i,z}) + |\theta_{0,t}| U_t(R_{0,t+1} - R_{0,z}) \right) \right),
\end{aligned}$$

where $\ell_{t+1} = \frac{(W_t - C_t) \theta_{0,t} R_{0,t+1} + W_t \ell_t}{(W_t - C_t) \theta'_{1,t} \mathbf{R}_{1,t+1} + W_t \ell_t}$.

At the last decision making period T , we have $p_T^s = 0$ and the boundary condition

$$\begin{aligned}
J_T(W_T) &= \max_{C_T, \theta_{1,T}} H \left(C_T, \left(\kappa \mathbb{E}_T (B_{T+1}^{1-\gamma}) \right)^{\frac{1}{1-\gamma}} \right), \\
&= \max_{C_T, \theta_{1,T}} H \left(C_T, \left(\kappa \mathbb{E}_T \left(((W_T - C_T) \theta'_{1,T} \mathbf{R}_{1,T+1})^{1-\gamma} \right) \right)^{\frac{1}{1-\gamma}} \right)
\end{aligned}$$

independent from ℓ_T . Note that the decision problem at time T can be separated as follows:

$$\begin{aligned}
J_T(W_T) &= \max_{C_T, \theta_{1,T}} \left(C_T^\rho + \beta \left(\kappa \mathbb{E}_T \left(((W_T - C_T) \theta'_{1,T} \mathbf{R}_{1,T+1})^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \right)^\rho \right)^{\frac{1}{\rho}} \\
&= \max_{C_T, \theta_{1,T}} \left(C_T^\rho + \beta \kappa^\rho (W_T - C_T)^\rho \left(\left(\mathbb{E}_T (\theta'_{1,T} \mathbf{R}_{1,T+1})^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \right)^\rho \right)^{\frac{1}{\rho}} \\
&= \max_{C_T} \left(C_T^\rho + \beta \kappa^\rho (W_T - C_T)^\rho \max_{\theta_{1,T}} \left(\left(\mathbb{E}_T (\theta'_{1,T} \mathbf{R}_{1,T+1})^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \right)^\rho \right)^{\frac{1}{\rho}}
\end{aligned}$$

If we set $D_T^* = \max_{\theta_{1,T}} \left(\left(\mathbb{E}_T (\theta'_{1,T} \mathbf{R}_{1,T+1})^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \right)^\rho$ then

$$J_T(W_T) = \max_{C_T} (C_T^\rho + \beta \kappa^\rho (W_T - C_T)^\rho D_T^*)^{\frac{1}{\rho}}$$

and the first order condition is

$$C_T^{\rho-1} - \beta \kappa^\rho (W_T - C_T)^{\rho-1} D_T^* = 0,$$

which implies

$$\alpha_T^* = \frac{C_T^*}{W_T} = \left[\left(\frac{1}{\beta \kappa^\rho D_T^*} \right)^{1/(\rho-1)} + 1 \right]^{-1}$$

and

$$J_T(W_T) = C_T^*{}^{\frac{\rho-1}{\rho}} W_T^{\frac{1}{\rho}}.$$

1.3.4 Computational aspects

We apply numerical dynamic programming to compute the optimal consumption and portfolio decisions over time from the investor's Bellman equation (see, for example, Judd (1998) and Rust (1996)). We consider an investor at time t who is in state S_t . In our model, the two state variables are W_t (the investor's wealth) and ℓ_t (the proportion of annuity payments at time t with respect to the overall wealth W_t). Generally, this state S_t could cover any kind of relevant information about the investor or market which would impact the investor's optimal behaviour.

During period t , the investor consumes C_t and invests her remaining wealth across some assets with a proportion specified by the vector θ_t and collects the constant payments from life annuities bought at time $t - 1$ or before. At the end of the period $t + 1$, the investor is in state W_{t+1} with a proportion of annuity payments with respect to her overall wealth ℓ_{t+1} . These are random variables because of the stochastic nature of the asset returns. The investor's optimal utility V_t , optimal consumption C_t and optimal portfolio θ_t are given by the Bellman equation which can be expressed as follows

$$\begin{aligned} V_t(W_t, \ell_t) &= \max_{C_t, \theta_t} H(C_t, \mathcal{U}_t(V_{t+1}(W_{t+1}, \ell_{t+1}), \mathcal{V}_{t+1}(C_t, \theta_t, W_t))), \\ (1.19) \quad &= \max_{C_t, \theta_t} J(C_t, W_t, \mathcal{U}_t(V_{t+1}(W_{t+1}, \ell_{t+1}), \mathcal{V}_{t+1}(C_t, \theta_t, W_t))), \end{aligned}$$

where H represents the combination of the utility derived from consumption in the current time period t , the current value of optimal utility $V_{t+1}(W_{t+1}, \ell_{t+1})$ and the sum of gains and losses from the narrowly framed assets $\mathcal{V}_{t+1}(C_t, \theta_t, W_t)$ at time $t + 1$. The function \mathcal{U}_t calculates the current value of next period's utility by summing up the certainty equivalent at time t of $V_{t+1}(W_{t+1}, \ell_{t+1})$ and the CPT value of gains and losses $\mathcal{V}_{t+1}(C_t, \theta_t, W_t)$ from the framed assets. The function which needs to be maximized at each step is $J(C_t, W_t, \mathcal{U}_t(V_{t+1}(W_{t+1}, \ell_{t+1}), \mathcal{V}_{t+1}(C_t, \theta_t, W_t)))$.

We observe from Equation (1.19) that its left part V_t is defined in terms of V_{t+1} on the right. The main idea to tackle this problem is that, given that we know V_{t+1} , V_t can be solved by maximizing J over C_t and θ_t . Specifically, we resort into a backward induction technique. Starting with T which is the investor's last year of survival, we assume that she leaves all her remaining wealth as bequests, therefore we know $V_T(W_T) = C_T^* \frac{\rho-1}{\rho} W_T^{\frac{1}{\rho}}$. Then, we calculate V_{T-1} by applying Equation (1.19) over some fixed points of state W_{T-1} and ℓ_{T-1} . For each selected state W_{T-1} and ℓ_{T-1} we then maximize over C_{T-1} and θ_{T-1} in order to find $V_{T-1}(W_{T-1}, \ell_{T-1})$. In order to ensure that we obtain global optima and not just local ones, we conduct a grid search over possible values of C_{T-1} and θ_{T-1} . We then identify the best region of this space to within a few percent and use a standard function optimization algorithm to locate the optimum within this sub-region to several decimal places. Given the values of W_{T-1} , ℓ_{T-1} , C_{T-1} and θ_{T-1} , we obtain J function by substituting all these values and perform integration where necessary.

Subsequently, we repeat the steps above to compute an estimate for V_{T-2} . Differently from before, where $V_T(W_T)$ can be obtained for any W_T , now we have just the estimates of V_{T-1} on a finite set of W_{T-1} and ℓ_{T-1} values. In order to estimate V_{T-1} based on the points we know, we interpolate and fit a spline function on a two dimensional surface over W_{T-1} and ℓ_{T-1} . After estimating

$V_{T-1}(W_{T-1}, \ell_{T-1})$, we calculate V_{T-2} and the optimal values of C_{T-2} and θ_{T-2} as in the previous procedure. By continuously repeating this process, we obtain an estimate of the investor's optimal utility, consumption and investment portfolio at each point in time.

1.4 Optimal asset allocation with annuities

In this section, we study investor's optimal annuitization and asset allocation strategy. Each year the investor allocates her post-consumption wealth across three assets. The first asset is a constant-life annuity with $R_{0,t+1} = \frac{L_t}{A_t}$ as in Equation (1.18). The second asset is risk-free bonds and has net returns $R_f - 1 = 2\%$. The third asset is risky stocks with gross returns $R_{2,t+1}$. We assume that the stocks gross returns is log-normally distributed, as follows

$$(1.20) \quad \log R_{2,t+1} = g_2 + \sigma_2 \epsilon_{2,t+1}.$$

The investor's wealth evolves according to

$$(1.21) \quad W_{t+1} = (W_t - C_t) (\theta_{0,t} R_{0,t+1} + \theta_{1,t} R_f + \theta_{2,t} R_{2,t+1}) + W_t \ell_t.$$

The three assets are narrowly framed where the investor holds preferences according to Equations (1.8)-(1.10), where $n = 2$ and $m = 0$. We assume $\rho = 1 - \gamma$, where γ is the parameter of risk aversion. Moreover, the reference gross returns $R_{i,z}$ used to specify U_t in Equation (1.8) is set to $R_f = 1.02$ and corresponds to the risk-free gross returns. For the stocks gross returns, we use drift rate and volatilities based on annual returns for the S&P 500 index from January 1946 to January 2009. These correspond to $g_2 = 6.15\%$ and $\sigma_2 = 15.49\%$.

As a base case, we compute the optimal annuitization and asset allocation strategy excluding loads for annuities ($\delta = 0$). The starting age is set to 20, the retirement age to 65 ($t=46$) and the maximum age to 100 ($T=81$). As in Horneff, Maurer, and Stamos (2008) we fit the Gompertz force of mortality to the 2000 Population Basic mortality table for US females. The estimated parameters are given by $m_f^{(s,a)} = 86.85$ and $b_f^{(s,a)} = 9.98$. We assume an asymmetry between the insurer's and the annuitant's mortality beliefs. The insurer's belief on the annuitant's mortality is based on an assumption that $v^a = 1$ in Equation (1.5)-(1.7). This assumption results in probability of surviving p_u^a in Equation (1.17). We assume that the annuitant's belief is different than the insurer. Upon purchasing annuity, the annuitant usually projects that she is healthier than most people in the population. Hence, for the annuitant, we take $v^s = 0.5$ with probability of surviving p_t^s . The preferences parameters are set as follows: coefficient of relative risk aversion $\gamma = 2$, discount factor $\beta = 0.98$. We also introduce a moderate bequest motives $\kappa = 2$.

In terms of the behavioural factors specification, we consider the degree of narrow framing to be $b_0 = 0.4$. We do not consider CPT in the base case, hence the parameter of loss aversion is $\lambda = 1$ and those of probability weighting are $\delta^+ = 1$ and $\delta^- = 1$. A summary of our model's parametric specification is reported in Table 1.1 below.

SUMMARY OF MODEL'S PARAMETRIC SPECIFICATION

No	Descriptions	Parameters	Values
1	Risk-free gross returns	R_f	1.02
2	Drift rates of the risky gross returns	g_2	6.15%
3	Volatilities of the risky gross returns	σ_2	15.49%
4	Discount factor	β	0.98
5	Coefficient of relative risk aversion	γ	2
6	Parameter accounts for bequests	κ	2
7	Parameter determining the shape of the force of mortality function	$m_f^{(s,a)}$	86.85
8	Parameter determining the shape of the force of mortality function	$b_f^{(s,a)}$	9.98
9	Parameter to calculate the force of mortality by the insurers	v^a	1
10	Parameter to calculate the annuitant's subjective force of mortality	v^s	0.5
11	Degree of narrow framing	b_0	0.4
12	Parameter of the probability weighting function (losses)	δ^+	1
13	Parameter of the probability weighting function (gains)	δ^-	1
14	Degree of loss aversion	λ	1

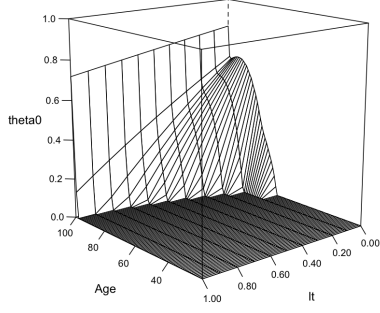
Table 1.1: This table shows our model's parametric specification. The second and third parameters are based on the annual returns of the S&P 500 index from January 1946 to January 2009. The seventh and eighth parameters are fitted using the Gompertz force of mortality to the 2000 Population Basic mortality table for US females. The last three parameters show that the base case do not consider behavioural factors such as probability weighting and loss aversion.

Figure 1.1 shows the share of post-consumption wealth allocated to annuities, bonds and stocks as a function of both existing annuity payments over wealth (ℓ_t) and age. This result enables us to analyze the age and annuity income effects for the various investment and annuitization policies. For example, at the initial stage of her adult life (age 20), an individual who never purchases annuities before ($\ell_t = 0$) would allocate 0% of her post-consumption wealth ($\theta_{0,1}$) into annuities. She would assign around $\theta_{1,1} = 25\%$ into bonds and $\theta_{2,1} = 75\%$ into stocks. The remaining wealth is used for consumption purposes only. In almost all ages from 20 to 69 years old, if we start with $\ell_t = 0$, annuities are fully crowded out by stocks and bonds, where each have a stable allocation of around 75% and 25%. Stocks are a very liquid asset as compared to annuities, which demands a higher compensation for the investor in terms of mortality credit. From the age of 70 onwards, there is a switch between the desire to allocate into bonds and stocks. The allocation to bonds diminishes and reaches zero over time until the age of around 99, while the one of stocks hikes up. Stocks are now held more in order to align the portfolio to the desired return and risk profile. When the individual reaches the age of retirement (age 70), her allocation into annuities increases progressively. At the age of 99, where she knows that her mortality credit is the highest before she dies for sure at 100, then she wants to hold very high level of annuities. In the other states, where the individual already accumulates some annuity payments from her previous annuity purchases and it reaches 10% or more over her total wealth ($\ell_t \geq 10\%$), she allocates nothing into annuities at almost any stage of her life (age 20 to 70). In fact, when $\ell_t \geq 30\%$, it is optimal for the investor to hold a lot of stocks at age 20 to 70. From the age of 70 onwards, the allocation to annuities hikes up as before until the investor reaches the age of 99 where she highly annuitizes.

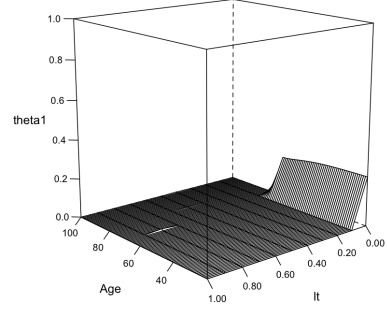
It is also useful to analyze the expected life-cycle profile when investors follow the optimal holdings derived above. The left graph in Figure 1.2 describes the expected optimal composition of total post-consumption wealth over time. The expectations are calculated by performing 300,000 Monte Carlo simulations using the optimal policies derived before in Figure 1.1. At the starting age of 20, the expected optimal allocation of the post-consumption wealth into bonds and stocks are around 25% and 75% respectively, while nothing is allocated into annuities. The expected optimal allocation into annuities stays at zero until the age of 67. From then on, it gradually rises up to 77% at the age of 99 years old. The expected optimal allocation into bonds is at 25% at the age of 21 and slowly decreasing till 0% at the age of 74 years old onwards. On the contrary, the expected optimal allocation into stocks is at 75% at the age of 21 and slowly hikes up till as high as 96.5% at 79 years old before it reaches 23% at the age of 99.

The right graph in Figure 1.2 presents the expected level of consumption, annuity income and liquid savings from stocks and bonds. The result shows that the optimal consumption over time is almost constant and then slowly increases. At the age of 20, it is assumed that the annuity income earned from the previous period is zero and the investor only starts allocating some of his post-consumption wealth into annuities. At the age of 21 to 67 years old, the annuity income earned from the existing post-consumption wealth invested into annuities are at 0% of the current wealth. The income gained from annuities only starts rising from the age of 68 because the investor starts allocating into annuities.

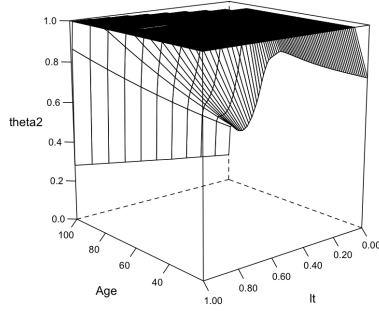
OPTIMAL ASSET ALLOCATION: BASE CASE



(a) allocation to annuities



(b) allocation to bonds



(c) allocation to stocks

Figure 1.1: We assume a female with maximum life-span age 20-100, no loads for annuities ($\delta = 0$), with mortality asymmetries, Relative Risk Aversion ($\gamma=2$), and bequest motives ($\kappa = 2$). The top left figure depicts share of post-consumption wealth allocated into annuities ($\theta_{0,t}$) and the top right is for bonds ($\theta_{1,t}$). The bottom figure describes the allocation into stocks ($\theta_{2,t}$). These figures are shown against the proportion of annuity payments with respect to the overall wealth (ℓ_t) at each age.

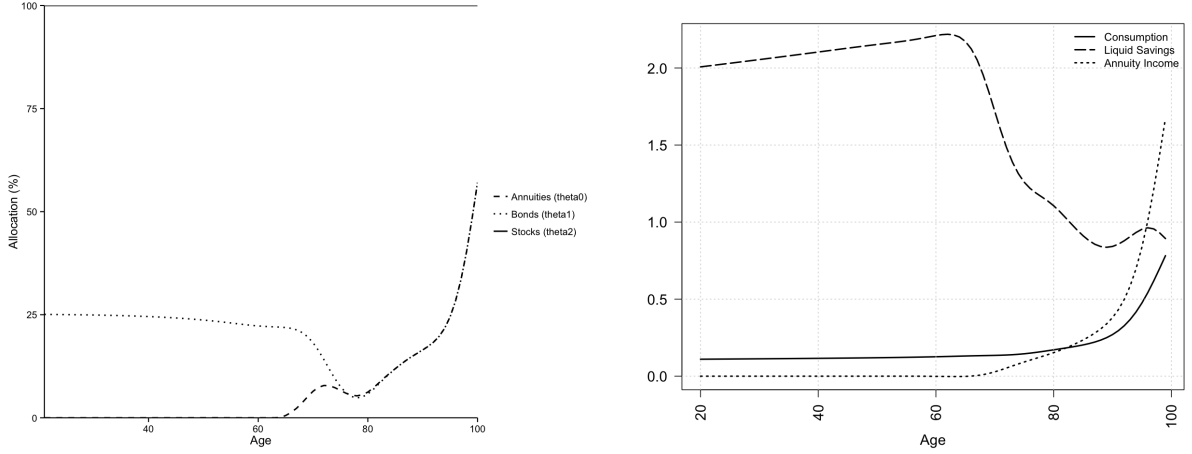


Figure 1.2: We calculate the expectations by performing 300,000 Monte Carlo simulations using the optimal policies derived above in Figure 1.1. The expected optimal asset allocation is the share of post-consumption wealth invested into annuities, stocks or bonds. The life-cycle profile consists of consumption, liquid savings from the value of the invested stocks and bonds and the annuity income (all the annuity payments received at each age). The model parameters are the same as in Table 1.1. We assume that the individual has no initial endowment at age 20.

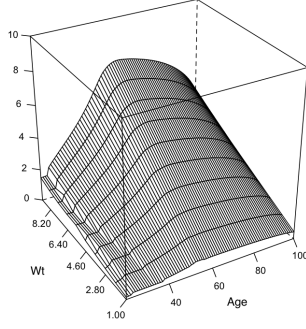
From then on, the annuity income rises until at the last age of 99 before she dies, the annuity income reaches its peak. These results suggest that it is not optimal for the investor to purchase annuities at the beginning of the period. It is only profitable to do so when she is at the later stage of her retirement (68 years old onwards), in order to take advantage of the higher mortality credit prior to her demise. The amount of liquid savings from stocks and bonds starts at around at a high level and then gradually drops from the age of 69 onwards.

1.4.1 The role of behavioural preferences in explaining the low annuity market participation

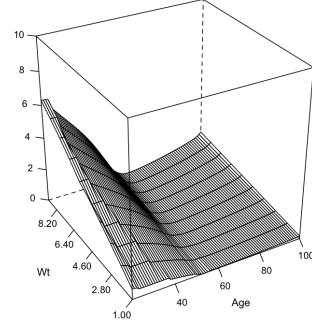
In this section we illustrate the importance of behavioural preferences in explaining the low annuity market participation. In order to achieve this we identify some reference models which are special cases of our model for comparison. The first reference model is the case where no behavioural preferences (No BP) is included ($b_0 = 0$), hence, we are back to the classical recursive utility preferences with bequests. Our first reference model is similar to the one of Horneff, Maurer, and Stamos (2008), with the difference that we do not include the stochastic labour income process for simplification. Indeed, if we replicate the optimal asset allocation and compare it with their results (refer to Horneff, Maurer, and Stamos (2008) page 3599), we get very similar results as shown in Figure 1.3.

The second reference model is the case with narrow framing ($b_0 = 0.4$), other behavioural preferences parameters (loss aversion ($\lambda = 2.25$) and probability weighting ($\delta^+ = 0.61$ and $\delta^- = 0.69$)). The

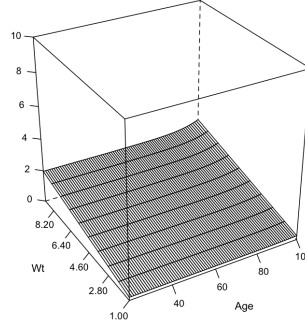
OPTIMAL ASSET ALLOCATION: THE FIRST REFERENCE MODEL ($b_0 = 0$)



(a) Annuity demand



(b) Bonds policy



(c) Stocks policy

Figure 1.3: We assume a female with maximum life-span age 20-100, no loads for annuities ($\delta = 0$), with mortality asymmetries, Relative Risk Aversion ($\gamma=2$), and bequest motives ($\kappa = 2$). The top left figure depicts the annuity demand ($\theta_{0,t}*(W_t - C_t)$) and the top right is the bonds policy ($\theta_{1,t}*(W_t - C_t)$). The bottom figure describes the allocation into stocks policy ($\theta_{2,t}*(W_t - C_t)$). These figures are shown against the wealth (W_t) at each age and $\ell_t = 0$.

third reference model presents the case with narrow framing ($b_0 = 0.4$) and loss aversion ($\lambda = 2.25$), but without probability weighting ($\delta^+ = 1$ and $\delta^- = 1$). The fourth one refers to the case with narrow framing ($b_0 = 0.4$) and probability weighting ($\delta^+ = 0.61$ and $\delta^- = 0.69$), but without loss aversion ($\lambda = 1$). Table 1.2 below shows how the share of post-consumption wealth invested into annuities at each age differs between the base case than those of the four reference models.

We observe that the first allocation to annuities, when the annuity payments is assumed to be $\ell_t = 0$ at the age of 20, is 0% for our base case and the third and fourth reference models. The first reference model with no behavioural preferences produce allocation to annuities which is around 17.8%. The second reference model, which includes a combination of narrow framing, loss aversion, and probability weighting, suggests a very high annuities allocation (around 95.1%). This implies that at the age of 20, when an investor is very loss averse and narrowly framed her assets, the decision to invest into the three assets are solely pinned down by the returns of the assets. In this case, the risk-free gross returns are fixed at $R_f = 2\%$ while the stocks gross returns are with mean $g_2 = 6.15\%$ and standard deviation of $\sigma_2 = 15.49\%$. On the other hand, annuities have returns which range at around 3%. Hence, an investor with a moderate risk appetite ($\gamma = 2$), has the tendency to allocate more into annuities than the other two assets. At the next age of 21, when the investor already invested some share of his post-consumption wealth into annuities at 20, she starts accumulating her annuity payments ($\ell_t \neq 0$) and therefore the returns of the annuities are not the only driver of the decision to invest into annuities anymore, hence its allocation drops to 7%.

Our base case suggests that it is optimal to invest 0% into annuities from 21 years old up until 67 years old. As compared to the first three reference models, the allocation into annuities at those ages is not consistently 0% and its slightly increases with age. The first reference model, where no behavioural preferences (No BP) is included ($b_0 = 0$), shows that it is optimal to invest between 3.2% to 16.6% from 21-67 years old. The second case, where narrow framing is introduced ($b_0 = 0.4$) with loss aversion ($\lambda = 2.25$) and probability weighting ($\delta^+ = 0.61$ and $\delta^- = 0.69$), depicts that it is optimal to invest between 5.7% to 12.7% from 21-67 years old. The third reference model, where we included narrow framing ($b_0 = 0.4$) and loss aversion ($\lambda = 2.25$), but not probability weighting ($\delta^+ = 1$ and $\delta^- = 1$), shows the optimal allocation into annuities ranging between 0% to 13.6%. The fourth reference model, with narrow framing ($b_0 = 0.4$) and probability weighting ($\delta^+ = 0.61$ and $\delta^- = 0.69$) but no loss aversion ($\lambda = 1$), is similar to the base case. It also suggests zero annuitization from 21 to 67 years old.

From this comparison we conclude that the base case model which includes narrow framing but no loss aversion and no probability weighting generates always zero annuity market participation from the earliest age of 20 years old up until 67 years old which is lower than all the reference models. Hence, other behavioural parameters such as loss aversion and probability weighting are not the main driver of the zero annuity market participation. The impact of probability weighting is very marginal while a high loss aversion may result in a very high annuities' allocation as early as 20 years old.

From 68 years old until 99 years old the base case shows that the share of post-consumption wealth invested into annuities are gradually increasing from 3.5% to 77%. These results are still relatively

low as compared to the reference models for the same age range. From 98 to 99 years old the base case suggests that the allocation into annuities should be the highest at 77%. The rationale is because the mortality credit is the highest before she dies for sure at 100. In a nutshell, the returns of the annuities are the main driver of the decision to invest into annuities towards the end, where the agent knows that she will earn the highest mortality credit before dying for sure. In the other age range, the main driver of zero and relatively lower annuity market participation is the degree of narrow framing.

Now we change the behavioural preferences parameters defined on the narrowly framed assets based on the CPT. Furthermore, we compare between the base case and an alternative scenario where the degree of narrow framing is lower than the base case ($b_0 = 0.2$). Table 1.3 below shows how the share of post-consumption wealth invested into annuities at each age of the base case differs from the alternative scenario.

We observe that the first allocation to annuities, when the annuity payments is assumed to be $\ell_t = 0$ at the age of 20 years old for the alternative case, is the same as in our base case (0%). The alternative case ($b_0 = 0.2$) suggests that it is optimal to invest 0% into annuities from 21 years old up until 60 years old. This is a less pronounced case of zero annuitization as compared to our base case. From 61 to 99 years old, the case where $b_0 = 0.2$ shows that the share of post-consumption wealth invested into annuities is gradually increasing from 1% to 66.3%. These results are lower as compared to the base case for the same age ranges. The rationale is because the mortality credit increases before she dies for sure at 100. These results show that the higher the degree of narrow framing, the longer the investor is holding zero annuitization.

Figure 1.4 describes a comparison of the the expected optimal allocation into annuities over time. At the starting age of 20 the expected optimal allocation of the post-consumption wealth into annuities is 0% for both cases. The interesting part is that when the investor has lower degree of narrow framing, her expected optimal allocation suggests zero annuitization up until the age of 60. The lower the degree of narrow framing, the gradual increment of annuitization from the age of 61 onwards becomes less evident than in the base case.

The left graph in Figure 1.5 describes the expected optimal composition of total post-consumption wealth over time. The expectations are calculated by performing 300,000 Monte Carlo simulations using the optimal policies derived with degree of narrow framing $b_0 = 0.2$, while the remaining parameters stay the same as in Table 1.1. At the starting age of 20 the expected optimal allocation of the post-consumption wealth into bonds and stocks reaches around 36% and 64% respectively, while nothing is allocated into annuities. The expected optimal allocation into annuities subsequently increases at 61 years old and reaches 66% at 99 years old. This differs from the base case where the increment happened only as late as the age of 67. It suggests that the investor starts purchasing annuities earlier if her degree of narrow framing is lower. The expected optimal allocation into stocks stays at 64% and reaches its peak at 96% at the age of 70 before it slowly goes down till 34% at 99 years old. On the other hand, the expected optimal allocation into bonds remains at 36% before it drops to zero from 74 years old onwards.

The right graph in Figure 1.5 presents the expected level of consumption, annuity income and

THE EXPECTED OPTIMAL ALLOCATION INTO ANNUITIES OVER TIME: AN ALTERNATIVE CASE OF $b_0 = 0.2$

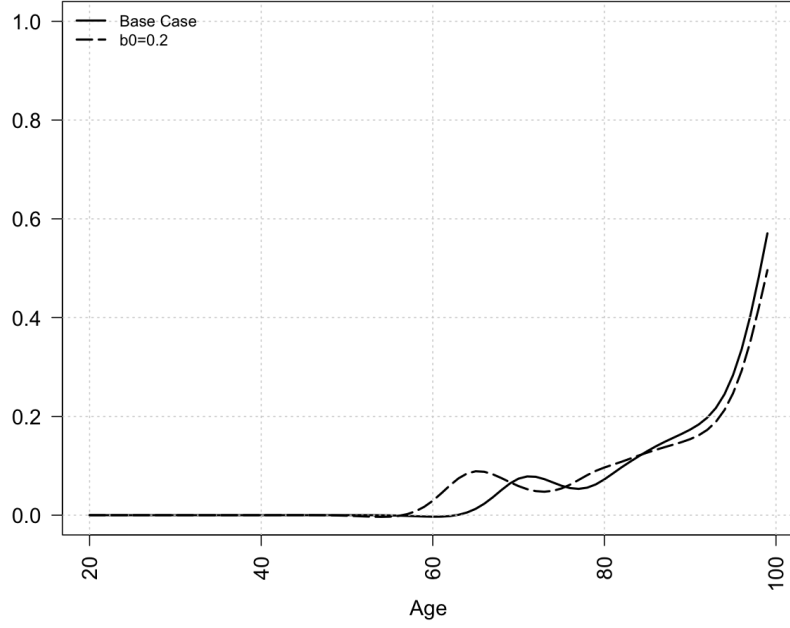


Figure 1.4: We consider the alternative case where $b_0 = 0.2$. The figure shows the expected optimal allocation into annuities over time. At the starting age of 20 the expected optimal allocation of the post-consumption wealth into annuities is 0% for both cases. The interesting part is, when the investor has lower degree of narrow framing, her expected optimal allocation suggests zero annuitization up until the age of 60 years old. The lower the degree of narrow framing, the gradual increment of annuitization from the age of 61 onwards becomes less evident than in the base case.

EXPECTED ASSET ALLOCATION AND EXPECTED LIFE-CYCLE PROFILE: AN ALTERNATIVE CASE OF $b_0 = 0.2$

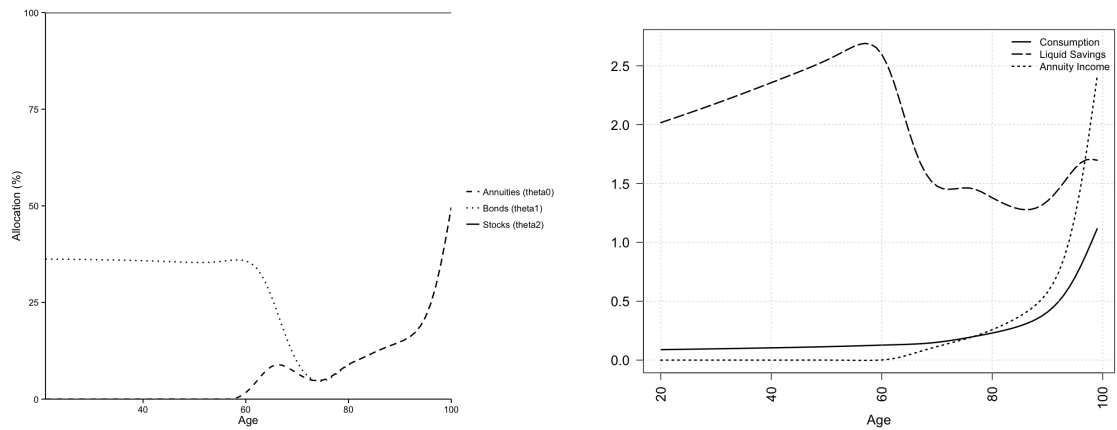


Figure 1.5: We consider the alternative case where $b_0 = 0.2$. The left figure depicts the expected asset allocation while the right one shows the expected life-cycle profile.

liquid savings from stocks and bonds for the case where $b_0 = 0.2$. The result shows that the optimal consumption over time is almost constant before slowly rising from 70 years old onwards. The annuity income earned from the previous period is zero and the investor only starts allocating some of his post-consumption wealth into annuities. The annuity income earned from the existing post-consumption wealth invested into annuities only starts increasing at the age of 61 years old. These results suggest that it is not optimal for the investor to purchase annuities at the beginning of the period. It is only profitable to gradually allocate into annuities close to the retirement age (61 years old onwards). The amount of liquid savings from stocks and bonds starts increasing until the age of 60 and gradually drops.

To summarize, the degree of narrow framing is the only driver in the investor's decision to hold zero annuities. Specifically, an investor who narrowly frames her assets presents the following behaviour: if her degree of narrow framing is higher, the gradual increment into full annuitization is much more evident. Moreover, an investor who has a higher degree of narrow framing tends to hold zero annuities much longer.

1.5 Robustness analysis

1.5.1 The role of bequests

In this section, we analyze whether bequests play a major role in explaining the low annuity market participation as suggested by many existing research such as Lockwood (2012). Column 3 and Column 9 in Table 1.4 below show how the share of post-consumption wealth invested into annuities at each age of the base case differs than those without bequests.

We observe that the first allocation to annuities when the annuity payments is assumed to be $\ell_t = 0$ at the age of 20 years old, for the case with no bequests, is equal to our base case (0%). This alternative case also suggests that it is optimal to invest 0% into annuities from 21 to 64 years old, similar to our base case. From 65 to 84 years old it shows that the share of post-consumption wealth invested into annuities is gradually increasing from 4.1% to 87.6%. From 85 to 99 years old it also suggests that the allocation into annuities should be 100%. The rationale is because the mortality credit is the highest before she dies for sure at 100. This also suggests that when the investor has no bequest motives, she would be more willing to hold annuities at a later stage of her life.

Figure 1.6 shows a comparison between the base case and an alternative scenario with no bequests. The chart describes a comparison of the the expected optimal allocation into annuities over time. At the starting age of 20, the expected optimal allocation of the post-consumption wealth into annuities is 0% for both cases. The interesting part is from 65 to 84 years old: the case with no bequests suggests that the expected optimal allocation into annuities should increase more as compared to the base case.

The left chart in Figure 1.7 describes the expected optimal composition of total post-consumption wealth over time. The expectations are calculated by performing 300,000 Monte Carlo simulations using the optimal policies derived with no bequests, while the remaining parameters stay the same

THE EXPECTED OPTIMAL ALLOCATION INTO ANNUITIES OVER TIME: ALTERNATIVE CASE WITH NO BEQUESTS ($\kappa = 0$)

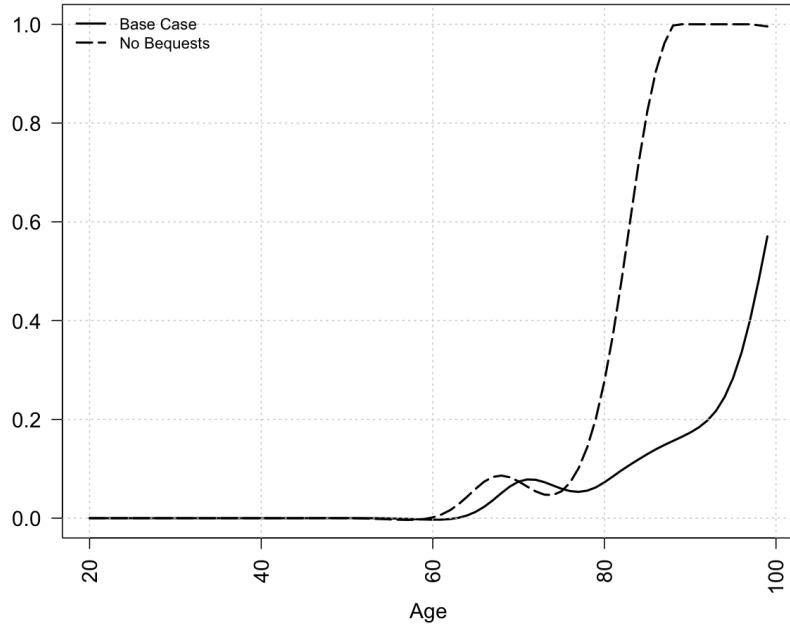


Figure 1.6: We consider the alternative case where the model has the same parameters as in Table 1.1 but with no bequests. The figure shows the expected optimal allocation into annuities over time. At the starting age of 20, the expected optimal allocation of the post-consumption wealth into annuities is 0% for both cases. The interesting part is from the age of 65 to 84: the case with no bequests suggests that the expected optimal allocation into annuities should increase more as compared to the base case.

EXPECTED ASSET ALLOCATION AND EXPECTED LIFE-CYCLE PROFILE: ALTERNATIVE CASE WITH NO
BEQUESTS ($\kappa = 0$)

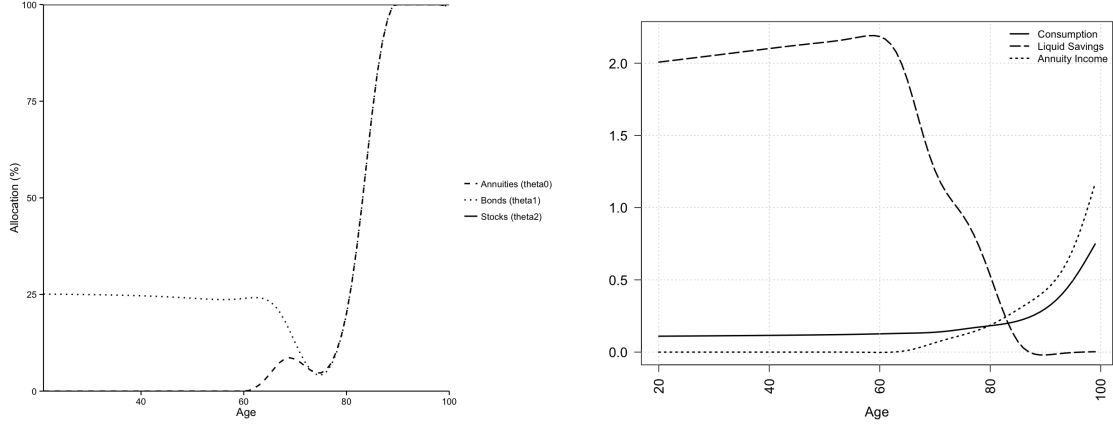


Figure 1.7: We consider the alternative case where the model have the same parameters as in Table 1.1, but with no bequests. The left figure depicts the expected asset allocation while the right one shows the expected life-cycle profile.

as in Table 1.1. At the starting age of 20, the expected optimal allocation of the post-consumption wealth into bonds and stocks reaches around 25% and 75% respectively, while nothing is allocated into annuities. The expected optimal allocation into annuities is gradually rising from around 4% to 88% at 84 years old before it reaches full annuitization from the age of 85 to 99 years old. This differs from the base case where the increment is less than the case with no bequests. In other words, an investor who has no bequests would like to annuitize around 30% more at later stage of her life as compared to one who wants to leave bequests. The expected optimal allocation into bonds is at 7.5% at the age of 67 and slowly decreasing till as low as 0% at 99 years old. On the contrary, the expected optimal allocation into stocks reaches its peak at 96% before it finally reaches 0% at 99 years old.

The right chart in Figure 1.7 presents the expected level of consumption, annuity income and liquid savings from stocks and bonds for the case with no bequests. The result shows that the optimal consumption over time is almost constant before slowly rising from 84 years old onwards. The annuity income earned from the previous period is zero and the investor only starts allocating some of his post-consumption wealth into annuities. The annuity income earned from the existing post-consumption wealth invested into annuities only starts increasing at the age of 64 years old. These results suggest that it is not optimal for the investor to purchase annuities at the beginning of the period. It is only profitable to gradually allocate into annuities close to the retirement age (64 years old onwards). The amount of liquid savings from stocks and bonds starts increasing until the age of 64 and gradually drops.

1.5.2 Asset returns specification

In this section, we change the returns specification of each asset separately such as annuity factor with administrative costs $\delta = 7.3\%$; $R_f = 1.05$; $g_2 = 4\%$ and $\sigma_2 = 10\%$. The purpose is to analyze whether they will play a major contribution in changing the annuity market participation. Column 4-6 and Column 10-12 in Table 1.4 below show how the share of post-consumption wealth invested into annuities at each age of the base case differs than those with different returns specification described above.

We observe that the first allocation to annuities, when the annuity payments are assumed to be $\ell_t = 0$ at the age of 20 years old, for the three alternative cases, is the same as our base case (0%). We consider the case of annuity factor with loadings/administrative costs $\delta = 7.3\%$ as in the empirical study by Mitchell (1999) for the annuity markets in the United States. This alternative case with loadings suggests that it is optimal to invest 0% into annuities from 21 years old up until 70 years old. From 71 years old until 99 years old, it shows that the share of post-consumption wealth invested into annuities is gradually increasing from 3.3% to 76%. This is a very similar result as our base case. The case where $R_f = 1.05$ shows that the optimal age to invest 0% into annuities is from 21 to 42 years old. From the age of 43 to 99 years old, it shows that the share of post-consumption wealth invested into annuities is gradually increasing from 2.9% to 40.6%. The case where $g_2 = 4\%$ and $\sigma_2 = 10\%$ shows that it is optimal to invest 0% into annuities from 21 years old up until 70 years old. The increment for the allocation into annuities from 71 to 99 years old is more similar to the base case (ranges from 5.4% to 77%). These results show that the different returns' specification have also a similar impact on the decision to hold zero annuities.

The graph in Figure 1.8 describes a comparison of the expected optimal allocation into annuities over time for the case of annuity factor with loadings/administrative costs $\delta = 7.3\%$. At the starting age of 20 the expected optimal allocation of the post-consumption wealth into annuities is 0% for all the cases. Only the case where $R_f = 1.05$ shows a slight difference than in the base case where the zero annuitization is much shorter. The interesting part is from the age of 71 to 99 years old where all the three alternative cases behave very similar to the base case.

The three graphs in Figure 1.9 describe the expected optimal composition of total post-consumption wealth over time for the three alternative cases. The expectations are calculated by performing 300,000 Monte Carlo simulations using the optimal policies derived for the three separate cases (annuity factor with loadings/administrative costs $\delta = 7.3\%$; $R_f = 1.05$; $g_2 = 4\%$ and $\sigma_2 = 10\%$), while the remaining parameters stay the same as in Table 1.1. The top left chart shows the case where the constant-life annuities are priced according to an annuity factor with loadings/administrative costs $\delta = 7.3\%$. We observe that it behaves similarly to the base case (the left graph in Figure 1.2). The main difference is that the gradual increment from zero annuitization happens as late as 70 years old and reaches 76.4% at 99 years old. The expected optimal allocation into bonds is close to 25% at the age of 20 and slowly decreases until as low as 0% from 76 to 99 years old. On the contrary, the expected optimal allocation into stocks is at 75% at the age of 20 and slowly drops until 23% at the age of 99.

THE EXPECTED OPTIMAL ALLOCATION INTO ANNUITIES OVER TIME: ALTERNATIVE CASES WITH DIFFERENT RETURNS SPECIFICATION

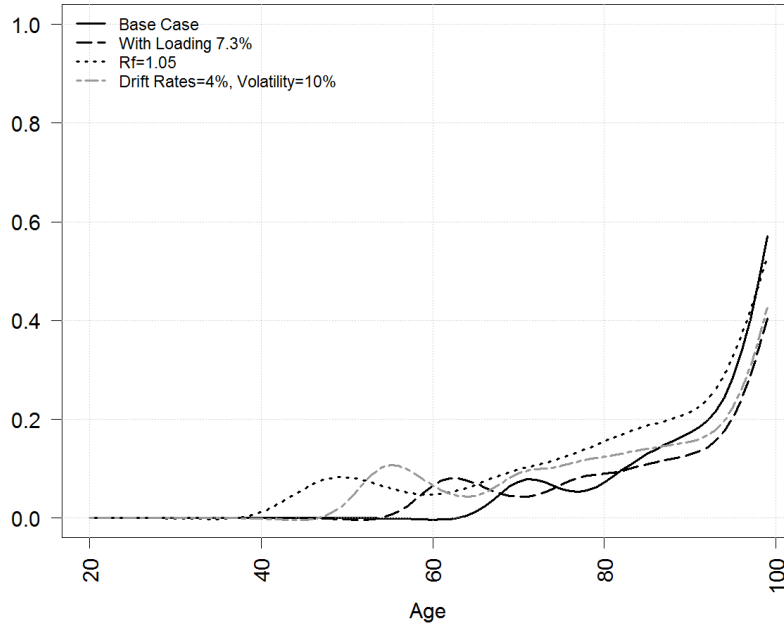


Figure 1.8: We consider the alternative case where we change the returns specification of each asset separately such as annuity factor with administrative costs $\delta = 7.3\%$, $R_f = 1.05$ and $g_2 = 4\%$ with $\sigma_2 = 10\%$. The figure shows the expected optimal allocation into annuities over time. At the starting age of 20 the expected optimal allocation of the post-consumption wealth into annuities is 0% for all the cases. Only the case where $R_f = 1.05$ shows a slight difference than in the base case where the zero annuitization is much shorter.

EXPECTED ASSET ALLOCATION WITH DIFFERENT RETURNS SPECIFICATIONS

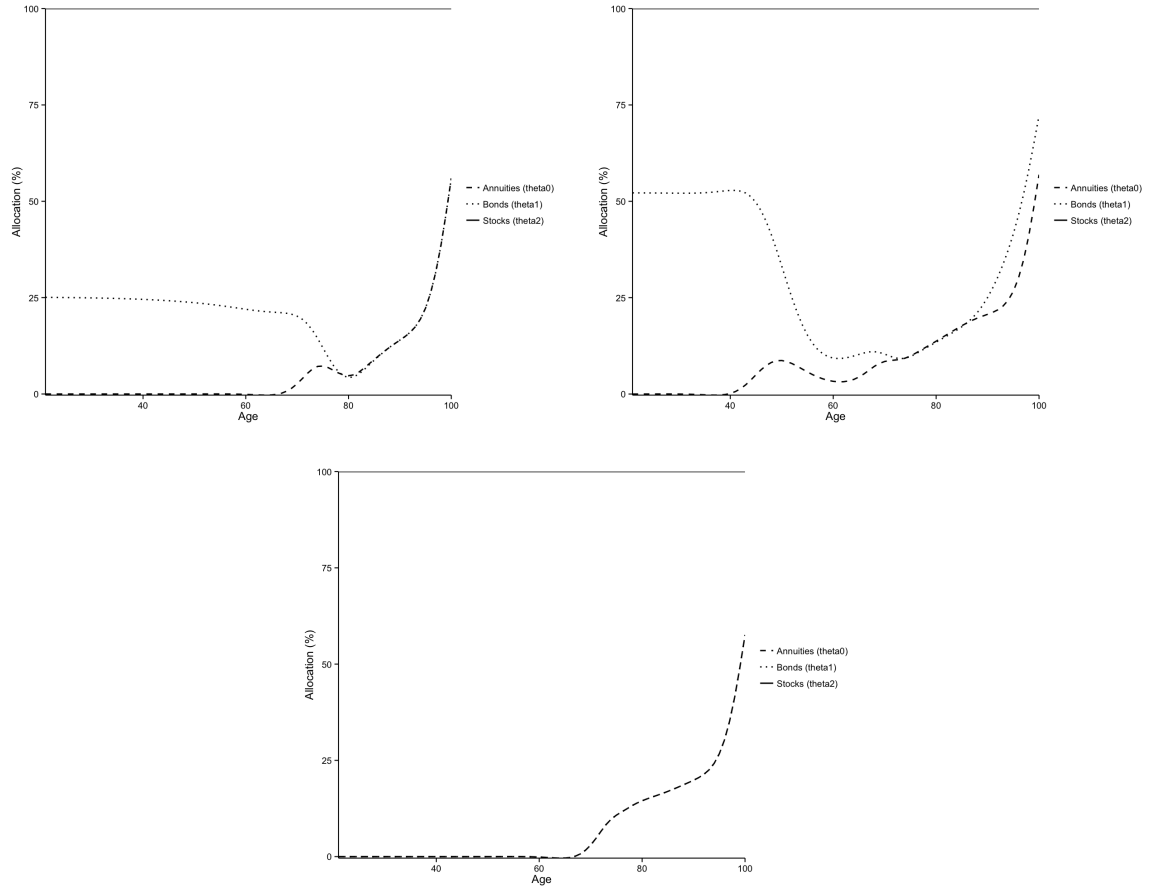


Figure 1.9: We consider the expected asset allocation for the alternative cases where we change the returns specification of each asset separately such as annuity factor with administrative costs $\delta = 7.3\%$; $R_f = 1.05$; $g_2 = 4\%$ and $\sigma_2 = 10\%$.

The top right chart depicts the case where the risk-free gross returns is higher than the base case ($R_f = 1.05$). We observe that the gradual increment from zero annuitization happens much earlier than in the previous case (as early as 43 years old) and reaches 77% at 99 years old. The expected optimal allocation into bonds is around 52% from the age of 20 and then decreases to around 5% at 99 years old. On the contrary, the expected optimal allocation into stocks is at 48% at the age of 20 and stays the same till 43 years old. At the age of 99 a slight allocation is dedicated to stocks (around 18%).

The bottom chart describes the case where the stocks gross returns have lower drift rates $g_2 = 4\%$ and volatility $\sigma_2 = 10\%$. We observe that the gradual increment from zero annuitization happens as late as 71 years old reaching 77% at 99 years old. The expected optimal allocation into bonds is always 0% from the age of 20 till 99 years old. On the contrary, the expected optimal allocation into stocks is at 100% at the age of 20 and stays the same till the age of 70. From then on it slowly drops till 23% at the age of 99.

To summarize, the robustness analysis on the three alternative scenarios (annuity factor with administrative costs $\delta = 7.3\%$; $R_f = 1.05$; $g_2 = 4\%$ and $\sigma_2 = 10\%$) shows that different asset returns specification do not have a substantial influence on the share of post consumption wealth invested into annuities. The expected optimal composition of total post-consumption wealth over time for the three alternative cases depicts a similar pattern as the base case. However, it provides some impact on the timing of annuitization. Upon adding administrative costs to the annuity factor, the investor decides to hold zero annuities for as long as 63-years old, a delay of around 3 years as compared to the base case. Increasing the risk-free gross returns further diminishes the desire to hold zero annuities. Lowering the drift rates and volatilities of the stocks gross returns provide a similar impact.

1.6 Conclusions

This paper discusses a theoretical model illustrating how behavioural preferences, including the Cumulative Prospect Theory (CPT) and narrow framing, formally explain the annuity market participation puzzle. We consider an intertemporal multi-period framework which maximizes the agent's utility to allocate her risks separately between consumption and three different assets: bonds, stocks and the constant-life annuity.

Our results show that the degree of narrow framing is the main driver of zero and relatively lower participation into annuities throughout the investor's life cycle. An investor who narrowly frames her assets presents the following characteristics when it comes to her decision to annuitize: if she has a high degree of narrow framing, she is very likely to annuitize only at a later stage of her life. On a contrary, if she is very loss averse combined with a high degree of narrow framing, she prefers to annuitize very early and postpones her decision to re-purchase annuities at later stage. With probability weighting, we assume she has the tendency to overweight the low probability of dying very soon after purchasing an annuity and to underweight the probability of outliving her resources if she does not annuitize. However this particular behavioural characteristics only present a marginal impact in the investor's

decision to annuitize.

Aside from the degree of narrow framing, classical factors such as bequest motives and the asset returns slightly influence the investor's degree of annuitization. We show that the degree of narrow framing combined with bequest motives dampens the desire to invest into annuities. A certain pattern of returns' specification provide similar share of post-consumption wealth invested into annuities as compared to the base case. However, it may change slightly the timing of annuitization.

SHARE OF POST-CONSUMPTION WEALTH INVESTED INTO ANNUITIES $\theta_{0,t}$ AT EACH AGE: COMPARISON BETWEEN THE BASE CASE AND THE REFERENCE MODELS ($b_0 = 0$; $b_0 = 0.4, \lambda = 2.25, \delta^+ = 0.61, \delta^- = 0.69$; $b_0 = 0.4, \lambda = 2.25, \delta^+ = 1, \delta^- = 1$;
 $b_0 = 0.4, \delta^+ = 0.61, \delta^- = 0.69, \lambda = 1$)

Age	Base Case	No BP	with NF, LA and PW	with NF and LA no PW	Age	Base Case	No BP	with NF, LA and PW	with NF and LA no PW	with NF and PW no LA
20	0.0	17.8	95.1	0.0	60	0.0	9.8	9.8	9.6	0.0
21	0.0	16.6	7.0	0.0	61	0.0	10.1	10.0	9.5	0.0
22	0.0	15.2	6.9	0.0	62	0.0	9.2	10.1	9.1	0.0
23	0.0	13.5	6.9	0.0	63	0.0	8.5	10.1	9.2	0.0
24	0.0	11.8	6.8	0.0	64	0.0	8.0	10.6	9.2	0.0
25	0.0	10.2	6.7	0.0	65	0.0	7.6	11.5	11.0	0.0
26	0.0	8.7	6.6	0.0	66	0.0	7.4	12.2	13.6	0.0
27	0.0	7.6	6.6	0.0	67	0.0	7.3	12.7	13.5	1.2
28	0.0	7.7	6.6	0.0	68	4.5	7.1	13.0	13.5	7.6
29	0.0	6.3	6.8	0.0	69	9.5	7.0	13.3	13.5	10.9
30	0.0	5.4	6.9	0.0	70	11.5	7.0	13.5	13.8	11.5
31	0.0	4.8	7.1	0.0	71	11.0	6.9	13.2	13.8	10.2
32	0.0	4.3	7.3	0.0	72	9.2	6.9	14.1	13.5	8.3
33	0.0	4.0	7.5	0.0	73	7.4	6.8	14.4	13.4	6.7
34	0.0	3.8	6.6	0.0	74	6.0	8.0	14.6	14.9	5.5
35	0.0	3.5	7.2	0.0	75	4.9	9.9	15.0	16.7	4.7
36	0.0	3.3	6.7	0.0	76	4.3	10.6	15.5	16.2	4.1
37	0.0	3.2	6.6	0.0	77	3.9	11.7	16.4	16.4	3.8
38	0.0	3.1	6.1	0.0	78	3.7	12.4	15.9	17.0	3.6
39	0.0	4.1	5.7	0.0	79	3.5	11.8	15.8	17.3	3.5
40	0.0	3.9	6.0	0.0	80	5.7	10.9	17.1	17.6	4.6
41	0.0	3.7	6.4	0.0	81	12.5	10.6	17.7	18.5	11.6
42	0.0	3.6	7.5	0.0	82	12.5	10.3	18.1	19.7	12.3
43	0.0	3.5	7.8	0.0	83	9.8	10.0	18.5	20.4	10.1
44	0.0	3.5	7.9	0.0	84	10.2	10.1	18.9	20.9	7.9
45	0.0	3.4	8.0	0.0	85	14.3	11.1	19.3	21.1	13.1
46	0.0	3.3	8.2	0.0	86	14.7	12.5	20.0	21.5	13.8
47	0.0	3.3	8.3	0.0	87	15.4	12.9	20.4	22.5	13.6
48	0.0	3.3	8.4	0.0	88	16.0	13.2	20.9	23.2	15.3
49	0.0	3.2	8.4	0.0	89	17.2	13.2	21.4	23.9	15.6
50	0.0	3.2	8.6	0.0	90	19.0	13.3	21.9	24.5	16.5
51	0.0	3.2	8.7	0.0	91	19.4	12.9	22.5	25.2	18.5
52	0.0	3.3	8.8	0.0	92	21.0	13.1	23.3	25.9	18.6
53	0.0	3.3	9.0	0.0	93	22.3	14.6	24.0	26.6	20.2
54	0.0	3.3	9.1	0.0	94	23.6	15.3	24.9	27.4	21.5
55	0.0	3.3	9.2	0.0	95	25.3	15.9	26.0	28.5	23.1
56	0.0	3.6	9.4	0.0	96	27.6	17.2	27.8	30.1	25.4
57	0.0	9.3	9.5	0.0	97	31.5	19.4	31.2	33.4	29.4
58	0.0	10.0	9.6	0.0	98	40.4	23.2	39.6	41.7	38.3
59	0.0	7.5	9.7	0.0	99	77.0	30.7	73.5	77.3	72.8

Table 1.2: Our base case suggests that it is optimal to invest 0% into annuities from 20 to 67 years old, while the four reference models depict that the allocation into annuities in those ages are never 0%. The main driver of the zero annuity market participation is only the degree of narrow framing.

SHARE OF POST-CONSUMPTION WEALTH INVESTED INTO ANNUITIES $\theta_{0,t}$ AT EACH AGE: COMPARISON BETWEEN THE BASE CASE AND AN ALTERNATIVE CASE ($b_0 = 0.2$)

Age	Base Case	$b_0 = 0.2$	Age	Base Case	$b_0 = 0.2$
20	0.0	0.0	60	0.0	0.0
21	0.0	0.0	61	0.0	1.0
22	0.0	0.0	62	0.0	7.1
23	0.0	0.0	63	0.0	10.8
24	0.0	0.0	64	0.0	12.0
25	0.0	0.0	65	0.0	11.5
26	0.0	0.0	66	0.0	9.8
27	0.0	0.0	67	0.0	8.1
28	0.0	0.0	68	4.5	6.7
29	0.0	0.0	69	9.5	5.8
30	0.0	0.0	70	11.5	5.1
31	0.0	0.0	71	11.0	4.7
32	0.0	0.0	72	9.2	4.5
33	0.0	0.0	73	7.4	4.3
34	0.0	0.0	74	6.0	4.2
35	0.0	0.0	75	4.9	4.0
36	0.0	0.0	76	4.3	3.8
37	0.0	0.0	77	3.9	7.0
38	0.0	0.0	78	3.7	11.5
39	0.0	0.0	79	3.5	11.9
40	0.0	0.0	80	5.7	10.4
41	0.0	0.0	81	12.5	8.8
42	0.0	0.0	82	12.5	7.7
43	0.0	0.0	83	9.8	12.2
44	0.0	0.0	84	10.2	13.0
45	0.0	0.0	85	14.3	13.1
46	0.0	0.0	86	14.7	12.7
47	0.0	0.0	87	15.4	14.9
48	0.0	0.0	88	16.0	14.7
49	0.0	0.0	89	17.2	15.5
50	0.0	0.0	90	19.0	16.3
51	0.0	0.0	91	19.4	18.0
52	0.0	0.0	92	21.0	17.9
53	0.0	0.0	93	22.3	19.1
54	0.0	0.0	94	23.6	20.5
55	0.0	0.0	95	25.3	21.8
56	0.0	0.0	96	27.6	24.0
57	0.0	0.0	97	31.5	27.8
58	0.0	0.0	98	40.4	35.9
59	0.0	0.0	99	77.0	66.3

Table 1.3: The alternative case ($b_0 = 0.2$) show that the investor has the tendency to hold zero annuities shorter than the base case (up till 60 years old respectively).

SHARE OF POST-CONSUMPTION WEALTH INVESTED INTO ANNUITIES $\theta_{0,t}$ AT EACH AGE: COMPARISON BETWEEN THE BASE CASE AND
FOUR ALTERNATIVE CASES (NO BEQUESTS, ANNUITY FACTOR WITH LOADINGS/ADMINISTRATIVE COSTS $\delta = 7.3\%$; $R_f = 1.05$; $g_2 = 4\%$
WITH $\sigma_2 = 10\%$)

Age	Base Case	No Bequest	with loadings $\delta = 7.3\%$	$R_f = 1.05$	$g_2 = 4\%$ and $\sigma_2 = 10\%$	Age	Base Case	No Bequest	with loadings $\delta = 7.3\%$	$R_f = 1.05$	$g_2 = 4\%$ and $\sigma_2 = 10\%$
20	0.0	0.0	0.0	0.0	0.0	60	0.0	0.0	0.0	3.4	0.0
21	0.0	0.0	0.0	0.0	0.0	61	0.0	0.0	0.0	3.3	0.0
22	0.0	0.0	0.0	0.0	0.0	62	0.0	0.0	0.0	3.2	0.0
23	0.0	0.0	0.0	0.0	0.0	63	0.0	0.0	0.0	3.2	0.0
24	0.0	0.0	0.0	0.0	0.0	64	0.0	0.0	0.0	3.1	0.0
25	0.0	0.0	0.0	0.0	0.0	65	0.0	5.8	0.0	2.9	0.0
26	0.0	0.0	0.0	0.0	0.0	66	0.0	11.4	0.0	4.8	0.0
27	0.0	0.0	0.0	0.0	0.0	67	0.0	13.3	0.0	10.0	0.0
28	0.0	0.0	0.0	0.0	0.0	68	4.5	12.3	0.0	11.5	0.0
29	0.0	0.0	0.0	0.0	0.0	69	9.5	10.0	0.0	10.6	0.0
30	0.0	0.0	0.0	0.0	0.0	70	11.5	7.8	0.0	9.2	0.0
31	0.0	0.0	0.0	0.0	0.0	71	11.0	6.2	6.8	8.0	8.5
32	0.0	0.0	0.0	0.0	0.0	72	9.2	5.1	10.8	7.3	14.1
33	0.0	0.0	0.0	0.0	0.0	73	7.4	4.4	11.4	6.8	14.7
34	0.0	0.0	0.0	0.0	0.0	74	6.0	4.1	9.9	8.0	11.6
35	0.0	0.0	0.0	0.0	0.0	75	4.9	3.8	7.7	12.4	8.0
36	0.0	0.0	0.0	0.0	0.0	76	4.3	5.5	5.7	12.8	5.4
37	0.0	0.0	0.0	0.0	0.0	77	3.9	15.1	4.2	12.7	17.6
38	0.0	0.0	0.0	0.0	0.0	78	3.7	15.5	3.3	12.7	18.1
39	0.0	0.0	0.0	0.0	0.0	79	3.5	12.7	2.7	12.1	10.8
40	0.0	0.0	0.0	0.0	0.0	80	5.7	23.2	2.4	15.7	14.9
41	0.0	0.0	0.0	0.0	0.0	81	12.5	26.3	2.2	15.3	17.9
42	0.0	0.0	0.0	0.0	0.0	82	12.5	35.7	8.4	15.8	14.1
43	0.0	0.0	0.0	2.9	0.0	83	9.8	52.9	12.3	16.7	15.9
44	0.0	0.0	0.0	5.4	0.0	84	10.2	87.6	9.8	17.6	17.3
45	0.0	0.0	0.0	7.3	0.0	85	14.3	100.0	6.6	19.1	17.3
46	0.0	0.0	0.0	8.7	0.0	86	14.7	100.0	13.1	19.5	17.8
47	0.0	0.0	0.0	9.5	0.0	87	15.4	100.0	13.9	20.0	19.3
48	0.0	0.0	0.0	9.7	0.0	88	16.0	100.0	12.7	21.0	20.0
49	0.0	0.0	0.0	9.4	0.0	89	17.2	100.0	15.7	21.8	20.9
50	0.0	0.0	0.0	8.8	0.0	90	19.0	100.0	16.1	22.5	21.9
51	0.0	0.0	0.0	7.8	0.0	91	19.4	100.0	17.7	23.2	22.8
52	0.0	0.0	0.0	6.8	0.0	92	21.0	100.0	18.3	24.0	23.8
53	0.0	0.0	0.0	6.0	0.0	93	22.3	100.0	19.9	24.9	24.8
54	0.0	0.0	0.0	5.2	0.0	94	23.6	100.0	21.3	25.8	25.9
55	0.0	0.0	0.0	4.7	0.0	95	25.3	100.0	23.1	26.9	27.2
56	0.0	0.0	0.0	4.2	0.0	96	27.6	100.0	25.5	28.7	29.1
57	0.0	0.0	0.0	3.9	0.0	97	31.5	100.0	29.6	32.1	32.6
58	0.0	0.0	0.0	3.6	0.0	98	40.4	100.0	38.8	40.6	41.2
59	0.0	0.0	0.0	3.5	0.0	99	77.0	100.0	76.4	77.0	77.4

Table 1.4: The alternative case with no bequests (column 3 and column 9) suggests that it is optimal to invest 0% into annuities from 20 to 64 years old, similar to the base case. From the age of 65 to 84 years old, it suggests that the expected optimal allocation into annuities should increase more than in the base case. The three alternative scenarios (column 4-6 and column 10-12) provide a similar pattern of the share of post-consumption wealth assigned into annuities as the base case. Upon adding administrative costs to the annuity factor, the investor decides to hold zero annuities for as long as 70 years old. Increasing the risk-free gross returns further shortens the desire to hold zero annuities only until 42 years old. Lowering the drift rates and volatilities of the stocks gross returns provides a similar impact.

Chapter 2

An asset pricing model with reference point adaptation

2.1 Introduction

The **leverage effect** or **risk premium effect** is one of the stylized facts in finance where negative news produces a larger increase in volatility as compared to positive news. This asymmetry impact of innovations on volatility has been explained by two main hypotheses. Firstly, the leverage hypothesis of Black (1976) suggests that the leverage of a firm (debt-equity ratio) induces an inverse relationship between the future stock volatility and stock price. For example, a fall in the firm's stock value relative to the market value of its debt results in a rise in its debt-equity ratio and increases its stock volatility. Secondly, the volatility feedback hypothesis of Campbell and Hentschel (1992) proposes that a large piece of news (either bad or good ones) results in direct and indirect effects. When bad (good) news arrives, it directly decreases (increases) the stock prices. On top of that, news arrival also sends signals which indirectly influences the discount rates. The arrival of bad or good news signals a more volatile period, thus discount rates increases and stock prices drop even further. On the contrary, the indirect effect of good news offsets some of the direct increase on stock prices. Hence, the volatility feedback implies that there is an asymmetry between the future volatility which results from good news as compared to bad ones. In the former case (good news), the future volatility increases less as compared to the latter one (bad news).

This paper presents a representative agent, preference-based asset pricing model with reference point adaptation to explain the leverage effect. Our modelling technique relates closely to the model of Barberis, Huang, and Santos (2001). Similarly to this research, the agents in our model derive utility both from consumption (as the traditional consumption based approach) and also from the fluctuations of wealth. The utility from financial wealth is derived from stocks returns' fluctuations as a consequence of the previous investment's gains or losses. Departing from Barberis, Huang, and Santos (2001), whose results leveraged on the dynamics of loss aversion, ours is unique in itself. We formally incorporate the concept of reference point adaptation while keeping the loss aversion

constant. We introduce the impact of prior outcomes in terms of the historical reference price of the stocks recalled today. We propose that the investor compares the realized stocks price with its dividends $P_t + D_t$ against her own reference price at t formulated in the past $t - 1$ in order to formulate her reference stocks price for the next period $t + 1$. There is an asymmetry in the investor's reference point adaptation because as she experiences a gain, the investor is very eager in updating her reference point. Conversely, she is more reluctant to lower her reference point after a loss. These rationale are in line with experimental evidences on reference points adaptation introduced by Arkes, Hirshleifer, Jiang, and Lim (2010), Arkes, Hirshleifer, Jiang, and Lim (2008), Baucells, Weber, and Welfens (2011).

The model of Barberis, Huang, and Santos (2001) is one of the asset pricing model which disentangles the utility gained by investors from consumption and fluctuations in the value of their financial wealth. Their main contribution is explaining the stock returns' behaviour by emphasizing on the role of loss aversion. In their model, loss aversion represents how investor is much more sensitive to reductions in her financial wealth than to increases. The dynamics of the loss aversion depends on prior gains or losses attained by the investment. After prior gains, the investor becomes less loss averse, as these previous gains serve as a buffer in case of any subsequent losses. Conversely, after prior losses, the investor becomes more loss averse, since these losses make him more careful and sensitive in case of further losses in the future. The dynamics of loss aversion drives the stock returns such that they are much more volatile than the underlying dividends. The results show that the conditional expected return is an increasing function of both the prior gains or losses. In the former case (prior gains), the conditional expected return is lower as compared to the latter (prior losses). One caveat is that the model does not produce the leverage effect due to the sensitivity of the conditional volatility to the dynamics of loss aversion.

McQueen and Vorkink (2004) build upon the model of Barberis, Huang, and Santos (2001) and Campbell and Hentschel (1992) to address the leverage effect. Departing from Barberis, Huang, and Santos (2001), McQueen and Vorkink (2004) defines a mental scorecard which measures how far away is the financial gains and losses relative to an expected level. When the value of the risky asset is below (above) its expected level, investors become more (less) risk averse. Furthermore, the utility increases from gains are smaller than the utility decreases from losses (loss aversion). The leverage effect generated by the model is strongly influenced by the exogenous formulation of the dynamics of the investor's mental scorecard. It imposes a well-structured link between the current and the next period (t and $t + 1$) and highly depends on two important features which are psychologically motivated. Firstly, the decay or memory parameter measures the degree of attentiveness investors pay to news. This exogenous parameter penalizes the scorecard today at t . Secondly, the scorecard's sensitivity to wealth shock is represented by a function of scorecard today t . This exogenous function influences the financial gains and losses relative to an expected level.

Our asset pricing model analyzes the reference point adaptation as some behavioural and experimental results in finance suggest that an investor always considers her reference points before making any decision. Kahneman and Tversky (1979) introduces the well known Prospect Theory and advocates the idea that people experience utility from gains and losses relative to a reference point. In

different fields a reference point commonly associates to the status quo, an expectation, social norms, or an aspiration level. Specifically in the experiment such as Baucells, Weber, and Welfens (2011), it is shown that there are two common features of the reference point: *adaptation* and *recursivity*. Adaptation means that the reference point is a function of past information, while recursivity implies that the previous reference point then determines the new reference point.

The initial idea of reference point adaptation emerged two decades ago as Shefrin and Statman (1985) introduced the disposition effect, which is the investor's tendency to sell winners and hold losers. They propose that the disposition effect is linked to the reference prices which are a function of past purchase prices of a financial asset. Odean (1998), Grinblatt and Keloharju (2001) and Feng and Seasholes (2005) use the weighted average price as the proxy for the reference price. Weber and Camerer (1998) and Frazzini (2006) apply the initial purchase price or the current one as their reference point. Kliger and Kudryavtsev (2008), Heath, Huddart, and Lang (1999) and Core and Guay (2001) conjecture that if no information on the prices is available, historical peaks are used as reference prices.

In recent years, however, the focus has shifted into having a deeper understanding on how the reference point changes over time, taking into account the properties of adaptation and recursivity. For example, Arkes, Hirshleifer, Jiang, and Lim (2008) perform an experiment to test the reference point adaptation based on payoff outcomes in the domain of security trading. Arkes, Hirshleifer, Jiang, and Lim (2010) analyze the reference point adaptation based on participants from China, Korea and the US to provide a cross-cultural comparison. Baucells, Weber, and Welfens (2011) conduct an experiment to measure and model how the purchase price of the stocks are set initially and subsequently updated based on a sequence of observable prices. All these research show that the magnitude of reference point adaptation is significantly greater after a gain as compared to a loss of equal size.

Our results are robust and in line with the empirical values gathered from the aggregate data. Differently from Barberis, Huang, and Santos (2001), with the dynamics of reference point which is consistent with the experimental research, we generate a leverage effect. Furthermore, our model produces stock returns which are high on average, with high volatility and low correlation with consumption growth. At the same time, we keep a low and stable riskless interest rate.

The remaining of this paper is organized as follows. Section 2.2 discusses some literature on asset pricing models which aim to generate moments matching the empirical data. Section 2.3 describes our theoretical model. Section 2.4 presents the numerical methodology solving the model. Section 2.5 describes a sensitivity analyses discussing the impact of the reference point adaptation in the model. Section 2.6 draws the conclusions.

2.2 Existing literature

Some other asset pricing models, which attempts to generate moments by matching the empirical data, focus on the idea of habit formation. That means that the utility depends on consumption relative to

the reference level of consumption (see Sundaresan (1989), Abel (1990), Constantinides (1990), Ferson and Constantinides (1991), and Campbell and Cochrane (1999)). The habit formation approach differs from the one of Barberis, Huang, and Santos (2001) as households care about gains and losses in consumption instead of disentangling the gains and losses in financial wealth. These models in general are able to quantitatively match the key empirical data with one caveat, risk aversion should be high in order to explain the high equity premium.

Inspired by the caveat of those models, Yogo (2008) then proposes a habit-based asset pricing model with low risk aversion. His model is able to explain the empirical data: the low real interest rate, the high equity premium and the countercyclical variation in the equity premium. He formulates a new utility function which evaluates gains and losses in terms of consumption relative to habit. He also embeds the habit formation in the context of Köszegi and Rabin (2006) reference-dependent model. This model presents a parsimonious way to view large-scale risk aversion and loss aversion. Risk aversion is represented by the curvature of consumption utility which describes the household's behaviour for large gambles. On the other hand, loss aversion refers to the magnitude of marginal utility for losses relative to gains, which depicts the household's behaviour for small gambles.

The model of Yogo (2008) differs from Barberis, Huang, and Santos (2001). In the latter, households have power utility with low risk aversion, hence, small fluctuations in consumption do not highly impact utility. The reason for high equity premium is solely driven by the fact that the investors emphasize on the fluctuations in financial wealth which is unrelated to consumption. In Yogo (2008) even small fluctuations in consumption do influence utility through the household's aversion to losses in consumption relative to habit. A high equity premium represents the reward that investors acquire for holding stocks, which provide low returns during recessions when consumption approaches or drops below habit. Despite all the attempts in the habit formation literature in order to mimic moments matching the empirical data, none of them focuses on the leverage effect as our paper.

Andries (2014) proposes a consumption-based asset pricing model with recursive preferences which incorporates loss aversion and solves for asset prices in closed-form. The results show that loss aversion affects the pricing of risk in two ways. Firstly, loss aversion incurs a level effect which results in an increasing risk premium as compared to the standard models. This feature further improves the model's ability to match time-series moments on asset prices. Secondly, a new feature in the model is that loss aversion affects prices in the cross-section. It creates nonlinearities features in expected excess returns as a function of the exposure to consumption shocks. In other words, the risk premium is higher for assets with low exposure to consumption shocks than those with large exposure to consumption shocks. This feature of loss aversion produces predictions which are consistent with the empirical data, such as a negative premium for skewness and a downward sloping term structure of market Sharpe ratios. Furthermore, the combined level and cross-sectional effects of loss aversion reconcile the trade-off between a high equity premium and the flat security market line observed in the data.

2.3 The model

2.3.1 A Lucas-type economy

As Lucas (1978), we model a representative agent whose objective is maximizing her time-additive utility by assigning her wealth into consumption, C_t , risky asset (stocks) S_t and risk-free asset, B_t , at each period t . The feasibility constraint is $C_t + S_t + B_t \leq Y_t + S_{t-1} R_t + B_{t-1} R_{f,t}$ where Y_t is an exogenous labour income and $S_{t-1} R_t$ ($B_{t-1} R_{f,t}$) is the value of the risky (risk-free) asset attained from $t - 1$ to t . The risk-free and risky assets deliver the gross returns $R_{f,t+1}$ and R_{t+1} , respectively. $R_{f,t+1}$ is known while its counterpart R_{t+1} is a random variable at period t .

Similar to Barberis, Huang, and Santos (2001), the investor maximizes utility from both consumption, $U(C_t)$ and the fluctuations in the value of her financial wealth, $F(W_{t+1})$,

$$(2.1) \quad \max_{C_t, S_t} E \left[\sum_{t=0}^{\infty} \rho^t U(C_t) + b_0 \bar{C}_t^{-\gamma} \rho^{t+1} F(W_{t+1}) \right],$$

where ρ^t is the subjective discount factor which depicts the investor's degree of the impatience, \bar{C}_t is the aggregate per capita consumption and $b_0 \bar{C}_t^{-\gamma}$ is the scaling factor controlling the degree of relative importance between the utility of financial wealth relative to the utility of consumption. If $b_0 = 0$, we are back to the classical consumption based asset pricing model without the utility of financial wealth.

Our simplifying assumption is that the risk-free asset is zero in net supply and the supply of the risky asset is normalized to one. We further assume that the investor only cares about the fluctuations of the value of her risky asset even though there are two financial assets. One justification for this assumption is that if the returns from the risk-free asset is known a priori, then the investor does not attain any utility from the changes in its value as compared to the changes in the risky asset values. We assume the two income economy since the historical correlation between aggregate consumption and dividend growth rates is weak. Dividends, D_t , and Y_t are non-storable and determined exogenously, thus, in aggregate $\bar{C}_t = Y_t + D_t$. Formally, we assume

$$(2.2) \quad \log \frac{\bar{C}_{t+1}}{\bar{C}_t} = g_c + \sigma_c \eta_{t+1},$$

$$(2.3) \quad \log \frac{D_{t+1}}{D_t} = g_D + \sigma_D \epsilon_{t+1},$$

where

$$\begin{pmatrix} \eta_t \\ \epsilon_t \end{pmatrix} \sim \text{i.i.d. } N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \omega \\ \omega & 1 \end{pmatrix} \right).$$

These assumptions makes $\log \frac{\bar{C}_t}{D_t}$ a random walk and enables the construction of a one-factor Markov equilibrium which will be discussed in Section 2.3.4.

2.3.2 The utility of financial wealth

Formally, the reference price of the risky asset formulated at t for $t + 1$ is denoted as $Ref_{t,t+1}^P$. We define the reference point of the stocks gross returns as the ratio between the reference price of the

risky asset formulated at t at $t + 1$ over its realized price at t , that is, $Ref_{t,t+1}^R = \frac{Ref_{t,t+1}^P}{P_t}$. We define the financial gains and losses relative to the new reference point of the stocks gross returns as follows

$$(2.4) \quad \tilde{X}_{t+1} = R_{t+1} - Ref_{t,t+1}^R.$$

A positive difference in Equation (2.4) represents how much the investor's risky investment is attaining an even higher gross returns as compared to the stocks gross returns reference point. Conversely, if the difference is negative, the investor's investment returns on risky asset is lower than she predicted.

Hence, depending on the new reference point, the utility of financial wealth fluctuations $F(W_{t+1})$ can be described as a function $v(\tilde{X}_{t+1}, S_t)$ which is formulated as

$$(2.5) \quad \begin{aligned} F(W_{t+1}) &= v(\tilde{X}_{t+1}, S_t) \\ &= \lambda(\tilde{X}_{t+1}) S_t \tilde{X}_{t+1}, \end{aligned}$$

where

$$(2.6) \quad \lambda(\tilde{X}_{t+1}) = \begin{cases} 1, & \tilde{X}_{t+1} \geq 0 \\ \lambda, & \tilde{X}_{t+1} < 0 \end{cases}.$$

$\lambda(\tilde{X}_{t+1})$ is the degree of utility/disutility that the investor experienced from the prior gains and losses of the stocks gross returns.

2.3.3 Reference point adaptation

In line with current experimental research on reference point adaptation, we relate the idea of a reference price with its past information. The novelty of our model is to capture the impact of prior outcomes in terms of the historical reference price of the stocks recalled today. We propose that the investor compares the realized stocks price with its dividends $P_t + D_t$ against her own reference price at t formulated in the past $t - 1$ in order to formulate her stocks' reference price for the next period $t + 1$. We assume the dynamics of the stocks' reference price adaptation is as follows

$$(2.7) \quad Ref_{t,t+1}^P = \begin{cases} Ref_{t-1,t}^P + k_g((P_t + D_t) - Ref_{t-1,t}^P), & (P_t + D_t) > Ref_{t-1,t}^P \\ Ref_{t-1,t}^P + k_l((P_t + D_t) - Ref_{t-1,t}^P), & (P_t + D_t) < Ref_{t-1,t}^P \\ Ref_{t-1,t}^P, & (P_t + D_t) = Ref_{t-1,t}^P \end{cases}.$$

The dynamics above is in accordance with the experimental results described in Arkes, Hirshleifer, Jiang, and Lim (2010), Arkes, Hirshleifer, Jiang, and Lim (2008), Baucells, Weber, and Welfens (2011). These experiments show that decision makers are more likely to update the reference point after gains than after losses. Moreover, the magnitude of the reference point adaptation is significantly greater after a gain as compared to a loss of equal size. Therefore, this implies that the degree of adaptation k_g is strictly greater than k_l . We consider three different cases in Equation (2.7). Firstly, when the investor fully adapts her reference price to be the current observed price plus its dividends at t , i.e., $Ref_{t,t+1}^P = P_t + D_t$, implies that $k_g = k_l = 1$. Secondly, after a gain the new reference point is set to

be larger than the current observed price plus its dividends at t , we obtain that $k_g > 1$. Conversely, after a loss the new reference point is taken to be smaller than the current observed price plus its dividends at t , thus $k_l < 1$.

One possible interpretation of this dynamics is that at time t , when the actual price with dividends $P_t + D_t$ is realized and compared with the reference price set previously, the investor gauges whether she obtains gains or losses and formulates the new reference price $Ref_{t,t+1}^P$. If she has gains $(P_t + D_t) > Ref_{t-1,t}^P$, she becomes more confident about her investment strategy and wants to achieve an even higher performance. Thus, she updates the new reference price to be higher than the previous period (i.e., $Ref_{t,t+1}^P > Ref_{t-1,t}^P$) by a degree of adaptation $k_g > 1 > k_l$. Conversely, if she has losses $(P_t + D_t) < Ref_{t-1,t}^P$, she becomes wary and cautious on her investment decision and she is satisfied for as long as the performance is not going to be worst than before. Hence, she updates her reference price by a degree of $k_l < 1$ which lowers it further (i.e., $Ref_{t,t+1}^R < Ref_{t-1,t}^R$). There is an asymmetry in her reference point adaptation because as she experiences gains she updates her reference price higher than in the case of losses. If she has no prior gains or losses $(P_t + D_t) = Ref_{t-1,t}^P$, she fully adapted her reference price to be the observed price plus its dividends at t , i.e., $Ref_{t,t+1}^P = P_t + D_t$.

2.3.4 Equilibrium

Since z_t is a single Markov state variable and based on the assumption on consumption and dividend processes in Section 2.3.1, we construct a one-factor Markov equilibrium. For convenience in modeling purposes, we define the state variable as $z_t = \frac{Ref_{t-1,t}^R}{R_t}$ and consider three separate cases, that is, $z_t < 1$ is when the investor experiences prior gains, $z_t = 1$ is when she has no prior gains or losses and $z_t > 1$ is the case of prior losses. The investor's prior gains or losses z_t determines how she formulates her new reference point on the stocks gross returns in the next period ($Ref_{t,t+1}^R$). In a one-factor Markov equilibrium, the Markov state variable z_t determines the distribution of the future stock returns. We assume that the price-dividend ratio is a function of the state variable z_t

$$(2.8) \quad \frac{P_t}{D_t} = f(z_t),$$

and show that for this economy we indeed find an equilibrium which satisfies this assumption. Furthermore, the distribution of the risky asset R_{t+1} depends on z_t and the function f as follows

$$(2.9) \quad R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{1 + \frac{P_{t+1}}{D_{t+1}}}{\frac{P_t}{D_t}} \frac{D_{t+1}}{D_t} = \frac{1 + f(z_{t+1})}{f(z_t)} \frac{D_{t+1}}{D_t} = \frac{1 + f(z_{t+1})}{f(z_t)} e^{g_D + \sigma_D \epsilon_{t+1}}$$

To summarize, the investor's maximization problem is described as follows

$$\max_{C_t, S_t} E \left[\sum_{t=0}^{\infty} \rho^t U(C_t) + b_0 \bar{C}_t^{-\gamma} \rho^{t+1} F(W_{t+1}) \right],$$

where

$$F(W_{t+1}) = \lambda(\tilde{X}_{t+1}) S_t \tilde{X}_{t+1},$$

$$\lambda(\tilde{X}_{t+1}) = \begin{cases} 1, & \tilde{X}_{t+1} \geq 0 \\ \lambda, & \tilde{X}_{t+1} < 0 \end{cases}.$$

Based on the definition of $Ref_{t,t+1}^R$ and the adaptation of the stocks' reference price in Equation (2.7), we obtain the dynamics of the reference point of the stocks gross returns as follows¹

$$(2.10) \quad Ref_{t,t+1}^R = \begin{cases} \left(\frac{1+f(z_t)}{f(z_t)} z_t \right) \left(1 + k_g \left(\frac{1}{z_t} - 1 \right) \right), & z_t < 1 \\ \left(\frac{1+f(z_t)}{f(z_t)} z_t \right) \left(1 + k_l \left(\frac{1}{z_t} - 1 \right) \right), & z_t > 1 \\ \left(\frac{1+f(z_t)}{f(z_t)} z_t \right), & z_t = 1 \end{cases}.$$

This dynamics shows that $Ref_{t,t+1}^R$ is a result of an interaction between two different terms which fully depends on the state variable z_t . Firstly, the terms $\frac{1+f(z_t)}{f(z_t)} z_t$ represents the discounted old reference point $Ref_{t-1,t}^R$ relative to the ratio of old over new the stock prices $\frac{P_{t-1}}{P_t}$. Secondly, the second terms $\left(1 + k_g \left(\frac{1}{z_t} - 1 \right) \right)$ depicts the investor's degree of willingness in adapting her stocks gross returns reference point $Ref_{t,t+1}^R$ by a factor of k_g or k_l depending on whether she experiences previous gains or losses respectively.

We assume the standard power utility, $U(C_t) = \frac{C_t^{1-\gamma}}{(1-\gamma)}$. The parameter γ refers to the constant relative risk aversion which represents the utility function's curvature and the rate of the intertemporal substitution. The first order condition of the maximization problem above results in the following Euler equation for the risky asset:

$$(2.11) \quad 1 = \rho \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} + b_0 F'(W_{t+1}) \right].$$

The Euler equation above intuitively means that the marginal utility cost of consuming one less unit of consumption and investing it on risky asset instead at t , should be equal to the expected marginal utility benefit from the returns of selling that investment in the next period $t+1$ which comes from both consumption and fluctuations in financial wealth. The marginal utility of financial wealth, $F'(W_{t+1}) = \lambda(\tilde{X}_{t+1}) [R_{t+1} - Ref_{t,t+1}^R]$, can be further substituted into Equation (2.3.4) to obtain

$$(2.12) \quad \begin{aligned} 1 &= \rho \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} + b_0 \lambda(\tilde{X}_{t+1}) [R_{t+1} - Ref_{t,t+1}^R] \right], \\ 1 &= \rho \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} + b_0 \lambda(\tilde{X}_{t+1}) R_{t+1} - b_0 \lambda(\tilde{X}_{t+1}) Ref_{t,t+1}^R \right], \\ 1 + \rho \cdot Ref_{t,t+1}^R b_0 \mathbb{E}_t [\lambda(\tilde{X}_{t+1})] &= \rho \mathbb{E}_t \left[\left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} + b_0 \lambda(\tilde{X}_{t+1}) \right\} R_{t+1} \right], \\ 1 &= \frac{\rho}{1 + \rho \cdot Ref_{t,t+1}^R b_0 \mathbb{E}_t [\lambda(\tilde{X}_{t+1})]} \mathbb{E}_t \left[\left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} + b_0 \lambda(\tilde{X}_{t+1}) \right\} R_{t+1} \right], \\ 1 &= \mathbb{E}_t [m_{t+1} R_{t+1}], \end{aligned}$$

¹Details of this dynamics conversion can be found in Section 2.7.

where the pricing kernel for the stock is $m_{t+1} = \kappa_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} + b_0 \lambda(\tilde{X}_{t+1}) \right]$ and $\kappa_t = \frac{\rho}{1 + \rho \cdot Ref_{t,t+1}^R b_0 \mathbb{E}_t[\lambda(\tilde{X}_{t+1})]}$. Moreover, since the utility of financial wealth depends only on the risky asset, the Euler equation for the risk-free asset is just equivalent to the Mehra and Prescott (1985) constant risk-free rate,

$$(2.13) \quad R_{f,t+1} = \frac{1}{\rho \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \right]}.$$

Therefore, we obtain the price-dividend ratio equation by combining Equation (2.12) and Equation (2.9) to get

$$(2.14) \quad \frac{P_t}{D_t} = \mathbb{E}_t \left[m_{t+1} \left(\frac{P_{t+1}}{D_{t+1}} + 1 \right) \frac{D_{t+1}}{D_t} \right],$$

which is equivalent to

$$(2.15) \quad \begin{aligned} f(z_t) &= \mathbb{E}_t \left[m_{t+1} (f(z_{t+1}) + 1) \frac{D_{t+1}}{D_t} \right], \\ &= \kappa_t e^{g_D - \gamma g_c + \frac{\gamma^2 \sigma_c^2 (1 - \omega^2)}{2}} \mathbb{E}_t \left[(1 + f(z_{t+1})) e^{(\sigma_D - \gamma \omega \sigma_c) \epsilon_{t+1}} \right] \\ &+ \kappa_t e^{g_D} b_0 \mathbb{E}_t \left[\lambda(\tilde{X}_{t+1}) (1 + f(z_{t+1})) e^{\sigma_D \epsilon_{t+1}} \right]. \end{aligned}$$

2.4 Numerical results

2.4.1 Parameter values

We assume the following parameter values in Table 2.1 in order to solve our model numerically. The

SUMMARY OF MODEL'S PARAMETER VALUES

Descriptions	Parameters	Values
Mean of consumption growth rate	g_c	2%
Mean of dividend growth rate	g_D	1%
Standard deviation of consumption growth rate	σ_c	2.1%
Standard deviation of dividend growth rate	σ_D	11.7%
Correlation coefficient between consumption and dividend growth	ω	0.15
Consumption utility power	γ	1.0
Discount factor	ρ	0.98
The degree of loss aversion	λ	2.25
The degree of reference point adaptation based on gains	k_g	1.4
The degree of reference point adaptation based on losses	k_l	0.3
Scaling factor	b_0	(range)

Table 2.1: This table shows the model parameters which are equal to the historical US values except for the last three parameters which are specific to our model.

first four parameters above are obtained from the work of Constantinides and Ghosh (2011). They use

the monthly data on prices and dividends and annual data on consumption from January 1929 through December 2009. The proxy for the market is the Center for Research in Security Prices (CRSP) value-weighted index of all stocks on the NYSE, AMEX and NASDAQ. The mean of the annual log dividend growth rate on the market portfolio (g_D) is 1.0% with volatility (σ_D) 11.7%. The consumption data are obtained from the Bureau of Economic Analysis. The annual log consumption growth (g_c) has a mean of 2.0% and a volatility (σ_c) of 2.1% over the sample period. The correlation of shocks to dividend growth and consumption growth $\omega = 0.15$ is an estimated value from Campbell (1999) who used the time series of US data from the past century. We set the values of the consumption utility power $\gamma = 1.0$ and the discount factor $\rho = 0.98$ as Barberis, Huang, and Santos (2001) in order to produce a sensible low value of the risk-free rate (in this case $R_f - 1 = 4.08\%$). The degree of loss aversion λ is estimated to be 2.25 following Tversky and Kahneman (1992). The parameter $b_0 \in [0, 0.4]$ is model specific in order to match certain moments of the postwar data such as the equity premium and the Sharpe ratio. The degree of reference point adaptation parameters in case of gains and losses (k_g and k_l respectively) are chosen as 1.4 and 0.3 in order to achieve convergence and stable solutions to the model while reflecting the same moments of postwar data (mean, standard deviation and the Sharpe ratio of log excess returns). In the robustness and sensitivity analysis in Section 2.5, we further discuss how different range of values of these two parameters k_g and k_l would affect our results.

2.4.2 Methodology

The key in solving this model is the price-dividend function in Equation (2.15) and understanding how the state variable z_t evolves over time. The main challenge is that the price-dividend ratio function $f(\cdot)$ appears on both right and left hand side of the equation. As a consequence, we first have to guess this function and substitute it into the right hand side of Equation (2.15), which automatically delivers an updated $f(\cdot)$ on the left hand side.

By definition, we know that

$$(2.16) \quad z_{t+1} = \frac{Ref_{t,t+1}^R}{R_{t+1}}.$$

Therefore, combining Equation (2.10), Equation (2.16) and Equation (2.9), we can re-formulate the dynamics of the state variable as follows

$$(2.17) \quad z_{t+1} (1 + f(z_{t+1})) = \begin{cases} \frac{(1+f(z_t)) z_t (1+k_g(\frac{1}{z_t}-1))}{e^{g_D+\sigma_D\epsilon_{t+1}}}, & z_t < 1 \\ \frac{(1+f(z_t)) z_t (1+k_l(\frac{1}{z_t}-1))}{e^{g_D+\sigma_D\epsilon_{t+1}}}, & z_t > 1 \\ \frac{(1+f(z_t)) z_t}{e^{g_D+\sigma_D\epsilon_{t+1}}}, & z_t = 1 \end{cases}.$$

From Equation (2.17) above, we observe that $z_{t+1} (1 + f(z_{t+1}))$ depends on both ϵ_{t+1} and $f(z_t)$. Again, we are facing the same problem that the price-dividend ratio function $f(\cdot)$ appears on both right and left hand side of the equation. Hence, both Equation (2.15) and Equation (2.17) are self-referential and have to be solved concurrently to find the solution to the model.

We use the same technique as Barberis, Huang, and Santos (2001). Firstly, we make a good guess on the solution for the Equation (2.15) and denote it as $f^{(0)}$. Based on this guessed $f^{(0)}$, we concurrently solve Equation (2.17) and denote this function of the state variable as $z_{t+1} = h^{(0)}(z_t, \epsilon_{t+1})$. $h^{(0)}$ describes how z_{t+1} is distributed conditional on z_t . Given $h^{(0)}$, we get a new candidate solution $f^{(1)}$ through the following recursion

$$(2.18) \quad \begin{aligned} f^{(i+1)}(z_t) &= \kappa_t e^{g_D - \gamma g_c + \frac{\gamma^2 \sigma_c^2 (1 - \omega^2)}{2}} \mathbb{E}_t \left[(1 + f^{(i)}(z_{t+1})) e^{(\sigma_D - \gamma \omega \sigma_c) \epsilon_{t+1}} \right] \\ &+ \kappa_t e^{g_D} \mathbb{E}_t \left[\lambda(\tilde{X}_{t+1}) (1 + f^{(i)}(z_{t+1})) e^{\sigma_D \epsilon_{t+1}} \right], \forall z_t. \end{aligned}$$

Subsequently, we can calculate a new $h = h^{(1)}$ which simultaneously solves for Equation (2.17) for the new $f = f^{(1)}$. Similarly, this $h^{(1)}$ provides a new candidate for $f = f^{(2)}$. These recursive processes are repeated continuously until we get a convergence $f^{(i)} \rightarrow f$, $h^{(i)} \rightarrow h$.

2.4.3 Stock prices

Figure 2.1 illustrates how the price-dividend ratio $f(z_t)$, which solves Equation (2.15), vary against the prior gains/losses z_t by considering the scaling factor $b_0 = 0.2$ with fixed $k_g = 1.4$ and $k_l = 0.3$ as the base case. The reference point of stock returns $Ref_{t,t+1}^R$ is inversely related to the price-dividend ratio $f(z_t)$ as described in Equation (2.10). $f(z_t)$ is an increasing function with a slightly gentler gradient in the cases of gains ($z_t < 1$) and a decreasing function in the case of losses ($z_t > 1$). This is because investors are more likely to update the reference point after gains than after losses $k_g > k_l$. Moreover, the magnitude the reference point adaptation of the stock returns $Ref_{t,t+1}^R$ is much more in the case of gains than losses of the same size. $f(z_t)$ is an inverse v-shaped function of z_t with a slight deflection at $z_t = 1$.

Figure 2.1 does not give us the range of price-dividend ratios that we observe in equilibrium. Hence, we have to know the equilibrium distribution of the state variable z_t which is described in Figure 2.2. We derive the histogram by drawing a long time series of $\{\epsilon_t\}_{t=1}^{50,000}$ of 50,000 independent samples from the standard normal distribution. Then, commencing with $z_0 = 1$, we use the function $z_{t+1} = h(z_t, \epsilon_{t+1})$ obtained from Equation (2.17) to generate a time series of all the 50'000 z_t 's.

We can also compute the returns from the generated time series of z_t period by period according to Equation (2.9). From these simulated returns, we obtain unconditional sample moments of stock returns which are depicted in Table 2.2 considering different values of b_0 and fixing $k_g = 1.4$ and $k_l = 0.3$. The extreme case where $b_0 = 0$ is the classical case which is considered by Mehra and Prescott (1985). This table shows that by modelling both the utility of consumption and fluctuations on the financial wealth and taking certain range values of b_0 , we obtain the asset returns moments which closely mimic the empirical values. Especially for the case of $b_0 = 0.2$ (our base case), the mean of the log excess returns are equal to the empirical value at 4.4. Similarly, the unconditional volatility of the stock returns trails the empirical value at 16.1. The Sharpe ratio matches the empirical result at 0.27.² From the table we observe that our model also describes a weak correlation between the

²Barberis, Huang, and Santos (2001) increases the degree of investor's loss aversion k in their model to significantly

THE PRICE-DIVIDEND RATIO $f(z_t)$ VERSUS PRIOR GAINS/LOSSES z_t : BASE CASE

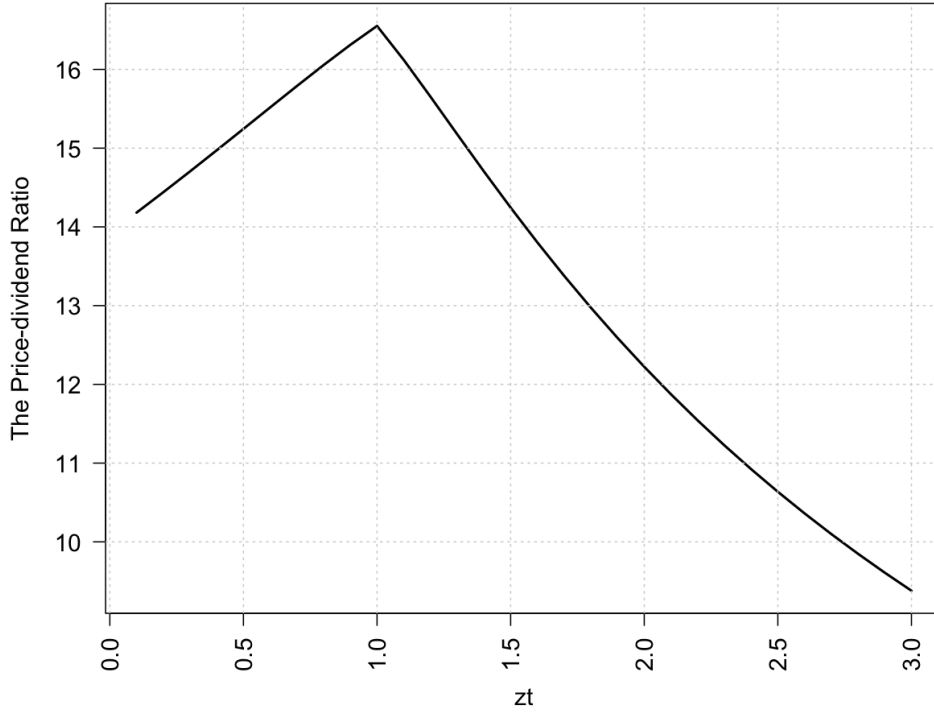


Figure 2.1: This graph describes how the price-dividend ratio $f(z_t)$, which solves Equation (2.15), varies against the prior gains/losses z_t . We consider the base case with scaling factor $b_0 = 0.2$, fixed $k_g = 1.4$ and $k_l = 0.3$. $f(z_t)$ is an inverse v-shaped function of z_t with a slight deflection at $z_t = 1$. The intuition is that the reference point of the stock returns $Ref_{t,t+1}^R$ is inversely related to the price-dividend ratio $f(z_t)$. If $k_g = 1.4$ and $k_l = 0.3$, $f(z_t)$ is an increasing function with a slightly gentler gradient in the cases of gains ($z_t < 1$) and a decreasing function in the case of losses ($z_t > 1$).

THE DISTRIBUTION OF THE STATE VARIABLE z_t

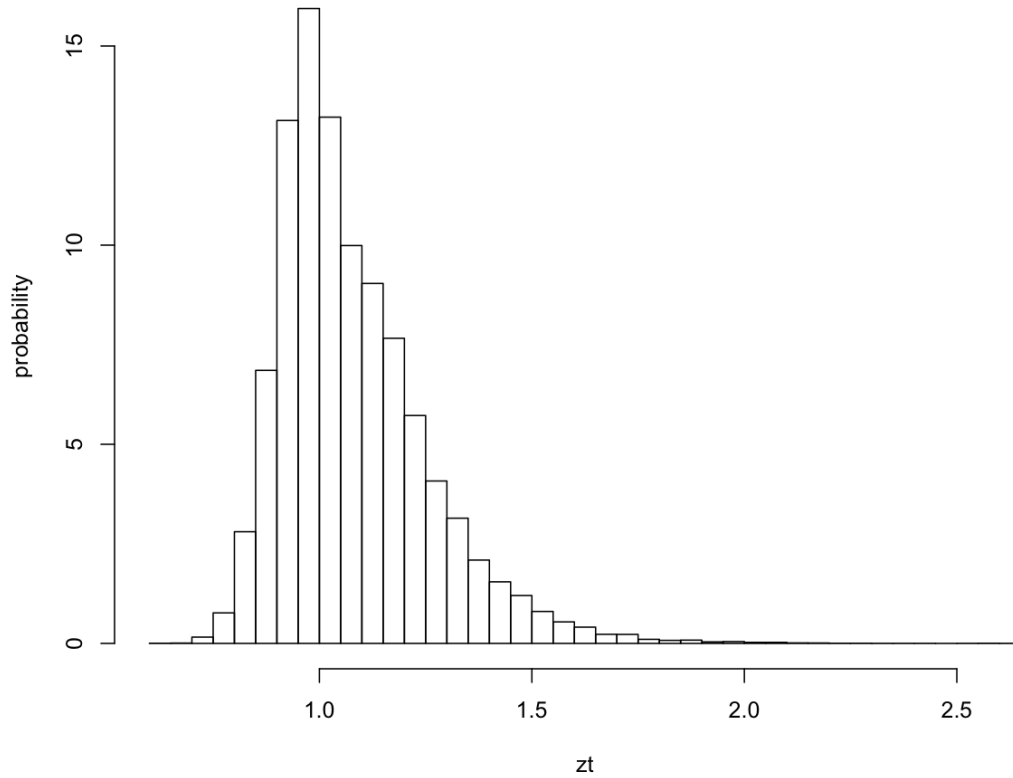


Figure 2.2: This histogram describes how the state variable (z_t) is distributed in equilibrium by considering the degree of reference point adaptation in the case of gains $k_g = 1.4$ and losses $k_l = 0.3$ and the scaling factor $b_0 = 0.2$. The histogram is obtained by drawing a long time series of $\{\epsilon_t\}_{t=1}^{50,000}$ of 50,000 independent samples from the standard normal distribution. Then, starting from $z_0 = 1$, we use the function $z_{t+1} = h(z_t, \epsilon_{t+1})$ obtained from Equation (2.17) to generate a time series of all the 50'000 z_t 's.

stock returns and consumption growth (0.15%).

One caveat of our model is that, similar to Barberis, Huang, and Santos (2001), it underestimates the volatility of the price-dividend ratio. One possible reason for this weakness is that, in our model we consider only a single factor (historical reference point of the stock returns) in generating the movement of the price-dividend ratio. A possible improvement may be to include other factors such as habit formation over consumption. However, this is beyond the scope of our current research.³

THE UNCONDITIONAL MOMENTS OF ASSET PRICES AND RETURNS WITH DIFFERENT b_0

	$b_0 = 0$	$b_0 = 0.1$	$b_0 = 0.2$	$b_0 = 0.3$	$b_0 = 0.4$	Empirical
		$k_g = 1.4$	$k_g = 1.4$	$k_g = 1.4$	$k_g = 1.4$	value
		$k_l = 0.3$	$k_l = 0.3$	$k_l = 0.3$	$k_l = 0.3$	
Log risk-free rate	4.08	4.08	4.08	4.08	4.08	0.6
Log excess stock returns						
Mean	0.03	2.00	4.37	7.06	10.06	5.6
Standard deviation	12.25	14.07	16.11	18.29	20.65	19.8
Sharpe ratio	0.00	0.14	0.27	0.39	0.49	0.31
Correlation w/ consumption growth	0.15	0.15	0.15	0.15	0.15	0.10
Price-dividend ratio						
Mean	41.46	23.89	16.02	11.68	8.98	29.4
Standard deviation	0.00	0.54	0.65	0.66	0.63	1.6
Loss aversion		2.25	2.25	2.25	2.25	

Table 2.2: This table shows the unconditional moments of asset returns expressed in annual percentages. The empirical values are based on the monthly data on prices and dividends and annual data on consumption from January 1929 through December 2009. The proxy for the market is the Center for Research in Security Prices (CRSP) value-weighted index of all stocks on the NYSE, AMEX and NASDAQ. k_g and k_l are the degree of reference point adaptation in the case of gains and losses respectively.

Figure 2.3 shows the conditional expected stock returns as a function of z_t , which is obtained by numerically integrating the return in Equation (2.9) over the conditional distribution of z_{t+1} given by Equation (2.17). From Equation (2.16), we define that the stocks gross returns are directly proportional to their reference point. We consider the base case with a scaling factor $b_0 = 0.2$ and fixed $k_g = 1.4$ and $k_l = 0.3$. We observe that $E_t(r_{t+1})$ is a decreasing function of z_t for the cases of gains ($z_t < 1$) with gentler gradients than increasing function in the case of losses ($z_t > 1$). The intuition is the following: $z_t < 1$ means that the stock returns realized at t is larger than the reference point of the stock returns formed at $t - 1$ ($R_t - Ref_{t-1,t}^R > 0$), which translates as a gain for the investor. After a gain at t , she becomes more willing to increase her stocks gross returns' reference point at

improve their results, while we keep the degree of reference point adaptation k_g and k_l the same.

³We already discussed similar research attempts which include habit formation in Section 2.2.

$t + 1$ by a factor of $k_g = 1.4$, hence the second terms in Equation (2.10) becomes large and more than 1. However, the larger the value of z_t in the case of gains ($z_t < 1$), the more $Ref_{t,t+1}^R$ decreases due to diminishing effect of the magnitude of adapted gains. This further results in a smaller magnitude of $E_t(r_{t+1})$.

Conversely, $z_t > 1$ refers to the stocks gross returns realized at t which is lower than its reference point formed at $t - 1$, that is, a loss ($R_t - Ref_{t-1,t}^R < 0$). After a loss at t , the investor is less likely to adapt her reference point for the stocks gross returns at $t + 1$, hence, her degree of reference point adaptation is small $k_l = 0.3 < k_g$. However, this is compensated by a larger discount from the ratio of the stock prices in Equation (2.10), as it is multiplied by a large $z_t > 1$. Overall, the reference point of the stocks gross returns at $t + 1$ increases as $z_t > 1$. The larger the value of z_t in the case of losses ($z_t > 1$) implies that the discount arises from the price ratios hikes up, dominating the not much adapted magnitude of losses. Hence, the $Ref_{t,t+1}^R$ increases which results in a larger magnitude of $E_t(r_{t+1})$. This result is unique as compared to other research, for example, Barberis, Huang, and Santos (2001) obtained conditional expected return as an increasing function of the state variable.

Figure 2.4 shows the conditional expected stock returns for a scaling factor $b_0 = 0.2$, fixed $k_g = 1.4$ and $k_l = 0.3$ plotted against $r_t = R_t - 1$. In our model we have $R_t = \frac{Ref_{t-1,t}^R}{z_t}$. Since we showed in Equation (2.10) that the dynamics of the stocks gross returns' reference point $Ref_{t,t+1}^R$ depends fully on z_t , we can take any arbitrary $Ref_{t-1,t}^R$ to illustrate the relationship between expected returns $E_t(r_{t+1})$ against r_t . We take $Ref_{t-1,t}^R = R_f$ and obtained the r_t values in the x-axis. The intuition is the same as above, however we are now looking at it from a different perspective. In this case, gains ($z_t < 1$) refer to the case when $r_t \in [0, 20\%]$, while losses ($z_t > 1$) depict the case when $r_t \in [-20\%, 0]$. We observe that $E_t(r_{t+1})$ is an increasing function of r_t for the cases of gains ($r_t \in [0, 20\%]$) with gentler gradients than decreasing function in the case of losses ($r_t \in [-20\%, 0]$). $r_t > 0$ means that the stock returns realized at t is larger than the reference point of the stock returns formed at $t - 1$ ($R_t > Ref_{t-1,t}^R = R_f$, in this case), which translates as a gain for the investor. After a gain at t , she becomes more willing to increase her stocks gross returns' reference point at $t + 1$ by a factor of $k_g = 1.4$, hence, the second terms in Equation (2.10) becomes large and more than 1. However, the larger the value of r_t in the case of gains, the higher $Ref_{t,t+1}^R$ as the magnitude of adapted gains is getting larger. Thus, the greater magnitude of $E_t(r_{t+1})$. On the other hand, $r_t < 0$ refers to the stocks gross returns realized at t which is lower than its reference point formed at $t - 1$, that is, a loss ($R_t - Ref_{t-1,t}^R = R_f < 0$). The smaller the value of r_t in the case of losses means that the discount arises from the price ratios increases, dominating the magnitude of losses which are not much adapted as $k_l = 0.3$. Hence, the $Ref_{t,t+1}^R$ increases which results in a larger magnitude of $E_t(r_{t+1})$.

Figure 2.5 shows the conditional volatility of the stock returns for the scaling factor $b_0 = 0.2$, fixed $k_g = 1.4$ and $k_l = 0.3$. We observe that $E_t(\sigma_{t+1})$ is a decreasing function of z_t for the cases of gains $z_t < 1$ with gentler gradients than increasing function in the case of losses $z_t > 1$. Upon facing higher prior gains (lower values of z_t for the range $z_t < 1$), the conditional volatility of the stock returns is relatively high. In the case of higher prior losses (higher values of z_t for the range $z_t > 1$), the conditional volatility of the stock returns is hiking up even more than in the case of

THE CONDITIONAL EXPECTED RETURNS $E_t(r_{t+1})$ VERSUS PRIOR GAINS/LOSSES z_t : BASE CASE

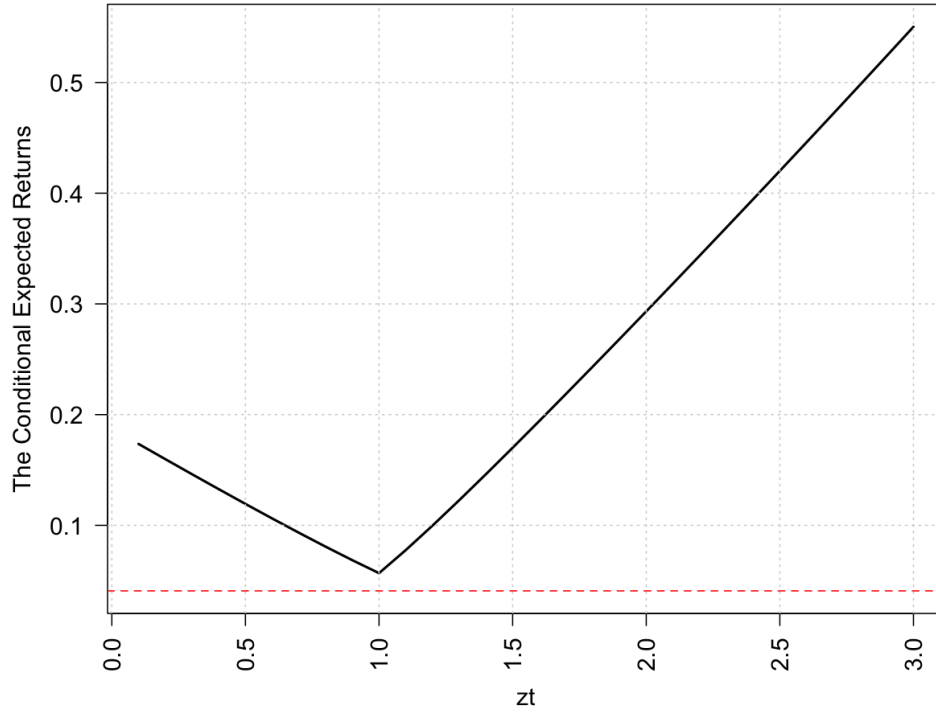


Figure 2.3: This graph describes the conditional expected returns $E_t(r_{t+1})$ versus prior gains/losses (z_t) for a scaling factor $b_0 = 0.2$, fixed $k_g = 1.4$ and $k_l = 0.3$. The parameter k_g and k_l control the degree of adaptation of the reference point upon prior gains/losses respectively. $E_t(r_{t+1})$ is a decreasing function of z_t for the cases of gains $z_t < 1$ with gentler gradients than increasing function in the case of losses $z_t > 1$. The dotted red line refers to the risk-free rate $R_f - 1 = 4.08\%$.

THE CONDITIONAL EXPECTED RETURNS $E_t(r_{t+1})$ VERSUS STOCK RETURNS r_t : BASE CASE

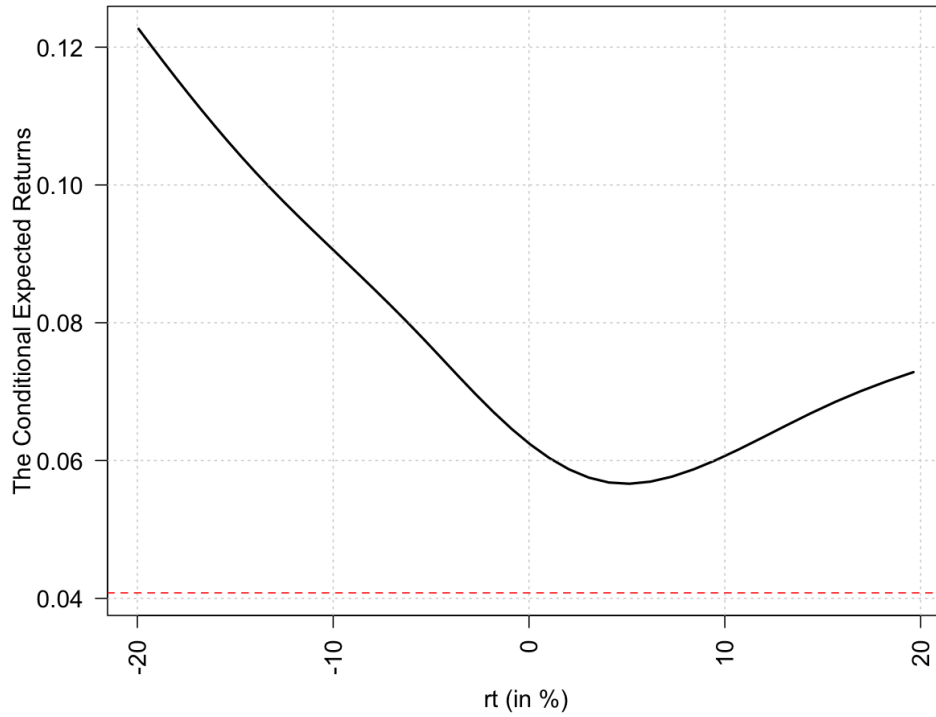


Figure 2.4: This graph describes the conditional expected returns $E_t(r_{t+1})$ versus stock returns at t (r_t) for a scaling factor $b_0 = 0.2$, fixed $k_g = 1.4$ and $k_l = 0.3$. The parameter k_g and k_l control the degree of adaptation of the reference point upon prior gains/losses respectively. $E_t(r_{t+1})$ is an increasing function of r_t for the cases of gains $r_t \in [0, 20\%]$ with gentler gradients than decreasing function in the case of losses $r_t \in [-20\%, 0]$. The dotted red line refers to the risk-free rate $R_f - 1 = 4.08\%$.

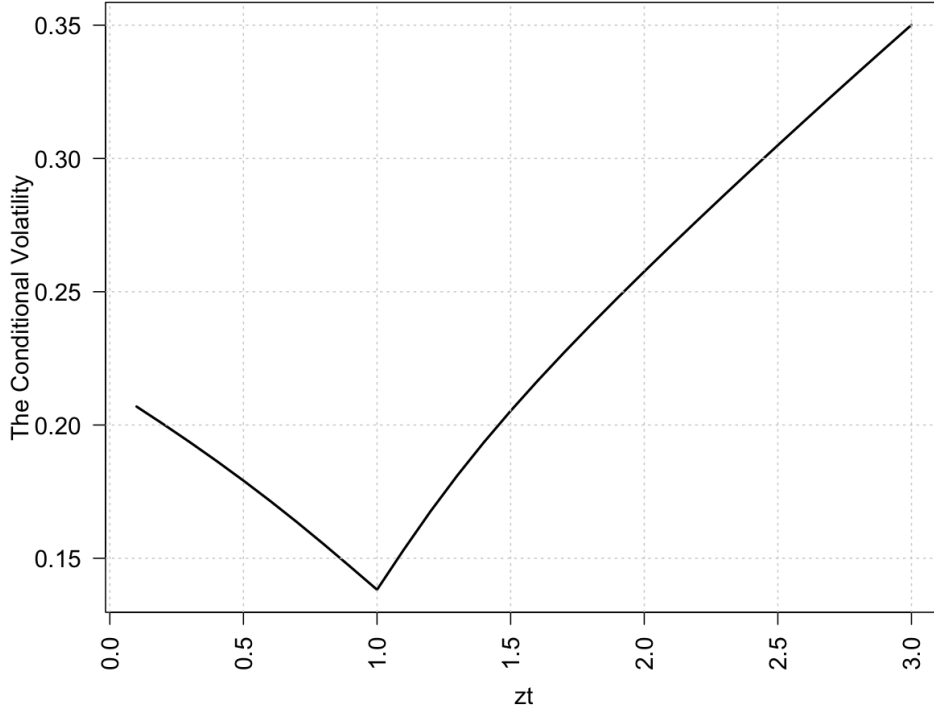


Figure 2.5: This graph describes the conditional volatility of stock returns $E_t(\sigma_{t+1})$ versus prior gains/losses (z_t) for the scaling factor $b_0 = 0.2$, fixed $k_g = 1.4$ and $k_l = 0.3$. The parameter k_g and k_l control degree of adaptation of the reference point upon prior gains/losses respectively. $E_t(\sigma_{t+1})$ is a decreasing function of z_t for the cases of gains $z_t < 1$ with gentler gradients than increasing function in the case of losses $z_t > 1$.

prior gains. The intuition is that the lower the magnitude of z_t for the range of $z_t < 1$, the higher $Ref_{t,t+1}^R$ due to increasing magnitude of adapted gains. Conversely, the higher the magnitude of z_t for the range $z_t > 1$, the higher the discount arisen from the price ratios, dominating the not much adapted magnitude of losses. Hence, the $Ref_{t,t+1}^R$ also increases by much more than in the case of gains. Hence, the asymmetry in this new reference point, as a consequence of interaction between the discount terms of the price ratios and the degree of reference point adaptation, produces an asymmetry in the volatility of the stocks gross returns. Our results are in line with the empirical research which shows that the volatility is to be higher during bad market condition than good ones. The bad market condition in our model is described as the situation where the aggregate investors are experiencing prior losses while the good market condition is the opposite spectrum where the investors are having prior gains.

Figure 2.6 shows the conditional volatility of the stock returns for the scaling factor $b_0 = 0.2$, fixed $k_g = 1.4$ and $k_l = 0.3$ plotted against r_t . We take $Ref_{t-1,t}^R = R_f$ and obtained the r_t values in the x-axis. We observe that $E_t(\sigma_{t+1})$ is an increasing function of r_t for the cases of gains $r_t \in [0, 20\%]$

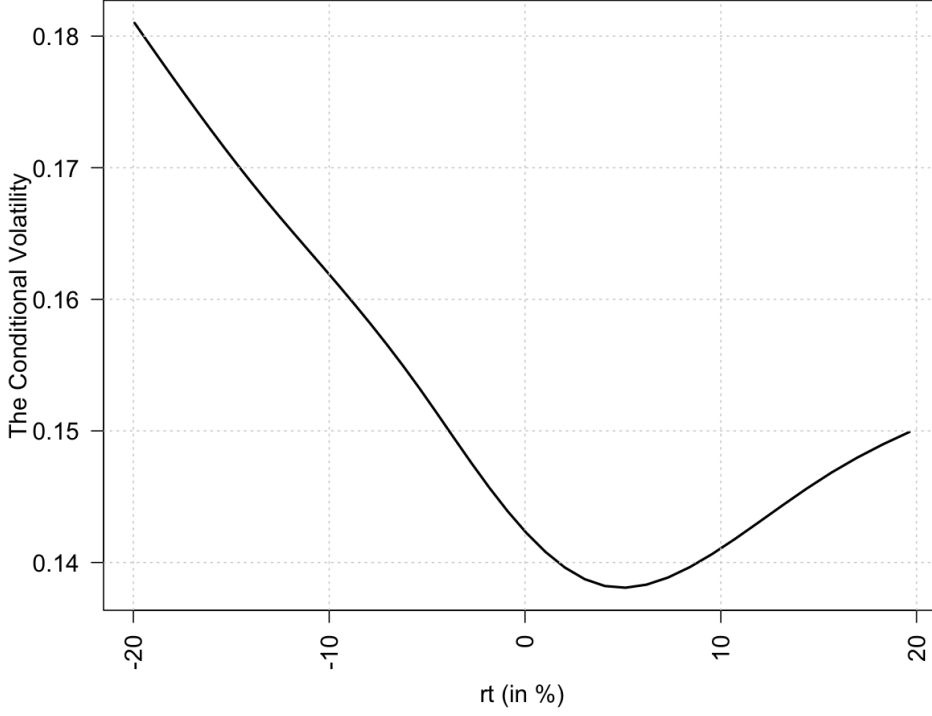


Figure 2.6: This graph describes the conditional volatility of stock returns $E_t(\sigma_{t+1})$ versus stock returns at t (r_t) for the scaling factor $b_0 = 0.2$, fixed $k_g = 1.4$ and $k_l = 0.3$. The parameter k_g and k_l control degree of adaptation of the reference point upon prior gains/losses respectively. $E_t(\sigma_{t+1})$ is an increasing function of r_t for the cases of gains $r_t \in [0, 20\%]$ with gentler gradients than decreasing function in the case of losses $r_t \in [-20\%, 0]$.

with gentler gradients than decreasing function in the case of losses $r_t \in [-20\%, 0]$. The asymmetry in our result is again unique as compared to other research, for example, Barberis, Huang, and Santos (2001) obtained conditional volatility of the stock returns which is again an increasing function of the state variable.

2.5 Sensitivity analysis

2.5.1 The reference point adaptation of the asset returns (the case of gains)

We now analyze the sensitivity of the stock returns characteristics to its reference point adaptation in the case of gains by changing the parameter k_g . The top left figure describes the reference point of the stock returns $Ref_{t,t+1}^R$ versus z_t . By focusing on the domain of gains $z_t < 1$, we observe that a larger value of k_g translates into a decreasing function of the reference point of the stock returns with respect to z_t with steeper gradient than the others. This means that $Ref_{t,t+1}^R$ is larger in the case of higher

THE SENSITIVITY OF THE STOCK RETURNS CHARACTERISTICS TO ITS REFERENCE POINT ADAPTATION IN
THE CASE OF GAINS k_g

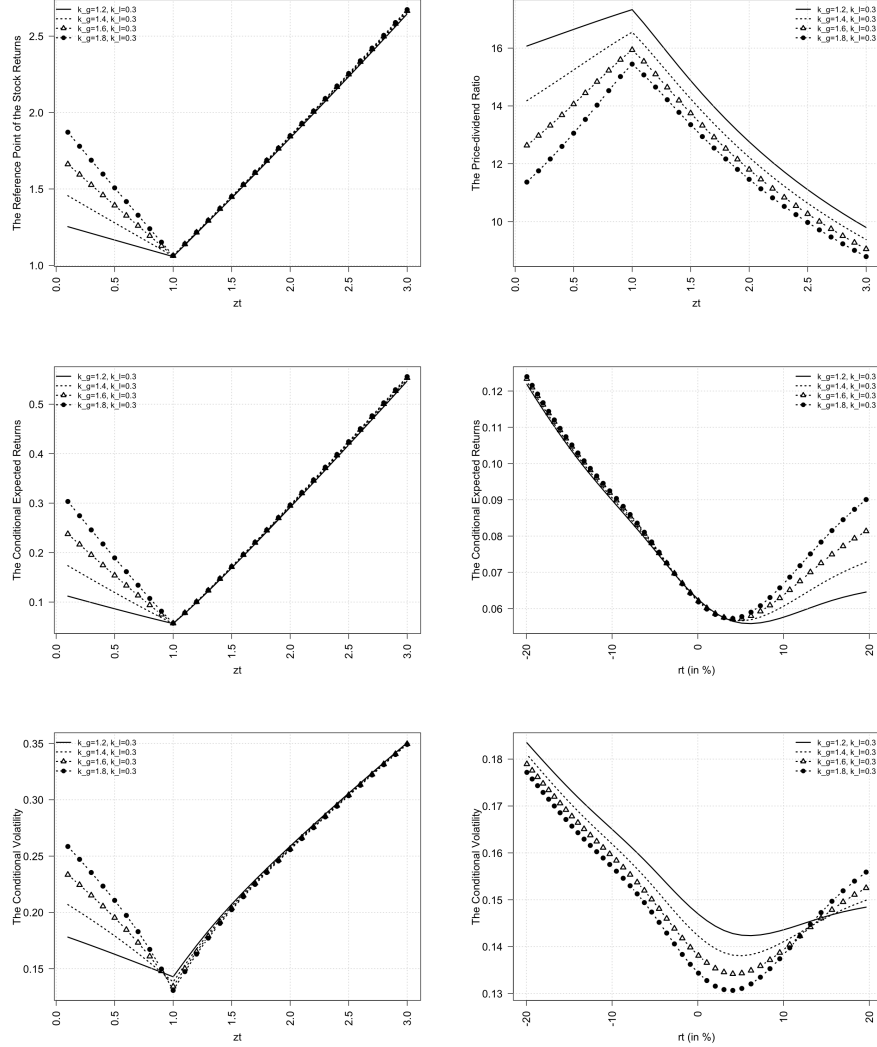


Figure 2.7: This figure shows the sensitivity of the stock returns characteristics to its reference point adaptation in the case of gains by changing the parameter k_g . The top left figure describes the reference point of the stock returns $Ref_{t,t+1}^R$ versus z_t . The top right figure depicts the price-dividend ratio $f(z_t)$ against the prior gains/losses z_t . The middle left figure shows the conditional expected stock returns versus the prior gains/losses z_t . The middle right figure depicts the conditional expected stock returns versus the stock returns r_t . The bottom left figure describes the conditional volatility of the stock returns versus the prior gains/losses z_t . The bottom right figure shows the conditional volatility of the stock returns against the stock returns r_t where the domain of gains is $r_t \in [0, 20\%]$.

magnitude of gains (lower value of z_t) and smaller otherwise. The rationale is as follows: a higher k_g in the domain of gains ($z_t < 1$) means that the investor becomes more willing to adapt her stocks gross returns reference point after experiencing previous gains. Mathematically this is achieved through an increase in the term $\left(1 + k_g\left(\frac{1}{z_t} - 1\right)\right)$ in Equation (2.10). Overall, the asymmetry of the reference point of the stock returns in the case of gains/losses becomes more pronounced as k_g increases.

In Figure 2.7, the top right chart depicts the price-dividend ratio $f(z_t)$, which solves Equation (2.15) against the prior gains/losses z_t . The reference point of stock returns $Ref_{t,t+1}^R$ is inversely related to the price-dividend ratio $f(z_t)$. This is described in the terms $\frac{1+f(z_t)}{f(z_t)} z_t$ in Equation (2.10) which represents the discounted old reference point $Ref_{t-1,t}^R$ relative to the ratio of old over new stock prices $\frac{P_{t-1}}{P_t}$. Therefore, in the domain of gains ($z_t < 1$), the price-dividend ratio is an increasing function of z_t . $f(z_t)$ gets smaller as the magnitude of z_t is low, while it is larger when z_t is high. The higher the value of k_g , the smaller the overall $f(z_t)$.

The middle left figure shows the conditional expected stock returns versus the prior gains/losses z_t . The $E_t(r_{t+1})$ is obtained by numerically integrating the return in Equation (2.9) over the conditional distribution of z_{t+1} given by Equation (2.17). From Equation (2.16), we define that the stocks gross returns are directly proportional to its reference point. Therefore, higher k_g means that the investor becomes more willing to adapt her stocks gross returns reference point, which subsequently increases the conditional expected stock returns. The middle right figure depicts the conditional expected stock returns versus the stock returns r_t . In this figure, the domain of gains refers to the case where $r_t \in [0, 20\%]$. As k_g increases, the conditional expected stock returns is an increasing function of r_t . The asymmetry of the conditional expected stock returns in the case of gains/losses becomes less pronounced as k_g hikes up.

The bottom left figure describes the conditional volatility of the stock returns versus the prior gains/losses z_t . It is shown that $E_t(\sigma_{t+1})$ is a decreasing function of z_t in the domain of gains $z_t < 1$. Higher k_g means that the investor becomes more willing to adapt her stocks gross returns reference point, which subsequently increases the conditional expected stock returns and also its conditional volatility. The bottom right figure shows the conditional volatility of the stock returns against the stock returns r_t where the domain of gains is $r_t \in [0, 20\%]$. Similarly to the graph of the conditional expected stock returns versus r_t , the asymmetry of the conditional volatility also becomes less evident as k_g gets larger.

We can also compute the returns from the generated time series of z_t period by period according to Equation (2.9). From these simulated returns, we obtain unconditional sample moments of stock returns which are depicted in Table 2.3 by considering different values of $k_g = 1.2, 1.4, 1.6, 1.8$, fixed $k_l = 0.3$ and $b_0 = 0.2$. This table shows that, as the degree of reference point adaptation in the case of gains increases (k_g is larger), we obtain the average of the equity premium increases and very close to the empirical value (around 5). The unconditional volatility also decreases and the Sharpe ratios are generally similar to the empirical values at around 0.3. The mean and the standard deviation of the price-dividend ratio also decreases to values which are relatively lower than the empirical one, as k_g increases.

THE UNCONDITIONAL MOMENTS OF ASSET PRICES AND RETURNS WITH DIFFERENT k_g

	$k_g = 1.2$ $k_l = 0.3$	$k_g = 1.4$ $k_l = 0.3$	$k_g = 1.6$ $k_l = 0.3$	$k_g = 1.8$ $k_l = 0.3$	Empirical value
Log risk-free rate	4.08	4.08	4.08	4.08	0.6
Log excess stock returns					
Mean	4.18	4.37	4.54	4.69	5.6
Standard deviation	16.68	16.11	15.63	15.23	19.8
Sharpe ratio	0.25	0.27	0.29	0.31	0.31
Correlation w/ consumption growth	0.15	0.15	0.15	0.15	0.10
Price-dividend ratio					
Mean	16.77	16.02	15.43	14.95	29.4
Standard deviation	0.74	0.65	0.59	0.55	1.6
Loss aversion	2.25	2.25	2.25	2.25	

Table 2.3: This table shows the sensitivity of the asset returns to the degree of reference point adaptation in case of gains. By considering different values of $k_g = 1.2, 1.4, 1.6, 1.8$, fixed $k_l = 0.3$ and $b_0 = 0.2$. The results show that as the degree of reference point adaptation in the case of gains increases (k_g is larger), we obtain the average of the equity premium increases and very close to the empirical value (around 5). The unconditional volatility also decreases and the Sharpe ratios are generally similar to the empirical values at around 0.3. The mean and the standard deviation of the price-dividend ratio also decreases to values which are relatively lower than the empirical one as k_g increases.

2.5.2 The reference point adaptation of the asset returns (the case of losses)

This section covers the sensitivity of the stock returns properties to its reference point adaptation in the case of losses by changing the parameter k_l . In Figure 2.8, the top left figure depicts the reference point of the stock returns $Ref_{t,t+1}^R$ versus z_t . Looking at the domain of losses $z_t > 1$, we observe that a larger value of k_l translates into an increasing function of the reference point of the stock returns with respect to z_t with flatter gradient than the rest. This means that $Ref_{t,t+1}^R$ is larger in the case of higher magnitude of losses (higher value of z_t) and smaller otherwise. The rationale is as follows: a higher k_l in the domain of losses ($z_t > 1$) means that the investor becomes much less willing to adapt her stocks gross returns reference point after experiencing previous losses. However, some compensation arises by the discount from the ratio of the stock prices in Equation (2.10), as it is multiplied by a large $z_t > 1$. Overall, the reference point of the stocks' gross returns at $t + 1$ increases as $z_t > 1$. The larger the value of z_t in the case of losses ($z_t > 1$) the higher the discount arisen from the price ratios, dominating the not much adapted magnitude of losses. Overall, the asymmetry of the reference point of the stock returns in the case of gains/losses become less pronounced as k_l increases. Apparently, as k_l increases, the discount from the ratio of the stock prices is not enough to create an increase in reference point which dominates the case of gains.

The top right chart depicts the price-dividend ratio $f(z_t)$, which solves Equation (2.15), against the prior gains/losses z_t . The reference point of stock returns $Ref_{t,t+1}^R$ is inversely related to the price-dividend ratio $f(z_t)$. This is described in the terms $\frac{1+f(z_t)}{f(z_t)} z_t$ in Equation (2.10) which represents the discounted old reference point $Ref_{t-1,t}^R$ relative to the ratio of old over new stock prices $\frac{P_{t-1}}{P_t}$. Therefore, in the domain of losses ($z_t > 1$), the price-dividend ratio is a decreasing function of z_t . $f(z_t)$ gets smaller as the magnitude of z_t is large, while it is larger when z_t is small. The higher the k_l , the larger the overall $f(z_t)$.

The middle left figure shows the conditional expected stock returns versus the prior gains/losses z_t . The $E_t(r_{t+1})$ is obtained by numerically integrating the return in Equation (2.9) over the conditional distribution of z_{t+1} given by Equation (2.17). From Equation (2.16), we define that the stocks gross returns are directly proportional to their reference point. Therefore, higher k_l means that the investor becomes less willing to adapt her stocks gross returns reference point, with compensated discount from the ratio of the stock prices. Hence, the conditional expected stock returns increases but not as much as in the case of gains. The middle right figure depicts the conditional expected stock returns versus the stock returns r_t . In this figure, the domain of losses refers to the case where $r_t \in [-20\%, 0]$. As k_l increases, the conditional expected stock returns is an increasing function of r_t . The asymmetry of the conditional expected stock returns in the case of gains/losses becomes less pronounced as k_l hikes up.

The bottom left figure describes the conditional volatility of the stock returns versus the prior gains/losses z_t . It is shown that $E_t(\sigma_{t+1})$ is an increasing function of z_t in the domain of losses $z_t > 1$. Higher k_l means that the investor becomes less willing to adapt her stocks gross returns reference point, with compensated discount from the ratio of the stock prices. Therefore, the conditional expected

THE SENSITIVITY OF THE STOCK RETURNS CHARACTERISTICS TO ITS REFERENCE POINT ADAPTATION IN
THE CASE OF LOSSES k_l

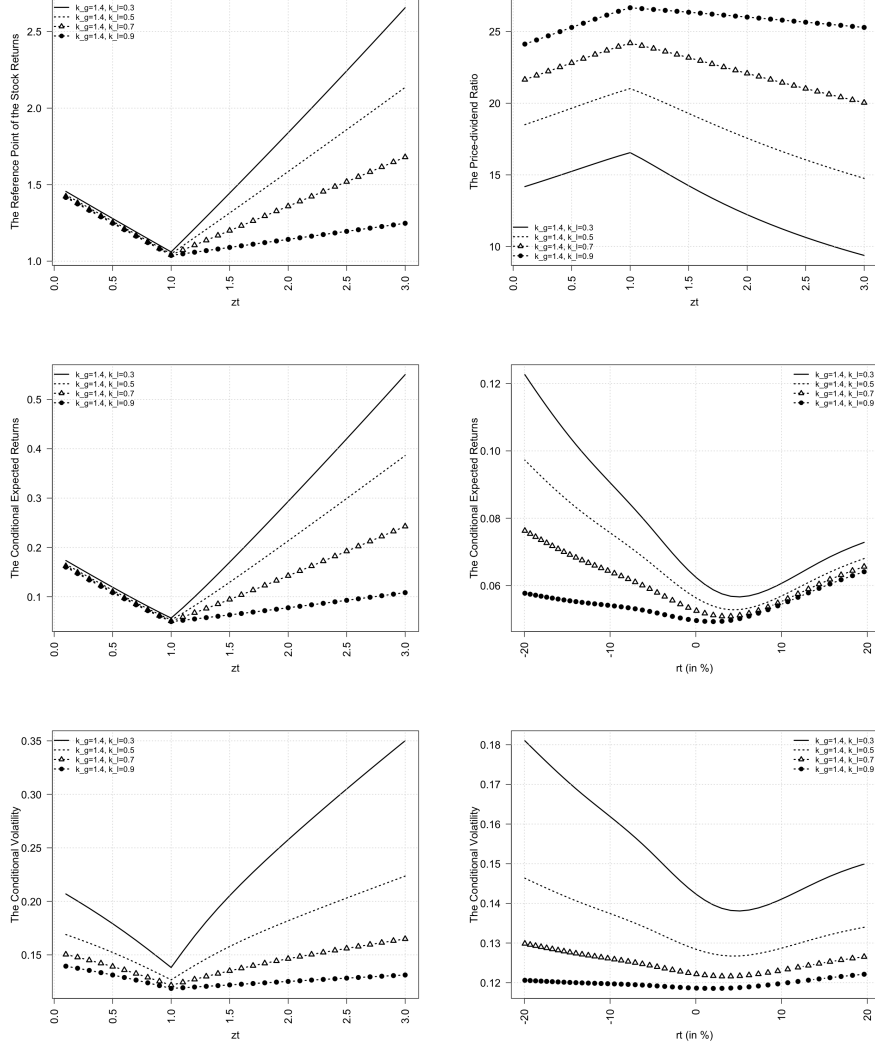


Figure 2.8: This figure shows the sensitivity of the stock returns characteristics to its reference point adaptation in the case of losses by changing the parameter k_l . The top left figure describes the reference point of the stock returns $Ref_{t,t+1}^R$ versus z_t . The top right figure depicts the price-dividend ratio $f(z_t)$ against the prior gains/losses z_t . The middle left figure shows the conditional expected stock returns versus the prior gains/losses z_t . The middle right figure depicts the conditional expected stock returns versus the stock returns r_t . The bottom left figure describes the conditional volatility of the stock returns versus the prior gains/losses z_t . The bottom right figure shows the conditional volatility of the stock returns against the stock returns r_t where the domain of losses is $r_t \in [-20\%, 0]$.

stock returns and also its conditional volatility subsequently increases although not as much as in the case of gains. The bottom right figure shows the conditional volatility of the stock returns against the stock returns r_t where the domain of losses is $r_t \in [-20\%, 0]$. Similarly to the graph of the conditional expected stock returns versus r_t , the asymmetry of the conditional volatility also becomes less evident as k_l gets larger.

We also calculate the returns from the generated time series of z_t period by period in line with Equation (2.9). From these simulated returns, we obtain unconditional sample moments of stock returns which are depicted in Table 2.4. It considers different values of $k_l = 0.3, 0.5, 0.7, 0.9$, fixed $k_g = 1.4$ and $b_0 = 0.2$. The results depict that as the degree of reference point adaptation in the case of losses increases (k_l is larger), we obtain an average equity premium which decreases by far than the empirical value. The unconditional volatility also decreases and the Sharpe ratios are relatively lower than the empirical values. The mean of the price-dividend ratio increases to a closer value as the empirical value, as k_l increases. On the other hand, its standard deviation decreases much lower than the empirical one.

THE UNCONDITIONAL MOMENTS OF ASSET PRICES AND RETURNS WITH DIFFERENT k_l

	$k_g = 1.4$ $k_l = 0.3$	$k_g = 1.4$ $k_l = 0.5$	$k_g = 1.4$ $k_l = 0.7$	$k_g = 1.4$ $k_l = 0.9$	Empirical value
Log risk-free rate	4.08	4.08	4.08	4.08	0.6
Log excess stock returns					
Mean	4.37	2.61	1.81	1.33	5.6
Standard deviation	16.11	13.74	12.76	12.21	19.8
Sharpe ratio	0.27	0.19	0.14	0.11	0.31
Correlation w/ consumption growth	0.15	0.15	0.15	0.15	0.10
Price-dividend ratio					
Mean	16.02	20.66	23.97	26.56	29.4
Standard deviation	0.65	0.39	0.21	0.11	1.6
Loss aversion	2.25	2.25	2.25	2.25	

Table 2.4: This table shows the sensitivity of the asset returns to the degree of reference point adaptation in case of losses. By considering different values of $k_l = 0.3, 0.5, 0.7, 0.9$, fixed $k_g = 1.4$ and $b_0 = 0.2$, we observe that, as the degree of reference point adaptation in the case of losses increases (k_l is larger), we obtain an average equity premium which decreases by far more than the empirical value. The unconditional volatility also decreases and the Sharpe ratios are relatively lower than the empirical values. The mean of the price-dividend ratio increases to a closer value as the empirical value, as k_l increases. On the other hand, its standard deviation decreases much lower than the empirical one.

2.5.3 The degree of disutility from prior losses

This section describes the sensitivity of the stock returns characteristics to the degree of utility/disutility that the investor experienced from the prior gains and losses of the stocks gross returns $\lambda(\tilde{X}_{t+1})$. Equation (2.6) shows that in the domain of gains ($z_t < 1$), the degree of utility experienced from gains is always equal to 1, while, in the domain of losses ($z_t > 1$), the degree of disutility from losses is a constant value of λ . In other words, to see the impact of different values of λ , we can focus on the domain of losses. In Figure 2.9, the top right chart depicts the price-dividend ratio $f(z_t)$, which solves Equation (2.15) against the prior gains/losses z_t . The price-dividend ratio $f(z_t)$ is inversely related to $\lambda(\tilde{X}_{t+1})$ as described in Equation (2.12). Therefore, in the domain of gains ($z_t < 1$), the price-dividend ratio is an increasing function of z_t since the $\lambda(\tilde{X}_{t+1})$ is fixed at 1. This results in a higher value of κ which enters in Equation (2.12). In the case of losses ($z_t > 1$), the price-dividend ratio is a decreasing one as $\lambda(\tilde{X}_{t+1}) \geq 1$. The higher the value of $\lambda(\tilde{X}_{t+1})$, the smaller the overall $f(z_t)$.

The top left figure describes the reference point of the stock returns $Ref_{t,t+1}^R$ versus z_t . The reference point of stock returns $Ref_{t,t+1}^R$ is inversely related to the price-dividend ratio $f(z_t)$. Therefore, the shape of the figure is perfectly the opposite of the top right one. In the domain of gains $z_t < 1$, we observe that $Ref_{t,t+1}^R$ is a decreasing function of z_t . In the domain of losses $z_t > 1$, a larger value of λ means that the increasing function of the reference point of the stock returns with respect to z_t has a steeper gradient than the remaining. The higher the value of $\lambda(\tilde{X}_{t+1})$, the larger the overall $Ref_{t,t+1}^R$.

The middle left figure shows the conditional expected stock returns versus the prior gains/losses z_t . The $E_t(r_{t+1})$ is obtained by numerically integrating the return in Equation (2.9) over the conditional distribution of z_{t+1} given by Equation (2.17). From Equation (2.16), we define that the stocks gross returns are directly proportional to its reference point. Therefore, higher λ subsequently increases the conditional expected stock returns. The middle right figure depicts the conditional expected stock returns versus the stock returns r_t . As λ increases, the conditional expected stock returns is an increasing function of r_t . The asymmetry of the conditional expected stock returns in the case of gains/losses becomes more pronounced as λ hikes up.

The bottom left figure describes the conditional volatility of the stock returns versus the prior gains/losses z_t . It is shown that $E_t(\sigma_{t+1})$ is a decreasing function of z_t in the domain of gains $z_t < 1$. Higher λ means that the conditional expected stock returns and also its conditional volatility increases. The bottom right figure shows the conditional volatility against the stock returns r_t . Similarly to the graph of the conditional expected stock returns versus r_t , the asymmetry of the conditional volatility also becomes more evident as λ gets larger.

We further compute the returns from the generated time series of z_t period by period according to Equation (2.9). From these simulated returns, we obtain unconditional sample moments of stock returns which are depicted in Table 2.5 by considering different values of $\lambda = 1, 1.5, 2.25, 3$, fixed $k_g = 1.4$, $k_l = 0.3$ and $b_0 = 0.2$. This table shows that, as the degree of disutility in the case of prior losses increases (λ is larger), we obtain the average of the equity premium increases and very close to

THE SENSITIVITY OF THE STOCK RETURNS CHARACTERISTICS TO $\lambda(\tilde{X}_{t+1})$

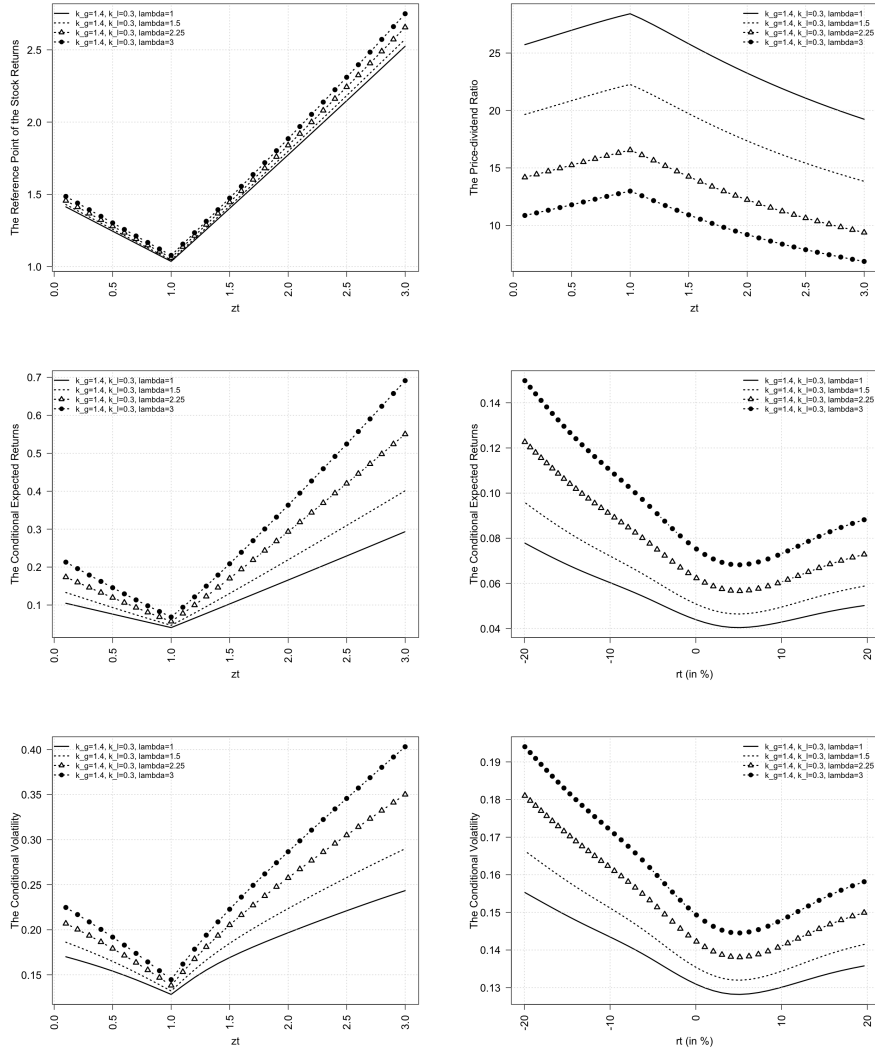


Figure 2.9: This figure shows the sensitivity of the stock returns characteristics to the degree of utility/disutility that the investor experienced from the prior gains and losses of the stocks gross returns $\lambda(\tilde{X}_{t+1})$. The top left figure describes the reference point of the stock returns $Ref_{t,t+1}^R$ versus z_t . The top right figure depicts the price-dividend ratio $f(z_t)$ against the prior gains/losses z_t . The middle left figure shows the conditional expected stock returns versus the prior gains/losses z_t . The middle right figure depicts the conditional expected stock returns versus the stock returns r_t . The bottom left figure describes the conditional volatility of the stock returns versus the prior gains/losses z_t . The bottom right figure shows the conditional volatility of the stock returns against the stock returns r_t .

the empirical value at $\lambda = 2.25$. The unconditional volatility also increases, while the Sharpe ratios decrease and reach a similar value at $\lambda = 2.25$ as the empirical one. The mean of the price-dividend ratio also decreases as λ increases and it is closest to the empirical value when $\lambda = 1$. The standard deviation of the price-dividend ratio is always hovering around 0.6.

THE UNCONDITIONAL MOMENTS OF ASSET PRICES AND RETURNS WITH DIFFERENT λ

	$k_g = 1.4$ $k_l = 0.3$	$k_g = 1.4$ $k_l = 0.3$	$k_g = 1.4$ $k_l = 0.3$	$k_g = 1.4$ $k_l = 0.3$	Empirical value
Log risk-free rate	4.08	4.08	4.08	4.08	0.6
Log excess stock returns					
Mean	1.39	2.53	4.37	6.37	5.6
Standard deviation	13.99	14.81	16.11	17.48	19.8
Sharpe ratio	0.10	0.17	0.27	0.36	0.31
Correlation w/ consumption growth	0.15	0.15	0.15	0.15	0.10
Price-dividend ratio					
Mean	27.89	21.72	16.02	12.47	29.4
Standard deviation	0.62	0.65	0.65	0.63	1.6
Loss aversion	1	1.5	2.25	3	

Table 2.5: This table shows the sensitivity of the asset returns to the degree of disutility in the case of prior losses. By considering different values of $\lambda = 1, 1.5, 2.25, 3$ and fixed $k_g = 1.4$, $k_l = 0.3$ and $b_0 = 0.2$, we observe that, as λ gets larger, we obtain the average of the equity premium increases and very close to the empirical value at $\lambda = 2.25$. The unconditional volatility also increases, while the Sharpe ratios decrease and reaches similar value at $\lambda = 2.25$ as the empirical one. The mean of the price-dividend ratio also decreases as λ increases and it is closest to the empirical value when $\lambda = 1$. The standard deviation of the price-dividend ratio is always hovering around 0.6.

2.5.4 The case of full adaptation of reference point $k_g = k_l = 1$

As described in Section 2.3.3, if the investor fully adapts her reference price to be the current observed price plus its dividends at t , i.e., $Ref_{t,t+1}^P = P_t + D_t$, then $k_g = k_l = 1$. We discuss the stock returns' characteristics in the case of full adaptation $k_g = k_l = 1$ with changes in the parameter λ . The top left picture (Figure 2.10) describes the reference point of the stock returns $Ref_{t,t+1}^R$ versus z_t . Based on Equation (2.10), the case where $k_g = k_l = 1$ implies that $Ref_{t,t+1}^R$ is a constant and equal to $\left(\frac{1+f(z_t)}{f(z_t)}\right)$ for all z_t . The rationale is that the investor does not care about prior gains or losses and therefore her degree of reference point adaptation only depends on the function of the price-dividend ratio $f(z_t)$. λ only amplifies the magnitude of the $Ref_{t,t+1}^R$. The higher the value of λ , the higher the magnitude of the reference point adaptation $Ref_{t,t+1}^R$.

In Figure 2.10, the top right chart depicts the price-dividend ratio $f(z_t)$, which solves Equation

THE CASE OF FULL ADAPTATION OF REFERENCE POINT $k_g = k_l = 1$ AND CHANGING λ

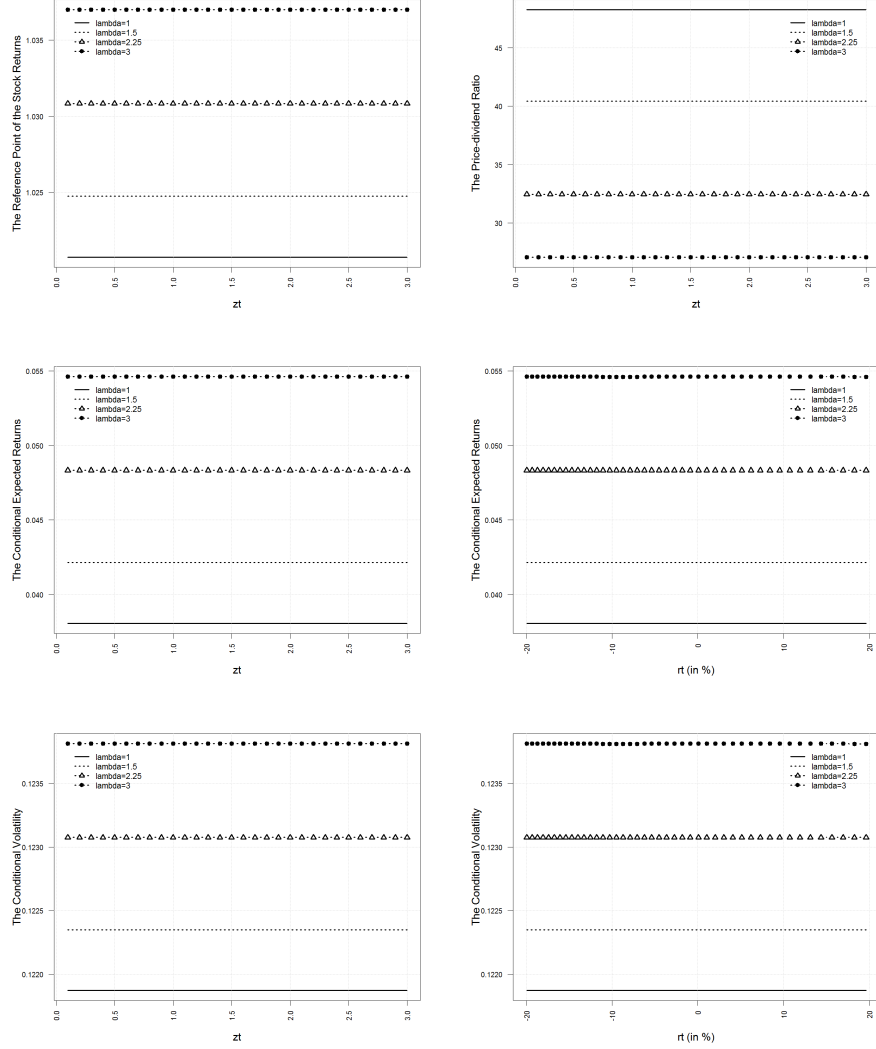


Figure 2.10: This figure shows the stock returns' characteristics in the case of full adaptation $k_g = k_l = 1$ with changes in the parameter λ . The top left figure describes the reference point of the stock returns $Ref_{t,t+1}^R$ versus z_t . The top right figure depicts the price-dividend ratio $f(z_t)$ against the prior gains/losses z_t . The middle left figure shows the conditional expected stock returns versus the prior gains/losses z_t . The middle right figure depicts the conditional expected stock returns versus the stock returns r_t . The bottom left figure describes the conditional volatility of the stock returns versus the prior gains/losses z_t . The bottom right figure shows the conditional volatility of the stock returns against the stock returns r_t where the domain of gains is $r_t \in [0, 20\%]$.

(2.15), against the prior gains/losses z_t . The reference point of the stock returns $Ref_{t,t+1}^R$ is inversely related to the price-dividend ratio $f(z_t)$. This is described in the terms $\frac{1+f(z_t)}{f(z_t)}$ in this case. Therefore, as the size of λ increases, the magnitude of the price-dividend ratio $f(z_t)$ gets smaller.

The middle left chart shows the conditional expected stock returns versus the prior gains/losses z_t . The $E_t(r_{t+1})$ is obtained by numerically integrating the returns in Equation (2.9) over the conditional distribution of z_{t+1} given by Equation (2.17). From Equation (2.16), we define that the stocks' gross returns are directly proportional to their reference point. Therefore, as λ becomes larger, the magnitude of $E_t(r_{t+1})$ also increases. The middle right figure depicts the conditional expected stock returns versus the stock returns r_t . Since the results are independent of prior gains or losses, as λ increases, the conditional expected stock returns also rises in magnitude. There is no asymmetry of the conditional expected stock returns as λ hikes up.

The bottom left figure describes the conditional volatility of the stock returns versus the prior gains/losses z_t . The bottom right figure shows the conditional volatility of the stock returns against the stock returns r_t where the domain of gains is $r_t \in [0, 20\%]$. Similarly to the graph of the conditional expected stock returns versus r_t , there is no asymmetry of the conditional volatility as λ increases. This is an evidence that, in this paper, the only factor which drives the asymmetry in the conditional volatility is purely the degree of investor's willingness to adapt her reference point (k_g and k_l in the case of gains or losses respectively), and not λ .

We also compute the returns from the generated time series of z_t period by period according to Equation (2.9). From these simulated returns we obtain the unconditional sample moments of stock returns which are depicted in Table 2.6 by considering different values of $\lambda = 1, 1.5, 2.25, 3$, fixed $k_g = k_l = 1$ and $b_0 = 0.2$. This table shows that, in the case where investor fully adapts her reference point ($k_g = k_l = 1$), we obtain the average of the equity premium increases, which is very far from the empirical value as λ increases. The unconditional volatility is almost constant and increases by a very small amount. The Sharpe ratios are generally far from the empirical values too. The mean of the price-dividend ratio have values which are relatively close to the empirical ones only when λ is high.

2.6 Conclusions

This paper integrates the idea of reference point adaptation into a representative agent, preference-based asset pricing model to explain the leverage effect. Although our modelling technique relates closely to the model of Barberis, Huang, and Santos (2001), ours is unique in itself as we formally incorporate the concept of reference point adaptation while keeping the loss aversion constant. We introduce the impact of prior outcomes in terms of the historical reference price of the stocks recalled today. We propose that the investor compares the realized stocks price with its dividends $P_t + D_t$ against her own reference price at t formulated in the past ($t - 1$) in order to formulate her reference stocks price for the next period $t+1$. There is an asymmetry in the investor's reference point adaptation because as she experiences a gain, she is very eager in updating her reference point. Conversely, she is more reluctant to lower her reference point after a loss. These rationales are in line with experimental

THE UNCONDITIONAL MOMENTS OF ASSET PRICES AND RETURNS WITH DIFFERENT
 $\lambda = 1, 1.5, 2.25, 3$, FIXED $k_g = k_l = 1$ AND $b_0 = 0.2$

	$k_g = 1$ $k_l = 1$	$k_g = 1$ $k_l = 1$	$k_g = 1$ $k_l = 1$	$k_g = 1$ $k_l = 1$	Empirical value
Log risk-free rate	4.08	4.08	4.08	4.08	0.6
Log excess stock returns					
Mean	-0.38	0.032	0.65	1.28	5.6
Standard deviation	12.21	12.26	12.33	12.40	19.8
Sharpe ratio	-0.03	0.003	0.05	0.10	0.31
Correlation w/ consumption growth	0.15	0.15	0.15	0.15	0.10
Price-dividend ratio					
Mean	48.27	40.44	32.45	27.04	29.4
Standard deviation	0	0	0	0	1.6
Loss aversion	1	1.5	2.25	3	

Table 2.6: This table shows that, in the case where investor fully adapts her reference point ($k_g = k_l = 1$), we obtain the average of the equity premium increases and very far to the empirical value as λ increases. The unconditional volatility is almost constant and increases by a very small amount. The Sharpe ratios are generally far from the empirical values too. The mean of the price-dividend ratio have values which are relatively close to the empirical one, only when λ is high.

evidences on reference points adaptation introduced by Arkes, Hirshleifer, Jiang, and Lim (2010), Arkes, Hirshleifer, Jiang, and Lim (2008), Baucells, Weber, and Welfens (2011).

Our results are robust and in line with the empirical values gathered from the aggregate data. Differently from Barberis, Huang, and Santos (2001), with the dynamics of reference point consistent with the experimental research, we generate a leverage effect. Furthermore, our model produces stock returns which are high on average, with high volatility and low correlation with consumption growth. At the same time, we keep a low and stable riskless interest rate.

One limitation of our model is that, similar to Barberis, Huang, and Santos (2001), it underestimates the volatility of the price-dividend ratio. One possible reason for this weakness is that in our model we consider only a single factor (historical reference point of the stock returns) in generating the movement of the price-dividend ratio. A possible improvement may be to include other factors such as habit formation over consumption. However, this is beyond the scope of our current research.

2.7 Appendix

Based on Equation (2.7), we have

$$Ref_{t,t+1}^P = \begin{cases} Ref_{t-1,t}^P + k_g((P_t + D_t) - Ref_{t-1,t}^P), & (P_t + D_t) > Ref_{t-1,t}^P \\ Ref_{t-1,t}^P + k_l((P_t + D_t) - Ref_{t-1,t}^P), & (P_t + D_t) < Ref_{t-1,t}^P \\ Ref_{t-1,t}^P, & (P_t + D_t) = Ref_{t-1,t}^P \end{cases}.$$

Divide Equation (2.7) throughout by P_t

$$\frac{Ref_{t,t+1}^P}{P_t} = \begin{cases} \frac{Ref_{t-1,t}^P}{P_t} + k_g \left(\frac{(P_t + D_t) - Ref_{t-1,t}^P}{P_t} \right), & \frac{(P_t + D_t)}{P_t} > \frac{Ref_{t-1,t}^P}{P_t} \\ \frac{Ref_{t-1,t}^P}{P_t} + k_l \left(\frac{(P_t + D_t) - Ref_{t-1,t}^P}{P_t} \right), & \frac{(P_t + D_t)}{P_t} < \frac{Ref_{t-1,t}^P}{P_t} \\ \frac{Ref_{t-1,t}^P}{P_t}, & \frac{(P_t + D_t)}{P_t} = \frac{Ref_{t-1,t}^P}{P_t} \end{cases}.$$

By definition $Ref_{t,t+1}^R = \frac{Ref_{t,t+1}^P}{P_t}$ and simplifying the first case when $\frac{(P_t + D_t)}{P_t} > \frac{Ref_{t-1,t}^P}{P_t}$, we obtain

$$\begin{aligned} Ref_{t,t+1}^R &= \frac{Ref_{t-1,t}^P}{P_{t-1}} \frac{P_{t-1}}{P_t} + k_g \left[\frac{(P_t + D_t) - Ref_{t-1,t}^P}{P_{t-1}} \right] \frac{P_{t-1}}{P_t} \\ &= Ref_{t-1,t}^R \frac{P_{t-1}}{P_t} + k_g (R_t - Ref_{t-1,t}^R) \frac{P_{t-1}}{P_t} \\ &= [Ref_{t-1,t}^R + k_g (R_t - Ref_{t-1,t}^R)] \frac{P_{t-1}}{P_t} \\ &= \left[Ref_{t-1,t}^R \left(1 + k_g \left(\frac{1}{z_t} - 1 \right) \right) \right] \left[\frac{f(z_{t-1})}{f(z_t)} \frac{D_{t-1}}{D_t} \right] \\ &= \left[1 + k_g \left(\frac{1}{z_t} - 1 \right) \right] \left[\frac{1 + f(z_t)}{f(z_{t-1})} \frac{f(z_{t-1})}{f(z_t)} z_t \right] \\ &= \left[1 + k_g \left(\frac{1}{z_t} - 1 \right) \right] \left[\frac{1 + f(z_t)}{f(z_t)} z_t \right] \end{aligned}$$

We apply the same method for the other two cases, hence, we have the dynamics of the stocks gross returns reference point as in Equation (2.10)

$$Ref_{t,t+1}^R = \begin{cases} \left[\frac{1+f(z_t)}{f(z_t)} z_t \right] \left[1 + k_g \left(\frac{1}{z_t} - 1 \right) \right], & z_t < 1 \\ \left[\frac{1+f(z_t)}{f(z_t)} z_t \right] \left[1 + k_l \left(\frac{1}{z_t} - 1 \right) \right], & z_t > 1 \\ \left[\frac{1+f(z_t)}{f(z_t)} z_t \right], & z_t = 1 \end{cases}.$$

Chapter 3

Alpha from news sentiment: a decision tree approach

3.1 Introduction

As the financial market in highly developed countries such as the US becomes more complex, identifying the source of alpha has been more challenging. We define alpha as the risk adjusted measure of the stock returns in a standard equity selection model. This model is a linear weighting of factors which summarize the characteristics of a stock, typically takes the form $r = \alpha + \beta_1 f_1 + \dots + \beta_k f_k + \epsilon$.¹ So far, one usually depends on quantitative signals such as Price/Earnings ratio and earnings revisions to generate alpha. Recent developments show that these quantitative signals generate better performance simply because they have more exposure to common stock risk factors over the past few years. For example, using empirical data from 1927 to 2005, Hwang and Rubesam (2008) argue that momentum phenomena disappeared during the period 2000 to 2005. Khandani and Lo (2011) show how a mean-reversion strategy that they used to analyze market behaviour lost profitability in the 12-year period from 1995 to 2007 (the daily return decreases rapidly from 1.35% in 1995 to 0.45% in 2002 and just 0.13% in 2007). Moreover, there is similarity in portfolio holdings before the quant meltdown in July-August 2007 because most of the quantitative asset managers implement classical modelling methodologies which are combined with similar data sources and risk models (see Ang (2008), Khandani and Lo (2011)). This phenomenon coupled with poor performance among asset managers in 2007-2009 triggers an ongoing debate on which modelling framework is most suitable and appropriate in improving performance.

This paper analyzes a new source of data (news-sentiment-driven Thomson Reuters MarketPsych Index (TRMI)) and attempts to find new factors which might be a source of alpha through time. We use a state-of-the-art technique in finding the predictive power of news sentiment from TRMI.

¹In Section 3.5.3 later on, we illustrate the performance evaluation of our CART trading strategy by adjusting to the CAPM, Fama-French three factor and Fama-French-Carhart four factor model's risk factors, which are related to market in excess of R_f (MKT- R_f), value (HML), size (SMB) and momentum (MOM).

We move away from the usual standard linear models and focus on one of the most commonly used “supervised learning” types of model, that is, Classification and Regression Trees (CART). The latter methodology is advantageous than the former because of its data-driven nature. Instead of defining a hypothetical relationship ex-ante and testing it ex-post, we allow the data to speak for itself and determine the model’s structure. As shown by Zhu, Philpotts, and Stevenson (2012), CART delivers robust performance as compared to the classical linear framework.

Our results show that the TRMI indeed delivers a promising alpha which provides a breeze of fresh air for quantitative investors. The first indication of good performance of the 10 sectors CART classifiers are described by the cumulative average hit rates which are on average around 58% over time. We also compare our results with those of the two benchmark models. The first benchmark model is the CART model without TRMI (only implements the technical, Fama French and classical sentiment indicators). The second one is the naive trader model which forecasts the signals randomly based on the following distribution: under-performing (-1) with probability 45%, neutral (0) with probability of 10%, and outperforming (1) with probability of 45%. These probabilities are drawn from the historical distribution of the three classes. The cumulative average hit rates of our model by far outperforms the two benchmark models (around 50% for the CART without TRMI and around 42% for the naive trader). Based on the total cumulative average hit rates as of July 2013, our model’s top 4 best performer sectors are Technology (64%), Healthcare (63%), Energy (61%) and Financials (60%). The bottom 4 worst performers are Basic Materials (53%), Non-Cyclical Consumer Goods and Services (54%), Utilities (55%) and Industrials (56%). These results suggest that each different sector has different degree of sensitivity towards news. In terms of variable importance, most of the technical indicators, Fama French factors and classical sentiment indicators do not play a big role in our CART classifiers. They appear occasionally in the bottom 5 of the rank of the variable importance. On the other hand, the TRMI variables such as Price Increase, Market Risk, Sentiment, Gloom (negative future outlook), Market Forecast, Optimism and Fear consistently dominate the weekly average top 10 most important variable for all the 10 sectors.

The two trading strategies (Long/Short and Long Only strategies), which are formed from these CART classifiers, perform well as compared to their benchmark (Buy and Hold S&P 500). The portfolio of Long/Short and Long Only strategies produce investor wealth which is constantly higher than the benchmark. The drawdowns for the Long/Short strategy are within acceptable range (the highest drawdowns was during November 2008 which reached around 28%). We analyze the performance of our base model’s trading strategies (Long/Short and Long Only) and compare them with our benchmark model (CART which implements the technical, Fama French and classical sentiment indicators, while excluding all the 23 TRMIs). We observe that our base model’s Long Only strategy generally outperforms the benchmark model and the Buy and Hold S&P 500. The yearly (5 year annualized) returns are around 2% (13%) higher. The yearly and 5 year annualized volatility are almost the same (around 3% higher). At the end, the Sharpe Ratio for both yearly and 5 year annualized performs better than the benchmark. Our base model’s Long/Short strategy also outperforms one of the benchmarks (CART without TRMI) under the same strategy, but it is not better than the Buy and Hold

S&P 500 or the previous Long Only strategies.

Our trading strategies' good performance is a new source of alpha which is not explainable by exposure to common stock risk factors. We adjust the performance of these strategies for different risk factors related to market in excess of R_f (MKT- R_f), value (HML), size (SMB) and momentum (MOM). Our base model's Long Only strategy's weekly returns is significantly positive with mean at around 34 bps and t-statistics of 1.887. Based on the CAPM model, our base model's Long Only strategy produces additional excess returns of around 21 bps on a weekly basis or around 11.5% per year on top of the returns expected from a portfolio with $\beta = 1$. The t-statistics of the alpha obtained is significant at 1-5% significance level. The adjusted $R^2 = 0.858$ means that 85.8% of the variance of the returns are explained by our model. By adding more factors as in Fama-French or Fama-French-Carhart, the additional alpha generated weekly is stable at 22 bps and its t-statistics is significant at the same level. The adjusted R^2 is around 0.866 for both models. Our base model's Long Only strategy definitely produces new source of alpha as compared to the same strategy from our benchmark model (CART without TRMI). None of the risk factor regression results of our benchmark model's Long Only strategy show any significant alpha values. Our base model's Long/Short strategy also generates significant alpha values (around 30-31 bps weekly) which outperform the benchmark model. The adjusted R^2 is very low which indicates that the variance of the returns are not explained well by this model.

Our base model is robust even if we differentiate some of its characteristics such as taking excess returns as the dependent variable or defining only two categories (outperforming or underforming). One particular setback of using the TRMI as a trading strategy is that the predictive power of these news sentiments are relatively short-lived. Upon taking a closer look at the monthly rebalancing, the performance gets worse. Hence, the information from this news sentiment data decays relatively quickly. The TRMI serves as an option for lower-frequency quantitative investors in enhancing their performance.

The remaining of this paper is organized as follows. Section 3.2 describes the current research on quantifying the content from the news through indices such as TRNA and its impact on the stock returns. Section 3.3 introduces the Thomson Reuters MarketPsych Index (TRMI) as our source of data for this research. Section 3.4 presents the Classification and Regression Trees (CART) methodology we implement in this paper. Section 3.5 describes the results of the predictive performance of the CART classifiers, the variable importance of the CART classifiers for each sector, the trading strategies constructed from these classifiers and robustness checks. Section 3.6 draws the conclusions.

3.2 Media content and stock returns

The idea that market movements are closely related to the news media is first introduced by Shiller (2000). His conjecture is that investors follow closely what described in the media even though it may just be pure speculation, hence, market sentiment is driven by the news' content. How do we quantify the qualitative information presented in the news media? For the past ten years, there are many

different methods which have been developed on sentiment analysis/opinion mining to attach value to qualitative information gathered from news texts. Pang and Lee (2008) describe a complete survey on these methods. Through these methodologies of content analysis, we can determine whether the author of the text meant to express a positive or negative opinion on the subject matter.

Research on sentiment analysis gradually emerges in the field of finance. It begins with the simplest approach of “bag of words” where sentiment score is calculated based on how often certain important key words appear. For example, Li (2007) analyze how often the words “risk” and “uncertain” appear in company annual reports. Davis, Piger, and Sedor (2006) count the words which are categorized into positive or negative language (based on the DICTION dictionary) from the texts of earning announcements. Tetlock (2007) shows that the number of negative words in the “Abreast of the Market” column of the Wall Street Journal predicts stock returns at the daily frequency from 1984 to 1999. Tetlock, Saar-Tsechansky, and Macskassy (2008) further analyze all the company-specific news from the Wall Street Journal and Dow Jones Newswire archive. All of these research studies show that content analysis from texts in the news/media have an important role in explaining the stock returns without taking into account other quantitative factors.

Even up until recently, the research on financial news and its content is still relevant and has a lot of traction. Fang and Peress (2009) is one of the first to document a cross-sectional relation between media coverage and security returns. They find that stocks with no media coverage earn higher returns than stocks with high media coverage even after controlling for well-known risk factors such as analyst forecast dispersion and idiosyncratic volatility. Tetlock (2010) uses the data on financial news events to test four predictions from an asymmetric information model of a firm’s stock price. Tetlock (2011) tests whether stock market investors appropriately distinguish between new and old information about firms. The staleness of a news story is defined as its textual similarity to the previous ten stories about the same firm. He finds that firms’ stock returns respond less to stale news and individual investors trade more aggressively on news when news is stale. Schumaker, Zhang, Huang, and Chen (2012) investigate how the choice of words and tone used by authors of financial news article correlate to measurable stock price movements. They use the Arizona Financial Text (AZFinText) system and a sentiment analysis tool. Garcia (2013) constructs a proxy for market sentiment by counting the number of positive and negative words from two financial columns from the New York Times (The columns are “Financial Markets” and “Topics in Wall Street”). He shows that the predictability of the stock returns using news content is concentrated during periods of recession.

As of today, research in finance commonly uses Thomson Reuters News Analytics (TRNA) data to represent news sentiment. TRNA measures the relevance, sentiment, novelty and volume of news. Leinweber and Sisk (2011) analyze event studies on a broad universe of US equities (by sector and capitalization) from 2003-2008 and conduct a portfolio simulation from 2006-2009. They show that both the event studies and portfolio simulation show evidence of exploitable alpha using the TRNA. Dziulinski (2011) uses TRNA to directly compare news and no-news stock returns. The paper estimates whether returns on positive, neutral and negative news days are significantly different from the average daily return for a large sample of US stocks over the period from January 2003 to August 2010. The

results show that positive news days have above-average returns and negative news days returns are below average, while the neutral news days are economically barely distinguishable from the average.

Differently from this research, we make use of a new dataset TRMI. In Section 3.5.3, we illustrate how this new dataset successfully produces simple trading strategies with weekly rebalancing which generates significant alpha. TRMI performs equally good as TRNA, with some novelties and advantages in the construction of the indices which are discussed in the next section.

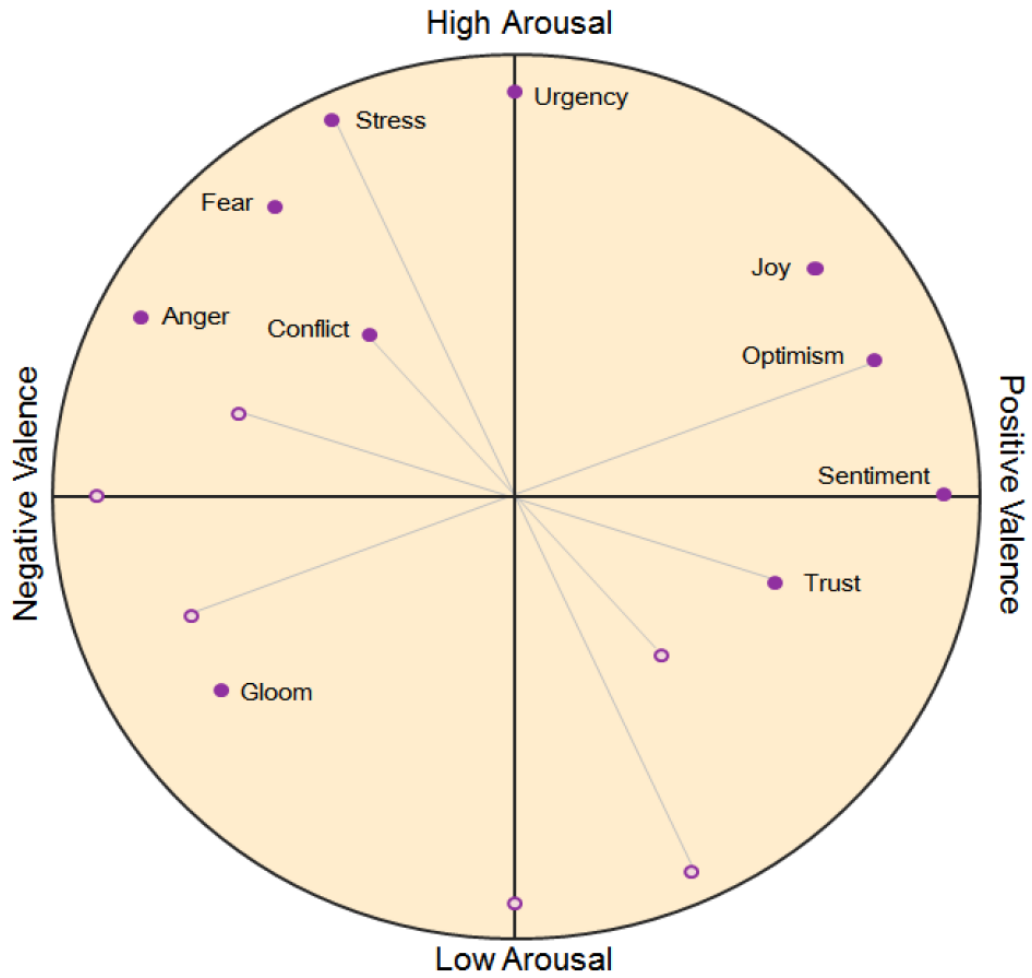
3.3 Thomson Reuters MarketPsych Index (TRMI)

Our news sentiment data is sourced from a multi-dimensional Thomson Reuters MarketPsych Index (TRMI), which is a joint collaboration between Thomson Reuters and MarketPsych LLC. TRMI is a novel and unique source of data as compared with TRNA. The indices are derived from a sophisticated algorithm which extracts complex meaning from text. The technique was designed to optimally quantify business-specific language for financial applications. TRMI deliver sentiments on 41 equity indices, 29 currencies, 34 commodities and 119 countries and it covers a large content collection, including tens of thousands of social media and news sources back to 1998. Unlike TRNA which only looks at the sentiment of company specific's stocks, TRMI considers the aggregated news sentiment which result in a sector and market level sentiment indices.

Traditional textual sentiment analysis usually results in one dimensional output that is a scale ranging from positive, neutral to negative. However, from a behavioural and psychological perspective, we know that human emotions are varied. Research in psychology (see Russell (1980)) commonly classifies human emotion into two dimensions: **valence** and **arousal**. Differently from TRNA, the sentiment data in TRMI is highly dimensional and includes scores on more than 50 sentiments and topics with wide range of entities. The TRMI captures the arousal dimension in the specific indices for Stress (slightly negative valence) and Urgency (neutral valence) as seen in Figure 3.1. Unipolar TRMIs (for example Fear, Anger, Joy and Gloom) are located in one quadrant of the circumplex and usually range between 0 and 1 and can fall below 1. Bipolar TRMIs (for example Trust, Sentiment, Optimism, Urgency, Stress and Conflict) consist of the net difference between psychological variables (PsychVar) values of equivalent meaning but opposite valence. For example, Sentiment is the net difference between positive and negative PsychVars. Therefore, the median Sentiment values are near zero and the range of such bipolar TRMI is -1 to 1.

The complex algorithms used in TRMI (MarketPsych Lexical Analysis) have specific features which present many advantages. For example, the algorithm differentiates models for news, social media forums, tweets, SEC filings and earnings conference call transcripts adapting to the idea that communication styles between these sources are inherently variative. Social media tend to have more flexibility from editorial oversight, hence the authors are usually more passionate and descriptive upon expressing their opinion as compared to news coming from more formal channels. The algorithms possess a correlate filter which ensures that only entities with the correct co-reference are included in entity identification. For example, when Twitter user tweets that "I am enjoying my breakfast oats,"

THE SENTIMENT-DERIVED TRMI PLOTTED ON THE AFFECTIVE CIRCUMPLEX



Source: MarketPsych LLC.

Figure 3.1: This figure describes several TRMI sentiments on the affective circumplex. Each dot represents the emotion's location on the circumplex. TRMI represents an emotion and its opposite are plotted with a thin grey line connecting the positive and negative poles.

the software will not count this as applicable to commodity “oats”. References to “oats” are counted only if they also contain other key identification which correlate such as “prices” and “futures”. It is calibrated to identify verb tenses in every phrase, including in instances when multiple verbs are present. It consider modifier words which change the meaning of a phrase or sentence by modifying its impact. For example, words or phrases which increase the significance of an adjective, e.g., “large”, is multiplicative on the weighting of the modified word.

MarketPsych provides an example of how they analyze the following sentence
“Analysts expect Mattel to report much higher earnings next quarter”
The language analyzer performs the following sequence

- (1) Associates ticker symbol MAT with company name Mattel,
- (2) Identifies “earnings” as an Earnings word in the lexicon,
- (3) Identifies “expect” as a future-oriented word and assigns future tense to the phrase,
- (4) Identifies “higher” as an Up-Word,
- (5) Multiplies “higher” by 2 due to presence of the modifier word “much”,
- (6) Associates “higher” (Up-Word) with “earnings” (Earnings) due to proximity.

The analysis algorithm will report as in Table 3.1.

Date	Time	Ticker	PsychVar	Score
20110804	15:00.123	MAT	<i>EarningsUp_f</i>	2

Source: MarketPsych LLC.

Table 3.1: We present the analysis algorithm result of the sentence *“Analysts expect Mattel to report much higher earnings next quarter”* in the example above. The raw score produced for *EarningsUp_f* is equal to 2.

The TRMI are computed for an asset or its constituents from news content scored in the past 24 hours. For example, consider that Mattel is a constituent of MarketPsych’s NASDAQ 100 index proxy asset (MPQQQ). At its most basic level, the sentiment of Mattel is aggregated with the sentiment of the other constituents of MPQQQ for the past 24 hours. This aggregate sentiment value is normalized by the **Buzz**, which is the sum of the absolute values of all TRMI-contributing sentiments, over the same period. The Buzz indicates how much something is being talked about in the news and social media. This ratio of aggregate sentiment to the Buzz is a single unipolar TRMI. However, most TRMI are not unipolar.

For a bipolar TRMI which consists of a net difference between two PsychVars, the algorithm performs an addition or subtraction operation on the two PsychVars and then normalizes it by Buzz. For example, to obtain EarningsForecast TRMI value for MPQQQ, the *EarningsUp_f* (*EarningsDown_f*)

score for Mattel is aggregated with all other $EarningsUp_f$ ($EarningsDown_f$) scores of the constituents of the MPQQQ. The aggregate $EarningsDown_f$ is then subtracted from the $EarningsUp_f$ and the result is divided by the Buzz value as follows

$$EarningsForecast(MPQQQ) = \frac{(EarningsUp_f(MPQQQ) - EarningsDown_f(MPQQQ))}{Buzz(MPQQQ)}$$

A company's PsychVars are included into the TRMI for all assets which have this company as a constituent. For example, if Mattel is a constituent for both the Consumer Goods sector and the NASDAQ 100 Index proxies, then Mattel's PsychVar scores will be incorporated into the TRMI for both. Similarly, a single PsychVar can contribute to multiple TRMI. For example, the $EarningsUp_f$ PsychVar in the above example is not only a constituent of EarningsForecast but also of Sentiment, Optimism and FundamentalStrength.

For our research, we focus on the asset class of US Equities for 10 economics sector-driven described in Table 3.2. The indices and their numerical ranges for US Equities are depicted in Table 3.3.

THE THOMSON REUTERS BUSINESS CLASSIFICATION (TRBC) OF 10 ECONOMIC SECTOR-DRIVEN
US EQUITIES

RIC	TRBC Description	TRBC Code
MPTRXENE	Energy	50
MPTRXMAT	Basic Materials	51
MPTRXIND	Industrials	52
MPTRXYCY	Cyclical Consumer Goods and Services	53
MPTRXNCY	Non-Cyclical Consumer Goods and Services	54
MPTRXFIN	Financials	55
MPTRXHLC	Healthcare	56
MPTRXTEC	Technology	57
MPTRXCOM	Telecommunication Services	58
MPTRXUTL	Utilities	59

Source: Thomson Reuters.

Table 3.2: This table describes the Thomson Reuters Business Classification (TRBC) of 10 Economic Sector-driven US Equities which are relevant for our research.

3.4 Methodology

3.4.1 Data set-up and adjustments

We gather the daily prices of the 10 economics sector-driven equities from 1st June 2000 until 31st July 2013 which is sourced from Thomson Reuters Datastream and calculate the weekly rate of returns of

each sector i at a particular week t which is equal to

$$(3.1) \quad r_{i,t} = \frac{S_{i,t} - S_{i,t-1}}{S_{i,t-1}},$$

where $S_{i,t}$ and $S_{i,t-1}$ are the daily price of each sector i at week t and $t - 1$ respectively.

Our dependent variable is the absolute rate of returns of each of the 10 economics sector-driven US equities, r , evaluated against a threshold, R . The threshold is a constant which is chosen to be 10 bps.² We set our classification of the dependent variable to follow these rules

$$(3.2) \quad \begin{aligned} r > R &\Rightarrow \text{Outperforming,} \\ |r| \leq R &\Rightarrow \text{Neutral,} \\ r < -R &\Rightarrow \text{Underperforming.} \end{aligned}$$

Our independent variables are all the 23 daily TRMI over the same period of time (1st June 2000 - 31st July 2013). We also include some common technical indicators (momentum 5 days, 30 days and 200 days)³, 4 Fama French factors (SMB, HML, MKT and MOM)⁴ and 6 widely used classical sentiment indicators⁵ to analyze whether they play significant role in explaining the absolute returns as compared to the 23 TRMI. We will show in this paper that most of these existing technical and classical sentiment indicators are dropped from the CART classifier and beaten by the TRMI.

3.4.2 Classification and Regression Tree (CART)

CART is a non-parametric tool which is designed to represent decision rules in a form of binary trees. In the field of economics and finance, CART has been implemented since almost thirty years ago where Frydman, Altman, and Kao (1985) analyze default risk. Then Kao and Shumaker (1999) uses it to explain relationships between macroeconomic variables and performance of timing strategies based on market, size and style. Sorensen, Miller, and Ooi (2000) implement it to partition assets into outperforming and under-performing assets and then compose a portfolio by uniformly weighted outperforming assets. Albanis and Batchelor (2000) compare different techniques to distinguish outperforming and under-performing assets and demonstrate the efficiency of CART.

Since we are dealing with TRMI news sentiment data which is huge in size and relatively new, CART is a possible candidate for our purpose to tackle the problem of multi-dimensionality. We conjecture that news sentiment (from a natural language processing perspective) will bring us to a non-linear analysis where there is complex dependencies between variables in our data. CART has been a preferred technique because it is non-parametric. Hence, we have flexibility and we are

²This threshold is chosen since from practitioners perspective, absolute returns which are close to zero are not worth betting on in the trading strategies.

³These data are calculated based on the daily returns obtained from Thomson Reuters Datastream.

⁴These data are obtained from Kenneth French website at <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>.

⁵These sentiment indicators are ABC News US Weekly Personal Finance Index, Weekly National Economy Index, Weekly Consumer Comfort Index, Thomson Reuters US Investors Intelligence Advisors Sentiment Bullish, Bearish and Correction. They are results of telephone interview of 1'000 adults US nationwide.

not restricted to make any assumptions concerning the model errors distribution. Variables are not required to be selected in advance. Especially in a situation where we have given subset of variables comprising a learning sample, CART is designed in such way that we automatically select the most significant variables. Hence, we do not need to worry even if at the beginning the learning sample may contain some irrelevant information. Due to its data-driven nature the model will identify the correct splits by itself and accounts for these disturbances. The method also has a very high degree of interpretability. CART efficiently compresses a large volume of data into a form which identifies its essential characteristics. The output is therefore easy to understand very quickly with a computation which is reasonably fast and efficient.⁶

As shown by Zhu, Philpotts, and Stevenson (2012) who extend the work of Sorensen, Miller, and Ooi (2000) on a considerably wider universe of stocks over a longer time period, the performance of portfolio formed from a CART-based model is quite robust during the period of 2007/2008 downturn in equities and the subsequent market recovery as compared to a more traditional linear framework. They point out that the methodology does not restrict us in assuming the stock returns to be normally distributed and at the same time offers a high degree of model diversification from more traditional approaches.

CART classifiers construction

Growing the tree

We consider the learning sample $\mathcal{L} = \{(x_1, j_1), \dots, (x_n, j_n)\}$. \mathbf{x}_i is a vector of features which belongs to a space \mathbb{X} . j_i is the relevant response which can be categorical or continuous, while n is the number of observations. The CART algorithm refers to a repetitive binary splitting procedure. The main idea is to repeatedly splitting subsets of \mathcal{L} into two descendant subsets. For a continuous variable x_i the splits take form $x_i < c$ versus $x_i \geq c$. On the other hand, a categorical variable serves in categorizing the relevant response into classes.

The CART classifiers are grown based on the basic idea of having each new subset to be purer and more homogenous than the previous set. In order to do so, the CART technique is to maximize the average purity of the two child nodes. Firstly we introduce $i(t)$ which represents the impurity of the node t and T is the performance of the classification tree. In each split s , the data is sent to node t_R or t_L and divided into proportion of p_R and p_L respectively. The proportions are equal to $p_L = \frac{p(t_L)}{p(t)}$ and $p_R = \frac{p(t_R)}{p(t)}$ respectively, where $p_L + p_R = 1$. Hence for every split we obtain a reduction in impurity which is equal to the impurity of the root node minus the proportion of impurity in each child

$$(3.3) \quad \Delta i(s, t) = i(t) - p_R i(t_R) - p_L i(t_L).$$

For any arbitrary node t and a set of splitting candidates S , the optimal split \mathcal{S}^* is taken based on

$$(3.4) \quad \mathcal{S}^* = \max_{s \in S} \Delta i(s, t).$$

⁶A good discussion the suitability of CART as compared to multiple linear regression and discriminate analysis can be found in Callahan and Sorensen (1991).

Hence, the optimal split is the one which achieves the greatest reduction in impurity. In CART, there are different measures of impurity which can be implemented to achieve the split.

Consider that we have a classification problem where we want to classify data into K classes. At each node t of this classification tree, there is the probability distribution $p_{tk}, \forall k = 1, \dots, K$ over all the K categories. The probabilities are results of the node proportions $p_{tk} = \frac{n_{tk}}{n_t}$, where n_{tk} is the number of observations in the k -th class and n_t is the sample size at node t .

The most commonly used splitting rules are the Gini Index

$$(3.5) \quad i(t) = \sum_{j \neq k} p_{tj} p_{tk} = 1 - \sum_k p_{tk}^2.$$

and entropy or information

$$(3.6) \quad i(t) = - \sum_k p_{tk} \log(p_{tk}).$$

where $0 \log(0) = 0$.

Pruning the tree

The idea of tree partitioning to reduce impurity does not guarantee that we will end up with a useful tree model. We will eventually get a maximal tree which has one observation or one class in each leaf, whichever comes first. Hence, we are facing the risk of overfitting tree which adapts too well to the features of the learning sample. Pruning the tree is a measure to improve the model's robustness by having a trade-off between the in-sample fitting and the out-of-sample accuracy.

Breiman and Stone (1984) suggests the cost-complexity procedure to prune the tree. Let T be a partially ordered subtree from a maximal tree grown without any pruning. $|\hat{T}|$ is the size of the subtree (the number of terminal nodes) which represents its complexity. The optimal tree is the one which minimizes the following cost-complexity measure

$$(3.7) \quad R_\alpha(T) = R(T) + \alpha |\hat{T}|,$$

where α is the complexity parameter which penalizes the size and $R(T)$ is the cost incurred as misclassification errors in the classification cases. If we have a small tree, then we can achieve a large $R(T)$. However, if we have a large tree with only one object per terminal node and class, then $R(T) = 0$ but $R_\alpha(T) \neq 0$ because of the complexity of the tree. Hence, we aim to minimize the cost complexity of the tree by searching for subtrees which may be eliminated in a nested partially ordered set of subtrees,

$$(3.8) \quad T_{max} \succ T_1 \succ T_2 \succ \dots \succ T_R = \{root\}.$$

In order to decide which subtree to eliminate we search for the weakest connection of a subtree by introducing a function $g_r(t)$ which is defined as

$$(3.9) \quad g_r(t) = \frac{R(t) - R(T_{rt})}{|\hat{T}_{rt}| - 1},$$

where t represents an internal node of one of the nested subtrees T_r and T_{rt} is its connection to the node t . The weakest connection in the tree T_r is the one which has the smallest value of the $g_r(t)$,

$$(3.10) \quad g_r(t_r^*) = \min_{t \in T_r} g_r(t).$$

We continuously implement the pruning until reaching the root node. This implies that for each new pruning level, a nested subtree is removed from the initial tree, $T_{r+1} = T_r - T_{t_r^*}$. Furthermore, at each new pruning level the complexity cost is updated as $\alpha_{r+1} = g_r(t_r^*)$ which means that there is an increasing sequence $\{\alpha_r\}$ for $r \geq 1$ where $\alpha_1 = 0$. According to the theorem, under subsequent conditions, the minimal cost complexity tree within the interval $\alpha_r \leq \alpha < \alpha_{r+1}$ is

$$(3.11) \quad T(\alpha) = T(\alpha_r) = T_r.$$

Given the sequence of both subtrees and cost complexity parameters, we arrive to the minimal cost complexity tree which exists within the sequence of trees. Next, we resort to the method of cross validation to find out the tree which accounts for the best pruning level.

Cross Validation

The main idea of cross-validation is that we use as much information from the learning sample L as possible by separating it into K subsets. In this research we use the commonly implemented 10-fold cross-validation where $K = 10$. We define the subset L_k as the test sample in the algorithm. The rest of the data $L^{(k)} = L - L_k$, where $k = 1, \dots, K$, will be used to create new trees $T_r^{(k)}$. Since we create sequences of trees and cost complexity parameters for both initial learning sample L and the K generated samples, at the end we will have $K + 1$ sequences of trees and parameters.

Since the superior tree T_r among any sequence of trees lies within an interval of complexity costs, we redefine this interval so as to make estimations of the misclassification. We then apply the geometric mean of the interval $\alpha_r^* = \sqrt{\alpha_r \alpha_{r+1}}$, which at the end enables us to get an estimation of the misclassification cost. As discussed, we would like to minimize the overall misclassification cost $R_\alpha(T)$ of the tree. In order to do so, we use the sequence of generated trees to derive the trees with the same complexity as the original one. Based on the theorem in Equation (3.11), the estimate of the cost complexity measure for each subtree T_r is equal to

$$(3.12) \quad \hat{R}(T_r) = \hat{R}(T(\alpha_r^*)).$$

This is done by finding all generated trees $T_r^{(k)}$ where α_r^* , which is associated with the tree T_r , lies within the generated interval $[\alpha_r^{(k)}, \alpha_{r+1}^{(k)}]$. When we found all these trees we then analyze the associated k th test set L_k in each generated tree $T_r^{(k)}$ and then create a vector which has the same amount of ones as the number of incorrect classifications and zeros as correct classifications. This procedure is done for all subtrees T_r and then we set $\hat{R}(T_r)$ as the mean of the vector which consists of zeros and ones. The best-pruned tree is the tree which possesses the least nodes, i.e., the greatest r value and within one standard error of the minimum $\hat{R}(T_r)$ for all r .

3.5 Results

We apply the CART methodology above to our training data from 1st June 2000 - 31st December 2006 and obtain 10 best-pruned CART classifiers (1 classifier for each sector). Each week we dynamically roll the training data and update the 10 CART classifiers accordingly (i.e., the training data is weekly rolling until July 2013). Table 3.4 depicts the complexity parameter (cp) table for one of the non-pruned dynamic tree of the Financial sector for the week of 5 July 2013. The nsplit (second column) shows the number of splits from the smallest tree with zero split to the largest one (57 splits) and the number of nodes is just equal to $1 + \text{nsplit}$. The rel error (third column) is calculated based on the ratio of the residual sum of squares for the tree with k terminal nodes over the residual sum of squares for the tree with 1 terminal node. Each of these trees is then examined using 10-fold cross-validation where the data are divided into 10 equal segments. The tree is built using 9 of the 10 segments and error is assessed on the tenth segment. This is repeated leaving off each segment in turn and errors are then averaged and scaled to give xerror (fourth column). In other words, the xerror is the cross validation relative error. The xstd (last column) is the variation between the 10 sub-sample estimates. We observe from this table that xerror are minimized at 0.669 where the number of split is 8 and the cp value is 0.017. Hence the size of the tree (number of terminal node) is equal to 9 ($1 + 8$).

The left side of Figure 3.2 illustrates this table in a chart where we observe that the xerror is minimized at around 0.669 where the size of the tree (number of terminal node) is 9. Hence, we prune the tree at this minimized xerror and obtain the best-pruned tree with only 9 terminal nodes (right side figure). This result also provides us with a sense of the hierarchy of importance between the different explanatory variables which we discuss further in Section 3.5.2.

3.5.1 Performance of the CART classifiers

In order to test the performance of the CART classifiers over time, we use all the 10 CART classifiers obtained weekly using the training data from 1st June 2000 - 31st December 2006 to predict the observed class of the sectors in the testing data starting from January 2007 until July 2013 weekly. Note that the training data has the same length each week. Our measure of performance is the hit rate, that is, the percentage when the CART classifiers are correctly predicting the classification of each sector as outperforming, neutral or under-performing.

Figure 3.3 describes the cumulative average hit rate for all 10 sectors which is updated dynamically each week. Paying attention on the result from January 2008 onwards, we observe that the cumulative average hit rates of the base case with TRMI are on average around 58% over time.⁷ We compare this result with two benchmark models. The first benchmark model is when we perform CART only using the technical, Fama French and classical sentiment indicators while excluding all the 23 TRMIs. The other benchmark model is when we perform CART as a naive trader which forecasts the signals randomly based on the following distribution: under-performing (-1) with probability 45%, neutral

⁷The first period of January 2007 only have one observation either correctly/wrongly predicted, hence, it is volatile. The number of predictions increase each week and the hit rate is cumulated weekly.

THE CP TABLE AND THE PRUNED TREE FOR THE FINANCIAL SECTOR FOR THE WEEK OF
5-JUL-2013

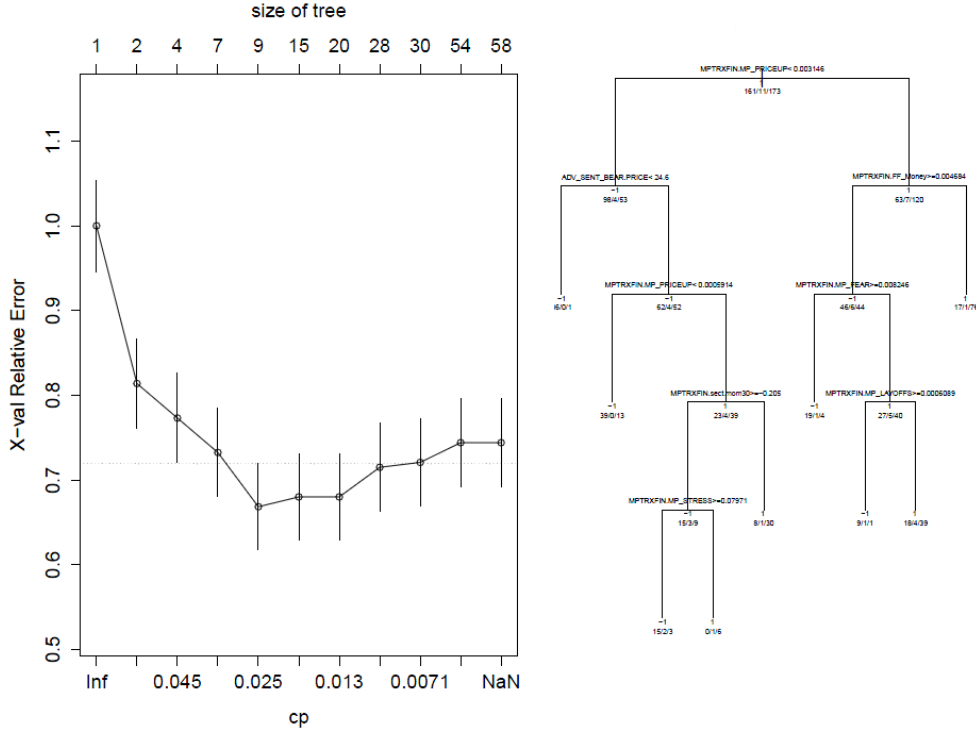


Figure 3.2: This figure describes the cp table and the pruned tree for the Financial sector for the week of 5-Jul-2013. The left side illustrates that the xerror is minimized at around 0.669 and the size of the tree (number of terminal node) is 9. Hence, we prune the tree at this minimized xerror and obtain the best-pruned tree with only 9 terminal nodes (right side figure).

(0) with probability of 10% and outperforming (1) with probability of 45%. These probabilities are drawn from the historical distribution of the three classes.

In the same Figure 3.3, we observe that the cumulative average hit rate of the benchmark model without TRMI is on average lower than the base model with TRMI (around 50% over time). Moreover, the cumulative average hit rate of the naive trader is on average even much lower than the base model with TRMI and the benchmark model without TRMI (around 42% over time). To summarize, our base case model with TRMI clearly outperforms the two benchmark models (without TRMI and naive trader) if we look at the percentage when the CART classifiers are correctly predicting the classification of each sector (as outperforming, neutral or under-performing).

Table 3.5 shows the total cumulative average hit rates for each of the 10 sectors as of July 2013 and compare these values with those of the two benchmark models. The results in this table depict that the total cumulative average hit rates for each of the 10 sectors are always higher for the base model with TRMI, as compared to the two benchmark models. Focusing on the base model with TRMI, the top 4 best performer sectors, where their total cumulative average hit rates are relatively higher than other

THE WEEKLY UPDATED CUMULATIVE AVERAGE HIT RATE FOR ALL 10 SECTORS

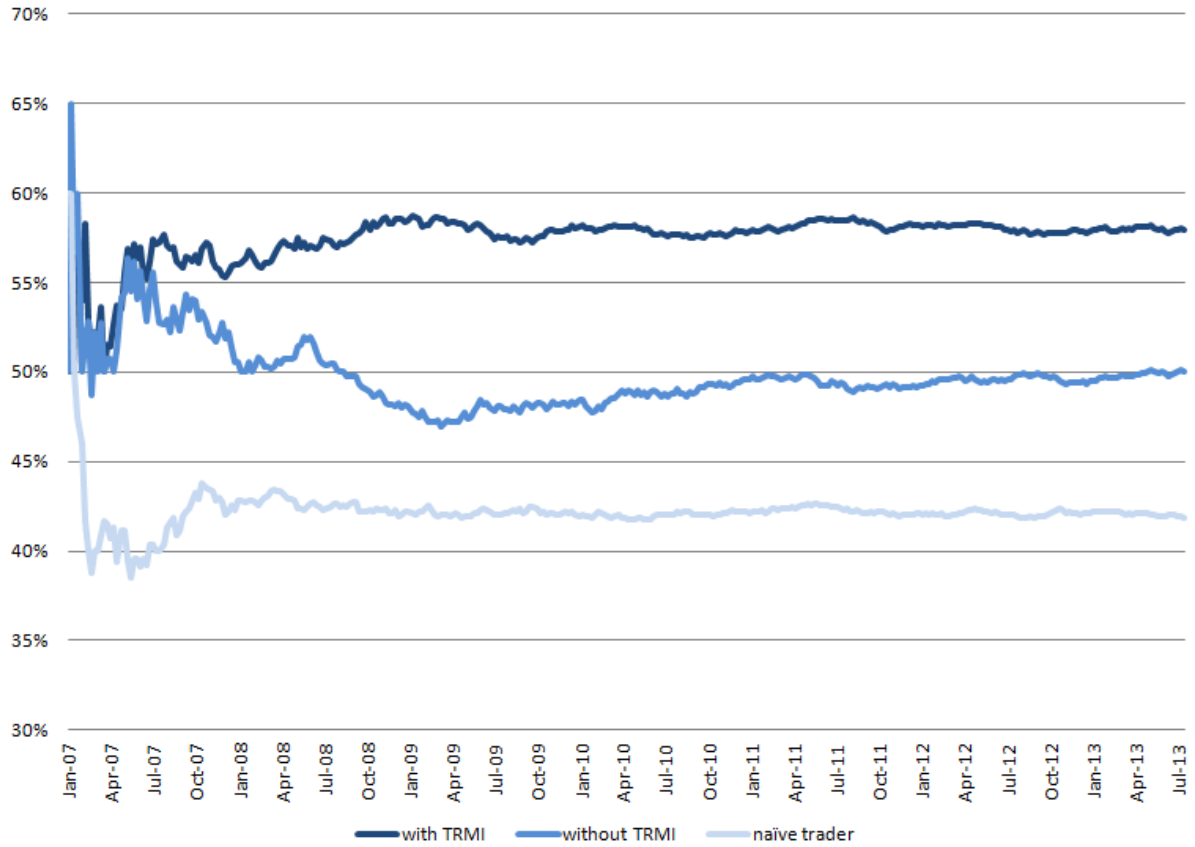


Figure 3.3: The cumulative average hit rates for all the 10 sectors (taking the average) for all three models (CART with TRMI, without TRMI and naive trader). The training data is from 1st June 2000 - 31st December 2006 (weekly rolling till July 2013). From January 2008 onwards, we observe that the cumulative average hit rates of the base case with TRMI are on average around 58% over time. The cumulative average hit rate of the benchmark model without TRMI is on average lower than the base model with TRMI (around 50% over time). Moreover, the cumulative average hit rate of the naive trader is on average even much lower than the base model with TRMI and the benchmark model without TRMI (around 42% over time).

sectors, are Technology (64%), Healthcare (63%), Energy (61%) and Financials (60%). The bottom 4 worst performer sectors, where their total cumulative average hit rates are relatively lower than other sectors, are Basic Materials (53%), Non-Cyclical Consumer Goods and Services (54%), Utilities (55%) and Industrials (56%). These results show that each different sector has different degree of sensitivity towards news.

3.5.2 The variable importance

The definition of variable importance is not a straightforward concept because it has to consider the complex interaction of one variable with the others. We define variable importance as the (absolute) reduction of the impurity/uncertainty of the dependent variable Y given the knowledge of X_i . It is the summation of the heterogeneity reduction due to the splits made by the variable along the whole tree, over J set of non-terminal nodes of t tree.⁸

Let d_{ij} be the decrease in the heterogeneity index allowed by the X_i variable at node $j \in J$. The X_i variable is used to split at node j if $d_{ij} > d_{kj}$ for all variables in the dataset, $k = 1, 2, \dots, p$, $k \neq i$. The variable importance of X_i for the t -th tree is measured by

$$(3.13) \quad \hat{V}I_{X_i}(t) = \sum_{j \in J} d_{ij} I_{ij},$$

where I_{ij} is the indicator function which equals to 1 if the i -th variable is used to split at node j and 0 otherwise. In gradient TreeBoost algorithm of Friedman (2001), a slight modification is used in Equation (3.13), that is, d_{ij} is replaced by d_{ij}^2 and the $\hat{V}I_{X_i}(t)$ is rescaled by assigning a value of 100 to the most influential variable.

Figure 3.4 shows the weekly average top 10 most important variable for all the 10 sectors. We observe that the TRMI which describes price increases net references to price decreases (MP PRICEUP) dominates the rank most of the times (9 out of the 10 sectors). Other dominant TRMIs which are always in the top 5 are Market Risk (MP MKTRISK), Sentiment (MP SENTIMENT), Gloom (MP GLOOM), Market Forecast (MP MKTFCST), Optimism (MP OPTIMISM) and Fear (MP FEAR). These results are consistent with some empirical and experimental research which analyze the effect of sentiment and specific emotions on the price movement. For example, Antoniou, Doukas, and Subrahmanyam (2013) empirically show that momentum profits arise only under optimism. Da, Engelberg, and Gao (2010) aggregate the search terms which reflect economic fear and find that there is short-term mean reversion in prices when the fear-related search terms increase in quantity. Lerner, Small, and Loewenstein (2004) examine the impact of specific emotions on the endowment effect, the tendency for selling prices to exceed buying or “choice” prices for the same object. Their experiment show that sadness/gloom reduce selling prices but increasing choice prices. If we translate this result in a larger market behaviour, we expect an increasing trading volume during a high period of gloom.

On the other hand, we also observe in Figure 3.4 that most of our technical indicators, Fama French factors and classical sentiment indicators are not playing an important role in our CART classifiers. These variables appear occasionally in the bottom 5 of the rank of the variable importance. These findings indicates that our TRMI variables are useful in predicting the future absolute returns of the sectors. The TRMI are suitable candidates to generate alpha rather than purely using the existing technical or classical sentiment indicators.

⁸See Breiman and Stone (1984).

THE WEEKLY AVERAGE TOP 10 MOST IMPORTANT VARIABLE FOR ALL 10 SECTORS



Figure 3.4: This figure describes the weekly average top 10 most important variable for all 10 sectors. The training data is from 1st June 2000 - 31st December 2006 (weekly rolling till July 2013). The y-axis describes the variable importance which is the summation of the heterogeneity reduction due to the splits made by the variable along the whole tree. We observe that the TRMI which describes price increases net references to price decreases (MP PRICEUP) dominates the variable importance rank most of the times (9 out of the 10 sectors). Other dominant TRMIs which are always in the top 5 are Market Risk (MP MKTRISK), Sentiment (MP SNTMENT), Gloom (MP GLOOM), Market Forecast (MP MKTFCST), Optimism (MP OPTIMISM) and Fear (MP FEAR).

3.5.3 The trading strategies

Based on the CART classifiers for the 10 sectors which are weekly updated (as results of the training data from 1st June 2000 - 31st December 2006, weekly rolling till July 2013), we form two simple trading strategies based on the signals we obtain from these classifiers on the testing data (period of January 2007 to July 2013). These trading strategies serve as a backtesting procedure to investigate how the TRMI can generate alpha. We use these CART classifiers to predict the signals of the 10 sectors absolute returns and classify them as 1 (“outperforming”), 0 (“neutral”), or -1 (“underperforming”). Each week, the portfolio is rebalanced based on the new classification of the sectors. The first strategy is “Long Only”: we go long the outperforming sectors and equally distributing the weights. The second strategy is “Long/Short”: we go long the outperforming sectors and go short the under-performing ones and do not invest if the sectors are neutral. The weights across Longs and Shorts are distributed so that the portfolio is market neutral (The net exposure which measures the market direction of the investments portfolio = 0). For both strategies, we consider the Buy and Hold S&P 500 (going long 100% on S&P 500) as a performance benchmark.

Our two strategies include commissions, fees and transaction costs. We follow the pricing structure of Interactive Brokers, which is one of the cheapest brokers in the market. We adopt a Fixed pricing model for the securities we trade, that is, the US equity sector ETFs. This model charges a fixed amount per share and includes all Interactive Brokers commissions, exchange and most regulatory fees with the exception of the transaction fees, which are passed through only on stock sales. For US ETFs commissions, exchange and regulatory fees amount USD 0.005 per share and transaction fees 0.000021 of the value of aggregate sales. To simplify our calculations, we approximate the total expenses including all commissions and fees at 0.01 per share.

Formally, the absolute return of the weekly portfolio is

$$(3.14) \quad r_p = \sum_{i=1}^N w_i r_i,$$

where r_i is the absolute return of sector i and w_i is the weight over total portfolio assign to each sector. The budget constraint is $\sum_{i=1}^{10} w_i = 1$. For the Long Only strategy, we do not allow short-selling, hence, the weight of each of the sector has to be non-negative $\forall i w_i \geq 0$. Conversely, for the Long/Short strategy, the weight can take a negative value.

Figure 3.5 and Figure 3.6 show the weekly returns of the rebalanced portfolio of the two strategies (Long/Short and Long Only respectively). We observe that both strategies provide returns which are more stable over time as compared to the benchmark (Buy and Hold S&P 500) (see the bottom part of the two Figures). The negative returns are generally less negative than the benchmark. This is because we use classification trees which allow us to minimize the influence of the large negative returns. Figure 3.7 and Figure 3.8 depict the distribution of the weekly returns of the rebalanced portfolio of the two strategies (Long/Short and Long Only respectively). Both strategies’ weekly returns are more or less normally distributed with the mass of the distribution is to the left and the upper tail is slightly fatter than the lower (positive skewness). On the other hand, the benchmark

show returns which are positive and negative almost equally distributed. By looking at the top graph in Figure 3.9, we clearly see that the portfolio of Long/Short and Long Only strategies have values which are constantly higher than the benchmark. The bottom graph at the same Figure 3.9 shows that the drawdowns (the changes in peak to trough of the investor's wealth) for the Long/Short strategy are within acceptable range (the highest drawdowns was during November 2008 which reached around 28%).

We analyze the performance of our base model's trading strategies (Long/Short and Long Only) and compare them with our benchmark model (CART which implements the technical, Fama French and classical sentiment indicators, while excluding all the 23 TRMIs). Table 3.6 below indicates the standard performance measures of our base model versus the benchmark model, taking into account the two simple trading strategies (Long Only and Long/Short). In addition, we also present the performance of the Buy and Hold S&P 500. We observe that our base model's Long Only strategy generally outperforms the benchmark model and the Buy and Hold S&P 500. The yearly (5 year annualized) returns are around 2% (13%) higher. The yearly and 5 year annualized volatility are almost the same (around 3% higher). At the end, the Sharpe Ratio for both yearly and 5 year annualized performs better than the benchmark. Our base model's Long/Short strategy also outperforms one of the benchmarks (CART without TRMI) under the same strategy, but it is not better than the Buy and Hold S&P 500 or the previous Long Only strategies.

We evaluate whether our trading strategies' good performance is mainly contributed by greater exposure to common stock risk factors, or is truly a new source of alpha. In order to achieve this purpose, we adjust the performance of these strategies for different risk factors: market in excess of R_f ($MKT-R_f$), value (HML), size (SMB) and momentum (MOM). Table 3.7 shows the results of our base model's Long Only and Long/Short strategy risk factor regressions (using the CAPM, Fama-French three factors and Fama-French-Carhart four factor models). We resort to the data provided by Kenneth French in his website.⁹ The CAPM model means that we regress the weekly returns obtained from our base model's against the market returns, both in excess of the risk free returns (R_f) over the same period of time (period of January 2007 until July 2013 weekly). The Fama-French three factors model perform similar operations by including additional factors: SMB and HML. The last model, Fama-French-Carhart four factors model then add one more factor which is the MOM.

We first analyze our base model's Long Only strategy. Its weekly returns is significantly positive with mean at around 34 bps and t-statistics of 1.887. Based on the CAPM model, our base model's Long Only strategy produces additional excess returns of around 21 bps on a weekly basis or around 11.5% per year on top of the returns expected from a portfolio with $\beta = 1$. The t-statistics of the alpha obtained is significant at 1-5% significance level. The adjusted $R^2 = 0.858$ means that 85.8% of the variance of the returns are explained by our model. By adding more factors as in Fama-French or Fama-French-Carhart, the additional alpha generated weekly is stable at 22 bps and its t-statistics is significant at the same level. The adjusted R^2 is around 0.866 for both models. Our base model's Long Only strategy definitely produces new source of alpha as compared to the same strategy from our

⁹See <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>

THE WEEKLY RETURNS OF THE REBALANCED PORTFOLIO OF LONG/SHORT STRATEGY AS
COMPARED TO THE BUY HOLD S&P 500 WITH WEEKLY REBALANCING

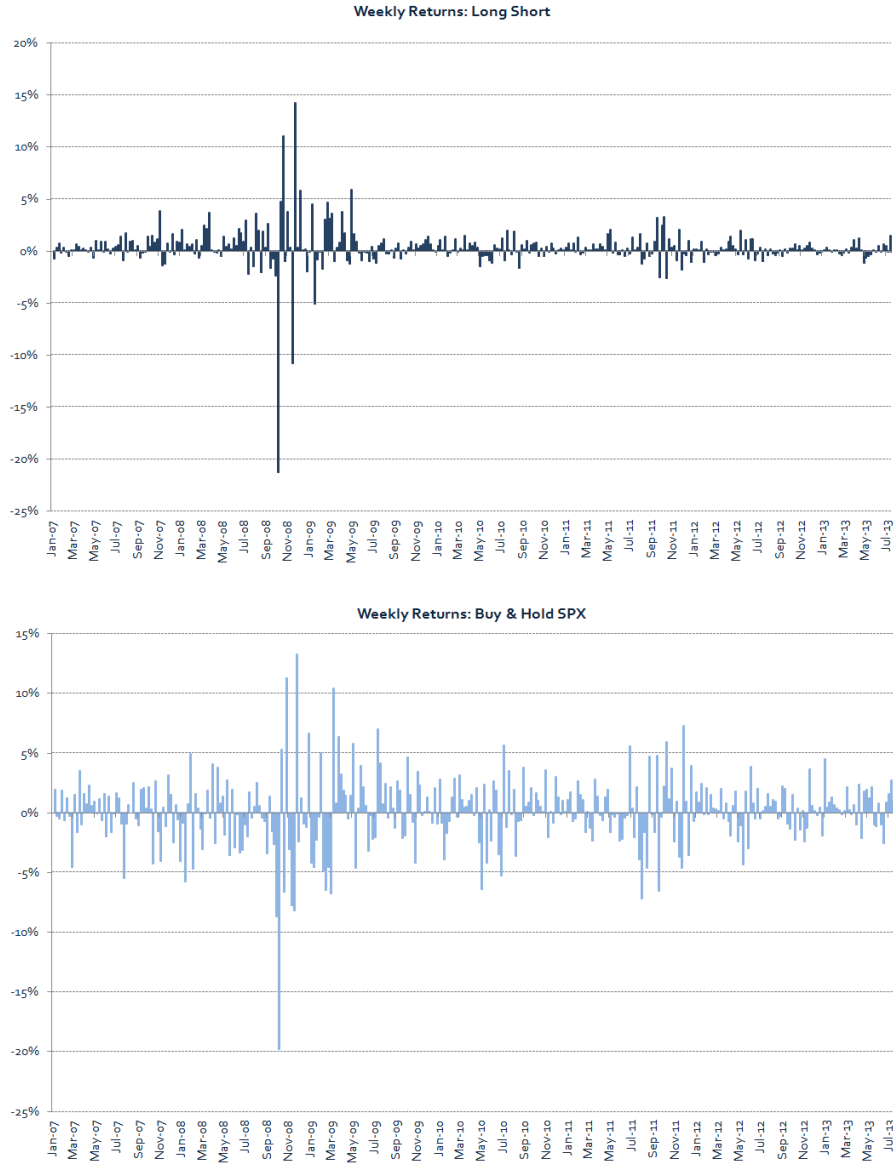


Figure 3.5: This figure describes the weekly returns of the rebalanced portfolio of Long/Short strategy as compared to the Buy Hold S&P 500 with weekly rebalancing. We observe that the Long/Short strategy provides returns which are more stable over time as compared to the benchmark (Buy and Hold S&P 500). The negative returns are generally less negative than the benchmark.

benchmark model (CART without TRMI). None of the risk factor regression results of our benchmark model's Long Only strategy show any significant alpha values.

Our base model's Long/Short strategy also generates significant alpha values (around 30-31 bps weekly) which outperform the benchmark model. The adjusted R^2 is very low which indicates that the variance of the returns are not explained well by this model. This is not surprising because unlike

THE WEEKLY RETURNS OF THE REBALANCED PORTFOLIO OF LONG ONLY STRATEGY AS COMPARED TO THE BUY HOLD S&P 500 WITH WEEKLY REBALANCING

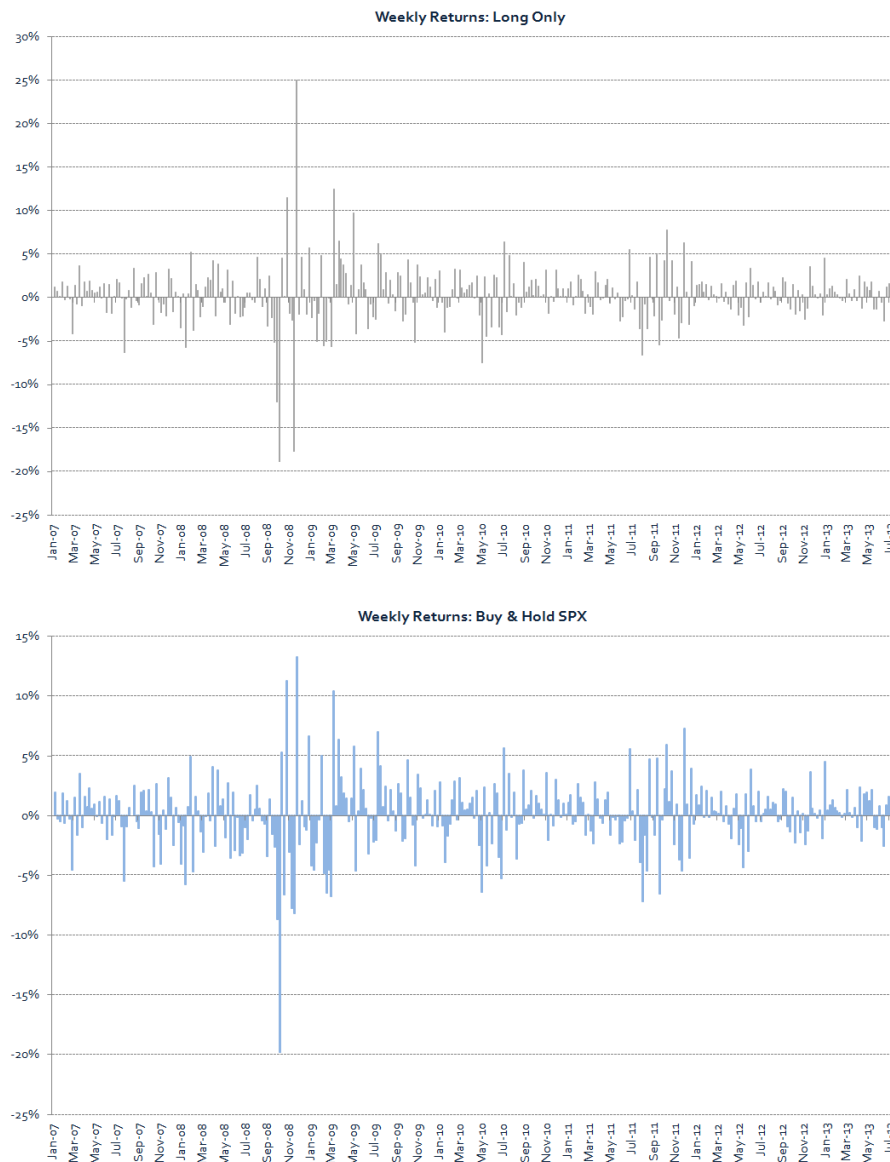


Figure 3.6: This figure describes the weekly returns of the rebalanced portfolio of Long Only strategy as compared to the Buy Hold S&P 500 with weekly rebalancing. Similar to the Long/Short strategy, we also observe that the Long Only strategy provides returns which are more stable over time as compared to the benchmark (Buy and Hold S&P 500). The negative returns are generally less negative than the benchmark.

the Long Only strategy, we usually do not foresee high correlations between the Long/Short strategy and the market as a whole (represented by the four risk factors).

Departing from these analysis, our first conclusion is that, our new dataset TRMI also produces simple trading strategies which are comparable to the existing research which uses TRNA in terms

THE DISTRIBUTION OF THE WEEKLY RETURNS OF THE REBALANCED PORTFOLIO OF LONG/SHORT STRATEGY AS COMPARED TO THE BUY HOLD S&P 500 WITH WEEKLY REBALANCING

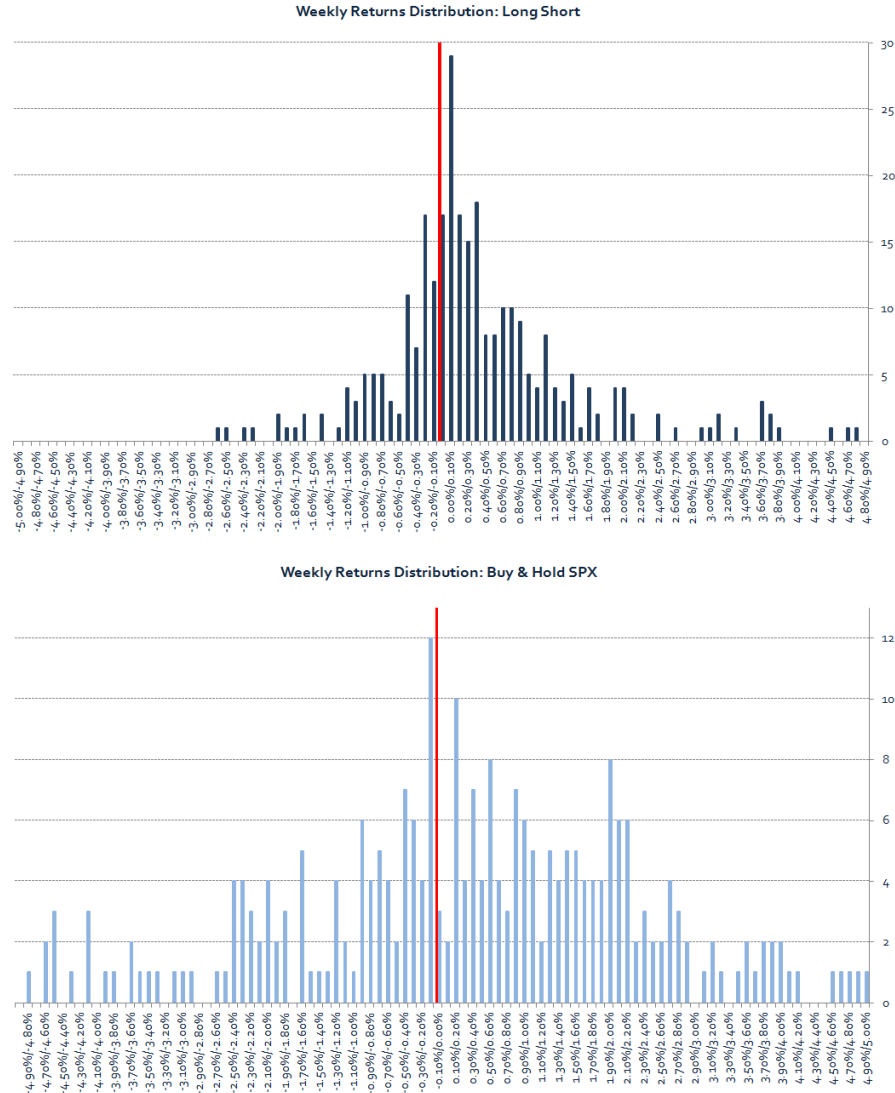


Figure 3.7: This figure describes the distribution of the weekly returns of the rebalanced portfolio of Long/Short strategy as compared to the Buy Hold S&P 500 with weekly rebalancing. The Long/Short strategy's weekly returns are more or less normally distributed with the mass of the distribution is to the left and the upper tail is slightly fatter than the lower (positive skewness). On the other hand, the benchmark show returns which are positive and negative almost equally distributed.

of generating good performance and alpha. For example, Leinweber and Sisk (2011) describes that their portfolio's annualized Sharpe ratio is 0.76 after transaction costs which is lower than our base model's strategies (Long Only: 2.07 ; Long/Short: 1.38). The portfolio's maximum drawdowns are around 60% in the period of February and July 2009. On the other hand, our base model's strategies maximum drawdowns are around 4-11% during the same period. Dzielinski (2011) implements equally

THE DISTRIBUTION OF THE WEEKLY RETURNS OF THE REBALANCED PORTFOLIO OF LONG ONLY
STRATEGY AS COMPARED TO THE BUY HOLD S&P 500 WITH WEEKLY REBALANCING

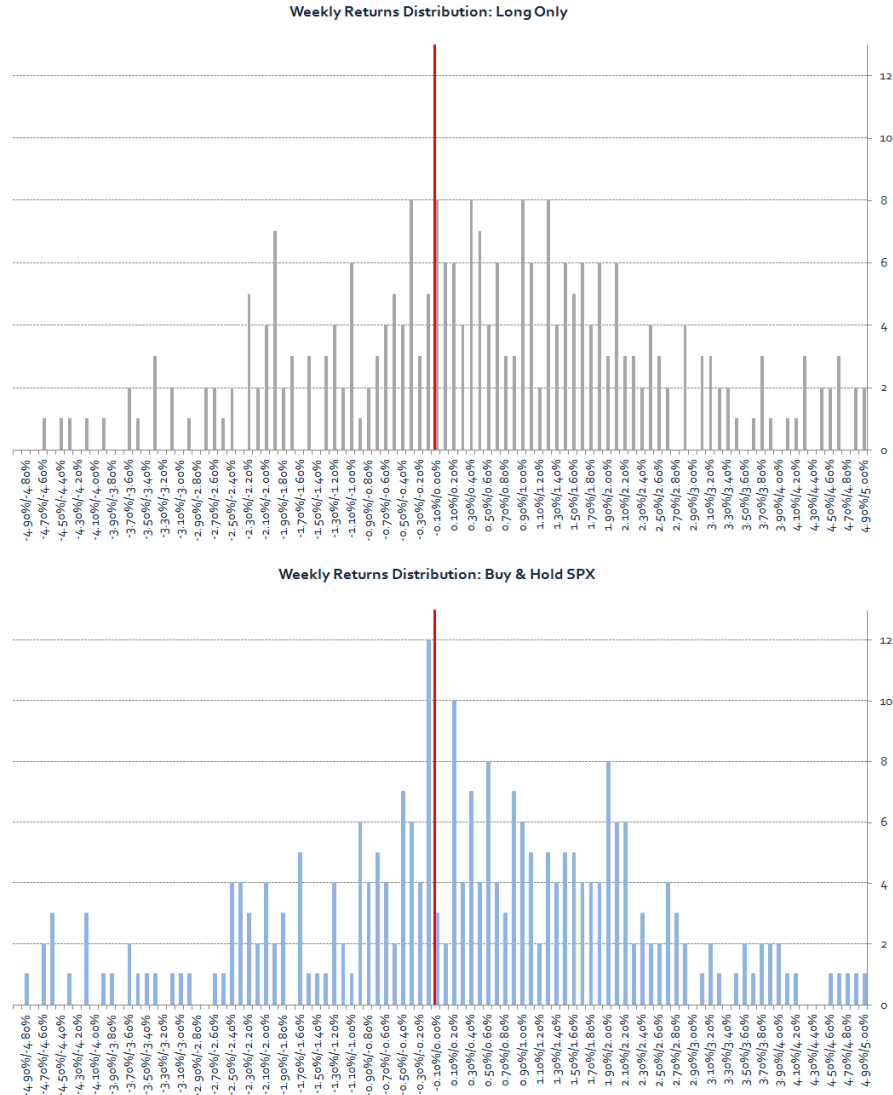


Figure 3.8: This figure describes the distribution of the weekly returns of the rebalanced portfolio of Long Only strategy as compared to the Buy Hold S&P 500 with weekly rebalancing. Similar to the Long/Short strategy, the Long Only strategy's weekly returns are more or less normally distributed with the mass of the distribution is to the left. The upper tail is slightly fatter than the lower (positive skewness). On the other hand, the benchmark show returns which are positive and negative almost equally distributed.

weighted portfolios by taking quintiles of the news sensitivity ranking from the TRNA. The portfolio is held for a month and then rebalanced. The monthly returns of this portfolio is significantly positive and quite persistent (mean: 0.95%, with t-statistics: 2.67). On a yearly basis, this alpha translates to be around 12%, similar to our base model's Long Only strategy.

THE INVESTOR'S WEALTH OF THE LONG/SHORT AND LONG ONLY STRATEGIES (COMPARED TO THE BUY HOLD S&P 500) AND THEIR DRAWDOWNS

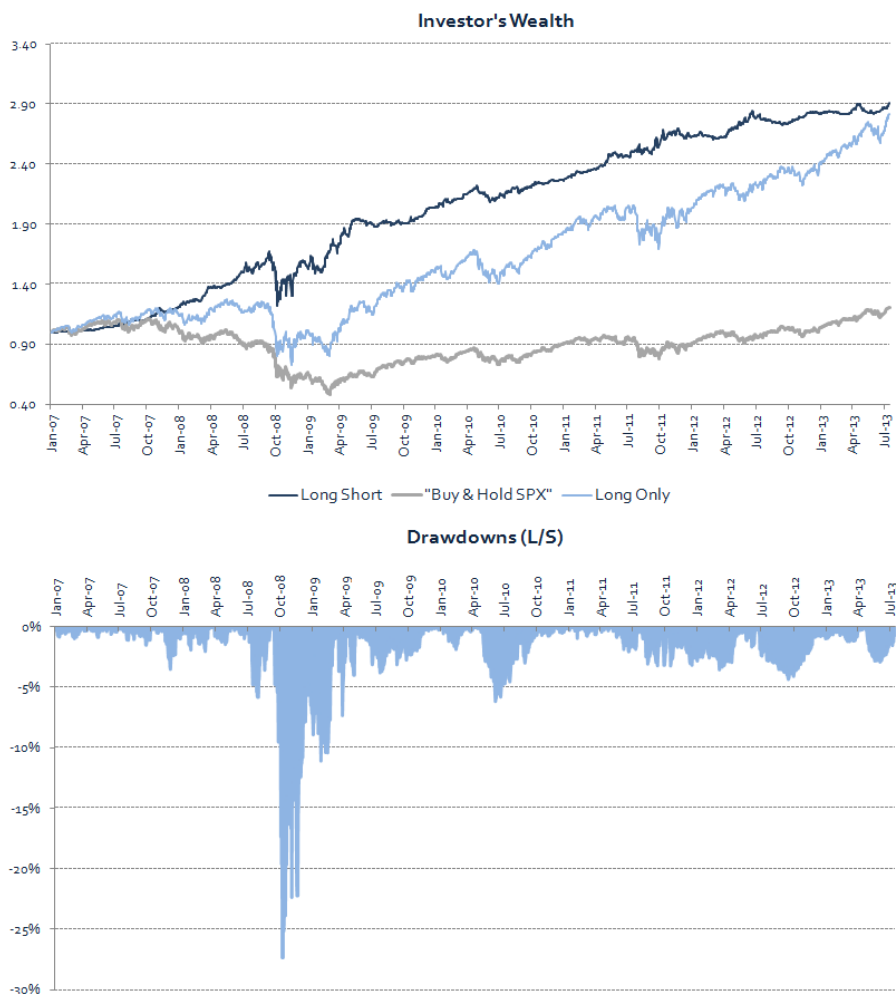


Figure 3.9: The top graph shows the investor's wealth of the Long/Short and Long Only strategies as compared to the Buy Hold S&P 500. The bottom one depicts the drawdowns for the Long/Short strategy. Both are obtained with weekly rebalancing. The top graph shows that the portfolio of Long/Short and Long Only strategies have values which are constantly higher than the benchmark. The bottom graph depicts the drawdowns (the changes in peak to trough of the investor's wealth) for the Long/Short strategy are within acceptable range (the highest drawdowns was during November 2008 which reached around 28%).

3.5.4 Robustness checks

Excess returns

For robustness checks, we analyze our results by changing the dependent variable as the rate of returns in excess to a benchmark (S&P 500) of each of the 10 economics sector-driven US equities, r ,

evaluated against a threshold, R . The threshold is a constant which is chosen to be 10 bps. We set our classification of the dependent variable to follow these rules

$$\begin{aligned}
 (3.15) \quad r > R &\Rightarrow \text{Outperforming,} \\
 |r| \leq R &\Rightarrow \text{Neutral,} \\
 r < -R &\Rightarrow \text{Underperforming.}
 \end{aligned}$$

Table 3.8 shows the total cumulative average hit rates for each of the 10 sectors as of July 2013 (weekly rebalanced and excess returns as the dependent variable). The results depict that the total cumulative average hit rates for each of the 10 sectors are on average to be 50%. The top 4 best performer sectors, where their total cumulative average hit rates are relatively higher than other sectors, are Energy (56%), Healthcare (53%), Financials (52%) and Technology (50%). The bottom 4 worst performer sectors, where their total cumulative average hit rates are relatively lower than other sectors, are Industrials (46%), Non-Cyclical Consumer Goods and Services (48%), Utilities (48%) and Cyclical Consumer Goods and Services (49%).

Performance-wise, the top graph in Figure 3.10 describes that both the portfolio of Long/Short and Long Only have values which are constantly higher than the benchmark. The bottom graph at the same Figure 3.10 shows that the drawdowns are even lower than in our base model (the highest drawdowns was during November 2008 which reached around 12%). We analyze the performance of the trading strategies as a result of CART with TRMI (weekly rebalanced and excess returns as the dependent variable) and comparing it with the base model (weekly rebalanced and absolute returns as the dependent variable). Table 3.9 below indicates the standard performance measures of these two models. We observe that our base model's Long Only strategy generally outperforms the alternative model (weekly rebalanced and excess returns as the dependent variable). The yearly (5 year annualized) returns are around 4% (7%) higher. The yearly and 5 year annualized volatility are almost the same (around 2% higher). The Sharpe Ratio for both yearly and 5 year annualized are better than the alternative model. Our base model's Long/Short strategy also outperforms the alternative one during the period of year. The 5 year annualized volatility of our base model is double than the alternative model for this strategy, hence, the annualized 5 year Sharpe ratio is much lower.

Table 3.10 compares the results of our Long/Short strategy risk factor regressions using the CAPM, Fama-French three factors and Fama-French-Carhart four factor models versus those from the CART with TRMI (weekly rebalanced and excess returns as the dependent variable). We first analyze our alternative model's Long Only strategy. Its weekly returns are not significantly positive with mean at around 22 bps and t-statistics of 1.299. Based on the CAPM model, this alternative model's Long Only strategy produces additional excess returns of around 9 bps on a weekly basis. However, the t-statistics of the alpha obtained is not significant. The adjusted $R^2 = 0.932$ means that 93.2% of the variance of the returns are explained by our model. By adding more factors as in Fama-French or Fama-French-Carhart, the additional alpha generated weekly are stable at 8 bps and its t-statistics is only significant at 5-10% level. The adjusted R^2 is around 0.932 for both models. Our alternative model's Long Only strategy produces slightly significant source of alpha which is still beaten by our

THE INVESTOR'S WEALTH OF THE LONG/SHORT AND LONG ONLY STRATEGIES (COMPARED TO THE BUY HOLD S&P 500) AND THEIR DRAWDOWNS: WEEKLY REBALANCED AND EXCESS RETURNS AS THE DEPENDENT VARIABLE

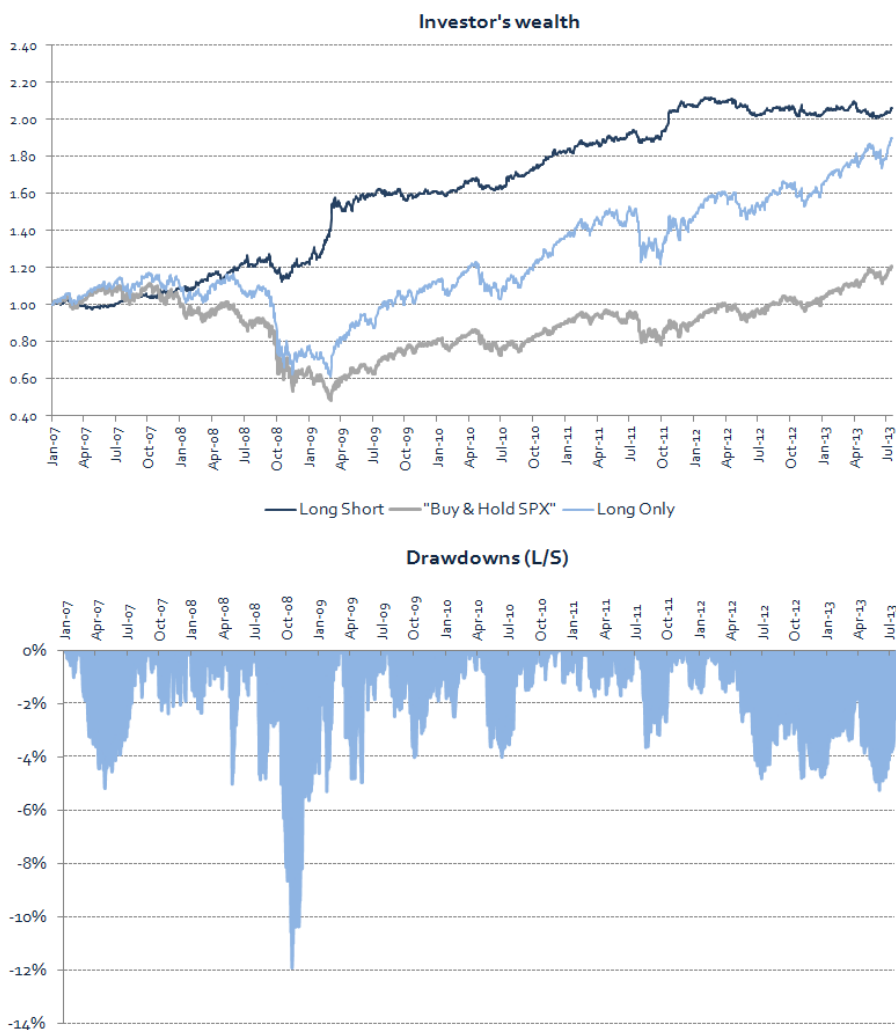


Figure 3.10: The top graph shows the investor's wealth of the Long/Short and Long Only strategies as compared to the Buy Hold S&P 500. The bottom one depicts the drawdowns for the Long/Short strategy. Both are obtained with weekly rebalancing and excess returns as the dependent variable. The top graph describes that both the portfolio of Long/Short and Long Only have values which are constantly higher than the benchmark. The bottom graph at the same Figure 3.10 show that the drawdowns are even lower than in our base model (the highest drawdowns was during November 2008 which reached around 12%).

base model with the same strategy.

On the other hand, our alternative model's Long/Short strategy performs better than the Long Only strategy. It generates significant alpha values (around 20-31 bps weekly) which is comparable

to the performance of our base model. The CAPM model for this strategy also shows that the β is close to zero, which indicates that systematic risk is fully diversified away. The adjusted R^2 is very low similar to our base model, for the same reason that we usually do not foresee high correlations between the Long/Short strategy and the market as a whole (represented by the four risk factors).

Classification into 2 categories: underperforming and outperforming

We investigate the robustness of our results by keeping the dependent variable as the absolute returns of each of the 10 economics sector-driven US equities, r and classified them into two categories as follows

$$(3.16) \quad \begin{aligned} r > 0 & \Rightarrow \text{Outperforming,} \\ r < 0 & \Rightarrow \text{Underperforming.} \end{aligned}$$

Table 3.11 shows the total cumulative average hit rates for each of the 10 sectors as of July 2013 (weekly rebalanced, absolute returns as the dependent variable, with only two categories). The results depict that the total cumulative average hit rates for each of the 10 sectors are on average to be 60%. The top 6 best performer sectors, where their total cumulative average hit rates are relatively higher than other sectors, are Energy (65%), Healthcare (64%), Technology (61%), Financials (60%), Telecommunications (60%) and Utilities (60%). The bottom 4 worst performer sectors, where their total cumulative average hit rates are relatively lower than other sectors, are Non-Cyclical Consumer Goods and Services (54%), Basic Materials (57%), Cyclical Consumer Goods and Services (58%) and Industrials (59%).

Performance-wise, the top graph in Figure 3.11 describes that both the portfolio of Long/Short and Long Only have values which are constantly higher than the benchmark. The bottom graph at the same Figure 3.11 shows that the drawdowns are much lower than in our base model (the highest drawdowns was during April 2009 which reached around 16%). We analyze the performance of the trading strategies as a result of CART with TRMI (weekly rebalanced, absolute returns as the dependent variable with only two categories) and compare it with the base model (weekly rebalanced, absolute returns as the dependent variable with three categories). Table 3.12 below indicates the standard performance measures of these two models. We observe that our base model's Long Only strategy performs equally well as the alternative model (weekly rebalanced, absolute returns as the dependent variable with three categories). The yearly and 5 year annualized returns are almost the same (1% higher). The yearly and 5 year annualized volatility are also similar (around 11% and 25% respectively). The Sharpe Ratio for both yearly and 5 year annualized are identical. Our base model's Long/Short strategy is also generally comparable to the alternative one in terms of these standard performance measures.

Table 3.13 compares the results of our Long/Short strategy risk factor regressions using the CAPM, Fama-French three factors and Fama-French-Carhart four factor models versus those from the CART with TRMI (weekly rebalanced, absolute returns as the dependent variable with only two categories). We first analyze our alternative model's Long Only strategy. Its weekly returns are significantly

THE INVESTOR'S WEALTH OF THE LONG/SHORT AND LONG ONLY STRATEGIES (COMPARED TO THE BUY HOLD S&P 500) AND THEIR DRAWDOWNS: WEEKLY REBALANCED, ABSOLUTE RETURNS AS THE DEPENDENT VARIABLE WITH ONLY TWO CATEGORIES

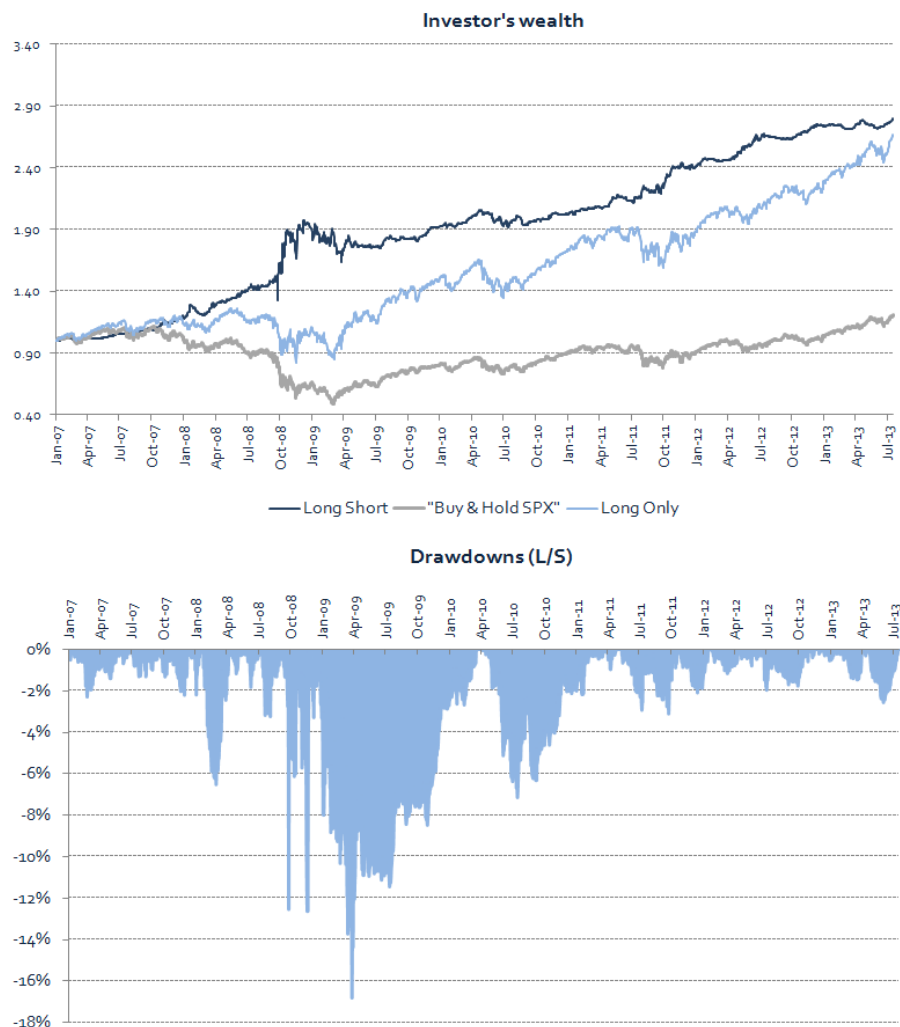


Figure 3.11: The top graph shows the investor's wealth of the Long/Short and Long Only strategies as compared to the Buy Hold S&P 500. The bottom one depicts the drawdowns for the Long/Short strategy. Both are obtained with weekly rebalancing and absolute returns as the dependent variable with only two categories. The top graph describes that both the portfolio of Long/Short and Long Only have values which are constantly higher than the benchmark. The bottom graph shows that the drawdowns are much lower than in our base model (the highest drawdowns was during April 2009 which reached around 16%).

positive with mean of around 32 bps and a t-statistics of 1.836. Based on the CAPM model, this Long Only strategy produces additional excess returns of around 19 bps on a weekly basis which is very close to the base model. The t-statistics of the alpha obtained is significant at 1-5% significance

level. The adjusted $R^2 = 0.854$. By adding more factors as in Fama-French or Fama-French-Carhart, the additional alpha generated weekly is stable at 20 bps and its t-statistics is significant at the same level. The adjusted R^2 is around 0.861 for both models. Our alternative model's Long Only strategy definitely produces new source of alpha which is comparable to the same strategy from our base model. Similarly, our alternative model's Long/Short strategy also generates significant alpha values (around 30-31 bps weekly). The adjusted R^2 is even lower than in the base model.

Monthly rebalancing

We further analyze the robustness of our results by changing the rebalancing frequency to monthly. Table 3.14 shows the total cumulative average hit rates for each of the 10 sectors as of July 2013 (monthly rebalanced). The results in this table depict that the total cumulative average hit rates for the 10 sectors on average to be 52%. The top 4 best performer sectors, where their total cumulative average hit rates are relatively higher than other sectors, are Technology (61.5%), Energy (61.5%), Industrials (60.3%) and Healthcare (53.8%). The bottom 4 worst performer sectors, where their total cumulative average hit rates are relatively lower than other sectors, are Non-Cyclical Consumer Goods and Services (42.3%), Financials (43.6%), Basic Materials (46.2%) and Cyclical Consumer Goods and Services (46.2%). This is an indication that the monthly rebalanced results provide relatively lower hit rate than the weekly ones.

Performance-wise, the top graph in Figure 3.12 describes that the portfolio of Long/Short has values which are constant over time and mostly higher than the benchmark. The Long Only strategies' investor's wealth is closely trailing and slightly higher than the benchmark. The bottom graph at the same Figure 3.12 show that the drawdowns are much higher than in the weekly rebalancing (around 12% without being able to recuperate after January 2009). We analyze the performance of the trading strategies as a result of CART with TRMI (monthly rebalanced) and comparing it with the base model (CART with TRMI weekly rebalanced). Table 3.15 below indicates the standard performance measures of these two models. We observe that our base model's Long Only strategy generally outperforms the alternative model (monthly rebalanced CART with TRMI). The yearly (5 year annualized) returns are around 4% (13%) higher. The yearly and 5 year annualized volatility are almost the same (around 4% higher). The Sharpe Ratio for both yearly and 5 year annualized are similar or better than the alternative model. Our base model's Long/Short strategy also outperforms the alternative one.

Table 3.16 compares the results of our Long/Short strategy risk factor regressions using the CAPM, Fama-French three factors and Fama-French-Carhart four factor models versus those from the alternative model: CART with TRMI (monthly rebalanced). We clearly see that both the Long Only and Long/Short strategies of our alternative model are beaten by our base model. None of the risk factor regression results of this model for both strategies show any significant alpha values. This is a first indication that the predictive power of the news sentiments generated by the TRMI is relatively short-lived. One possible explanation is that most investors usually have short memory when it comes to information or news. Moreover, in a longer time horizon, all the news which are gathered become common knowledge to the market and hence, it may just be a representation of the other risk factors

THE INVESTOR'S WEALTH OF THE LONG/SHORT AND LONG ONLY STRATEGIES (COMPARED TO THE BUY HOLD S&P 500) AND THEIR DRAWDOWNS: MONTHLY REBALANCED

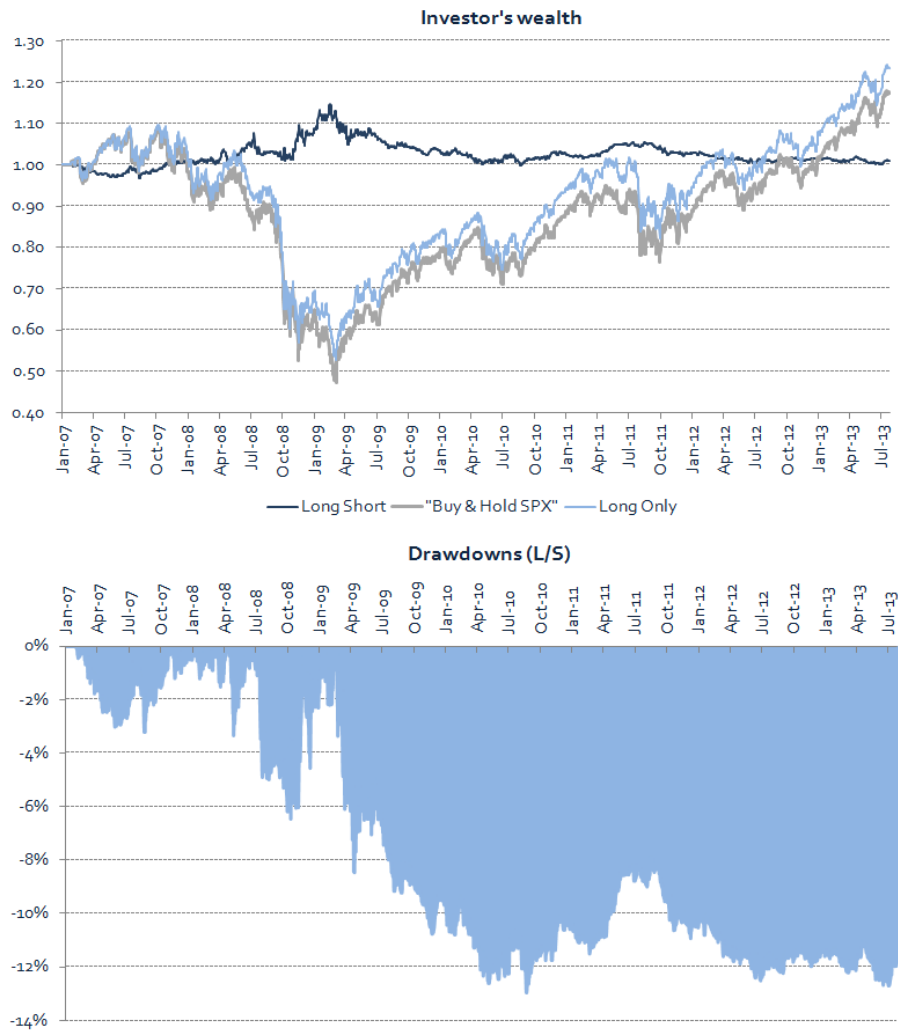


Figure 3.12: The top graph shows the investor's wealth of the Long/Short and Long Only strategies as compared to the Buy Hold S&P 500. The bottom one depicts the drawdowns for the Long/Short strategy. Both are obtained with monthly rebalancing. The top graph describes that the portfolio of Long/Short is not increasing much over time and is underperforming the benchmark on the full period. The Long Only strategies' investor's wealth are closely trailing and slightly higher than the benchmark. The bottom graph at the same Figure 3.12 show that the drawdowns are much higher than in the weekly rebalancing (around 12% without being able to recuperate after January 2009)

(MKT- R_f , SMB, HML and MOM). Similar to the base case, the adjusted R^2 for the Long Only strategy is generally very high (around 97%), while those of the Long/Short strategy are fairly low (around 14-19%).

3.6 Conclusions

This paper aims at using the state-of-the art news-sentiment-driven TRMI and implement the supervised learning methodology CART to identify a new source of alpha in the complexity of the financial market. Our results show that we reach this objective. The TRMI indeed produces promising alpha which provides a new source of ideas for quantitative investors. The first indication of good performance of the 10 sectors CART classifiers are described by the cumulative average hit rates which are on average around 58% over time. The cumulative average hit rates of our model by far outperforms the two benchmark models: the model without TRMI (around 50%) and the naive trader model (around 42%). Based on the total cumulative average hit rates as of July 2013, our model's top 4 best performer sectors are Technology (64%), Healthcare (63%), Energy (61%) and Financials (60%). The bottom 4 worst performers are Basic Materials (53%), Non-Cyclical Consumer Goods and Services (54%), Utilities (55%) and Industrials (56%). These results suggest that each different sector has different degree of sensitivity towards news.

Most of our technical indicators, Fama French factors and classical sentiment indicators are not playing an important role in our CART classifiers. These variables appear occasionally in the bottom 5 of the rank of the variable importance. Conversely, our TRMI variables such as Price Increase, Market Risk, Sentiment, Gloom, Market Forecast, Optimism and Fear consistently dominate the weekly average top 10 most important variable for all the 10 sectors.

The two trading strategies (Long/Short and Long Only strategies), which are formed from these CART classifiers, perform well as compared to their benchmark (Buy and Hold S&P 500). The portfolio of Long/Short and Long Only strategies produce investor wealth which is constantly higher than the benchmark. The drawdowns for the Long/Short strategy are within acceptable range (the highest drawdowns was during November 2008 which reached around 28%). We analyze the performance of our base model's trading strategies (Long/Short and Long Only) and compare them with our benchmark model (CART which implements the technical, Fama French and classical sentiment indicators, while excluding all the 23 TRMIs). We observe that our base model's Long Only strategy generally outperforms the benchmark model and the Buy and Hold S&P 500. The yearly (5 year annualized) returns are around 2% (13%) higher. The yearly and 5 year annualized volatility are almost the same (around 3% higher). At the end, the Sharpe Ratio for both yearly and 5 year annualized performs better than the benchmark. Our base model's Long/Short strategy also outperforms one of the benchmarks (CART without TRMI) under the same strategy but it is not better than the Buy and Hold S&P 500 or the previous Long Only strategies.

Our trading strategies' good performance is a new source of alpha which is not explainable by exposure to common stock risk factors. We adjust the performance of these strategies for different risk factors: market in excess of R_f ($MKT-R_f$), value (HML), size (SMB) and momentum (MOM). Our base model's Long Only strategy's weekly returns is significantly positive with mean at around 34 bps and t-statistics of 1.887. Based on the CAPM model, our base model's Long Only strategy produces additional excess returns of around 21 bps on a weekly basis or around 11.5% per year on

top of the returns expected from a portfolio with $\beta = 1$. The t-statistics of the alpha obtained is significant at 1-5% significance level. The adjusted $R^2 = 0.858$ means that 85.8% of the variance of the returns are explained by our model. By adding more factors as in Fama-French or Fama-French-Carhart, the additional alpha generated weekly is stable at 22 bps and its t-statistics is significant at the same level. The adjusted R^2 is around 0.866 for both models. Our base model's Long Only strategy definitely produces new source of alpha as compared to the same strategy from our benchmark model (CART without TRMI). None of the risk factor regression results of our benchmark model's Long Only strategy show any significant alpha values. Our base model's Long/Short strategy also generates significant alpha values (around 30-31 bps weekly) which outperform the benchmark model. The adjusted R^2 is very low which indicates that the variance of the returns are not explained well by this model.

Our base model is robust even if we differentiate some of its characteristics such as, taking excess returns as the dependent variable or defining only two categories (outperforming or underperforming). Our alternative model (excess returns as the dependent variable) produces a Long/Short strategy which performs better than the Long Only strategy and generates significant alpha values (around 20-31 bps weekly) comparable to the performance of our base model. The CAPM model for this strategy also shows that the β is close to zero, which indicates that systematic risk is fully diversified away. Our alternative model (classification with only two categories) results in Long Only and Long/Short strategy which generate significant alpha values (around 30-31 bps weekly).

The caveat of implementing the TRMI as a trading strategy is that the predictive power of these news sentiments is relatively short-lived. Evidently, if we analyze the monthly rebalanced results, the performance is not as good as in the weekly strategies. This is not surprising as behavioural research show that most investors usually have short memory when it comes to information or news. Hence, the information from this news sentiment data decays relatively quickly. Moreover, in a longer time horizon, all the news which are gathered become common knowledge to the market and hence, it may just be a representation of the other risk factors (MKT- R_f , SMB, HML and MOM). Nevertheless, this paper serves as an option for lower-frequency quantitative investors in enhancing their performance.

THE 23 TRMI INDICES FOR THE EQUITY ASSET CLASS CARRY SIX SIGNIFICANT DIGITS PAST THE DECIMAL POINT

Index	Description	Range
Sentiment	24 hour rolling average score of references in news and social media to...	
Optimism	overall positive references, net of negative references	-1 to 1
Gloom	optimism, net of references to pessimism	-1 to 1
Fear	gloom and negative future outlook	0 to 1*
Joy	fear and anxiety	0 to 1*
Anger	happiness and affection	0 to 1*
Innovation	anger and disgust	0 to 1*
Trust	innovativeness	0 to 1*
Violence	trustworthiness, net of references connoting corruption	-1 to 1
Conflict	violence and war	0 to 1*
Stress	disagreement and swearing net of agreement and conciliation	-1 to 1
Urgency	distress and danger	0 to 1*
FundamentalStrength	urgency and timeliness, net of references to tardiness and delays	-1 to 1
MarketRisk	positivity about accounting fundamentals, net of references to negativity about accounting fundamentals positive emotionality and positive expectations net of negative emotionality and negative expectations. Includes factors from social media found characteristic of speculative bubbles higher values indicate greater bubble risk. Also known as the Bubbleometer.	-1 to 1
PriceForecast	predictions of asset price rises, net of references to predictions of asset price drops	-1 to 1
EarningsForecast	expectations about improving earnings, less those of worsening earnings	-1 to 1
Mergers	merger or acquisition activity	0 to 1*
Layoffs	staff reductions and layoffs	0 to 1*
Litigation	litigation and legal activity	0 to 1*
AnalystRating	upgrade activity, net of references to downgrade activity	-1 to 1
Uncertainty	uncertainty and confusion	0 to 1*
Price	price increases, net of references to price decreases	-1 to 1
Volatility	volatility in market prices or business conditions net of stability	-1 to 1

Source: MarketPsych LLC.

Table 3.3: This table describes the 23 TRMI indices for the equity asset class which carry six significant digits past the decimal point. Negative numbers have a leading minus (-) sign.

THE CP TABLE FOR THE FINANCIAL SECTOR FOR THE WEEK OF 5-JUL-2013

CP	nsplit	rel error	xerror	xstd
0.262	0	1.000	1.000	0.054
0.047	1	0.738	0.814	0.053
0.044	3	0.645	0.773	0.053
0.035	6	0.512	0.733	0.052
0.017	8	0.442	0.669	0.051
0.015	14	0.337	0.680	0.051
0.012	19	0.262	0.680	0.051
0.009	27	0.169	0.715	0.052
0.006	29	0.151	0.721	0.052
0.003	53	0.012	0.744	0.052
Inf	57	0.000	0.744	0.052

Table 3.4: This table describes the non-pruned dynamic tree of the Financial sector for the week of 5 July 2013. The nsplit (second column) shows the number of splits from the smallest tree with zero split to the largest one (57 splits) and the number of nodes is just equal to $1 + \text{nsplit}$. The rel error (third column) is calculated based on the ratio of the residual sum of squares for the tree with k terminal nodes over the residual sum of squares for the tree with 1 terminal node. Each of these trees is then examined using 10-fold cross-validation where the data are divided into 10 equal segments. The tree is built using 9 of the 10 segments and error is assessed on the tenth segment. This is repeated leaving off each segment in turn and errors are then averaged and scaled to give xerror (fourth column). The xstd (last column) is the variation between the 10 sub-sample estimates. We observe from this table that xerror are minimized at 0.669 where the number of split is 8 and the cp value is 0.017. Hence the size of the tree (number of terminal node) is equal to 9 ($1 + 8$).

THE TOTAL CUMULATIVE AVERAGE HIT RATES FOR ALL 10 SECTORS AS OF JULY 2013 (WEEKLY REBALANCED)

Sector	Base model with TRMI	Benchmark model without TRMI	Benchmark model Naive traders
Energy	61%	51%	44%
Basic Materials	53%	53%	37%
Industrials	56%	51%	42%
Cyclical Consumer Goods and Services	57%	44%	40%
Non-Cyclical Consumer Goods and Services	54%	51%	47%
Financials	60%	47%	39%
Healthcare	63%	52%	46%
Technology	64%	51%	42%
Telecommunication Services	57%	49%	40%
Utilities	55%	53%	42%

Table 3.5: This table describes the total cumulative average hit rates for all 10 sectors as of July 2013. The training data is from 1st June 2000 - 31st December 2006 (weekly rolling till July 2013). The results depict that the total cumulative average hit rates for each of the 10 sectors are always higher for the base model with TRMI, as compared to the two benchmark models. Focusing on the base model with TRMI, the top 4 best performer sectors, where their total cumulative average hit rates are relatively higher than other sectors, are Technology (64%), Healthcare (63%), Energy (61%) and Financials (60%). The bottom 4 worst performer sectors, where their total cumulative average hit rates are relatively lower than other sectors, are Basic Materials (53%), Non-Cyclical Consumer Goods and Services (54%), Utilities (55%) and Industrials (56%). These results show that each different sector has different degree of sensitivity towards news.

THE PERFORMANCE MEASURES OF THE TRADING STRATEGIES (LONG ONLY AND LONG/SHORT): BASE MODEL VS BENCHMARK MODEL

Strategies	Model	Returns (1 year)	Returns (5 year ann.)	Volatility (1 year)	Volatility (5 year ann.)	Sharpe Ratio (1 year)	Sharpe Ratio (5 year ann.)	Number of trades
Long Only	Base model (with TRMI)	26.43%	19.08%	10.63%	25.97%	2.07	1.70	1670
	Benchmark model (without TRMI)	24.20%	5.97%	9.86%	21.29%	2.09	0.64	2061
Long/Short	Base model (with TRMI)	5.02%	14.03%	3.40%	16.09%	1.38	1.53	3177
	Benchmark model (without TRMI)	-3.31%	-4.73%	2.24%	5.50%	1.68	-2.32	2808
Buy and Hold S&P 500		24.20%	6.11%	10.64%	22.91%	1.87	0.82	1

Table 3.6: This table describes the standard performance measures of our base model versus the benchmark model, taking into account the two simple trading strategies (Long Only and Long/Short). The last row shows the performance of the Buy and Hold S&P 500. We observe that our base model's Long Only strategy generally outperforms the benchmark model and the Buy and Hold S&P 500. The yearly (5 year annualized) returns are around 2% (13%) higher. The yearly (5 year annualized) volatility are almost the same (around 3% higher). At the end, the Sharpe Ratio for both yearly and 5 year annualized performs better than the benchmark. Our base model's Long/Short strategy also outperforms one of the benchmarks (CART without TRMI) under the same strategy, but it is not better than the Buy and Hold S&P 500 or the previous Long Only strategies.

THE RISK FACTOR REGRESSIONS (LONG ONLY AND LONG/SHORT STRATEGY): BASE MODEL VS BENCHMARK MODEL

Long Only	Time series mean	Time series mean	CAPM	CAPM	Fama-French three factors	Fama-French three factors	Fama-French three factors	Fama-French-Carhart four factors	Fama-French-Carhart four factors
	with TRMI	CART without TRMI	CART with TRMI	CART without TRMI	CART with TRMI	CART without TRMI	CART with TRMI	CART without TRMI	CART without TRMI
Alpha	0.0034 ^o (1.887)	0.0006 (0.361)	0.0021* (3.021)	-0.0007 (-2.572)	0.0022* (3.279)	-0.0007 (-2.490)	0.0022* (3.296)	-0.0007 (-2.486)	-0.0007 (-2.486)
MKT- R_f	-	-	1.025** (45.41)	1.004* (107.80)	0.996** (35.57)	1.009** (93.78)	0.985** (36.06)	1.009** (90.64)	1.009** (90.64)
SMB	-	-	-	-	-0.152* (-2.639)	-0.125** (-5.348)	-0.146 (-2.534)	-0.125** (-5.326)	-0.125** (-5.326)
HML	-	-	-	-	0.177** (3.439)	0.031 (1.478)	0.134 (2.266)	0.031 (1.281)	0.031 (1.281)
MOM	-	-	-	-	-	-	-0.045 (-1.480)	-0.0001 (-0.006)	-0.0001 (-0.006)
No of obs	342	342	342	342	342	342	342	342	342
Adj. R^2	-	-	0.858	0.971	0.866	0.973	0.866	0.866	0.973
Long Short	Time series mean	Time series mean	CAPM	CAPM	Fama-French three factors	Fama-French three factors	Fama-French three factors	Fama-French-Carhart four factors	Fama-French-Carhart four factors
	with TRMI	CART without TRMI	CART with TRMI	CART without TRMI	CART with TRMI	CART without TRMI	CART with TRMI	CART with TRMI	CART without TRMI
Alpha	0.0032* (2.889)	-0.0008 (-2.137)	0.0029* (2.768)	-0.0009 (-2.372)	0.0031* (3.059)	-0.0009 (-2.312)	0.0031* (3.058)	-0.0009 (-2.311)	-0.0009 (-2.311)
MKT- R_f	-	-	0.174** (4.970)	0.054** (4.231)	0.158** (3.869)	0.040* (2.610)	0.154** (3.650)	0.041 (2.590)	0.041 (2.590)
SMB	-	-	-	-	-0.364** (-4.106)	-0.010 (-0.300)	-0.362** (-4.065)	-0.011 (-0.317)	-0.011 (-0.317)
HML	-	-	-	-	0.209* (2.634)	0.059 (1.985)	0.192 (2.111)	0.063 (1.855)	0.063 (1.855)
MOM	-	-	-	-	-	-	-0.017 (-0.360)	0.005 (0.265)	0.005 (0.265)
No of obs	342	342	342	342	342	342	342	342	342
Adj. R^2	-	-	0.065	0.047	0.129	0.053	0.126	0.051	0.051

Table 3.7: This table shows the results of our base model's Long Only and Long/Short strategy risk factor regressions (using the CAPM, Fama-French three factors and Fama-French-Carhart four factor models). The t-statistics are in parentheses. * * *, ** , * and o indicate statistical significance at the 0-0.1%, 0.1-1%, 1-5% and 5-10% level respectively. Our base model's Long Only strategy's weekly returns is significantly positive with mean at around 34 bps and t-statistics of 1.887. Based on the CAPM model, it produces additional excess returns of around 21 bps on a weekly basis with a significant t-statistics. The adjusted $R^2 = 0.858$ means that 85.8% of the variance of the returns are explained by our model. By adding more factors as in Fama-French or Fama-French-Carhart, the additional alpha generated weekly is stable at 22 bps and its t-statistics is significant at the same level. The adjusted R^2 is around 0.866 for both models. Our base model's Long Only strategy definitely produces new source of alpha as compared to the same strategy from our benchmark model (CART without TRMI). None of the risk factor regression results of our benchmark model's Long Only strategy shows any significant alpha values. The Long/Short strategy also generates significant alpha values (around 30-31 bps weekly) which outperforms the benchmark. The adjusted R^2 is very low which indicates that the variance of the returns are not explained well by this model.

THE TOTAL CUMULATIVE AVERAGE HIT RATES FOR ALL 10 SECTORS AS OF JULY 2013 (WEEKLY REBALANCED AND EXCESS RETURNS AS THE DEPENDENT VARIABLE)

Sector	Base model with TRMI (weekly rebalanced and excess returns as the dependent variable)
Energy	56%
Basic Materials	48%
Industrials	46%
Cyclical Consumer Goods and Services	49%
Non-Cyclical Consumer Goods and Services	48%
Financials	52%
Healthcare	53%
Technology	50%
Telecommunication Services	49%
Utilities	48%

Table 3.8: This table describes the the total cumulative average hit rates for all 10 sectors as of July 2013 (excess returns as the dependent variable). The training data is from 1st June 2000-31st December 2006 (weekly rolling till July 2013). The results depict that the total cumulative average hit rates for each of the 10 sectors are on average around 50%. The top 4 best performer sectors, where their total cumulative average hit rates are relatively higher than other sectors, are Energy (56%), Healthcare (53%), Financials (52%) and Technology (50%). The bottom 4 worst performer sectors, where their total cumulative average hit rates are relatively lower than other sectors, are Industrials (46%), Non-Cyclical Consumer Goods and Services (48%), Utilities (48%) and Cyclical Consumer Goods and Services (49%).

THE PERFORMANCE MEASURES OF THE TRADING STRATEGIES (LONG ONLY AND LONG/SHORT): BASE MODEL (ABSOLUTE RETURNS)
VS CART WITH TRMI (EXCESS RETURNS)

Strategies	Model	Returns (1 year)	Returns (5 year ann.)	Volatility (1 year)	Volatility (5 year ann.)	Sharpe Ratio (1 year)	Sharpe Ratio (5 year ann.)	Number of trades
Long Only	Base model	26.43%	19.08%	10.63%	25.97%	2.07	1.70	1670
	CART with TRMI (absolute returns)							
	CART with TRMI (excess returns)	22.92%	12.55%	10.93%	23.78%	1.69	1.37	1580
Long/Short	Base model	5.02%	14.03%	3.40%	16.09%	1.38	1.53	3177
	CART with TRMI (absolute returns)							
	CART with TRMI (excess returns)	1.22%	11.30%	4.43%	8.72%	0.05	2.74	2977

Table 3.9: This table describes the standard performance measures of our base model versus the CART with TRMI (excess returns), taking into account the two simple trading strategies (Long Only and Long/Short). We observe that our base model's Long Only strategy generally outperforms the alternative model (weekly rebalanced and excess returns as the dependent variable). The yearly (5 year annualized) returns are around 4% (7%) higher. The yearly and 5 year annualized volatility are almost the same (around 2% higher). The Sharpe Ratio for both yearly and 5 year annualized are better than the alternative model. Our base model's Long/Short strategy also outperforms the alternative one during the period of year. The 5 year annualized volatility of our base model is double than the alternative model for this strategy, hence, the annualized 5 year Sharpe ratio is much lower.

THE RISK FACTOR REGRESSIONS (LONG/SHORT AND LONG ONLY STRATEGY): BASE MODEL (ABSOLUTE RETURNS) VS CART WITH TRMI (EXCESS RETURNS)

Long Only		Time series mean	Time series mean	CAPM	CAPM	Fama-French three factors	Fama-French three factors	Fama-French-Carhart four factors	Fama-French-Carhart four factors
	Time series mean	CART with TRMI (absolute returns)	CART with TRMI (excess returns)	CART with TRMI (absolute returns)	CART with TRMI (excess returns)	CART with TRMI (absolute returns)	CART with TRMI (excess returns)	CART with TRMI (absolute returns)	CART with TRMI (excess returns)
Alpha	0.0034° (1.887)	0.0022 (1.299)	-	0.0021* (3.021)	0.0009 (1.996)	0.0022* (3.279)	0.0008° (1.939)	0.0022* (3.296)	0.0008° (1.931)
MKT- R_f	-	-	-	1.025** (45.41)	0.988** (68.55)	0.996** (35.57)	1.005** (57.99)	0.985** (36.06)	1.012** (56.60)
SMB	-	-	-	-	-	-0.152* (-2.639)	0.0039 (0.105)	-0.146 (-2.534)	0.000004 (0.001)
HML	-	-	-	-	-	0.177** (3.439)	-0.069 (-2.062)	0.134 (2.266)	-0.041 (-1.067)
MOM	-	-	-	-	-	-	-	-0.045 (-1.480)	-0.029 (1.479)
No of obs	342	342	342	342	342	342	342	342	342
Adj. R^2	-	-	-	0.858	0.932	0.866	0.932	0.866	0.933
Long Short		Time series mean	Time series mean	CAPM	CAPM	Fama-French three factors	Fama-French three factors	Fama-French-Carhart four factors	Fama-French-Carhart four factors
	Time series mean	CART with TRMI (absolute returns)	CART with TRMI (excess returns)	CART with TRMI (absolute returns)	CART with TRMI (excess returns)	CART with TRMI (absolute returns)	CART with TRMI (excess returns)	CART with TRMI (absolute returns)	CART with TRMI (excess returns)
Alpha	0.0032* (2.889)	0.0020* (3.169)	-	0.0029* (2.768)	0.0020* (3.163)	0.0031* (3.059)	0.0019* (3.076)	0.0031* (3.058)	0.0019* (3.075)
MKT- R_f	-	-	-	0.174** (4.970)	-0.0006 (-0.027)	0.158** (3.869)	0.019 (0.797)	0.154** (3.650)	0.033 (1.298)
SMB	-	-	-	-	-	-0.364** (-4.106)	0.190** (3.580)	-0.362** (-4.065)	0.182** (3.443)
HML	-	-	-	-	-	0.209* (2.634)	-0.155* (-3.265)	0.192 (2.111)	-0.099° (-1.827)
MOM	-	-	-	-	-	-	-	-0.017 (-0.360)	0.058 (2.081)
No of obs	342	342	342	342	342	342	342	342	342
Adj. R^2	-	-	-	0.065	-0.003	0.129	0.065	0.126	0.074

Table 3.10: This table compares the results of our Long/Short strategy risk factor regressions using the CAPM, Fama-French three factors and Fama-French-Carhart four factor models versus those from the CART with TRMI (weekly rebalanced and excess returns as the dependent variable). The t-statistics are in parentheses. * * *, ** , * and ° indicate statistical significance at the 0.0.1%, 0.1-1%, 1-5% and 5-10% level respectively. Our alternative model's Long Only strategy produces weekly returns which is not significant (mean: 22 bps and t-statistics: 1.299). Based on the CAPM model, it produces 9 bps weekly with t-statistics which is not significant. Based on Fama-French or Fama-French-Carhart, the additional alpha generated weekly is stable at 8 bps and weekly and its t-statistics is only significant at 5-10% level. Our alternative model's Long Only strategy produces slightly significant source of alpha which is still beaten by our base model with the same strategy. The Long/Short strategy performs better than the Long Only strategy. It generates significant alpha values (around 20-31 bps weekly) which is performance of our base model.

THE TOTAL CUMULATIVE AVERAGE HIT RATES FOR ALL 10 SECTORS AS OF JULY 2013 (WEEKLY REBALANCED, ABSOLUTE RETURNS AS THE DEPENDENT VARIABLE WITH ONLY TWO CATEGORIES)

Sector	Base model with TRMI (weekly rebalanced, absolute returns as the dependent variable with only two categories)
Energy	56%
Basic Materials	48%
Industrials	46%
Cyclical Consumer Goods and Services	49%
Non-Cyclical Consumer Goods and Services	48%
Financials	52%
Healthcare	53%
Technology	50%
Telecommunication Services	49%
Utilities	48%

Table 3.11: This table describes the the total cumulative average hit rates for all 10 sectors as of July 2013 (absolute returns as the dependent variable with only two categories). The training data is from 1st June 2000-31st December 2006 (weekly rolling till July 2013). The results depict that the total cumulative average hit rates for each of the 10 sectors are on average around 60%. The top 6 best performer sectors, where their total cumulative average hit rates are relatively higher than other sectors, are Energy (65%), Healthcare (64%), Technology (61%), Financials (60%), Telecommunications (60%) and Utilities (60%). The bottom 4 worst performer sectors, where their total cumulative average hit rates are relatively lower than other sectors, are Non-Cyclical Consumer Goods and Services (54%), Basic Materials (57%), Cyclical Consumer Goods and Services (58%) and Industrials (59%).

THE PERFORMANCE MEASURES OF THE TRADING STRATEGIES (LONG ONLY AND LONG/SHORT): BASE MODEL (THREE CATEGORIES)
VS CART WITH TRMI (TWO CATEGORIES)

Strategies	Model	Returns (1 year)	Returns (5 year ann.)	Volatility (1 year)	Volatility (5 year ann.)	Sharpe Ratio (1 year)	Sharpe Ratio (5 year ann.)	Number of trades
Long Only	Base model	26.43%	19.08%	10.63%	25.97%	2.07	1.70	1670
	CART with TRMI (3 classes)							
	CART with TRMI (2 classes)	26.60%	18.08%	10.79%	25.01%	2.09	1.69	1723
Long/Short	Base model	5.02%	14.03%	3.40%	16.09%	1.38	1.53	3177
	CART with TRMI (3 classes)							
	CART with TRMI (2 classes)	5.19%	14.25%	3.09%	14.04%	1.68	1.76	3198

Table 3.12: This table describes the standard performance measures of our base model versus the CART with TRMI (absolute returns as the dependent variable with only two categories), taking into account the two simple trading strategies (Long Only and Long/Short) and the benchmark (Buy and Hold S&P 500). We observe that our base model's Long Only strategy performs equally well as the alternative model (weekly rebalanced, absolute returns as the dependent variable with three categories). The yearly and 5 year annualized returns are almost the same (1% higher). The yearly and 5 year annualized volatility are almost the same (around 11% and 25% respectively). The Sharpe Ratio for both yearly and 5 year annualized are identical. Our base model's Long/Short strategy is also generally comparable to the alternative one in terms of these standard performance measures.

THE RISK FACTOR REGRESSIONS (LONG/SHORT AND LONG ONLY STRATEGY): BASE MODEL (THREE CATEGORIES) VS CART WITH TRMI (TWO CATEGORIES)

Long Only		Time series mean	CART	CART	CAPM	Fama-French three factors	Fama-French three factors	Fama-French-Carhart four factors	Fama-French-Carhart four factors
		CART	with TRMI (3 classes)	with TRMI (3 classes)	CART	with TRMI (3 classes)	CART	with TRMI (3 classes)	CART
Alpha		0.0034 ^o (1.887)	0.0032 ^o (1.836)	0.0021* (3.021)	0.0019* (2.853)	0.0022* (3.279)	0.0020* (3.064)	0.0022* (3.296)	0.0020* (3.062)
MKT- R_f		-	-	1.025** (45.41)	0.987*** (44.73)	0.996** (35.57)	0.940** (36.15)	0.985** (36.06)	0.938** (34.86)
SMB		-	-	-	-	-0.152* (-2.639)	-0.050 (-0.878)	-0.146 (-2.534)	-0.048 (-0.849)
HML		-	-	-	-	0.177** (3.439)	0.206** (4.090)	0.134 (2.266)	0.196** (3.370)
MOM		-	-	-	-	-	-	-0.045 (-1.480)	-0.010 (-0.370)
No of obs	342	342	342	342	342	342	342	342	342
Adj. R^2	-	-	0.858	0.854	0.866	0.861	0.861	0.866	0.861
Long Short		Time series mean	CART	CART	CAPM	Fama-French three factors	Fama-French three factors	Fama-French-Carhart four factors	Fama-French-Carhart four factors
		CART	with TRMI (3 classes)	with TRMI (3 classes)	CART	with TRMI (3 classes)	CART	with TRMI (3 classes)	CART
Alpha		0.0032* (2.889)	0.0029* (3.076)	0.0029* (2.768)	0.0029* (3.003)	0.0031* (3.059)	0.0030* (3.202)	0.0031* (3.058)	0.0030* (3.195)
MKT- R_f		-	-	0.174** (4.970)	0.059 ^o (1.848)	0.158** (3.869)	0.0039 (0.102)	0.154** (3.650)	0.006 (0.154)
SMB		-	-	-	-	-0.364** (-4.106)	-0.108 (-1.307)	-0.362** (-4.065)	-0.109 (-1.317)
HML		-	-	-	-	0.209* (2.634)	0.263** (3.562)	0.192 (2.111)	0.272* (3.202)
MOM		-	-	-	-	-	-	-0.017 (-0.360)	0.0095 (0.219)
No of obs	342	342	342	342	342	342	342	342	342
Adj. R^2	-	-	0.065	0.007	0.129	0.046	0.126	0.043	0.043

Table 3.13: This table compares the results of our Long/Short strategy risk factor regressions using the CAPM, Fama-French three factors and Fama-French-Carhart four factor models versus those from the CART with TRMI (weekly rebalanced, absolute returns as the dependent variable with only two categories). Our alternative model Long Only strategy's weekly returns is significantly positive with mean at around 32 bps and t-statistics of 1.836. Based on the CAPM model, this Long Only strategy produces additional excess returns of around 19 bps weekly and is very close to the base model. The t-statistics of the alpha obtained is significant at 1-5% significance level. By adding more factors as in Fama-French or Fama-French-Carhart, the additional alpha generated weekly is stable at 20 bps and its t-statistics is significant at the same level. Similarly, our alternative model's Long/Short strategy also generates significant alpha values (around 30-31 bps weekly).

THE TOTAL CUMULATIVE AVERAGE HIT RATES FOR ALL 10 SECTORS AS OF JULY 2013 (MONTHLY REBALANCED)

Sector	Base model with TRMI (monthly rebalanced)
Energy	62%
Basic Materials	46%
Industrials	60%
Cyclical Consumer Goods and Services	46%
Non-Cyclical Consumer Goods and Services	42%
Financials	44%
Healthcare	54%
Technology	62%
Telecommunication Services	53%
Utilities	49%

Table 3.14: This table describes the the total cumulative average hit rates for all 10 sectors as of July 2013. The training data is from 1st June 2000-31st December 2006 (monthly rolling till July 2013). The results depict that the total cumulative average hit rates for each of the 10 sectors are on average around 52%. The top 4 best performer sectors, where their total cumulative average hit rates are relatively higher than other sectors, are Technology (62%), Energy (62%), Industrials (60%) and Healthcare (54%). The bottom 4 worst performer sectors, where their total cumulative average hit rates are relatively lower than other sectors, are Non-Cyclical Consumer Goods and Services (42%), Financials (44%), Basic Materials (46%) and Cyclical Consumer Goods and Services (46%).

THE PERFORMANCE MEASURES OF THE TRADING STRATEGIES (LONG ONLY AND LONG/SHORT): BASE MODEL (WEEKLY REBALANCED)
VS CART WITH TRMI (MONTHLY REBALANCED)

Strategies	Model	Returns (1 year)	Returns (5 year ann.)	Volatility (1 year)	Volatility (5 year ann.)	Sharpe Ratio (1 year)	Sharpe Ratio (5 year ann.)	Number of trades
Long Only	Base model	26.43%	19.08%	10.63%	25.97%	2.07	1.70	1670
	CART with TRMI (weekly rebalanced)							
	CART with TRMI (monthly rebalanced)	22.51%	5.87%	9.75%	21.24%	2.09	1.02	427
Long/Short	Base model	5.02%	14.03%	3.40%	16.09%	1.38	1.53	3177
	CART with TRMI (weekly rebalanced)							
	CART with TRMI (monthly rebalanced)	-0.53%	-0.42%	1.63%	4.92%	1.68	-0.16	569

Table 3.15: This table describes the standard performance measures of our base model versus the CART with TRMI (monthly rebalanced), taking into account the two simple trading strategies (Long Only and Long/Short). We observe that our base model's Long Only strategy generally outperforms this alternative model. The yearly (5 year annualized) returns are around 4% (13%) higher. The yearly (5 year annualized) volatility are almost the same (around 4% higher). The Sharpe Ratio for both yearly and 5 year annualized are similar or better than the alternative model. Our base model's Long/Short strategy also outperforms the alternative one.

THE RISK FACTOR REGRESSIONS (LONG/SHORT AND LONG ONLY STRATEGY): BASE MODEL VS CART WITH TRMI (MONTHLY REBALANCED)

Long Only		Time series mean	Time series mean	CAPM	CAPM	Fama-French three factors	Fama-French three factors	Fama-French-Carhart four factors	Fama-French-Carhart four factors
		CART with TRMI (weekly rebalanced)	CART with TRMI (monthly rebalanced)	CART with TRMI (weekly rebalanced)	CART with TRMI (monthly rebalanced)	CART with TRMI (weekly rebalanced)	CART with TRMI (monthly rebalanced)	CART with TRMI (weekly rebalanced)	CART with TRMI (monthly rebalanced)
Alpha		0.0034 ^o (1.887)	0.003 (0.551)	0.0021* (3.021)	0.0022* (3.279)	-0.001 (-1.007)	0.0022* (3.296)	-0.001 (-1.084)	-0.001 (-1.084)
MKT- R_f		-	-	1.025** (45.41)	0.996** (35.57)	0.920** (45.64)	0.985** (43.48)	0.956** (40.78)	0.956** (40.78)
SMB		-	-	-	-0.152* (-2.639)	-0.117 (-2.287)	-0.146 (-2.534)	-0.116 (-2.257)	-0.116 (-2.257)
HML		-	-	-	0.177** (3.439)	-0.092 (-2.208)	0.134 (2.266)	-0.096 (-2.229)	-0.096 (-2.229)
MOM		-	-	-	-	-	-0.045 (-1.480)	-0.008 (-0.416)	-0.008 (-0.416)
No of obs	342	78	78	342	342	78	342	342	78
Adj. R^2	-	-	0.858	0.964	0.866	0.969	0.866	0.969	0.968
Long Short		Time series mean	Time series mean	CAPM	CAPM	Fama-French three factors	Fama-French three factors	Fama-French-Carhart four factors	Fama-French-Carhart four factors
		CART with TRMI (weekly rebalanced)	CART with TRMI (monthly rebalanced)	CART with TRMI (weekly rebalanced)	CART with TRMI (monthly rebalanced)	CART with TRMI (weekly rebalanced)	CART with TRMI (monthly rebalanced)	CART with TRMI (weekly rebalanced)	CART with TRMI (monthly rebalanced)
Alpha		0.0032* (2.889)	-0.0001 (-0.111)	0.0029* (2.768)	0.0002 (0.193)	0.0031* (3.059)	-0.00002 (-0.016)	0.0031* (3.058)	-0.00003 (-0.026)
MKT- R_f		-	-	0.174** (4.970)	-0.088** (-3.639)	0.158** (3.869)	-0.072 (-2.630)	0.154** (3.650)	-0.065 (-2.245)
SMB		-	-	-	-0.364** (-4.106)	0.067 (1.056)	-0.362** (-4.065)	0.065 (1.024)	0.065 (1.024)
HML		-	-	-	0.209* (2.634)	-0.135 (-2.636)	0.192 (2.111)	-0.125 (-2.354)	-0.125 (-2.354)
MOM		-	-	-	-	-	-0.017 (-0.360)	0.018 (0.733)	0.018 (0.733)
No of obs	342	78	342	78	342	78	342	342	78
Adj. R^2	-	-	0.065	0.137	0.129	0.193	0.126	0.193	0.188

Table 3.16: This table compares the results of our Long/Short strategy risk factor regressions using the CAPM, Fama-French three factors and Fama-French-Carhart four factor models versus those from the CART with TRMI (monthly rebalanced). The t-statistics are in parentheses. *, **, * and ^o indicate statistical significance at the 0-0.1%, 0.1-1%, 1-5% and 5-10% level respectively. We clearly see that both the Long Only and Long/Short strategies of our alternative model are beaten by our base model. None of the risk factor regression results of this model for both strategies show any significant alpha values. This is a first indication that the predictive power of the news sentiments generated by the TRMI is relatively short-lived. Similar to the base case, the adjusted R^2 for the Long Only strategy is generally very high (around 97%), while those of the Long/Short strategy are fairly low (around 14-19%).

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